$(0,2)$ STRING COMPACTIFICATIONS:
THE HIGGS MECHANISM

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ABSTRACT

We present the Higgs mechanism in (0,2) compactifications. The existence of a vector bundle data duality (VBDD) in $(0,2)$ compactifications which is present at the Landau-Ginzburg point allows us to connect in a smooth manner theories with different gauge groups with the same base manifold and same number of effective generations. As we move along the Kahler moduli space of the theories with E_6 gauge group, some of the gauginos pick up masses and break the gauge group to $SO(10)$ or $SU(5)$.

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1 Introduction

The heterotic string is believed to be the correct description of particle physics. A Part of this theory is compactified to a three complex dimension Kahler manifold with vanishing Ricci tensor, a Calabi-Yau manifold, and much effort has been invested in understanding how this part of the theory affects the remaining four dimensional theory which should describe the low energy effective field theory. In particular, the internal manifold will determine the number of generations of the low energy effective field theory, while the choice of the complex structure deformations and Kahler structure deformations determine the Yukawa couplings. Thus a thorough understanding of the internal theory is needed to relate the possible vacuum configurations of this theory, which is determined by the structure of the manifold. In [4] [2] [3], $(2,2)$ strings propagating on birationally distinct manifolds were related with the use of mirror symmetry [7]. Of phenomenological importance are the $(0,2)$ compactifications because they admit gauge groups of smaller rank than E_6, the only gauge group admitted by $(2,2)$ compactifications. Recently [10], it has been shown that two $(0,2)$ models propagating on different pairs (M, E), M being the base manifold and E the vector bundle, having different Euler characters for the base manifolds are dual. The duality transformation exchanges the gauge group moduli with the gravitational moduli. This duality occurs for both models when the Kahler class is negative, or equivalently, at the Landau-Ginzburg phase.

Here we present a different duality where two pairs (M, E) which have same base manifold M but different vector bundles, are dual theories. It is shown that this duality takes place at the Landau-Ginzburg point. The rank of the gauge groups of the two pairs will be different. One will have an E_6 space time gauge group while the other will have an $SO(10)$ space time gauge group. In order to achieve this duality one of the models will have to exhibit a Higgs mechanism as we shall see.

This article is organized as follows. Section two reviews the Calabi-Yau Landau-Ginzburg [5] correspondence presented in [2] for $(2,2)$ models. It is in this section

that we show that in the Calabi-Yau region where instantons effects are small, the description of the model needs the information of both the hypersurface and the tangent bundle. The information they carry are equivalent. In the Landau-Ginzburg region, where the instanton effects are large, and the perturbative treatment of them is not possible, we show that the information of the hypersurface is lost and the only data that remains is that of the tangent bundle. In section three we review the $(0, 2)$ gauged linear σ models, deformations [2] of (2,2) models and as well as the Higgs (0,2) model. In section four, we present examples of models in which the gauge group E_6 is broken to $SO(10)$ with out changing the number of net generations. In section 5 we show how Calabi-Yau manifolds with E_6 are connected in a contunous manner to the same Calabi-Yau manifold with $SU(5)$ gauge group preserving the number of effective generations. The last section has some conclusions.

2   Calabi-Yau/Landau-Ginzburg Phases of $(2, 2)$ Models

As pointed out [2], it is possible to relate a string propagating on a Calabi-Yau manifold to Landau-Ginzburg orbifolds by means of a linear σ model. We review this relation, by following the construction in [2].

The $N = 2$ supersymmetric linear σ model in two dimensions is constructed by dimensionally reduced N=1 supersymmetry in four dimensions. These models contain the following fields. One $U(1)$ gauge vector multiplet A whose bosonic vector has field components a_1, a_2. Of importance will be the presence of the auxiliary field D in this multiplet. There will also be chiral superfields X_i, $i = 1, ..., n$ which transform in the fundamental of $U(1)$ and which are charged with respect to the $U(1)$ gauge field. Their bosonic component will be denoted by x_i. Finally, there will be other chiral superfields P_j which are charged and also transform in the fundamental of $U(1)$, its bosonic component we will denote by p_j.

