The Isovector Quadrupole-Quadrupole Interaction Used in Shell Model Calculations

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Abstract

An interaction $-\chi Q \cdot Q(1 + B\vec{\tau}(1) \cdot \vec{\tau}(2))$ is used in a shell model calculation for ^{10}Be. Whereas for $B = 0$ the 2_1^+ state is two-fold degenerate, introducing a negative B causes an 'isovector' 2^+ state to come down to zero energy at $B = -0.67$ and an $S = 1$ $L = 1$ triplet ($J = 0^+,\ 1^+,\ 2^+$) to come down to zero energy at $B = -0.73$. These are undesirable properties, but a large negative B is apparently needed to fit the energy of the isovector giant quadrupole resonance.

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I. INTRODUCTION

In this work, we wish to deal with a mismatch which occurs when the schematic quadrupole-quadrupole interaction, including an isospin-dependent part, is used as a particle-hole interaction in *R.P.A.* calculations or is used as a particle-particle interaction in shell model calculations. Concerning the former, we have the reviews of Bayman [1] and of Bes and Sorensen [2] which show that the $Q \cdot Q$ interaction plus pairing can explain the low lying 2^+ vibrational states in even-even nuclei. These are also well described in the books of Bohr and Mottelson [3] and of Soloviev [4]. Bes, Broglia and Nilsson [5], Bohr and Mottelson [3] and Suzuki and Rowe [6] also use the $Q \cdot Q$ interaction for high frequency modes and note that, in order to explain the large splitting between the isoscalar and isovector giant quadrupole resonances, one needs a strong isospin-dependent term $Q \cdot Q\vec{\tau}(1) \cdot \vec{\tau}(2)$. Using the parametrization $V = -\chi Q \cdot Q(1 + B\vec{\tau}(1) \cdot \vec{\tau}(2))$ (where $\vec{\tau}(1) \cdot \vec{\tau}(2) = 1$ for $T = 1$ states and -3 for $T = 0$ states), Bohr and Motteslon [3] state that B is equal to -3.6, very large indeed. Soloviev [4] quotes the formula $B^{(\lambda)} = -0.5(2\lambda + 3)$ which equals -3.5 for $\lambda = 2$, but in actual calculations he uses $-0.2(2\lambda + 3)$. In Ref. [5] it is noted however that if $B = -3.6$ in a large space including $\Delta N = 2$ excitations, then if one truncates to a $\Delta N = 0$ space one should use a value which is much smaller in magnitude $B = -0.6$. More recent references include those of Hamamoto and Nazarewicz [7] and of Nojarov, Faessler and Dingfelder [8–10]. The latter authors made a critical study of the parameter B and concluded that it should have a smaller magnitude than was previously used. They use $B = -2$ in Ref. [10] then compare this favoured value with other values e.g. $B = -3.6$ and $B = -0.6$.

On the other hand, the $Q \cdot Q$ interaction has been used as a particle-particle interaction as well, especially by Elliott with his $SU(3)$ scheme [11]. In the $s - d$ shell this interaction is used to explain rotational behaviour in many nuclei e.g. ^{20}Ne, ^{22}Ne and ^{24}Mg. The model, as shown by Elliott [11] and by Harvey [12], also helps explain deviations from the extreme rotational model due to the truncation effects in the shell model. However, Elliott [11] uses a $Q \cdot Q$ interaction *without* an isovector term (i.e. with $B = 0$). One may well wonder what would happen to his scheme if we introduced a large $\vec{\tau}(1) \cdot \vec{\tau}(2)$ term.

In this work we consider precisely this problem but we work in the p shell, where things are even simpler than in the $s - d$ shell, and consider the case of ^{10}Be. We choose this nucleus because it is strongly deformed ($\beta = 1.12$ according to the tables of Raman et. al. [13]) and also because it is an $N \neq Z$ nucleus. For such a nucleus we can have isovector transitions from the ground state which don't change the overall isospin. Such transitions are very important for our considerations.

