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Strangeness in Hadronic Stellar Matter

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Abstract

We examine the presence of strangeness-bearing components, hyperons and kaons, in dense neutron star matter. Calculations are performed using relativistic mean field models, in which both the baryon-baryon and kaon-baryon interactions are mediated by meson exchange. Results of kaon condensation are found to be qualitatively similar to previous work with chiral models, if compatibility of the kaon optical potentials is required. The presence of strangeness, be it in the form of hyperons or kaons, implies a reduction in the maximum mass and a relatively large number of protons, sufficient to allow rapid cooling to take place. The need to improve upon the poorly-known couplings of the strange particles, which determine the composition and structure of neutron stars, is stressed. We also discuss generic problems with effective masses in mean field theories.

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I. INTRODUCTION

The physical state and internal constitution of neutron stars chiefly depends on the nature of strong interactions. Although the composition and the equation of state (EOS) of neutron star matter are not yet known with certainty, QCD based effective Lagrangians have opened up intriguing possibilities. Among these is the possible existence of matter with strangeness to baryon ratio, $|S|/B$, of order unity. Strangeness may occur in the form of fermions, notably the Λ and Σ^- hyperons, or as a Bose condensate, such as a K^- meson condensate, or in the form of strange quarks in a mixed phase of hadrons and quarks. *All these alternatives involve negatively charged matter*, which if present in dense matter, results in important consequences for neutron stars (see, for example, Ref. [1]). For example, the appearance of strangeness-bearing components results in protoneutron (newly born) stars having larger maximum masses than catalyzed (older, neutrino-free) neutron stars, a reversal from ordinary nucleons-only matter. This permits the existence of metastable protoneutron stars that could collapse to black holes during their deleptonization [2]. In older stars, the presence of such components also implies rapid cooling of the star's interior via the direct Urca processes [3]. Interpretation of the surface temperatures of neutron stars in conjunction with different possibilities for the star's core cooling is currently a topic of much interest [4].

Our objective here is to investigate kaon condensation in dense neutron star matter allowing for the explicit presence of hyperons. The fact that hyperons can significantly influence neutron star structure has been emphasized by Glendenning [5] and Ellis et al. [6]. With respect to kaons, the suggestion of Kaplan and Nelson [7] that, above some critical density, the ground state of baryonic matter might contain a Bose-Einstein condensate of negatively charged kaons has generated a flurry of activity examining the effective chiral Lagrangian approach and exploring the astrophysical consequences, e.g. [8–11]. In chiral $SU(3)_L \times SU(3)_R$ the baryons are directly coupled to the kaons. This leads to a strong attraction between K^- mesons and baryons which increases with density and lowers the energy of the zero-momentum state. A condensate forms when this energy becomes equal to the kaon chemical potential, μ.

In cold catalyzed (neutrino-free) dense neutron star matter containing only nucleons, μ is related to the electron (or muon) and nucleon chemical potentials by

$$\mu = \mu_n - \mu_p = \mu_e = \mu_\mu \, , \tag{1}$$

due to chemical equilibrium in the reactions

$$n \leftrightarrow p + e^- + \bar{\nu}_e \, , \qquad e^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_e \qquad and \qquad n \leftrightarrow p + K^- \, . \tag{2}$$

Typically, the critical density for condensation is $\sim (3-4)n_0$ in nucleons-only matter (where n_0 denotes equilibrium nuclear matter density), although it is model and parameter dependent. It is usually the case that a density of this order is less than the central density in a neutron star, so a K^- condensate is expected to be present in the core region.

However, many calculations of dense matter [1,5,6] indicate that hyperons, starting with the Σ^- and Λ, begin to appear at densities $\sim (2-3)n_0$. The requirement of chemical equilibrium in the weak processes yields

$$\begin{aligned} \mu_\Lambda &= \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n \\ \mu_{\Sigma^-} &= \mu_{\Xi^-} = \mu_n + \mu_e \\ \mu_p &= \mu_{\Sigma^+} = \mu_n - \mu_e \, . \end{aligned} \tag{3}$$