The Lagrangian for the liner σ model in terms of these superfields will be [2]

$$L = L_{kin} + L_W + L_{gauge} + L_{D,\theta} \tag{1}$$

with

$$
\begin{aligned}
L_{kin} &= \int d^2y d^4\theta \sum_i \bar{X}_i e^{2Q_i A} X_i + \bar{P}_j e^{2Q_j A} P_j \\
L_W &= -\int d^2y d^2\theta P_j W^j(X) + h.c. \\
L_{gauge} &= -\frac{1}{4} \int d^y d^4\theta \bar{\mathcal{A}}\mathcal{A} \\
L_{D,\theta} &= \frac{it}{2\sqrt{2}} \int d^2y d\theta^+ d\bar{\theta}^- \bar{\mathcal{A}}|_{\theta^-=\bar{\theta}^+=0} - \frac{i\bar{t}}{2\sqrt{2}} \int d^2y d\theta^- d\bar{\theta}^+ \bar{\mathcal{A}}|_{\theta^+=\bar{\theta}^-=0}
\end{aligned} \tag{2}
$$

Here, $\mathcal{A}$ is the field strength of the supergauge field. The potentials W_j, $j = 1, ..., n-4$, will be homogeneous transverse polynomials of degree d_j in the fields X_i. $t = \theta + ir$ will turn out to be a good coordinate over the moduli space of Khaler structure deformations. Performing the superspace integration and integrating out the auxiliary fields of the chiral and gauge superfields we obtain the following bosonic potential

$$
\begin{aligned}
U &= \sum_j |W_j(x)|^2 + \sum_i |\sum_j p_j \frac{\partial W_j}{\partial x_i}|^2 + |\sigma|^2(\sum_i |q_i|^2|x_i|^2 + |q_j|^2|p_j|^2) \\
&+ \frac{1}{2}(|\sum_i q_i|x_i|^2 + q_j|p_j|^2 - r)^2
\end{aligned} \tag{3}
$$

where σ is the scalar component of the A vector multiplet. The last term in U must vanish because it is proportional to D^2, since only if it vanishes will supersymmetry be preserved. If we consider the situation in which the charges q_i are all positive, the q_j's are all negative, and $\sum_j q_j = -\sum_i q_i$ then when $r >> 1$ all x_i cannot vanish and since we have chosen W_j's to be transverse it follows that the p_j's must vanish. Also a_1 and a_2 pick a mass of order $\sqrt{r}$ and remain excluded from the massless spectrum. The last term in (3) will then describe a manifold which after moding out by the $U(1)$ symmetry induced by the gauge field, will be a weighted complex projective space of complex dimension $n - 1$, $WCP^{n-1}_{q_1,...,q_n}$ with Kahler class proportional to r.

3

The vanishing of the first term in (3) will define a hypersurface, given by the locii of $W_j(x)$, embedded in $WCP_{q_1,...,q_n}^{n-1}$. We will call this manifold M. Since the sum of the degrees of the polynomials W_j has been chosen to be equal to the sum of the weights of $WCP_{q_1,...,q_n}^{n-1}$, the first Chern class will vanish implying that M is Ricci flat, a condition needed to allow for a conformal field theory on a compact manifold with complex dimension 3. This way we can construct a string propagating on a Calabi Yau manifold starting from a linear σ model.

Let us analyze the case in which $r << -1$. Inspection of (3) subject to the condition that supersymmetry be preserved shows that the p_j's cannot vanish simultaneously. By transversality of W_j we conclude that all x_i must vanish and that a_1 and a_2 pick up a mass of order $\sqrt{r}$ and drop out of the massless spectrum once more. The fields p_j describe a $WCP_{\{q_j\}}^{n-5}$ whose Kahler class is of order $\mathcal{O}(\sqrt{r})$. We may now expand about the classical solutions. We find that the fields x_i remain massless for our particular case and that we may integrate out p_j by setting it to its expectation value. What remains after rescaling some of the fields is shown below

$$U = \sum_i |\sum_j \frac{\partial W^j}{\partial x_i}|^2. \tag{4}$$

or in superspace coordinates

$$L_{W_{eff}} = -\int d^2y \, d^2\theta \, W^j(X) \tag{5}$$

which is a Hibrid Landau-Ginzburg (HLG) superpotential which we believe to be a special point in the enlarged Kahler structure moduli space of the Calabi-Yau model for the $r >> 0$ limit. We then arrive to a Landau-Ginzburg orbifold model with the following form after a rescaling of the fields

$$L_1 = \int d^2y \, d^4\theta \sum_i \bar{X}_i X_i - \int d^2y \, d^2\theta \sum_i W^i(X). \tag{6}$$

In going through the point $r = 0$, the size of the Calabi-Yau shrinks to zero leaving us with a singular manifold. However, the Kahler moduli space has as complex coordinate $t = r + i\theta$. Thus we may analytically continue about $r = 0$ by considering

a path with $\theta \neq 0$ and arrive at the Landau-Ginzburg phase without encountering any singularities [1].

