

# Quantization of field theories in the presence of boundaries

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**Summary:** *This paper reviews the progress made over the last five years in studying boundary conditions and semiclassical properties of quantum fields about 4-real-dimensional Riemannian backgrounds. For massless spin- $\frac{1}{2}$  fields one has a choice of spectral or supersymmetric boundary conditions, and the corresponding conformal anomalies have been evaluated by using zeta-function regularization. For Euclidean Maxwell theory in vacuum, the mode-by-mode analysis of BRST-covariant Faddeev-Popov amplitudes has been performed for relativistic and non-relativistic gauge conditions. For massless spin- $\frac{3}{2}$  fields, the contribution of physical degrees of freedom to one-loop amplitudes, and the 2-spinor analysis of Dirac and Rarita-Schwinger potentials, have been obtained. In linearized gravity, gauge modes and ghost modes in the de Donder gauge have been studied in detail. This program may lead to a deeper understanding of different quantization techniques for gauge fields and gravitation, to a new vision of gauge invariance, and to new points of view in twistor theory.*

## 1. Introduction

The way in which quantum fields respond to the presence of boundaries is responsible for many interesting physical effects such as, for example, the Casimir effect, and the quantization program of spinor fields, gauge fields and gravitation in the presence of boundaries is currently leading to a better understanding of modern quantum field theories. The motivations for this investigation come from at least three areas of physics and mathematics, i.e. [1-15]

(i) *Cosmology.* One wants to understand what is the quantum state of the universe, and how to formulate boundary conditions for the universe [16].

(ii) *Field Theory*. It appears necessary to get a deeper understanding of different quantization techniques in field theory, i.e. the reduction to physical degrees of freedom before quantization, or the Faddeev-Popov Lagrangian method, or the Batalin-Fradkin-Vilkovisky extended phase space. Moreover, perturbative properties of supergravity theories and conformal anomalies in field theory deserve further thinking, especially within the framework of semiclassical evaluation of path integrals in field theory via zeta-function regularization.

(iii) *Mathematics*. A (pure) mathematician may regard quantum cosmology as a problem in cobordism theory, and one-loop quantum cosmology as a relevant application of the theory of eigenvalues in Riemannian geometry, of self-adjointness theory, and of the analysis of asymptotic heat kernels for manifolds with boundary.

On using zeta-function regularization [1-15], the  $\zeta(0)$  value yields the scaling of quantum amplitudes and the one-loop divergences of physical theories. The choices to be made concern the quantization technique, the background 4-geometry, the boundary 3-geometry, the boundary conditions respecting Becchi-Rouet-Stora-Tyutin invariance and local supersymmetry, the gauge condition, the regularization algorithm [9]. By the latter, we mean that  $\zeta(0)$  can be evaluated by means of analytic and direct techniques, i.e. Laplace transform of the heat kernel and Euler-Maclaurin formula, or Laplace and Watson transforms, or zeta-function at large  $x$ , or BKKM formalism [1-15,17]. However,  $\zeta(0)$  can also be obtained in a geometric (though indirect) way, by evaluating the corresponding coefficient in the Schwinger-DeWitt asymptotic expansion.

The following sections describe recent progress on these issues, starting from massless spin- $\frac{1}{2}$  fields, and then focusing on Euclidean Maxwell theory, spin- $\frac{3}{2}$  potentials and linearized gravity. Open problems are presented in section 6.

## 2. Massless spin- $\frac{1}{2}$ fields

The early analysis in [1-3] studied a massless spin- $\frac{1}{2}$  field at one-loop about flat Euclidean 4-space bounded by a 3-sphere of radius  $a$ . By virtue of the first-order nature of the Dirac operator, an elliptic operator mapping elements of primed spin-space to unprimed spin-space (and the other way around) in even dimension, one has a choice of *spectral* or *local* boundary conditions. Using two-component spinor notation as in [1-3], the massless spin- $\frac{1}{2}$  field is represented by a pair of independent spinor fields  $(\psi^A, \tilde{\psi}^{A'})$ . Their expansion on a family of 3-spheres centred on the