II. CALCULATIONS

We perform p shell calculations for states in ^{10}Be using the interaction

$$V = -\chi Q \cdot Q(1 + B\vec{\tau}(1) \cdot \vec{\tau}(2)) \tag{1}$$

with $\chi = 0.36146$. We study the behaviour of selected states for various negative values of B, the coefficient of the isovector $Q \cdot Q$ interaction.

An attractive feature of the multipole-multipole interaction $O^{L\tau} \cdot O^{L\tau}$ is that direct particle-hole matrix elements of the form $\langle (P_B H_B^{-1})^{JT} \; V \; (P_A H_A^{-1})^{JT} \rangle$ vanish unless $J = L$ and $T = \tau$ (the exchange terms are usually taken to be zero). Thus if one adjusts the parameters of one (JT) mode pictured as an $R.P.A.$ state $\Psi^{JT} = \sum\{X_{PH}(PH^{-1})^{JT} - Y_{PH}(PH^{-1})^{\dagger\; JT}\}$, then the other modes $(JT)' \neq (JT)$ are not affected.

The expressions that we use for the particle-particle matrix elements are as follows:

$$\begin{aligned}\Big\langle [j_A(1) j_B(2)]^{JT} \mid -\chi Q \cdot Q[(1 + B\vec{\tau}(1)\cdot\vec{\tau}(2))] \mid \{[j_C(1) j_D(2)]^{JT} - [j_C(2) j_D(1)]^{JT}\}\Big\rangle &= \\ -\chi[1 + (\delta_{T,1} - 3\delta_{T,0})B][1 + (-1)^{j_C + j_D + J + T} P_{j_C j_D}] \left(\frac{2j_A + 1}{2j_C + 1}\right)^{\frac{1}{2}}& \\ \times \langle \Psi^{j_A}[Y^L r^L \Psi^{j_C}]^{j_A}\rangle \langle \Psi^{j_B}[Y^L r^L \Psi^{j_D}]^{j_B}\rangle U(\; j_A \;\; L \;\; J \;\; j_D;\; j_C \;\; j_B \;)&\end{aligned}$$

where we define our singly reduced matrix elements by the following convention:

$$(L \; j_1 \; M \; m_1 | j_2 \; m_2)\langle \Psi^{J_2}[O^L \Psi^{J_1}]^{J_2}\rangle = \langle \Psi^{J_2}_{M_2} \; O^L_M \; \Psi^{J_1}_{M_1}\rangle$$

In the above U is the unitary Racah coefficient. Note that the entire dependence on J, the total angular momentum of the two particles, is contained in the above U coefficient.

The expression for the *direct* part of the particle-hole interaction is:

$$\begin{aligned}\langle (j_A j_B^{-1})^{JT} \; |V| \; (j_C j_D^{-1})^{JT}\rangle &= -\chi(\delta_{T,0} + B\delta_{T,1})[4(2j_A + 1)(2j_C + 1)]^{\frac{1}{2}}\delta_{J,L} \\ &\times \langle \Psi^{j_B}[Y^L r^L \Psi^{j_A}]^{j_B}\rangle \langle \Psi^{j_C}[Y^L r^L \Psi^{j_D}]^{j_C}\rangle\end{aligned}$$

In the above expression we have made the isospin dependence as explicit as possible. We see that for particle-hole states the $T = 0$ shift is proportional to $-\chi$ and the $T = 1$ to $-\chi B$. We can give a simplified derivation of the value of B using the quadrupole giant resonance data presented in Suzuki and Rowe's work [6]. The isoscalar quadrupole resonance is at $\frac{63}{A^{\frac{1}{3}}}$ MeV. The unperturbed energy of these giant resonances is $2\hbar\omega = \frac{82}{A^{\frac{1}{3}}}$ MeV. Assuming a Tamm-Dancoff model, the ratio of isovector to isoscalar shifts is

$$\frac{-\chi B}{-\chi} = B = \frac{141 - 82}{63 - 82} = -3.1$$

A. The $B = 0$ Limit

We now consider shell model calculations for ^{10}Be using the interaction of Eq.(1). For $B = 0$ we simply have the interaction $-\chi Q \cdot Q$. In the p shell, as for any spin-isospin independent interaction, the states are classified by the orbital symmetry values $[f_1\ f_2\ f_3]$ as well as by the quantum numbers L, S and T [14]. The energies are given by the Elliott formula [11] for $SU(3)$ with $\lambda = f_1 - f_2$ and $\mu = f_2 - f_3$:

