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Higher-Derivative Gravitation in Bosonic and Superstring Theories

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Abstract

A discussion of the number of degrees of freedom, and their dynamical properties, in higher-derivative gravitational theories is presented. The complete non-linear sigma model for these degrees of freedom is exhibited using the method of auxiliary fields. As a by-product we present a consistent non-linear coupling of a spin-2 tensor to gravitation. It is shown that non-vanishing $(C_{\mu\nu\alpha\beta})^2$ terms arise in $N = 1, D = 4$ superstring Lagrangians due to one-loop radiative corrections with light field internal lines.

1 Bosonic Gravitation

The usual Einstein theory of gravitation involves a symmetric tensor $g_{\mu\nu}$ whose dynamics is determined by the Lagrangian

$$\mathcal{L} = \frac{1}{2\kappa^2} \mathcal{R} \quad (1)$$

The diffeomorphic gauge group reduces the number of degrees of freedom from ten down to six. Einstein's equations further reduce the degrees of freedom to two, which correspond to a physical spin-2 massless graviton. Now let us consider an extension of Einstein's theory by including terms in the action which are quadratic in the curvature tensors. This extended Lagrangian is given by

$$\mathcal{L} = \frac{1}{2\kappa^2} \mathcal{R} + \alpha \mathcal{R}^2 + \beta (C_{\mu\nu\alpha\beta})^2 + \gamma (\mathcal{R}_{\mu\nu})^2 \quad (2)$$

where \mathcal{R}^2 , $(C_{\mu\nu\alpha\beta})^2$, and $(\mathcal{R}_{\mu\nu})^2$ are a complete set of CP-even quadratic curvature terms. The topological Gauss-Bonnet term is given by

$$\text{GB} = (C_{\mu\nu\alpha\beta})^2 - 2(\mathcal{R}_{\mu\nu})^2 + \frac{2}{3} \mathcal{R}^2 \quad (3)$$

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Therefore, we can write

$$\mathcal{L} = \frac{1}{2\kappa^2}\mathcal{R} + a\mathcal{R}^2 - b(C_{\mu\nu\alpha\beta})^2 + c \text{ GB} \quad (4)$$

In this case, it can be shown [1] that there is still a physical spin-2 massless graviton in the spectrum. However, the addition of the \mathcal{R}^2 term introduces a new physical spin-0 scalar, ϕ , with mass $m = 1/\sqrt{12a\kappa^2}$. Similarly, the $(C_{\mu\nu\alpha\beta})^2$ term introduces a spin-2 symmetric tensor, $\phi_{\mu\nu}$, with mass $m = 1/\sqrt{4b\kappa^2}$ but this field, having wrong sign kinetic energy, is ghost-like. The GB term, being a total divergence, is purely topological and does not lead to any new degrees of freedom. The scalar ϕ is perfectly physical and can lead to very interesting new physics [2]. The new tensor $\phi_{\mu\nu}$, however, appears to be problematical. There have been a number of attempts to show that the ghost-like behavior of $\phi_{\mu\nu}$ is illusory, being an artifact of linearization [3]. Other authors have pointed out that since the mass of $\phi_{\mu\nu}$ is near the Planck scale, other Planck scale physics may come in to correct the situation [4]. In all these attempts, the gravitational theories being discussed were not necessarily consistent and well defined. However, in recent years, superstring theories have emerged as finite, unitary theories of gravitation. Superstrings, therefore, are an ideal laboratory for exploring the issue of the ghost-like behavior of $\phi_{\mu\nu}$, as well as for asking whether the scalar ϕ occurs in the superstring Lagrangian. Hence, we want to explore the question “*Do quadratic gravitation terms appear in the $N = 1, D = 4$ superstring Lagrangian?*”

Before doing this, however, we would like to present further details about the emergence of the new degrees of freedom in quadratic gravitation. We begin by adding to Einstein gravitation, quadratic terms associated with the scalar curvature only. That is, we consider the action

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\mathcal{R} + \frac{1}{6}m^{-2}\mathcal{R}^2 \right) \quad (5)$$