It is suggestive the fact that the information that remains in the Landau-Ginzburg phase of (3) is that of the tangent bundle. On the other hand, in the Calabi-Yau phase we find that the information that remains is that of the hypersurface and the tangent bundle (through the mass term for the fermions). Thus if we construct two $(2, 2)$ models which have the same tangent bundle but with different embedding in the same ambient space they will be dual because they will have the same Landau-Ginzburg phase. This situation is impossible to realize in the context of $(2, 2)$ models. The hypersurface embedded in the ambient space and its tangent bundle are in one-to-one correspondence. However, as we shall see, this is not the case in $(0, 2)$ models. They allow different models to have the same vector bundle data F_{ij}.

3   Construction of $(0, 2)$ Models

3.1   The (0,2) Multiplets

As noted in [9], one may construct $(0, 2)$ models which flow from a Calabi-Yau phase to a Landau-Ginzburg phase by extending in a natural manner the work of [2]. For this we must introduce a set of chiral superfields X_i. In components, they are written, following the convention of [2], as

$$X = x + \sqrt{2}\theta^+\psi_+ - i\theta^+\bar{\theta}^+(D_0 + D_1)x, \tag{7}$$

It has the property

$$\bar{\mathcal{D}}_+ X = 0, \tag{8}$$

where $\bar{\mathcal{D}}_+$ is the gauge covariant derivative in superspace which satisfies

$$\bar{\mathcal{D}}_+ = \frac{\partial}{\partial\theta^+} - i\bar{\theta}^+(\partial_0 + \partial_1 + i(a_0 + a_1)). \tag{9}$$

Of course, the fields a_i belong to a $U(1)$ vector multiplet A which has the following expansion

$$A = a_0 - a_1 - 2i\theta^+\bar{\lambda}_- - 2i\bar{\theta}^+\lambda_- + 2\theta^+\bar{\theta}^+ D, \tag{10}$$

in the Wess-Zumino gauge.

We must also make use of Fermi superfields Γ_b whose components are

$$\Gamma_b = \gamma_b - \sqrt{2}\theta^+ l_b - i\theta^+\bar{\theta}^+(D_0 + D_1)\gamma_b. \tag{11}$$

These Fermi superfields will have the property

$$\bar{\mathcal{D}}_+\Gamma_b = 0. \tag{12}$$

3.2 (0,2) Models which are not Deformations of (2,2) Compatifications

We may now write the following action

$$
\begin{aligned}
L &= L_{gauge} + L_{chiral} + L_{Fermi} + L_{D,\theta} + L_W \\
L_{gauge} &= \frac{1}{8}\int d^2y d\theta^+ d\bar{\theta}^+ \bar{\mathcal{A}}\mathcal{A} \\
L_{chiral} &= \frac{-i}{2}\int d^2y d^2\theta(\bar{X}_i(\mathcal{D}_0 - \mathcal{D}_1)X_i + \bar{P}_j(\mathcal{D}_0 - \mathcal{D}_1)P_j) \\
L_{Fermi} &= -\frac{1}{2}\int d^2y d^2\theta \bar{\Gamma}_b\Gamma_b
\end{aligned}
$$

$$
\begin{aligned}
L_{D,\theta} &= \frac{it}{2}\int d^2y d\theta^+ \mathcal{A}|_{\bar{\theta}+=0} + h.c., \\
L_W &= \frac{-1}{\sqrt{2}}\int d^2y d\theta^+ \Gamma_b J^b|_{\bar{\theta}+=0} + h.c. \tag{13}
\end{aligned}
$$

where $\mathcal{A} = [\bar{\mathcal{D}}_+, \partial_0 - \partial_1 + iA]$. After performing the supercoordinate integration we arrive at the following bosonic potential [2]

$$U(x_i) = \frac{1}{2}(\sum_i q_i|x_i|^2 + \sum_a q_j|p_j|^2 - r)^2 + \sum_b |J_b|^2. \tag{14}$$

6

The first term is proportional to D^2, where D is the auxiliary field of the vector multiplet.