$$E = \bar{\chi}[-4(\lambda^2 + \mu^2 + \lambda\mu) + 3(\lambda + \mu) + 3L(L+1)] \tag{2}$$

where $\bar{\chi} = \frac{5b^4}{32\pi}\chi$ and b is the harmonic oscillator length parameter ($b^2 = \frac{\hbar}{m\omega}$). The values of χ and $\bar{\chi}$ for ^{10}Be are respectively 0.36146 and 0.1286. The ground state has quantum numbers [4 2 0] $S = 0$ $L = 0$ $T = 1$; $J = 0^+$. The first excited state is doubly degenerate: $L = 2$ $S = 0$ [4 2 0]; $J = 2_1^+,\ 2_2^+$, and the excitation energy is $18\bar{\chi}$. We also consider the next excited states arising from two degenerate orbital symmetry states [3 3 0] and [4 1 1]. The other quantum numbers are $L = 1$ $S = 1$ $T = 1$. The J values are therefore 0^+, 1^+ and 2^+. In other words, for each orbital symmetry we have a triplet of states. The excitation energy is $30\bar{\chi}$. These states cannot be reached from the ground state by the $M1$ operator. There are several other states, one of which is the scissors mode state with quantum numbers $L = 1$ $S = 0$ ($J = 1^+$). This state has an excitation energy of $66\bar{\chi}$ and orbital symmetry $[f] = [3\ 2\ 1]$. Of particular interest is the fact that the $T = 1$ and $T = 2$ scissors mode states are degenerate in energy for the above interaction $-\chi Q \cdot Q$. Note that the scissors mode is not the lowest 1^+ state; the aforementioned $L = 1$ $S = 1$ $J = 1^+$ states lie lower.

B. The Dependence on B

The main thrust of the paper is in this section. Having noted in the introduction that a large and negative value of B is needed to fit the splitting of isovector and isoscalar giant quadrupole resonances, we will now study what happens to selected states in ^{10}Be when a finite negative B is introduced. The results are presented in Fig. 1.

We first focus on the two $J = 2^+$ states, which for $B = 0$ are degenerate and lie lowest. This is not the case experimentally; the 2_1^+ state is at 3.368 MeV and the 2_2^+ state at 5.960 MeV the splitting being largely due to a spin-orbit interaction. Both 2^+ states have the same orbital symmetry as the $J = 0^+$ ground state $[f] = [4\ 2\ 0]$. We see from Fig.1 that as B is made negative the degeneracy is removed with one state going rapidly down towards zero energy and the other rising in energy. The behaviour is not precisely linear, but the linear approximation (which would hold if there were no admixtures of states of different symmetry) is remarkably good for negative B. A linear fit to the behaviour for negative B is as follows:

$$\frac{E_A}{\bar{\chi}} = 18 - 26.935|B|$$

with E_A vanishing at $B = -0.668$.

The fact that the 2^+_A state comes down towards zero already is a signal of a very peculiar behaviour. We find even more peculiar behaviour if we look at the transition rates in Table I. For electric quadrupole excitations we define $B(E2, e_p, e_n)$ as the transition rate when effective charges e_p and e_n are used for the proton and neutron respectively.

In Table I we list the isoscalar transition rate $B(E2, 1, 1)$ and the isovector transition rate $B(E2, 1, -1)$. Note that the state $|2_A\rangle$ comes down in energy as B becomes more negative and is excited from the ground state only by the isovector operator, whilst the state $|2^+_B\rangle$ goes up in energy as B becomes more negative and is excited only by the isoscalar operator. This behaviour, which is shown in Fig.1, clearly goes against experiment. The lowest 2^+ states in essentially all nuclei, although they may have some isovector part, are dominantly isoscalar.