The equations of motion derived from this action are of fourth order and their physical meaning is somewhat obscure. These equations can be reduced to second order, and their physical content illuminated, by introducing an auxiliary field ϕ . The action then becomes

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\mathcal{R} + \frac{1}{6}m^{-2}\mathcal{R}^2 - \frac{1}{6}m^{-2} \left[\mathcal{R} - 3m^2 \{e^\phi - 1\} \right]^2 \right) \quad (6)$$

$$= \int d^4x \sqrt{-g} \left(e^\phi \mathcal{R} - \frac{3}{2}m^2 [e^\phi - 1]^2 \right) \quad (7)$$

Note that the ϕ equation of motion sets the square bracket in equation (6) to zero. Hence, action (7) with the auxiliary field ϕ is equivalent to the original action (5). Now, let us perform a Weyl rescaling of the metric

$$g_{\mu\nu} = e^{-\phi} \bar{g}_{\mu\nu} \quad (8)$$

It follows that

$$\sqrt{-g} = e^{-2\phi} \sqrt{-\bar{g}} \quad (9)$$

$$\mathcal{R} = e^\phi \left(\overline{\mathcal{R}} + 3\overline{\nabla}^2 \phi - \frac{3}{2} [\overline{\nabla} \phi]^2 \right) \quad (10)$$

where $\overline{\nabla}_\lambda \overline{g}_{\mu\nu} = 0$. Therefore,

$$\begin{aligned} \sqrt{-g} e^\phi \mathcal{R} &= \sqrt{-\overline{g}} \left(\overline{R} + 3\overline{\nabla}^2 \phi - \frac{3}{2} [\overline{\nabla} \phi]^2 \right) \\ -\frac{3}{2} m^2 \sqrt{-g} (e^\phi - 1)^2 &= -\frac{3}{2} m^2 \sqrt{-\overline{g}} (1 - e^{-\phi})^2 \end{aligned} \quad (11)$$

and the action becomes

$$\mathcal{S} = \int d^4x \sqrt{-\overline{g}} \left(\overline{\mathcal{R}} - \frac{3}{2} [\overline{\nabla} \phi]^2 - \frac{3}{2} m^2 [1 - e^{-\phi}]^2 \right) \quad (12)$$

where we have dropped a total divergence term. It follows that the higher-derivative pure gravity theory described by action (5) is equivalent to a theory of normal Einstein gravity coupled to a real scalar field ϕ . It is important to note that, with respect to the metric signature $(-, +, +, +)$ we are using, the kinetic energy term for ϕ has the correct sign and, hence, that ϕ is not ghost like. Also, note that a unique potential energy function

$$V(\phi) = \frac{3}{2} m^2 (1 - e^{-\phi})^2 \quad (13)$$

emerges which has a stable minimum at $\phi = 0$. We conclude that $\mathcal{R} + \mathcal{R}^2$ gravitation with metric $g_{\mu\nu}$ is equivalent to $\overline{\mathcal{R}}$ gravitation with metric $\overline{g}_{\mu\nu}$ plus a non-ghost real scalar field ϕ with a fixed potential energy and a stable vacuum state. The property that ϕ is non-ghost like is sufficiently important that we will present yet another proof of this fact. This proof was first presented in [2]. If we expand the metric tensor as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (14)$$

then the part of action (5) quadratic in $h_{\mu\nu}$ is given by

$$\mathcal{S} = \int d^4x \left[\frac{1}{4} h^{\mu\nu} \left(\nabla^2 \left\{ P_{\mu\nu\rho\sigma}^{(2)} - 2P_{\mu\nu\rho\sigma}^{(0)} \right\} + 2m^{-2} (\nabla^2)^2 P_{\mu\nu\rho\sigma}^{(0)} \right) h^{\rho\sigma} \right] \quad (15)$$

where $P_{\mu\nu\rho\sigma}^{(2)}$ and $P_{\mu\nu\rho\sigma}^{(0)}$ are transverse projection operators for $h_{\mu\nu}$. Inverting the kernel yields the propagator