The above relations show that, in equilibrium, there exist only two independent chemical potentials, μ_n and μ_e, reflecting the conservation of baryon number and electric charge. The remaining condition is that of overall charge neutrality, namely

$$\sum_B q_B n_B - n_K - n_e - n_\mu = 0 \, , \tag{4}$$

where q_B is the charge and n_B is the number density of baryon species B. With increasing density the concentration of negatively charged hyperons rises so that fewer electrons are required to maintain charge neutrality. Consequently, the rate of increase of the electron density n_e and the chemical potential μ_e slows, and in many cases it begins to drop. Since $\mu_e = \mu$ governs the onset of kaon condensation, the question of whether condensation occurs in the presence of hyperons in stellar matter is raised. In a previous work [12], we addressed this issue on the basis of the Kaplan-Nelson Lagrangian for the kaon-baryon interactions and a Walecka-type relativistic field theoretical approach for the baryon-baryon interactions. Using various models for the latter, it was found that (i) the condensate threshold is sensitive to the behavior of the scalar density; the more rapidly it increases with baryon density, the lower is the threshold density for condensation, (ii) the presence of hyperons, particularly the Σ^-, shifts the threshold for K^- condensation to a higher density, and (iii) in the mean field approach, with hyperons, the condensate amplitude grows sufficiently rapidly that the nucleon effective mass vanishes at a finite baryon density, perhaps signalling strangeness-driven chiral restoration at high baryon density.

These findings have also raised further questions [13]. Among these are issues related with (i) whether it is consistent to use the Walecka type Lagrangian for the baryon-baryon interactions and the chiral Kaplan-Nelson Lagrangian for the kaon-baryon interactions, and (ii) whether or not qualitatively similar results would be obtained in more traditional approaches to kaon-baryon interactions. Here we address these issues by utilizing a traditional meson-exchange picture which can be used to generate the kaon-baryon interactions [14] as well as the nucleon-nucleon interaction [15]. In this case the kaons interact directly with the meson fields, which we take here to be the σ, ω and ρ, and these in turn interact with the baryons. This meson exchange picture meshes more naturally with the Walecka approach, which is usually used in the baryon sector. We note that the earlier discussion of kaon condensation using a similar approach by Schaffner et. al. [16] was confined to nuclear matter, where $\mu = \mu_n - \mu_p = 0$, whereas in neutron star matter in which weak interactions are in equilibrium [17], μ typically increases with density, at least up to the density where other hadronic negative charges appear in matter, reaching values on the order of 200 MeV.

In Sec. II, we give the formalism and discuss the results obtained in the traditional meson-exchange model. A comparison of the meson-exchange approach with previous work employing the Kaplan-Nelson Lagrangian is made in Sec. III to identify the common threads in the formalism and similarities in the results; see also the recent discussions of Brown and Rho [18] for the case where hyperons are absent. In Sec. IV, we offer a critique of the model; in particular we discuss the sensitivity to the poorly-known hyperon couplings and difficulties that can arise with the effective masses when hyperons are present. Our conclusions are presented in Sec. V.

II. MESON-EXCHANGE MODEL

A. Theory

The total hadron Lagrangian is written as the sum of the baryon and the kaon Lagrangians, $\mathcal{L}_H = \mathcal{L}_B + \mathcal{L}_K$. In the baryon sector, we employ a relativistic field theory model of the Walecka type [19]. We consider all charge states of the baryon octet $B = n, p, \Lambda, \Sigma^+, \Sigma^-, \Sigma^0, \Xi^-$ and Ξ^0 (we shall use the symbol N for a nucleon). Explicitly,

$$\begin{aligned}\mathcal{L}_B =& \sum_B \bar{B}\left(i\gamma^\mu\partial_\mu - g_{\omega B}\gamma^\mu\omega_\mu - g_{\rho B}\gamma^\mu b_\mu\cdot t - M_B + g_{\sigma B}\sigma\right)B \\ &- \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{\zeta}{4!}g_{\omega N}^4(\omega_\mu\omega^\mu)^2 \\ &- \tfrac{1}{4}B_{\mu\nu}\cdot B^{\mu\nu} + \tfrac{1}{2}m_\rho^2 b_\mu\cdot b^\mu \\ &+ \tfrac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tfrac{1}{2}m_\sigma^2\sigma^2 - \tfrac{1}{3}bM(g_{\sigma N}\sigma)^3 - \tfrac{1}{4}c(g_{\sigma N}\sigma)^4. \end{aligned} \tag{5}$$