origin takes the form [1-3]

$$\psi^A = \frac{\tau^{-\frac{3}{2}}}{2\pi} \sum_{n=0}^{\infty} \sum_{p,q=1}^{(n+1)(n+2)} \alpha_n^{pq} \left[ m_{np}(\tau) \rho^{nqA} + \tilde{r}_{np}(\tau) \bar{\sigma}^{nqA} \right] \quad , \quad (2.1)$$

$$\tilde{\psi}^{A'} = \frac{\tau^{-\frac{3}{2}}}{2\pi} \sum_{n=0}^{\infty} \sum_{p,q=1}^{(n+1)(n+2)} \alpha_n^{pq} \left[ \tilde{m}_{np}(\tau) \bar{\rho}^{nqA'} + r_{np}(\tau) \sigma^{nqA'} \right] \quad , \quad (2.2)$$

where  $\alpha_n^{pq}$  are block-diagonal matrices with blocks  $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ , and the  $\rho$ - and  $\sigma$ -harmonics obey the identities described in [1-3]. In the classical theory, since the field is massless, the modes  $m_{np}(\tau)$  and  $r_{np}(\tau)$  are regular  $\forall \tau \in [0, a]$ , while the modes  $\tilde{m}_{np}(\tau)$  and  $\tilde{r}_{np}(\tau)$  behave as negative powers of the Euclidean-time coordinate, and hence are singular at the origin of Euclidean 4-space. Thus, to find a smooth solution of the classical boundary-value problem, only half of the fermionic field can be fixed at the boundary, corresponding to the modes  $m_{np}$  and  $r_{np}$  multiplying harmonics having positive eigenvalues for the intrinsic 3-dimensional Dirac operator at the boundary [1-3]. This part of the spin- $\frac{1}{2}$  field is denoted with the label (+), while the singular part is denoted by the label (-). The expansions (2.1)-(2.2) are then re-expressed as

$$\psi^A = \psi_{(+)}^A + \psi_{(-)}^A \quad , \quad (2.3)$$

$$\tilde{\psi}^{A'} = \tilde{\psi}_{(-)}^{A'} + \tilde{\psi}_{(+)}^{A'} \quad . \quad (2.4)$$

In the case of spectral boundary conditions, we impose [1,3]

$$\left[ \psi_{(+)}^A \right]_{\partial M} = 0 \quad , \quad \left[ \tilde{\psi}_{(+)}^{A'} \right]_{\partial M} = 0 \quad . \quad (2.5)$$

In the case of local boundary conditions, motivated by local supersymmetry and supergravity multiplets [1-2], we require instead that the spinor field defined as (here  $\epsilon \equiv \pm 1$ )

$$\Phi^{A'} \equiv \sqrt{2} \epsilon n_A^{A'} \psi^A - \epsilon \tilde{\psi}^{A'} \quad , \quad (2.6)$$

should vanish at the boundary, i.e.

$$\left[ \Phi^{A'} \right]_{\partial M} = 0 \quad . \quad (2.7)$$

In [1-2] the existence was proved of self-adjoint extensions of the Dirac operator subject to local boundary conditions, after generalizing a result for complex scalar

fields due to von Neumann. Remarkably, for massless spin- $\frac{1}{2}$  fields, one finds  $\zeta(0) = \frac{11}{360}$  in the case of flat Euclidean 4-space bounded by a 3-sphere, both for spectral and local boundary conditions, although the spectra are different. This result was first proved in [1-3], and then confirmed with the help of the more powerful technique used in [4-5]. It seems to reflect a symmetry of the classical boundary-value problem, as shown in [18]. In [18] we have also worked out the boundary term necessary in the action functional when the spinor field defined in (2.6) is not set to zero at the boundary. The corresponding Euclidean action is [18]

$$I_E = \frac{i}{2} \int_M \left[ \tilde{\psi}^{A'} \left( \nabla_{AA'} \psi^A \right) - \left( \nabla_{AA'} \tilde{\psi}^{A'} \right) \psi^A \right] \sqrt{\det g} d^4 x \\ + \frac{i\epsilon}{2} \int_{\partial M} \Phi^{A'} \epsilon n_{AA'} \psi^A \sqrt{\det h} d^3 x \quad . \quad (2.8)$$