$$\begin{aligned} \Delta_{\mu\nu\rho\sigma}^{-1} &= \left(\nabla^2 P_{\mu\nu\rho\sigma}^{(2)} + 2m^{-2} \nabla^2 \left[\nabla^2 - m^2 \right] P_{\mu\nu\rho\sigma} \right)^{-1} \\ &= \frac{1}{\nabla^2} \left(P_{\mu\nu\rho\sigma}^{(2)} - \frac{1}{2} P_{\mu\nu\rho\sigma}^{(0)} \right) + \frac{1}{2(\nabla^2 - m^2)} P_{\mu\nu\rho\sigma}^{(0)} \end{aligned} \quad (16)$$

The term proportional to $(\nabla^2)^{-1}$ corresponds to the usual two helicity massless graviton. However, the term proportional to $(\nabla^2 - m^2)^{-1}$ represents the propagation of a real scalar field with positive energy and, hence, not a ghost. This corresponds to the results obtained using the auxiliary field above. We would like to point out that there may be very interesting

physics associated with the scalar field ϕ . For example, as emphasized in [5], ϕ may act as a natural inflaton in cosmology of the early universe.

Now let us consider Einstein gravity modified by quadratic terms involving the Weyl tensor only. That is, consider the action

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\mathcal{R} - \frac{1}{2} m^{-2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right) \quad (17)$$

Using the identity

$$C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} = GB + 2 \left(\mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} - \frac{1}{3} \mathcal{R}^2 \right) \quad (18)$$

where GB is the topological Gauss-Bonnet combination defined in (3), the action becomes

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\mathcal{R} - m^{-2} \left[\mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} - \frac{1}{3} \mathcal{R}^2 \right] \right) \quad (19)$$

where we have dropped a total divergence. The fourth order equations of motion can be reduced to second order equations by introducing an auxiliary symmetric tensor field $\phi_{\mu\nu}$. Using this field, the action can be written as

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\mathcal{R} - G_{\mu\nu} \phi^{\mu\nu} + \frac{m^2}{4} [\phi_{\mu\nu} \phi^{\mu\nu} - \phi^2] \right) \quad (20)$$

where $\phi = \phi_{\mu\nu} g^{\mu\nu}$ and $G_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R}$ is the Einstein tensor. Note that the $\phi_{\mu\nu}$ equation of motion is

$$\phi_{\mu\nu} = 2m^{-2} \left(\mathcal{R}_{\mu\nu} - \frac{1}{6} g_{\mu\nu} \mathcal{R} \right) \quad (21)$$

Substituting this into (20) gives back the original action (19). As it stands, action (20) is somewhat obscure since the $G_{\mu\nu} \phi^{\mu\nu}$ term mixes $g_{\mu\nu}$ and $\phi_{\mu\nu}$ at the quadratic level. They can, however, be decoupled by a field redefinition. First write the above action as

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\left[\left[1 + \frac{1}{2} \phi \right] g^{\mu\nu} - \phi^{\mu\nu} \right] \mathcal{R}_{\mu\nu} + \frac{m^2}{4} [\phi_{\mu\nu} \phi^{\mu\nu} - \phi^2] \right) \quad (22)$$

Now transform the metric as

$$\sqrt{-\bar{g}} \bar{g}^{\mu\nu} = \sqrt{-g} \left(\left[1 + \frac{1}{2} \phi \right] g^{\mu\nu} - \phi^\mu{}_\alpha g^{\alpha\nu} \right) \quad (23)$$

or, equivalently

$$\begin{aligned} g_{\mu\nu} &= (\det A)^{-1/2} A_\mu{}^\alpha \bar{g}_{\alpha\nu} \\ A_\mu{}^\alpha &= \left(1 + \frac{1}{2} \phi \right) \delta_\mu{}^\alpha - \phi_\mu{}^\alpha \end{aligned} \quad (24)$$