We will consider the case in which $J_b = \sum_j P_j F_{ji}$, $b = i = 1, ..., n$ and $J_b = W_j$, $b = j = n, ..., 2n-4$, and $\Gamma_b = \Lambda_i$, $b = i = 1, ..., n$ and $\Gamma_b = \Sigma_j$, $b = j = n, ..., 2n-4$, and denote the charges of the Λ_i by n_i and the charges of the Σ_j's by $-d_j$.

The bosonic potential of the model will then be

$$U(x, p) = \sum_j |W_j|^2 + \sum_i |\sum_j p^j F_j^i| + \frac{1}{2}(\sum_i q_i |x_i|^2 + q_j |p_j|^2 - r)^2 \qquad (15)$$

Let us then study this potential in the limit in which $r >> 1$ for the case in which all the charges q_i are positive and all the charges q_j are negative. The vanishing of the D-term, equivalently the vanishing of the last term in (15), implies for this case, that the fields x_i cannot vanish simultaneously. Since the F_j^i vanish simultaneously only at the origin it implies that all the p_j's must vanish. As an end result, we arrive at a σ model which has a target space given by the locii of the hypersurfaces W_j embedded in $WCP_{q_1, ..., q_n}$. In order for this manifold M to be a Calabi-Yau manifold, it must satisfy

$$\sum_i q_i - \sum_j d_j = 0. \qquad (16)$$

where d_j are the degree of the homogeneous polynomials W_j and q_i are the charges of the chiral fields X_i with respect to the gauge field. The natural question which arises is what role do the polynomials F_j^i play. For this we must study the left moving massless fermions λ_i's. As shown in [6], the fermions which couple to these polynomials transform as sections of a holomorphic vector bundle E over the manifold M on which the string propagates. As all the vector bundle data F_{ij} cannot vanish simultaneously, we find that j linear combinations of the λ_i's pick up a mass through the term in the Lagrangian

$$\bar{\psi}_{p_j} \lambda_i F^{i,j} \qquad (17)$$

where $\bar{\psi}_{p_j}$ is the fermionic component of the chiral multiplet P_j.

The linear combinations which pick up a mass can be read off the sequence

$$0 \to E \to \oplus_{a=1}^{r+1} \mathcal{O}(n_a) \to^{\otimes F_{ij}(x)} \mathcal{O}(m) \to 0, \tag{18}$$

with $\sum_j |q_j| = \sum_i n_i$. Thus, we see that the polynomials F_{ij} define the vector bundle E over M. If we desire to construct a stable vector bundle which is not a deformation of TM, there are certain conditions we must impose.

The first condition demands that the vector bundle $E \to M$ must yield an anomaly free theory. This implies [6] that the second Chern class, $c_2(E)$, must be equivalent to the second Chern class of the tangent bundle TM. Then, in order to have spinors defined over this vector bundle, the first Chern class of E must be a multiple of two[2]. We will take this class to vanish because it will also guarantee that the Donaldson-Uhlenbeck-Yau condition [6] is satisfied, a condition needed for E to exist. These two conditions impose respectively the constraints

$$c_2(E) = c_2(T) \tag{19}$$

$$c_1(E) = 0. \tag{20}$$

These may be also formulated in the form

$$\sum_j q_j^2 - \sum_i n_i^2 = \sum_j d_j^2 - \sum_j q_j^2 \tag{21}$$

$$\sum_k |q_j| - \sum_i n_i = 0. \tag{22}$$

The first condition implies that two different Calabi-Yau manifolds which have the same vector bundle data could be dual as we shall soon show.

In the limit in which $r >> 0$ we find a Calabi-Yau phase in which the left moving fermions which couple to the polynomials F_{ij} transform as sections of the vector bundle E.

At $r << 0$, in the LGO phase, we encounter an effective superpotential given by

$$W_{eff} = \sum_j \Sigma_j W^j + \sum_a P^j F_{ij} \Lambda^i \tag{23}$$

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[2]$c_1(E)$ must also be positive otherwise E is never stable

This potential differs in the first term from the potential encountered in the (2,2) models. It is this term which does not allow us to formulate the Higgs mechanism in a straightforward manner. Rather we should find a way to make the fermions Σ_j massive.