### 3. Euclidean Maxwell theory

We are interested in the mode-by-mode analysis of BRST-covariant Faddeev-Popov amplitudes, which relies on the expansion of the electromagnetic potential in harmonics on the boundary 3-geometry. In the case of 3-sphere boundaries, one has [1,6-9]

$$A_0(x, \tau) = \sum_{n=1}^{\infty} R_n(\tau) Q^{(n)}(x) \quad , \quad (3.1)$$

$$A_k(x, \tau) = \sum_{n=2}^{\infty} \left[ f_n(\tau) S_k^{(n)}(x) + g_n(\tau) P_k^{(n)}(x) \right] \quad , \quad (3.2)$$

where  $Q^{(n)}(x)$ ,  $S_k^{(n)}(x)$  and  $P_k^{(n)}(x)$  are scalar, transverse and longitudinal vector harmonics on  $S^3$  respectively.

Magnetic boundary conditions set to zero at the boundary the gauge-averaging function, the tangential components of the potential, and the ghost field, i.e.

$$\left[ \Phi(A) \right]_{\partial M} = 0 \quad , \quad \left[ A_k \right]_{\partial M} = 0 \quad , \quad [\epsilon]_{\partial M} = 0 \quad . \quad (3.3)$$

Moreover, electric conditions set to zero at the boundary the normal component of the potential, the partial derivative with respect to  $\tau$  of the tangential components of the potential, and the normal derivative of the ghost field, i.e.

$$\left[ A_0 \right]_{\partial M} = 0 \quad , \quad \left[ \partial A_k / \partial \tau \right]_{\partial M} = 0 \quad , \quad \left[ \partial \epsilon / \partial n \right]_{\partial M} = 0 \quad . \quad (3.4)$$

One may check that these boundary conditions are compatible with BRST transformations, and do not give rise to additional boundary conditions after a gauge transformation.

By using zeta-function regularization and flat Euclidean backgrounds, the effects of relativistic gauges are as follows [1,6-8].

(i) In the Lorentz gauge, the mode-by-mode analysis of one-loop amplitudes agrees with the results of the Schwinger-DeWitt technique, both in the 1-boundary case (i.e. the disk) and in the 2-boundary case (i.e. the ring).

(ii) In the presence of boundaries, the effects of gauge modes and ghost modes *do not* cancel each other.

(iii) When combined with the contribution of physical degrees of freedom, i.e. the transverse part of the potential, this lack of cancellation is exactly what one needs to achieve agreement with the results of the Schwinger-DeWitt technique.

(iv) Thus, physical degrees of freedom are, by themselves, insufficient to recover the full information about one-loop amplitudes.

(v) Moreover, even on taking into account physical, non-physical and ghost modes, the analysis of relativistic gauges different from the Lorentz gauge yields gauge-invariant amplitudes only in the 2-boundary case.

(vi) Gauge modes obey a coupled set of second-order eigenvalue equations (section 6). For some particular choices of gauge conditions it is possible to decouple such a set of differential equations, by means of two functional matrices which diagonalize the original operator matrix.