Here M_B is the vacuum baryon mass, the ρ-meson field is denoted by b_μ, the quantity t denotes the isospin operator which acts on the baryons, and the field strength tensors for the vector mesons are given by the usual expressions:– $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$. The nucleon mass, $M = 939$ MeV, is included in the penultimate term to render b dimensionless. In the Lagrangian we have included "non-linear" σ^3 and σ^4 terms so that a reasonable compression modulus can be achieved for equilibrium nuclear matter in the mean field approximation. In some cases, we will also explore the influence of a "non-linear" vector interaction of the form $(\omega_\mu\omega^\mu)^2$ since this helps to achieve a satisfactory description of the properties of finite nuclei in the mean field approximation [20]. More general couplings between the scalar and vector fields as well as isovector non-linear couplings will be examined in a later work.

For the kaon sector, we take a Lagrangian which contains the usual kinetic energy and mass terms, along with the meson interactions,

$$\mathcal{L}_K = \partial_\mu K^+ \partial^\mu K^- - (m_K^2 - g_{\sigma K} m_K \sigma) K^+ K^- \\ + i\left[g_{\omega K}\omega^\mu + g_{\rho K} b^\mu\right]\left(K^+\partial_\mu K^- - K^-\partial_\mu K^+\right) . \tag{6}$$

Here b^μ denotes the ρ^0 field and m_K is the vacuum kaon mass (which is present in the fourth term so that $g_{\sigma K}$ is dimensionless). The scalar interaction term can be combined with the kaon mass into an effective kaon mass defined by

$$m_K^{*\,2} = m_K^2 - g_{\sigma K} m_K \sigma . \tag{7}$$

We shall treat the kaons in the mean field approximation, writing [9] the time dependence of the fields $K^\pm = \frac{1}{\sqrt{2}} f\theta e^{\pm i\mu t}$; thus, θ gives the condensate amplitude. For the baryons, we shall consider calculations at the mean field level. We need to calculate the potential, Ω, of the grand canonical ensemble at zero temperature. It is straightforward to obtain

$$\frac{\Omega}{V} = \tfrac{1}{2}(f\theta)^2[m_K^{*\,2} - 2\mu(g_{\omega K}\omega_0 + g_{\rho K} b_0) - \mu^2] + \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{3}bM(g_{\sigma N}\sigma)^3 \\ + \tfrac{1}{4}c(g_{\sigma N}\sigma)^4 - \tfrac{1}{2}m_\omega^2\omega_0^2 - \frac{\zeta}{4!}(g_{\omega N}\omega_0)^4 - \tfrac{1}{2}m_\rho^2 b_0^2 + \sum_B \frac{1}{\pi^2}\int_0^{k_{FB}} dk\, k^2 (E_B^* - \nu_B) . \tag{8}$$

Here V is the volume, $E_B^* = \sqrt{k^2 + M_B^{*2}}$, and the baryon effective masses are $M_B^* = M_B - g_{\sigma B}\sigma$. The chemical potentials μ_B are given in terms of the effective chemical potentials, ν_B, by

$$\mu_B = \nu_B + g_{\omega B}\omega_0 + g_{\rho B} t_{3B} b_0 , \tag{9}$$

where t_{3B} is the z-component of the isospin of the baryon. The relation to the Fermi momentum k_{FB} is provided by $\nu_B = \sqrt{k_{FB}^2 + M_B^{*2}}$.

The thermodynamic quantities can be obtained from the grand potential in Eq. (8) in the standard way; thus the baryon number density $n_B = k_{FB}^3/(3\pi^2)$, while for kaons

$$n_K = (f\theta)^2(\mu + g_{\omega K}\omega_0 + g_{\rho K} b_0) \, . \tag{10}$$

The pressure $P = -\Omega/V$ and the energy density $\varepsilon = -P + \sum_B \mu_B n_B + \mu n_K$. The meson fields are obtained by extremizing Ω, giving

$$\begin{aligned} m_\omega^2 \omega_0 &= -\frac{\zeta}{6} g_{\omega N}^4 \omega_0^3 + \sum_B g_{\omega B} n_B - (f\theta)^2 \mu g_{\omega K} \\ m_\rho^2 b_0 &= \sum_B g_{\rho B} t_{3B} n_B - (f\theta)^2 \mu g_{\rho K} \\ m_\sigma^2 \sigma &= -bMg_{\sigma N}^3 \sigma^2 - c g_{\sigma N}^4 \sigma^3 + \sum_B g_{\sigma B} n_B^s + \tfrac{1}{2}(f\theta)^2 g_{\sigma K} m_K \, . \end{aligned} \tag{11}$$