We now look at other selected states. The states which for $B = 0$ have quantum numbers $S = 1$ $L = 1$ $f = [3\ 3]$ and $[f] = [4\ 1\ 1]$ (two degenerate configurations) lead to two sets of triplets with total angular momenta $J = 0^+,\ 1^+$ and 2^+. When a finite negative B is turned on, the J degeneracy is maintained but the degeneracy between the two sets of triplets is removed. Both sets come down in energy, and in the linear approximation we get:

$$\frac{E_C}{\bar{\chi}} = 30 - 41.160|B|$$

with E_C vanishing at $B = -0.729$, and

$$\frac{E_D}{\bar{\chi}} = 30 - 31.850|B|$$

with E_D vanishing at $B = -0.942$. We show the behaviour of the state $|C\rangle$ as a function of B in Fig.1. This figure shows clearly the linear collapse of this state as well as the isovector 2^+ state $|A\rangle$ as a function of negative B.

Note that the $J = 0^+,\ 1^+,\ 2^+$ triplet $|C\rangle$ vanishes at a value of B very close to that for the $J = 2^+$ state $|A\rangle$. The values are $B = -0.729$ and $B = -0.668$ respectively. Thus, care must be taken not to confuse the 2^+ states of each configuration in this region of B and beyond. There is a small region of B from -0.67 to -0.84 where the state $J = 2^+_A$ is the lowest state. But then the triplet $J = 0_C^+,\ 1_C^+,\ 2_C^+$, although starting from a higher energy at $B = 0$, has a slope of larger magnitude than the 2_A^+ state, and ultimately becomes the ground state for $B \leq -0.84$.

We finally look at the states with orbital symmetry [4 2 1]. One of these states is the $L = 1\ S = 0\ T = 1$ scissors mode state and is therefore of special interest [15], [16]. Equally of interest is the other part of the scissors mode strength $L = 1\ S = 0\ T = 2$. The behaviour as a function of B is shown in Fig.2 and Table II. For $B = 0$ the $T = 1$ and $T = 2$ scissors are degenerate in energy [17,14], and there are four degenerate states in all for each T. As B is made negative two $T = 1$ states come down in energy and two come up. We give formulae for only the two extreme states -one going down the fastest and one going up the fastest (respectively):

$$\frac{E_E}{\bar{\chi}} = 66 - 48.362|B|$$

$$\frac{E_F}{\bar{\chi}} = 66 + 23.761|B|$$

By looking at the $M1$ rates at Table II, we see that the state $J = 1_F^+\ T = 1$ and $J = 1_G^+\ T = 2$ form the scissors modes -they get *all* the isovector orbital strength. The state $|D\rangle$ which goes down in energy has no isovector orbital strength. In Fig.2 we show the behaviour, as a function of B, of the $T = 1$ and $T = 2$ scissors modes. Note that whereas for $B = 0$ the two are degenerate, for negative B the $T = 2$ strength comes below the $T = 1$ strength, another peculiar result.

We show now in Table II the isoscalar and isovector (scissors mode) orbital magnetic dipole rates. The transitions are to one state. From $B = 0$ to $B = -0.6$ the isovector rate to $T = 1$ final states increases from 0.0890 μ_N^2 to 0.230 μ_N^2 i.e. an isovector $Q \cdot Q$ interaction with negative B causes the scissors mode strength to increase. Conversly, for positive B, the strength decreases with increasing B. It should be noted that for $B = 0$ the strength is $\frac{9}{32\pi} = 0.0890\mu_N^2$. It should also be noted that in this limit the $T = 2$ scissors mode is degenerate in energy with the $T = 1$ mode, and the strength to $T = 2$ is $\frac{15}{32\pi}\ \mu_N^2$. The ratio is $\frac{(2T+1)_{T=2}}{(2T+1)_{T=1}} = \frac{5}{3}$. The isoscalar orbital rate starts at *zero* for $B = 0$ and increases with negative B to a finite albeit very small value i.e. for $B = -0.6$ $B(M1) \uparrow_{isoscalar\ \ orbital} = 2.52 \times 10^{-3}\ \mu_N^2$.