3.3 (0,2) Compactifications As Deformations Of (2,2) Compactifications

Another possibility which we will use are deformations of (2,2) models. These models have also been treated in [2]. They have in addition to the fields used in the previous $(0,2)$ model a fermi multiplet which contains the field that was the scalar superpartner σ of the gauge field A in the (2,2) model. This field will appear with its fermionic partner $\bar{\beta}$ in the supermultiplet ϵ. In addition, the fermi fields Γ_b will have a different expansion than those previously used in the above $(0,2)$ model. They will satisfy

$$\bar{\mathcal{D}}_+\Gamma_b = q_b\epsilon\phi_b. \tag{24}$$

where ϕ_b is a chiral field X_i or P_j and q_b the charge of the chiral field. With these modifications we arrive at the following bosonic potential

$$\begin{aligned}
U(x,p) &= \sum_j |W_j|^2 + \sum_i |\sum_j p_j F_i^j|^2 + \frac{1}{2}(\sum_i q_i|x_i|^2 + q_j|p_j|^2 - r)^2 \\
&\quad + |\sigma|^2(\sum_i |q_i|^2|x_i|^2 + \sum_j |q_j|^2|p_j|^2).
\end{aligned} \tag{25}$$

We see that the last term in (25) is the one which allows us to claim that this model will yield a (2,2) deformation provided that we introduce a fermionic gauge symmetry associated to the multiplet ϵ. This can only be done if

$$\sum_i X_i F_j^i = d_j W_j \tag{26}$$

holds.

Let us then, study this potential in the limit in which $r >> 1$. In this limit, we find that the fermions Λ_i transform as sections of the deformed tangent bundle

9

over a Calabi-Yau manifold defined by the vanishing of the W_j's on $WCP_{q_1,\ldots,q_n}^{n-1}$. The vector bundle over this manifold will be stable. As in the previous subsection j linear combinations of the Λ_i's will pick up a mass through the mass term

$$\bar{\psi}_{p_j}\lambda_i F^{ij}. \tag{27}$$

But another linear combination will also pick up a mass though the term

$$\bar{\beta}\lambda_i\bar{\phi}^i \tag{28}$$

due to the presence of the multiplet ϵ.

Thus, given two pairs (M, E) where M is a manifold and E is a vector bundle, with the same vector bundle data F^{ij} we see that if one is a (2,2) deformation it will have a rank 3 (E_6 gauge group) while if the second is not a (2,2) deformation it will have a stable rank 4 ($SO(10)$ gauge group). Can such pair be found to be dual? The answer is yes as we shall soon see.

In the $r << 0$ limit we find a HLO theory whose superpotential is

$$\sum_{i,j} P^j F_{ji}\Lambda^i. \tag{29}$$

In this case, the fermions Σ_j have picked up a mass with the fermion component of the multiplet ϵ which contains the scalar partner of the gauge field.

3.4 Higgs (0,2) Compactifications

Another possibility is to add additional gauge fields, chiral fields and fermi fields to the lagrangian (13). The field content of the model is summarized below.

Our old gauge field A will be acompanied by other gauge fields B_j. Then, the charge of the fields for the case in which we have two hypersurfaces of degree $d_j, j = 1, 2$ are given by the array $(q_A; q_{B_1}, q_{B_2})$.

The field content and charges are as follows.

Chiral primaries X_i with charges $(q_i; 0, 0)$. These are the fields which are used to define the CY manifold and they are present in (0,2) models. Chiral primaries P_j