Here n_B^s denotes the baryon scalar density

$$n_B^s = \frac{1}{\pi^2} \int_0^{k_{FB}} dk \, k^2 \frac{M_B^*}{E_B^*} \, . \tag{12}$$

Notice that the condensate contributes directly to the equations of motion (11), whereas in chiral models the contribution appears in the effective chemical potentials and effective masses. Further discussion of the two models is given in Sec. III below.

The condensate amplitude, θ, is also found by extremizing Ω. This yields the solutions $\theta = 0$ (no condensate), or, if a condensate exists, the equation [21]

$$\mu^2 + 2\mu(g_{\omega K}\omega_0 + g_{\rho K} b_0) - {m_K^*}^2 = 0 \, . \tag{13}$$

The roots of this equation are the energies of the zero-momentum K^- and K^+ states,

$$\omega^\pm = \sqrt{(g_{\omega K}\omega_0 + g_{\rho K} b_0)^2 + {m_K^*}^2} \pm (g_{\omega K}\omega_0 + g_{\rho K} b_0) \, , \tag{14}$$

so Eq. (13) amounts to setting the chemical potential equal to the energy of the lowest (K^-) state.

Eq. (13) can be used to simplify the expressions for pressure and energy density:

$$P = -\tfrac{1}{2}m_\sigma^2\sigma^2 - \tfrac{1}{3}bM(g_{\sigma N}\sigma)^3 - \tfrac{1}{4}c(g_{\sigma N}\sigma)^4 + \tfrac{1}{2}m_\omega^2\omega_0^2 + \frac{\zeta}{4!}(g_{\omega N}\omega_0)^4 + \tfrac{1}{2}m_\rho^2 b_0^2$$
$$+ \sum_B \frac{1}{3\pi^2}\int_0^{k_{FB}} dk\,\frac{k^4}{E_B^*} \tag{15}$$
$$\varepsilon = (f\theta)^2 {m_K^*}^2 + \tfrac{1}{2}m_\sigma^2\sigma^2 + \tfrac{1}{3}bM(g_{\sigma N}\sigma)^3 + \tfrac{1}{4}c(g_{\sigma N}\sigma)^4 + \tfrac{1}{2}m_\omega^2\omega_0^2 + \frac{\zeta}{8}(g_{\omega N}\omega_0)^4 + \tfrac{1}{2}m_\rho^2 b_0^2$$
$$+ \sum_B \frac{1}{\pi^2}\int_0^{k_{FB}} dk\,k^2 E_B^*\,. \tag{16}$$

Note that by virtue of Eq. (13), the first term in the expression for the thermodynamical potential Eq. (8) vanishes, and so the pressure due to the kaons is contained entirely in the meson fields via their field equations (11).

To complete the thermodynamics, leptonic contributions to the total energy density and pressure, which are given adequately by the standard free gas expressions, must be added to Eq. (15) and Eq. (16).

B. Coupling Constants

In the effective Lagrangian approach adopted here, knowledge of three distinct sets of coupling constants is required for numerical computations. These are the nucleon, hyperon and kaon couplings associated with the exchange of σ, ω and ρ mesons. In what follows, we consider each of these in turn.