Under this transformation

$$\mathcal{R}_{\mu\nu} = \bar{\mathcal{R}}_{\mu\nu} - \bar{\nabla}_\mu C^\alpha{}_{\alpha\nu} + \bar{\nabla}_\alpha C^\alpha{}_{\mu\nu} + C^\alpha{}_{\mu\nu} C^\beta{}_{\alpha\beta} - C^\alpha{}_{\mu\beta} C^\beta{}_{\nu\alpha} \quad (25)$$

where $\bar{\nabla}_\lambda \bar{g}_{\mu\nu} = 0$ and

$$\begin{aligned} C^\alpha{}_{\mu\nu} &= \frac{1}{2} (X^{-1})^{\alpha\beta} (\bar{\nabla}_\mu X_{\nu\beta} + \bar{\nabla}_\nu X_{\mu\beta} - \bar{\nabla}_\beta X_{\mu\nu}) \\ X_{\mu\nu} &= g_{\mu\nu} = (\det A)^{-1/2} A_\mu{}^\alpha \bar{g}_{\alpha\nu} \end{aligned} \quad (26)$$

Inserting these transformations into the above and dropping a total divergence, the action becomes [6]

$$\mathcal{S} = \int d^4x \sqrt{-\bar{g}} \left[\bar{\mathcal{R}} + \bar{g}^{\mu\nu} (C^\alpha{}_{\mu\nu} C^\beta{}_{\beta\alpha} - C^\alpha{}_{\mu\beta} C^\beta{}_{\nu\alpha}) - \frac{m^2}{4} (\det A)^{-1/2} (\phi_{\mu\nu} \phi^{\mu\nu} - \phi^2) \right] \quad (27)$$

Note that the action for $\phi_{\mu\nu}$ is a complicated non-linear sigma model since $C = C(X)$ and $X = X(\phi)$. It is useful to consider the kinetic energy part of the action expanded to quadratic order in $\phi_{\mu\nu}$ only. It is found to be

$$\mathcal{S}_\phi^{\text{quad}} = \int d^4x \sqrt{-\bar{g}} \left(\frac{1}{4} \bar{\nabla}^\alpha \phi^{\mu\nu} \bar{\nabla}_\alpha \phi_{\mu\nu} - \frac{1}{2} \bar{\nabla}^\alpha \phi^{\mu\nu} \bar{\nabla}_\mu \phi_{\nu\alpha} + \frac{1}{2} \bar{\nabla}_\mu \phi^{\mu\nu} \bar{\nabla}_\nu \phi - \frac{1}{4} \bar{\nabla}^\alpha \phi \bar{\nabla}_\alpha \phi \right) \quad (28)$$

This action is clearly the curved space generalization of the Pauli-Fierz action for a spin-2 field except that every term has the wrong sign! This implies, of course, that $\phi_{\mu\nu}$ propagates as a ghost. It is interesting to note that the action (27) is invariant under the gauge transformation

$$\begin{aligned} \phi'_{\mu\nu} &= \phi_{\mu\nu} + \bar{\nabla}_{(\mu} \xi_{\nu)} - C_{\mu\nu}^\alpha \xi_\alpha \\ \bar{g}'_{\mu\nu} &= \left(1 + \frac{1}{2} \phi' \right)^{-1} \left[\frac{\det^{1/2} A'}{\det^{1/2} A} \left(1 + \frac{1}{2} \phi \right) \bar{g}_{\mu\nu} + (\phi'_{\mu\nu} - \phi_{\mu\nu}) \right] \end{aligned} \quad (29)$$

This insures that the above action describes a consistent coupling of a spin-2 symmetric tensor field $\phi_{\mu\nu}$ to Einstein gravitation at the full non-linear level [7]. We conclude, therefore, that $\bar{\mathcal{R}} + C^2$ gravitation with metric $g_{\mu\nu}$ is equivalent to $\bar{\mathcal{R}}$ gravity with metric $\bar{g}_{\mu\nu}$ plus a ghost-like symmetric tensor field $\phi_{\mu\nu}$ with a consistent non-linear coupling to gravity and a fixed potential energy. The physics in the field $\phi_{\mu\nu}$ is obscured by its ghost-like nature. However, its ghost nature can be altered by yet higher-derivative terms, such as those one would expect to find generated in superstring theories. Therefore, at long last, we turn to our discussion of quadratic supergravitation in superstring theory.