(vii) For arbitrary choices of relativistic gauges, gauge modes remain coupled. The explicit proof of gauge invariance of quantum amplitudes becomes a problem in homotopy theory. Hence there seems to be a deep relation between the Atiyah-Patodi-Singer theory of Riemannian 4-manifolds with boundary [19], the zeta-function, and the BKKM function [17]:

$$I(M^2, s) \equiv \sum_{n=n_0}^{\infty} d(n) n^{-2s} \log [f_n(M^2)] \quad . \quad (3.5)$$

In (3.5),  $d(n)$  is the degeneracy of the eigenvalues parametrized by the integer  $n$ , and  $f_n(M^2)$  is the function occurring in the equation obeyed by the eigenvalues by

virtue of boundary conditions, after taking out fake roots. The analytic continuation of (3.5) to the whole complex- $s$  plane is given by [17]

$$“I(M^2, s)” = \frac{I_{\text{pole}}(M^2)}{s} + I^R(M^2) + O(s) \quad , \quad (3.6)$$

and enables one to evaluate  $\zeta(0)$  as [17]

$$\zeta(0) = I_{\log} + I_{\text{pole}}(\infty) - I_{\text{pole}}(0) \quad , \quad (3.7)$$

$I_{\log}$  being the coefficient of  $\log(M)$  appearing in  $I^R$  as  $M \rightarrow \infty$ .

#### 4. Spin- $\frac{3}{2}$ potentials

In the early analysis in [1-5,10], it was found that the contribution of physical degrees of freedom to the full  $\zeta(0)$  for gravitinos is equal to  $-\frac{289}{360}$  both for spectral and local boundary conditions, in the case of flat Euclidean 4-space bounded by a 3-sphere. More recently, attention has been focused on some basic properties of Dirac and Rarita-Schwinger potentials for spin  $\frac{3}{2}$ , motivated by the attempt of Roger Penrose to define twistors as spin- $\frac{3}{2}$  charges [11-14]. Following [11-14], Tables I and II present primary and secondary potentials in the Dirac and Rarita-Schwinger schemes, with their gauge transformations. In [11-12] the two-component spinor analysis of the four potentials of the totally symmetric and independent field strengths for spin  $\frac{3}{2}$  has been applied to the case of a 3-sphere boundary. It has been shown that the Breitenlohner-Freedman-Hawking reflective boundary conditions:

$$2^s \epsilon n^{AA'} \dots \epsilon n^{LL'} \phi_{A\dots L} = \pm \tilde{\phi}^{A'\dots L'} \quad \text{at } \partial M \quad (4.1)$$

can only be imposed in a flat Euclidean background, for which the gauge freedom in the choice of the potentials remains.

More recently, the Rarita-Schwinger field equations have been studied in four-real-dimensional Riemannian backgrounds with boundary [13-14], subject to the Luckock-Moss-Poletti boundary conditions compatible with local supersymmetry:

$$\sqrt{2} \epsilon n_A^{A'} \psi^A_i = \pm \tilde{\psi}^{A'}_i \quad \text{at } \partial M \quad . \quad (4.2)$$

Gauge transformations on the potentials are compatible with the field equations providing the background is Ricci-flat, in agreement with previous results in the literature. However, the preservation of boundary conditions under such gauge transformations leads to a restriction of the gauge freedom. The recent construction by Penrose of secondary potentials which supplement the Rarita-Schwinger

potentials, jointly with the boundary conditions (4.2), has been shown to imply that the background 4-geometry is further restricted to be totally flat [13].

## 5. Linearized gravity

A detailed mode-by-mode study of perturbative quantum gravity about a flat Euclidean background bounded by two concentric 3-spheres, including non-physical degrees of freedom and ghost modes, leads to one-loop amplitudes in agreement with the covariant Schwinger-DeWitt method [15]. This calculation provides the generalization of the previous analysis of fermionic fields and electromagnetic fields [1-9]. The basic idea is to expand the metric perturbations  $h_{00}$ ,  $h_{0i}$  and  $h_{ij}$  on a family of 3-spheres centred on the origin, and then use the de Donder gauge-averaging function in the Faddeev-Popov Euclidean action. The resulting eigenvalue equation for metric perturbations about a flat Euclidean background:

$$\square h_{\mu\nu} + \lambda h_{\mu\nu} = 0 \quad , \quad (5.1)$$

gives rise to seven coupled eigenvalue equations for metric perturbations. On considering also the ghost 1-form  $\varphi_\mu$ , and imposing mixed boundary conditions on metric and ghost perturbations [15]