1. Nucleon couplings

The nucleon-meson coupling constants are determined by adjusting them to reproduce properties of equilibrium nuclear matter. These are the saturation density and binding energy, the symmetry energy coefficient, the compression modulus and the Dirac effective mass at saturation. There is a considerable range of uncertainty in two of the empirical values that are to be fitted, the compression modulus and the Dirac effective mass, and correspondingly we consider different sets of coupling constants to cover this range. The constants determined in this way are given in Table 1. The sets marked H are from Heide [22] and those labelled GM are from Glendenning and Moszkowski [23]. We also consider one set labelled B91 with the non-linear ω coupling [20] for which the parameter $\zeta = 0.02364$. This has the advantage that one can achieve a value of $M_N^*/M_N \sim 0.6$, as favored by nuclei (in particular, spin-orbit splittings and the charge density distributions), and a reasonable compression modulus with a positive value of the coefficient c so that the scalar potential is bounded from below. We also list here for future use set HS81 taken from Ref. [24]. In addition, Table 1 gives the scalar and vector fields at equilibrium in nuclear matter, $S = g_{\sigma N}\sigma$ and $V = g_{\omega N}\omega_0$, which are relevant for the calculation of the kaon optical potential.

2. Hyperon couplings

Following Glendenning and Moszkowski [23], we constrain the coupling constants of the Λ hyperon by requiring that the correct binding energy be obtained for the lowest Λ level in nuclear matter at saturation. Defining $x_{\sigma\Lambda} = g_{\sigma\Lambda}/g_{\sigma N}$, with analogous definitions for the ω and ρ couplings, this gives

$$-28 = x_{\omega\Lambda} g_{\omega N}\omega_0 - x_{\sigma\Lambda} g_{\sigma N}\sigma \ , \tag{17}$$

in units of MeV. We adopt the value $x_{\sigma\Lambda} = 0.6$, as suggested in Ref. [23] on the basis of fits to hypernuclear levels and neutron star properties; $x_{\omega\Lambda}$ is then determined. We also choose $x_{\rho\Lambda} = x_{\sigma\Lambda}$, since the alternative, $x_{\rho\Lambda} = x_{\omega\Lambda}$, gives very similar results. Our choices are listed in Table 2. In a recent analysis of Σ^- atoms, Mareš et al. [25] find reasonable fits with $x_{\omega\Sigma} = \frac{2}{3}$ and 1, and $x_{\sigma\Sigma} = 0.54$ and 0.77, the larger values yielding a slightly better fit to the data. To begin with, we use the values in Table 2, which are close to the smaller values of Mareš et al., for all the hyperons and comment upon different couplings for the Σ and Ξ later in Sec. IV.

3. Kaon couplings

In order to investigate the effect of a kaon condensate on the equation of state in high-density baryonic matter, the kaon-meson coupling constants have to be specified. Empirically-known quantities can be used to determine these constants, but it is important to keep in mind that laboratory experiments give information only about the kaon-nucleon interaction in free space or in nuclear matter (matter with a proton fraction $x = \frac{1}{2}$ at an equlibrium density of $n_0 = 0.153\ fm^{-3}$). On the other hand, the physical setting in this work is matter in the dense interiors of neutron stars, i.e. infinite matter containing baryons (nucleons and possibly hyperons) and leptons in β-equilibrium, that has a different composition and spans a wide range in densities (up to central densities $\sim 8n_0$). As a consequence, kaon-meson couplings as determined from experiments might not be appropriate to describe the kaon-nucleon interaction in neutron star matter, and the particular choices of coupling constants should be regarded as parameters that have a range of uncertainty.

One possibility of experimentally determining the strength of the kaon-nucleon interaction is the analysis of phase shift data. An analysis of KN scattering data using a meson-exchange model [14] was used to determine couplings of nucleons and kaons to $\sigma,\ \omega,\ and\ \rho$ mesons. This yielded

$$\begin{aligned} G^{\sigma}_{KN} &= \frac{g_{\sigma N} g_{\sigma K}}{m_\sigma^2} = 2.444\ fm^2 \\ G^{\omega}_{KN} &= \frac{g_{\omega N} g_{\omega K}}{m_\omega^2} = 4.981\ fm^2 \\ G^{\rho}_{KN} &= \frac{g_{\rho N} g_{\rho K}}{m_\rho^2} = 1.301\ fm^2\ , \end{aligned} \tag{18}$$

with our Lagrangian conventions. Since only the ratio g/m enters the formalism, it is not necessary to specify the masses, and the kaon ratios are listed in Table 3.