2 Superspace Formalism

In the Kähler (Einstein frame) superspace formalism, the most general Lagrangian for Einstein gravity, matter and gauge fields is

$$\mathcal{L}_E = -\frac{3}{2\kappa^2} \int d^4\theta E[K] + \frac{1}{8} \int d^4\theta \frac{E}{R} f(\Phi_i)_{ab} W^{\alpha a} W_\alpha{}^b + \text{h.c.} \quad (30)$$

where we have ignored the superpotential term which is irrelevant for this discussion. The fundamental supergravity superfields are R and $W_{\alpha\beta\gamma}$, which are chiral, and $G_{\alpha\dot{\alpha}}$, which is Hermitian. The bosonic \mathcal{R}^2 , $(C_{mnpq})^2$ and $(\mathcal{R}_{mn})^2$ terms are contained in the highest components of the superfields $\bar{R}R$, $(W_{\alpha\beta\gamma})^2$ and $(G_{\alpha\dot{\alpha}})^2$ respectively. One can also define the superGauss-Bonnet combination

$$\text{SGB} = 8(W_{\alpha\beta\gamma})^2 + (\bar{\mathcal{D}}^2 - 8R)(G_{\alpha\dot{\alpha}}^2 - 4\bar{R}R) \quad (31)$$

The bosonic Gauss-Bonnet term is contained in the highest chiral component of SGB. It follows that the most general quadratic supergravity Lagrangian is given by

$$\mathcal{L}_Q = \int d^4\theta E \left[\Sigma(\bar{\Phi}_i, \Phi_i)\bar{R}R + \frac{1}{R}g(\Phi_i)(W_{\alpha\beta\gamma})^2 + \Delta(\bar{\Phi}_i, \Phi_i)(G_{\alpha\dot{\alpha}})^2 + \text{h.c.} \right] \quad (32)$$

3 (2,2) Symmetric Z_N Orbifolds

Although our discussion is perfectly general, we will limit ourselves to orbifolds, such as Z_4 , which have (1,1) moduli only. The relevant superfields are the dilaton, S , the diagonal moduli T^I , which we'll denote as T^I , and all other moduli and matter superfields, which we denote collectively as ϕ^i . The associated Kähler potential is

$$\begin{aligned} K &= K_0 + Z_{ij}\bar{\phi}^i\phi^j + \mathcal{O}((\bar{\phi}\phi)^2) \\ \kappa^2 K_0 &= -\ln(S + \bar{S}) - \sum (T^I + \bar{T}^I) \\ Z_{ij} &= \delta_{ij} \prod (T^I + \bar{T}^I)^{q_i} \end{aligned} \quad (33)$$

The tree level coupling functions f_{ab} and g can be computed uniquely from amplitude computations and are given by

$$f_{ab} = \delta_{ab}k_a S, \quad g = S \quad (34)$$

There is some ambiguity in the values of Δ and Σ due to the ambiguity in the definition of the linear supermultiplet. We will take the conventional choice

$$\Delta = -S, \quad \Sigma = 4S \quad (35)$$

It follows that, at tree level, the complete Z_N orbifold Lagrangian is given by $\mathcal{L} = \mathcal{L}_E + \mathcal{L}_Q$ where

$$\mathcal{L}_Q = \frac{1}{4} \int d^4\theta \frac{E}{R} S \text{SGB} + \text{h.c.} \quad (36)$$