with charges $(q_j; 0, 0)$. P_j are also present in all (0,2) models. Chiral primaries G_j with charges $(e_j; 0, 0)$. These fields are not present in the previous (0,2) models. The charges e_j will be fixed by demanding gauge invariance of the superpotential with respect to the gauge field A. Chiral primaries Y_j with charges $(b_{j_1}; \beta_{j_1}, 0)$, $(b_{j_2}; 0, \beta_{j_2})$. These fields are not present in the previous (0,2) models. Chiral primaries S_j with charges $((-b_{j_1} - 2e_{j_1})/2; -\beta_{j_1}/2, 0)$, $((-b_{j_2} - 2e_{j_2})/2; 0, -\beta_{j_2}/2)$. These fields are not present in the previous (0,2) models. Fermi fields Λ_i with charges $(n_i; 0, 0)$. Some of these fields will remain massless in the CY phase and will define the vector bundle. These fields are present in the previous (0,2) models. Fermi fields Σ_j with charges $(-d_j; 0, 0)$. These fields will remain massive in the LGO phase. They are present in the previous model but are massless in the LGO phase and couple to the hypersurface polynomials. Fermi field Ξ_j with charges $(-d_{j_1} - e_{j_1}; -\beta_{j_1}, 0)$, $(-d_{j_2} - e_{j_2}; 0, -\beta_{j_2})$. These fields will remain massive in the HLG phase. They are not present in the previous (0,2) models. Fermi fields Υ_j with charges $((-b_{j_1} + 2e_{j_1})/2; -\beta_{j_1}/2, 0)$, $((-b_{j_2} + 2e_{j_2})/2; 0, -\beta_{j_2}/2)$. The additional fields we have introduced will remain massive in the HGO and Calabi-Yau phases. They are not present in the previous (0,2) models. With these choices of charges we find that the cancelation of all the anomalies reduces to the ones previously encountered in other (0,2) models, $c_2(E) = c_2(TM)$. Since the charges of the additional fields are functions of b_j and β_j which are arbitrary constants, the charges are quite arbitrary, although it is required that the central charge of the theory in the HLO phase as well as the Calabi-Yau phase be consistent with the compactification.

The complete action is

$$
\begin{aligned}
L &= L_{kinetic} + L_{D,\theta} + L_W \\
L_{D,\theta} &= \frac{it}{2} \int d^2y \, d\theta^+ (\mathcal{A}|_{\bar{\theta}^+ = 0} - \sum_j \mathcal{B}_j)|_{\bar{\theta}^+ = 0}) + h.c. \\
L_W &= \int d^2y \, d\theta^+ (\sum_{ij} \Lambda_i P_j F^{ij} + \sum_j \Sigma_j W_j \\
&\quad + \sum_j (\Sigma^j M_{jk}(P) G^k + \Xi_j Y_j G_j + \Upsilon_j S_j Y_j))|_{\bar{\theta}^+ = 0} + h.c. \quad (30)
\end{aligned}
$$

The matrix $M(P)$ has entries which are holomorphic functions of P_j with appropiate charges to guarantee gauge invariance of the superpotential. Its determinant does not vanish anywhere over $WCP^3_{q_{j_1},...}$. For appropriate values of b_j and β_j, in the limit $r >> 0$ we find that the fields, which were not present in the previously studied (0,2) models, become massive. In addition, all chiral primaries have vanishing expectation values with the exception of Y_j, and thus decouple from the remaining fields. The effective action for the massless fields is a non linear σ model whose vector bundle is determined by the polynomials F_{ij} only, as was the case in the previous subsection, since there λ_i's cannot couple to the ϵ multiplet which is absent from this model. Thus, the bundle defining data F_{ij} will give rise to a rank 3 gauge bundle for (2,2) models but the same data in the model presented in this section will define a rank 4 vector bundle as was the case in the previous subsection.

On the other hand, for $r << 0$ we find that the fields Σ_j become massive and we arrive at an HLG phase for the massless fields whose superpotential is given by

$$L = \sum_{ij} \Lambda_j P_i F^{ij}. \tag{31}$$

The fermi fields Ξ_j remain massless and their charges are quite arbitrary. In fact, their charges can vanish.

4 Higgs Compactifications: $E_6 - SO(10)$ Case

The best example to study E_6 breaking to $SO(10)$ is the quintic. To exhibit this breaking we must consider a linear σ model used in subsection 3.4. Since we will consider the quintic we will have $j = 1$ only. Thus, we will drop this index.