With these couplings and the field strengths in nuclear matter at saturation, we can determine the value of the optical potential felt by a single kaon in infinite nuclear matter for the present model. Lagrange's equation for an s-wave K^- with a time dependence

$K^- = k^-(x)\ e^{-iEt}$, where $E = \sqrt{p^2 + m_K^2}$ is the asymptotic energy, is obtained [26] from Eq. (6) as

$$[\nabla^2 + E^2 - m_K^2]\ k^-(x) = [-2(g_{\omega K}\omega_0 + g_{\rho K} b_0)E - g_{\sigma K} m_k \sigma]\ k^-(x)$$
$$= 2\ m_K\ U_{opt}^K\ k^-(x)\ . \tag{19}$$

In nuclear matter, $b_0 = 0$, so for a kaon with zero momentum ($E = m_K$) the optical potential is

$$U_{opt}^K \equiv S_{opt}^K + V_{opt}^K = -\tfrac{1}{2} g_{\sigma K}\sigma - g_{\omega K}\omega_0\ . \tag{20}$$

The value of U_{opt}^K for the different models are contained in Table 4. Friedman et al. have recently reanalyzed the kaonic atom data [27] examining a more general parameterization of U_{opt}^K in nuclear matter than the standard $t_{eff}\rho$ approximation. They were able to obtain a better fit with a kaon optical potential whose real part had a depth of $-200 \pm 20\ MeV$. The coupling constants adjusted to the parameters obtained from phase shift measurements lead to a value of the kaon optical potential close to this value. Note that the ratio of the $K\omega$ coupling to the $N\omega$ coupling, $x_{\omega K} = g_{\omega K}/g_{\omega N}$, is close to one, whereas $x_{\sigma K}$ and $x_{\rho K}$ are close to the value of $\frac{1}{3}$ which is suggested by naive quark counting.

C. Results

Since the kaon coupling constants are uncertain, some orientation is gained by plotting the threshold density for condensation as a function of $g_{\sigma K}/m_\sigma$ and $g_{\omega K}/m_\omega$ (the threshold is less sensitive to $x_{\rho K}$, which we fix to be $\frac{1}{3}$). Fig. 1 shows contours of the critical density ratios n_{crit}/n_0 for kaon condensation in matter containing nucleons, while Fig. 2 shows the corresponding results for matter containing nucleons and hyperons. Comparison of these figures shows that the threshold density is higher when hyperons are present, and this is particularly marked for smaller values of the couplings. For orientation, the values in Table 3 are roughly $g_{\sigma K}/m_\sigma \sim 0.7$ and $g_{\omega K}/m_\omega \sim 2$, so while condensation will occur in the range $u = 2-3$ when hyperons are absent, their presence may increase u quite substantially depending on the precise value of $g_{\sigma K}/m_\sigma$. When we choose parameters as in Sec. III below, $g_{\omega K}/m_\omega \sim 0.7$, yielding a higher threshold, particularly when hyperons are allowed.

In the remainder of this section, we adopt the kaon couplings of Table 3 and first discuss the nucleons-only case, followed by the case where hyperons are also allowed.

1. Matter containing nucleons and leptons

In Fig. 3 we display our results for matter containing nucleons and leptons for a representative case, parameter set GM2 with kaon couplings from Table 3. The particle fractions, $Y_i = n_i/n$, are shown in panel 1 of Fig. 3. The proton fraction becomes much closer to the neutron fraction once kaons are present, and for high u they are essentially equal. It can be seen that the threshold condition for kaon condensation, Eq. (13), is fulfilled at a density of $2.6n_0$. (The dashed lines in Fig. 3 show the behavior if kaons are excluded.) In panel 2 of this figure, the energies of the zero-momentum kaon states, $\omega^\pm$, are plotted as a function of the ratio of baryon density to equilibrium nuclear matter density, i.e. $u = \sum_B n_B/n_0 \equiv n/n_0$. We see that the ω^- energy drops with increasing density and meets the chemical potential μ at threshold. The effective kaon mass m_K^* does not vary greatly; so, referring to Eq. (14), the density-dependent contributions, dominated by the term containing the ω meson, are critical in obtaining condensation. Panel 3 (right scale) shows that the condensate amplitude, θ, rises rapidly at threshold and then slowly approaches a maximum value of $\sim 40°$; this is smaller than in chiral models. The effect of the condensate on the ω and σ fields (panel 2) follows from the field equations (11), whereas the behavior of the ρ field is dominated by the changes in the neutron–proton ratio. The proton charge is balanced by an approximately equal number of K^- mesons beyond threshold, since the lepton contributions rapidly become negligible. Thus the magnitude of the strangeness/baryon, $|S|/B$, in panel 3 is $\sim \frac{1}{2}$ once kaons condense. Panel 4 of Fig. 3 shows that the total pressure and energy density are reduced when kaons are present.