$$\left[ h_{ij} \right]_{\partial M} = \left[ h_{0i} \right]_{\partial M} = \left[ \varphi_0 \right]_{\partial M} = 0 \quad , \quad (5.2)$$

$$\left[ \frac{\partial h_{00}}{\partial \tau} + \frac{6}{\tau} h_{00} - \frac{\partial}{\partial \tau} (g^{ij} h_{ij}) \right]_{\partial M} = 0 \quad , \quad (5.3)$$

$$\left[ \frac{\partial \varphi_i}{\partial \tau} - \frac{2}{\tau} \varphi_i \right]_{\partial M} = 0 \quad , \quad (5.4)$$

the analysis in [15] has shown that the full  $\zeta(0)$  vanishes in the 2-boundary problem, while the contributions of ghost modes and gauge modes *do not* cancel each other, as it already happens for Euclidean Maxwell theory (section 3).

## 6. Open problems

The main open problem seems to be the explicit proof of gauge invariance of one-loop amplitudes for relativistic gauges, in the case of flat Euclidean space bounded by two concentric 3-spheres. For this purpose, one may have to show that, for coupled gauge modes,  $I_{\log}$  and the difference  $I_{\text{pole}}(\infty) - I_{\text{pole}}(0)$  are not affected by a change in the gauge parameters. Three steps are in order:

- (i) To relate the regularization at large  $x$  used in [1] to the BKKM regularization.
- (ii) To evaluate  $I_{\log}$  from an asymptotic analysis of coupled eigenvalue equations.
- (iii) To evaluate  $I_{\text{pole}}(\infty) - I_{\text{pole}}(0)$  by relating the analytic continuation to the whole complex- $s$  plane of the difference  $I(\infty, s) - I(0, s)$ , to the analytic continuation of the zeta-function.

The last step may involve a non-local transform relating the BKKM function to the zeta-function, and a non-trivial application of the Atiyah-Patodi-Singer theory of Riemannian 4-manifolds with boundary [19]. In other words, one might have to prove that, *in the 2-boundary problem only*,  $I_{\text{pole}}(\infty) - I_{\text{pole}}(0)$  resulting from coupled gauge modes is the residue of a meromorphic function, invariant under a smooth variation in the gauge parameters of the matrix of elliptic self-adjoint operators appearing in the system [8]

$$\widehat{\mathcal{A}}_n g_n + \widehat{\mathcal{B}}_n R_n = 0 \quad , \quad \forall n \geq 2 \quad , \quad (6.1)$$

$$\widehat{\mathcal{C}}_n g_n + \widehat{\mathcal{D}}_n R_n = 0 \quad , \quad \forall n \geq 2 \quad . \quad (6.2)$$

Here, denoting by  $\gamma_1, \gamma_2, \gamma_3$  three dimensionless parameters which enable one to write the most general gauge-averaging function, and by  $\alpha$  the positive dimensionless parameter occurring in the Faddeev-Popov Euclidean action, one has [8]

$$\widehat{\mathcal{A}}_n \equiv \frac{d^2}{d\tau^2} + \frac{1}{\tau} \frac{d}{d\tau} - \frac{\gamma_3^2 (n^2 - 1)}{\alpha \tau^2} + \lambda_n \quad , \quad (6.3)$$

$$\widehat{\mathcal{B}}_n \equiv -\left(1 + \frac{\gamma_1 \gamma_3}{\alpha}\right) (n^2 - 1) \frac{d}{d\tau} - \left(1 + \frac{\gamma_2 \gamma_3}{\alpha}\right) \frac{(n^2 - 1)}{\tau} \quad , \quad (6.4)$$

$$\widehat{\mathcal{C}}_n \equiv \left(1 + \frac{\gamma_1 \gamma_3}{\alpha}\right) \frac{1}{\tau^2} \frac{d}{d\tau} + \frac{\gamma_3}{\alpha} (\gamma_1 - \gamma_2) \frac{1}{\tau^3} \quad , \quad (6.5)$$

$$\widehat{\mathcal{D}}_n \equiv \frac{\gamma_1^2}{\alpha} \frac{d^2}{d\tau^2} + \frac{3\gamma_1^2}{\alpha} \frac{1}{\tau} \frac{d}{d\tau} + \left[ \frac{\gamma_2}{\alpha} (2\gamma_1 - \gamma_2) - (n^2 - 1) \right] \frac{1}{\tau^2} + \lambda_n \quad . \quad (6.6)$$