The energy-weighted $M1$ orbital strength also increases as B is varied. The combined $T = 1$ and $T = 2$ energy-weighted strength in the range $0 > B > -0.6$ is given by the approximate linear formula

$$EWS(B) = EWS(B = 0)(1\ -\ 1.7B)$$

.

III. BEYOND THE CROSS OVER REGION

When B becomes less than -0.67, the state $|A\rangle$ which is a $J = 2^+$ 'isovector state' becomes the ground state. It so remains in the range $-0.67 > B > -0.84$. For $B < -0.84$ the ground state becomes a triplet $S = 1$ $L = 1$ $J = 0^+$, 1^+, 2^+emanating from some combination of the states of orbital symmetry [3 3 0] and 4 1 1] (these orbital states are degenerate in energy at $B = 0$).For B sufficiently negative, the nature of the ground state will again change.

But let us focus on the region of B for which the triplet above is the ground state ($B < -0.84$). Besides the striking fact that the ground state *is* a triplet, what other evidence do we have of a 'phase transition' relative to the case where the orbital symmetry was [4 2 0]? Let us consider the case $B = -1.0$ and look again at tables I and II below the horizontal double lines.

From table III we see that the main *isovector* $E2$ strength is from the $J = 0^+$ to the $J = 2^+$ member of the ground state triplet i.e. a zero-energy transition. Although the $B(E2)$ is substantial, -6.80 e^2fm^4, the rate would be zero if it is indeed a zero-energy transition. The isoscalar $E2$ strength is now split almost evenly between a low-energy state at 0.97 MeV and a high-energy state at 8.7 MeV. This is quite different from the case $B > -0.67$, where all the strength was concentrated in one state.

Most interestingly a rather large isoscalar orbital $M1$ strength emerges (0.119 μ_N^2). It is again a 'zero energy' transition, however from $L = 1$ $S = 1$ $J = 0^+$ to $L = 1$ $S = 1$ $J = 1^+$. Recall that for $B = 0$ the isoscalar orbital strength is zero because the ground state has $L = 0$. The isovector scissors mode strength from $J = 0^+$ $T = 1$ to the $J = 1^+$ $T = 1$ states is now fragmented into two parts -a low-energy part at 0.97 MeV with $B(M1)_{orbital} = 0.13$ μ_N^2 and a high-energy part at 13.3 MeV with $B(M1)_{orbital} = 0.27$ μ_N^2. For $B > -0.67$ all the 1^+, $T = 1$ strength went only to one state.

IV. CLOSING REMARKS

We have shown that an isovector quadrupole-quadrupole interaction $-\chi BQ(1)\cdot Q(2)\vec{\tau}(1)\cdot \vec{\tau}(2)$ with a large negative B yields very undesirable properties in p shell model calculations of ^{10}Be. On the other hand, such interactions appear to be needed to give correct splittings of the isovector and isoscalar giant quadrupole resonances. From our analysis of ^{10}Be in a p shell calculation, it appears that the best fit is obtained with $B = 0$ (Elliott $SU(3)$ model) -even a small *positive* B might be acceptable.

For $B \simeq -0.7$ we get two sets of states collapsing to zero energy -first an isovector $J = 2^+$ state, and then an $L = 1$ $S = 1$ triplet $J = 0^+,\ 1^+,\ 2^+$. This behaviour is undesirable - no known nuclei behave in this way. Perhaps a remedy to this dilemma is to introduce momentum-dependent quadrupole terms in the interaction as Elliott had done [11]. This enabled him to have an interaction which did not connect the $\Delta N = 0$ with the $\Delta N = 2$ space. We would choose these so that the $\Delta N = 0$ isovector quadrupole interaction is much weaker than the $\Delta N = 2$ part.