The complete action is

$$
\begin{aligned}
L &= L_{kinetic} + L_{D,\theta} + L_W \\
L_{D,\theta} &= \frac{it}{2} \int d^2y d\theta^+ (\mathcal{A}|_{\bar{\theta}+=0} - \sum_j \mathcal{B})|_{\bar{\theta}+=0}) + h.c.
\end{aligned}
$$

$$L_W = \int d^2 y d\theta^+ (\sum_i \Lambda_i P F^i + \sum \Sigma W$$
$$+ (\Sigma PG + \Xi YG + \Upsilon SY))|_{\bar{\theta}^+ = 0} + h.c. \tag{32}$$

The bosonic potential reads

$$U = |W + pg|^2 + |p|^2 \sum_i |F_i|^2 + |y|^2 |g|^2 + |y|^2 |s|^2$$
$$+ (\sum_i q_i |x_i|^2 + e|g|^2 + b|y|^2 - (b + 2e)/2|s|^2 + q_p |p|^2 - r)^2$$
$$+ (-\beta/2|s|^2 + \beta|y|^2 - r)^2. \tag{33}$$

The polynomial W is homogeneous of degree 5, and $x_i F^i = W$. The charge $q_p = -5$ is negative and the charges $q_i = 1$ are all positive. The charges of the Λ_i's are equal to the charges of the X_i's. The charge of Σ is -5 as required by gauge invariance. The charge e is determined by demanding gauge invariance of the superpotential. With these choices of charges the gauge anomalies for the A and B fields, as well as the mixed anomaly, cancel. However, we have not set the charge of the field Ξ. We will take the charge of this field to vanish. This will fix the charges of Y and S.

For large and positive r we see that y cannot vanish and thus s vanishes. Similarly, g must vanish. For very large β we find that the x_i cannot vanish simultneously and given the fact that the F_i vanish simultaneously only at the origin, p must vanish. The end result is a hypersurface, given by the vanishing of W, embedded in CP^4. Given the degree of W we find a manifold with vanishing Ricci tensor. The ambient space of the manifold is determined by the vanishing of the second line in (33) after moding out by the $U(1)$ symmetry introduced by the gauge field A. The $U(1)$ symmetry introduced by the gauge field B fixes the phase of the field y. One linear combination of the λ_i's picks up a mass with the fermionic component of the P multiplet. The left moving fermions then transform as sections of the vector bundle determined by the F_i. Given our choice of data, the vector bundle will be an extension of the tangent bundle $E = T \oplus \mathcal{O}$. It has rank 4 and the gauge group associated to it is $SO(10)$. The anomaly cancellation reduces to

$$c_2(E) = c_2(T)$$

13

which is the standard form of the anomaly cancellation for a vector bundle over a Calabi-Yau manifold. The fields g, s, y, ξ, v are all massive in this phase. The central charge of this phase is [6] $(10, 9)$. This model is in agreement with [12] where the existence of E_6 breaking directions in (0,2) compactifications where found for the quintic.

For negative and large values of r, we find that s cannot vanish. Which implies that y vanishes. These two fields along with v become massive. The gauge symmetry of B is used to set the phase of s. The vanishing of the D-term of the field A implies that p does not vanish. By transversality of the F_i's all x_i vanish. This in turn forces g to vanish. The fermions Λ_i remain massless while σ picks a mass with the fermionic component of G. Perhaps the most important fermion is ξ. This fellow remains free, massless and it is uncharged. The effective superpotential for the Landau-Ginzburg action found in this phase is

$$\int \Lambda_i F^i. \tag{34}$$

This action is nothing but the effective superpotential for the E_6 Landau-Ginzburg phase of the quintic. It has central charge (9,9). However, since there is also a free complex left moving fermion, the total charge of the model is $(10, 9)$ as it was in the Calabi-Yau phase. The complex free fermion will join the remaining 8 real free fermions. In all we will have in this phase 10 free fermions which together with the left moving $U(1)$ generate E_6. Thus we have continuously gone from the Calabi-Yau phase of a quintic with $SO(10)$ gauge group to the Landau-Ginzburg quintic with E_6 gauge group. This in turn is connected in a continuous manner to the Calabi-Yau phase of the same quintic [8]. It has recently been shown that the phases with E_6 gauge group are stable against world sheet instanton effects [11]. As we have gone in a continuous manner from the E_6 quintic to the $SO(10)$ quintic we may expect no additional instanton contributions which may destabilize the $SO(10)$ phase. We thus expect to have a stable $SO(10)$ phase for the quintic.