Other relevant research problems are the mode-by-mode analysis of one-loop amplitudes for gravitinos, including gauge modes and ghost modes studied within the Faddeev-Popov formalism, and the study of gauge transformations for the secondary potentials  $\rho_{A'}^{CB}$  and  $\theta_A^{C'B'}$  which supplement Rarita-Schwinger potentials (section 4). In that model it is not yet clear whether there is an underlying global theory, what parts of the curvature are the obstructions to defining a global theory, what are the key features of the global theory (if it exists). Moreover, it appears

necessary to understand whether one can define twistors as charges for spin  $\frac{3}{2}$  in a Riemannian background which is not Ricci-flat, and whether one can reconstruct the Riemannian 4-world from the resulting twistor space, or from whatever mathematical structure is going to replace twistor space.

Last, but not least, the mode-by-mode analysis of linearized gravity in the de Donder gauge in the 1-boundary case, the unitary gauge for linearized gravity, and the mode-by-mode analysis of one-loop amplitudes in the case of curved backgrounds, appear to be necessary to complete the picture outlined so far. The recent progress on problems with boundaries, however, seems to strengthen the evidence in favour of new perspectives being in sight in quantum field theory and quantum cosmology.

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TABLE I

	Dirac Potentials	Gauge Freedom
Primary	$\phi_{ABC} = \nabla_{CC'} \Gamma_{AB}^{C'}$	$\widehat{\Gamma}_{AB}^{C'} \equiv \Gamma_{AB}^{C'} + \nabla_A^{C'} \nu_B$
Primary	$\tilde{\phi}_{A'B'C'} = \nabla_{CC'} \gamma_{A'B'}^C$	$\widehat{\gamma}_{A'B'}^C \equiv \gamma_{A'B'}^C + \nabla_{A'}^C \tilde{\nu}_{B'}$
Secondary	$\Gamma_{AB}^{C'} = \nabla_{BB'} \Lambda_A^{B'C'}$	$\widehat{\Lambda}_A^{B'C'} \equiv \Lambda_A^{B'C'} + \nabla_A^{B'} \tilde{\chi}^{C'}$
Secondary	$\gamma_{A'B'}^C = \nabla_{BB'} \rho_{A'}^{BC}$	$\widehat{\rho}_{A'}^{BC} \equiv \rho_{A'}^{BC} + \nabla_{A'}^B \chi^C$

TABLE II

	Rarita-Schwinger Potentials	Gauge Freedom
Primary	$\psi_{A\mu} = \Gamma_{AB}^{C'} e_{C'\mu}^B$	$\widehat{\Gamma}_{BC}^{A'} \equiv \Gamma_{BC}^{A'} + \nabla_B^{A'} \nu_C$
Primary	$\widetilde{\psi}_{A'\mu} = \gamma_{A'B'}^C \epsilon_C^{B'\mu}$	$\widehat{\gamma}_{B'C'}^A \equiv \gamma_{B'C'}^A + \nabla_{B'}^A \lambda_{C'}$
Secondary	$\Gamma_{AB}^{C'} = \nabla_{B'B} \theta_A^{C'B'}$	$\widehat{\theta}_A^{A'B'} \equiv \theta_A^{A'B'} + \nabla_A^{A'} \mu^{B'}$
Secondary	$\gamma_{A'B'}^C = \nabla_{BB'} \rho_{A'}^{CB}$	$\widehat{\rho}_{A'}^{AB} \equiv \rho_{A'}^{AB} + \nabla_{A'}^A \xi^B$

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