Another suggestion is to bring effective mass into the picture when analyzing the separation of the isoscalar and isovector giant quadrupole resonances. The unperturbed energy, rather than being $2\hbar\omega$, is now $\frac{2\hbar\omega}{m^*}$. The energy of the isoscalar quadrupole resonance, rather than being $\sqrt{2}\hbar\omega$ is now $\sqrt{2}\frac{\hbar\omega}{(\frac{m^*}{m})^{\frac{1}{2}}}$ [3,18–20]. Using the 'empirical' energies $\frac{63}{A^{\frac{1}{3}}}$ and $\frac{141}{A^{\frac{1}{3}}}$ for the isoscalar and isovector quadrupole resonances respectively, we now modify the estimate of B in Sec. 2 as

$$B = \frac{141 - \frac{82}{m^*}}{63 - \frac{82}{m^*}}$$

With $\frac{m^*}{m} = 0.8$ we get $B = -0.97$; with $\frac{m^*}{m} = 0.7$ we get $B = -0.41$. These are much smaller in magnitude than the value $B = -3.1$ we obtained with $\frac{m^*}{m} = 1$. Furthermore, in a $\Delta N = 0$ space, the magnitude of B will be even smaller due to renormalization effects [5]. This argument is admittedly somewhat hybrid, but we believe it corresponds more closely to what happens when realistic interactions are used.

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TABLES

TABLE I. The energies and $B(E2,e_p,e_n)\uparrow$ (in e^2fm^4) to the first two 2^+ states as a function of B

	2^+_A(isovector) [a]		2^+_B(isoscalar) [a]	
B	E (MeV)	$B(E2,1,-1)$	E (MeV)	$B(E2,1,1)$
0.6	3.49	42.04	4.02	57.85
0.4	3.41	39.02	3.03	63.75
0.2	3.00	35.03	2.48	67.33
0.0	2.32	33.40	2.32	65.09
-0.2	1.57	30.26	2.44	67.71
-0.4	0.87	29.49	2.77	66.45
-0.6	0.21	29.11	3.21	65.06

[a]The value of $B(E2,1,1)$ to the state $|2^+_A\rangle$ is zero. The value of $B(E2,1,-1)$ to the state $|2^+_B\rangle$ is zero.

TABLE II. The magnetic dipole *orbital* strengths (in μ_N^2) -both isoscalar and isovector- as a function of B

	ISOSCALAR		ISOVECTOR			
	E	$1^+,\ T=1$	E	$1^+,\ T=1(scissors)$	E	$1^+,\ T=2$
	(MeV)	$B(M1)\uparrow_{orbital}$	(MeV)	$B(M1)\uparrow_{orbital}$	(MeV)	$B(M1)\uparrow_{orbital}$
0.6	6.04	0	9.27	0.0040	11.74	0.0390
0.4	5.55	0	8.59	0.0043	10.23	0.0728
0.2	4.89	0	8.34	0.0370	9.16	0.1012
0.0	3.86	0	8.49	0.0895 ($\frac{9}{32\pi}$)	8.49	0.1492 ($\frac{15}{32\pi}$)
-0.2	2.72	0	8.93	0.1431	8.10	0.1776
-0.4	1.64	0	9.56	0.1922	7.91	0.2019
-0.6	0.62	0	10.31	0.2314	7.84	0.2284

TABLE III. Beyond the Cross Over Region, $B=-1.0$

$J=2^+,\ T=1$		
$E^*(MeV)$	$B(E2,1,1)$ (e^2fm^4)	$B(E2,1,-1)$ (e^2fm^4)
0.00	0	6.7120
0.96	15.93	0
8.68	21.68	0
$J=1^+,\ T=1$		
$E^*(MeV)$	$B(M1)_{orbital}$ ISOSCALAR (μ_N^2)	$B(M1)_{orbital}$ ISOVECTOR (μ_N^2)
0.00	0.1194	0
0.96	0	0.1368
13.3	0	0.2672

Figure Captions

Figure (1): The excitation energies of selected states in ^{10}Be as a function of the isovector quadrupole interaction parameter B. The solid line is for the 2^+_A state (isovector 2^+), the dashed line is for the 2^+_B state (isoscalar 2^+) and the dot-dashed for the $S=1$ $L=1$ triplet ($J = 0^+_C,\ 1^+_C,\ 2^+_C$).

Figure (2): Same as Figure 1 but for the $J=1^+$ $T=1$ scissors mode branch (solid line) and for the $J=1^+$ $T=2$ scissors mode branch (dashed line).