This is in agreement with the statements made in [9] where it was argued that the

$SO(10)$ quintic in its LGO phase was unstable. This is because the argument of [9] was phrased at the $SO(10)$ LGO point where there is a quantum symmetry associated to the discrete group which orbifolds the LG theory. This symmetry which prevents gauginos of picking a mass at the LGO point is no longer present at the $SO(10)$ Calabi-Yau phase which is continuosly connected to the E_6 LGO phase. Furthermore, from the above analysis it follows that the mass of the massless states depends on the Kahler moduli. This is not the case in (2,2) compactifications which have E_6 gauge group. However, (2,2) compactifications have an $N = 2$ special geometry which prevents the massless states from picking up a mass. In (0,2) compatifications we find that the special geometry is absent and that there are additional moduli: gauge moduli. These moduli, can in principle, mediate between the Kahler moduli and the massless states in non trivial ways to make some of these massive [12].

5 Higgs Compactifications: $E_6 - SU(5)$ Case

The simplest examples to consider are models in which the gauge group E_6 breaks down to $SU(5)$ without changing the number of effective generations are along the lines of the quintic. However, we must use complete intersection Calabi-Yau manifold and use two more additional gauge fields B_1 and B_2. The action is

$$
\begin{aligned}
L &= L_{kinetic} + L_{D,\theta} + L_W \\
L_{D,\theta} &= \frac{it}{2} \int d^2y d\theta^+ (\mathcal{A}|_{\bar{\theta}^+=0} - \sum_j \mathcal{B}_j)|_{\bar{\theta}^+=0}) + h.c. \\
L_W &= \int d^2y d\theta^+ (\sum_{ij} \Lambda_i P_j F^{ij} + \sum_j \Sigma_j W_j \\
&\quad + \sum_j (\Sigma^j M_{jk}(P)G^k + \Xi_j Y_j G_j + \Upsilon_j S_j Y_j))|_{\bar{\theta}^+=0} + h.c.
\end{aligned}
\tag{35}
$$

We will take the charges of the x_i $i = 1, ..., 6$ to be unity. The charges of P_j $j = 1, 2$ will be -3 for both. The homogeneous polynomials W_j will be of degree 3, thus fixing the charges of the fermi fields Σ_j. The charges of the fermi fields Ξ_j will vanish. The

polynomials F_{ij} will be of degree 2 and will satisfy

$$x_i F^{ij} = W^j.$$

For large and positive r we find a three complex dimension Calabi-Yau phase given by the embedding of W_j in CP^5 with a gauge group $SU(5)$. The central charge of this model is (11,9). The five massless left moving fermions λ_i's which did not pick up a mass through the mass term

$$\lambda_i \bar{\psi}_{p_j} F^{ij}$$

transform as sections of the vector bundle which is an extension of the tangent bundle

$$E = T \oplus \mathcal{O} \oplus \mathcal{O}.$$

The rank of it will be five and the gauge group associated to it will then be $SU(5)$. All the fields which are charged with respect to the B_j gauge fields are massive. The G_j's are also massive.

For large and negative values of r we find a Hybrid Landau-Ginzburg phase (HLG). The mechanism to make the fermions Σ_j massive is present provided the holomorphic mass matrix $M(p)$ has a nonvanishing determinant. These fermions are replaced by the chargeless fermions ξ_j which become massless in this phase. The superpotential for the action of the compactification is

$$\int \sum_j \Lambda_i F^{ij}. \tag{36}$$

It has charge (9,9). With the complex Ξ_j the central charge goes up to (11,9) as in the Calabi-Yau phase. It is the phase of the E_6 Hibrid Landau-Ginzburg. It is continuously connected to the E_6 Calabi-Yau phase given by the embedding of W_j in CP^5. As the free complex fermions Ξ_j are uncharged they may join the remaining free real fermions to construct an E_6 gauge group. We then have, continuously connected a Calabi-Yau phase with $SU(5)$ gauge group to an HLG phase with E_6 gauge group. The latter is continuously connected to the Calabi-Yau phase with E_6 gauge group.

The same manifold but different gauge groups and vector bundles are continuosly connected through the mechanism presented here.

6   Conclusion

We have succeeded in formulating the Higgs mechanism in (0,2) compactifications. This was achieved by introducing additional fields to the ones which usually make their appearence in the literature. More work is needed to determine the stability of the $SO(10)$ and $SU(5)$ Calabi-Yau phases.

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