Using this Lagrangian, we now compute the one-loop moduli-gravity-gravity anomalous threshold correction [8]. This must actually be carried out in the conventional (string frame)

superspace formalism and then transformed to Kähler superspace [9]. We also compute the relevant superGreen-Schwarz graphs. Here we will simply present the result. We find that

$$\begin{aligned} \mathcal{L}_{\text{massless}}^{1\text{-loop}} = & \frac{1}{24(4\pi)^2} \sum \left[h^I \int d^4\theta (\bar{\mathcal{D}}^2 - 8R) \bar{R} R \frac{1}{\partial^2} D^2 \ln(T^I + \bar{T}^I) \right. \\ & + (b^I - 8p^I) \int d^4\theta (W_{\alpha\beta\gamma})^2 \frac{1}{\partial^2} D^2 \ln(T^I + \bar{T}^I) \\ & \left. + p^I \int d^4\theta (8(W_{\alpha\beta\gamma})^2 + (\bar{\mathcal{D}}^2 - 8R)((G_{\alpha\dot{\alpha}})^2 - 4\bar{R}R)) \frac{1}{\partial^2} D^2 \ln(T^I + \bar{T}^I) + \text{h.c.} \right] \end{aligned} \quad (37)$$

where

$$\begin{aligned} h^I &= \frac{1}{12} (3\gamma_T + 3\vartheta_T q^I + \varphi) \\ b^I &= 21 + 1 + n_M^I - \dim G + \sum (1 + 2q_I^i) - 24\delta_{GS}^I \\ p^I &= -\frac{3}{8} \dim G - \frac{1}{8} - \frac{1}{24} \sum 1 + \xi - 3\delta_{GS}^I \end{aligned} \quad (38)$$

The coefficients γ_T and ϑ_T , which arise from moduli loops, and φ and ξ , which arise from gravity and dilaton loops, are unknown. However, as we shall see, it is not necessary to know their values to accomplish our goal. Now note that if $h^I \neq 0$ then there are non-vanishing \mathcal{R}^2 terms in the superstring Lagrangian. If $b^I - 8p^I \neq 0$ then the Lagrangian has C^2 terms. Coefficient $p^I \neq 0$ merely produces a Gauss-Bonnet term. With four unknown parameters what can we learn? The answer is, a great deal! Let us take the specific example of the Z_4 orbifold. In this case, the Green-Schwarz coefficients are known [10]

$$\delta_{GS}^{1,2} = -30, \quad \delta_{GS}^3 = 0 \quad (39)$$

which gives the result

$$b^{1,2} = 0, \quad b^3 = 11 \times 24 \quad (40)$$

Now, let us try to set the coefficients of the $(C_{\mu\nu\alpha\beta})^2$ terms to zero simultaneously. This implies that

$$b^I = 8p^I \quad (41)$$

for $I = 1, 2, 3$ and therefore that

$$p^{1,2} = 0, \quad p^3 = 33 \quad (42)$$

From this one obtains two separate equations for the parameter ξ given by

$$\xi = \frac{3}{8} \dim G + \frac{1}{8} + \frac{1}{24} \sum 1 - 90 \quad (43)$$

for $I = 1, 2$ and

$$\xi = \frac{3}{8} \dim G + \frac{1}{8} + \frac{1}{24} \sum 1 \quad (44)$$

for $I = 3$. Clearly these two equations are incompatible and, hence, it is impossible to have all vanishing $(C_{\mu\nu\alpha\beta})^2$ terms in the 1-loop corrected Lagrangian of Z_4 orbifolds. We find that the same results hold in other orbifolds as well.

4 Conclusion

We conclude that non-vanishing $(C_{\mu\nu\alpha\beta})^2$ terms are generated by light field loops in the $N = 1, D = 4$ Lagrangian of Z_N orbifold superstrings. It is conceivable that loops containing the heavy tower of states might cancel these terms, but we find no reason, be it duality or any other symmetry, for this to be the case [11]. This is presently being checked by a complete genus-one string calculation [12]. We conjecture that cancellation will not occur. Unfortunately, since γ_T , ϑ_T and φ are unknown, we can say nothing concrete about the existence of \mathcal{R}^2 terms in the Lagrangian. This issue will also be finally resolved in the complete superstring calculation.

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