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On the anomaly of nonlocal symmetry in the chiral QED

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ABSTRACT

We show that the anomaly of nonlocal symmetry can be canceled by the well-known Wess-Zumino acton which cancels the gauge anomay in the two-dimensional chiral electrodynamics.

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Quantum gauge symmetries are very important to build modern quatum field theories. Recently, Lavelle and McMullan [1] have shown that there exists a distinct nonlocal symmetry which is different from the usual gauge symmetry in the quantum electrodynamics (QED). The original gauge symmetry is implemented by the Becci-Rouet-Stora-Tyutin (BRST) symmetry [2] in the gauge fixed action. The nonlocal symmetry, which the gauge fixing term is invariant, is dual to the BRST symmetry. The generalized Lorentz covariant symmetry has discussed in Ref.[3]. On the other hand, the classical gauge symmetry or BRST symmetry may be spoiled by the anomaly after quantization in the chiral QED, which has a chiral gauge coupling [4]. Therefore, it is natural to study whether the nonlocal symmetry may be anomalous in chiral QED or not.

In this Brief Report, we show that the anomaly of the nonlocal symmetry exists, and this can be canceled by the well-known Wess-Zumino action [5] in the exactly solvable two-dimensional chiral QED.

The chiral QED is defined by the gauge invariant Lagrangian as follows

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu - ieA_\mu\frac{1-\gamma_5}{2})\psi, \tag{1}$$

and we choose the Lorentz invariant gauge fixing term and the corresponding ghost action,

$$\mathcal{L}_{GF} + \mathcal{L}_{Ghost} = -\frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - i\partial_\mu\bar{c}\partial^\mu c. \tag{2}$$

Then, the total classical action $\mathcal{L}_C = \mathcal{L}_0 + \mathcal{L}_{GF} + \mathcal{L}_{Ghost}$ is BRST invariant under the following transformations as

$$\begin{aligned} &\delta_B\psi = iec\psi, \quad \delta_B\bar{\psi} = ie\bar{\psi}c \\ &\delta_B A_\mu = -\tfrac{1}{e}\partial_\mu c, \quad \delta_B c = 0, \quad \delta_B\bar{c} = \tfrac{i}{e\alpha}\partial_\mu A^\mu. \end{aligned} \tag{3}$$

It is also invariant under the nonlocal transformation [1] as follows

$$\delta^\perp\psi = e\left(\frac{\partial_0}{\nabla^2}c\right)\psi, \quad \delta^\perp\bar{\psi} = e\bar{\psi}\frac{\partial_0}{\nabla^2}\bar{c},$$

$$
\begin{aligned}
&\delta^{\perp} A_0 = i\bar{c}, \quad \delta^{\perp} A_1 = i\frac{\partial_1\partial_0}{\nabla^2}\bar{c}, \\
&\delta^{\perp}\bar{c} = 0, \quad \delta^{\perp} c = -A_0 + \frac{\partial_1\partial_0}{\nabla^2}A_1 + \frac{e}{\nabla^2}j_0 \ ,
\end{aligned} \tag{4}
$$

where $j^0 = e\bar{\psi}\gamma^0\frac{1-\gamma_5}{2}\psi$.

Let us now consider the quantum effective action to investigate the anomaly of the nonlocal symmetry at the quantum level. The effective action, which incorporates one fermionic loop [4], is exactly calculated as

$$
\mathcal{L}_{eff} = \frac{1}{2}ae^2 A_\mu A^\mu - \frac{e^2}{2}A_\mu \frac{(g^{\mu\nu}+\epsilon^{\mu\nu})\partial_\nu\partial_\rho(g^{\rho\sigma}-\epsilon^{\rho\sigma})}{} A_\sigma \ , \tag{5}
$$

where the constant a is a regularization ambiguity. Then the BRST anomaly of the total quantum effective action, $\mathcal{L}_Q = \mathcal{L}_{eff} + \mathcal{L}_{GF} + \mathcal{L}_{Ghost}$ is given by

$$
\delta_B \mathcal{L}_Q = e\left[\epsilon^{\mu\nu}\partial_\mu A_\nu + (a-1)\partial_\mu A^\mu\right]c. \tag{6}
$$

On the other hand, the nonlocal transformations are defined by exactly the same forms in Eq. (4) except for the ghost field c by the following rule

$$
\delta^{\perp} c = -A_0 + \frac{\partial_1\partial_0}{\nabla^2}A_1 + \frac{e}{\nabla^2}J_0 \ , \tag{7}
$$

where $J_0 = aeA_0 - e(\partial_0+\partial_1)^{\underline{1}}(g^{\mu\nu}-\epsilon^{\mu\nu})\partial_\mu A_\nu$. Then, the anomaly of the nonlocal transformation is given by

$$
\delta^{\perp}\mathcal{L}_Q = ie^2\bar{c}\frac{\partial_0}{\nabla^2}\left[\epsilon^{\mu\nu}\partial_\mu A_\nu + (a-1)\partial_\mu A^\mu\right]. \tag{8}
$$

It is interesting to note that the nonlocal symmetry can be broken after quantization similar to the BRST case in the two-dimensional chiral QED. To recover this symmetry, it is sufficient to add the following Wess-Zumino action $\mathcal{L}_{WZ}$ to the effective action $\mathcal{L}_Q$ [5],

$$
\mathcal{L}_{WZ} = \frac{1}{2}(a-1)\partial_\mu\theta\partial^\mu\theta + e\left[(a-1)g^{\mu\nu}+\epsilon^{\mu\nu}\right]\partial_\mu\theta A_\nu. \tag{9}
$$

Then, after the integration of the θ field, the resulting Lagrangian is given by

$$
\mathcal{L} = -\frac{1}{4}F_{\mu\nu}{}^{-1}(+m^2)F^{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - i\partial^\mu\bar{c}\partial_\mu c \ , \tag{10}
$$

where $m^2 = \frac{ae^2}{a-1}$ for $a > 1$. This Lagrangian is manifestly invariant under the nonlocal transformation as well as the BRST transformation.

In conclusion, we have shown that the nonlocal symmetry may be anomalous after quantization in the chiral QED. However, it is sufficient to add the well-known Wess-Zumino action to recover the nonlocal symmetry. It would be interesting to study the nonlocal anomaly related to the chiral anomaly in higher dimensions and its geometrical meaning.

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References

[1] M. Lavelle and David McMullan, Phys. Rev. Lett. **71**, 3758 (1993).

[2] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1(1979).

[3] Z. Tang and D. Finkelstein, Phys. Rev. Lett. **73**, 3055 (1994).

[4] R. Jackiw and Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985).

[5] K. Harada and I. Tsutsui, Phys. Lett. **B 183**, 311 (1987).