Finally, in the upper part of Table 5, the gross properties of neutron stars are given for the various equations of state. The critical density ratios lie in a narrow range, $u_{crit} \sim 2.5-3$, so that a significant region of the star will contain kaons. This softens the equation of state causing a reduction in the maximum mass by 4–10%. The precise value depends on the magnitude of the ω repulsion at high density, which is governed by the coupling constants of Table 1. This also affects the changes in the central density; usually this is increased by kaons, but in the B91 case there is a small reduction.

2. Matter containing nucleons, hyperons and leptons

We now consider the case where hyperons are allowed to be present in addition to nucleons. The results displayed in Fig. 4 can be compared with those of Fig. 3 where hyperons were excluded. The particle fractions are shown in panel 1. The first strange particle to appear is the Σ^-, since the somewhat higher mass of the Σ^- is compensated by the electron chemical potential in the equilibrium condition of the Σ^- (see Eq. (3)). Since the Σ^- carries a negative charge, it causes the lepton fractions to drop. This means that the chemical potential μ is reduced, requiring a smaller value of ω^- for kaon condensation which results in a higher threshold density. This is evident from panel 2. Also shown in panel 2 are the changes in the meson fields arising from kaon condensation, and these are much smaller than in the absence of hyperons. This arises partly from the reduction in the condensate amplitude (the maximum value of θ is $\sim 20°$, see panel 3) and partly from changes in the baryon fractions (see panel 1).

Immediately above threshold the kaon fraction rises dramatically to reach a maximum of $0.1-0.2$ per baryon. Since the kaons carry negative charge, charge neutrality for the system leads to a small drop in the Σ^- fraction, and the lepton concentrations become even smaller. By contrast, the fraction of the neutral Λ is little influenced by kaon condensation. In fact this is the largest fraction at large values of u, with roughly comparable amounts of n, p, Σ^- and a relatively small kaon presence. Thus, the hyperons dominate the strangeness/baryon, $|S|/B$, of ~ 0.6 at the highest density considered (panel 3).

The pressure and energy density are displayed in panel 4 of Fig. 4. A comparison with the corresponding panel of Fig. 3 shows the effect of hyperons, which, for a given baryon density, leads to significantly smaller pressures and larger energy densities. Panel 4 of Fig. 4 also shows that when hyperons are present the effects of a kaon condensate are rather small. The change in the energy density due to kaons receives positive contributions from the mesons, and a large negative contribution from the baryons. The lepton contribution is negligible. The net result is, as it must be, a reduction in the energy density; but it is at most only 0.2%. Turning to the change in the pressure arising from condensation, we first note that since we are plotting against density, rather than chemical potential, there is no requirement as to the sign. In fact, we see that at the lower densities the pressure is lowered (softer EOS), whereas at the higher densities it increases (stiffer EOS). This arises from competition between the negative σ and ρ meson contributions and the ω meson contribution, which is positive.

The neutron star properties for matter containing nucleons, hyperons and leptons are given in the lower part of Table 5. Here, a single asterisk indicates that the neutron effective mass becomes zero before reaching the expected central density; kaons have not condensed prior to this point. Further discussion is given in Sec. IV below. For the other cases, and excluding kaons for the moment, we see that the softening effect of hyperons causes a reduction of $\sim 0.5 M_\odot$ and a corresponding increase in the central density, as is well known [5,6]. If we include kaons in the calculation, condensation takes place within the star only for models GM2 and GM3, and it does so at a higher density than when hyperons are absent. The reduction in the maximum neutron star mass due to the presence of kaons amounts to only about $0.01 M_\odot$. The change in the central density is likewise small.

Thus, in this model, the influence of the hyperons is decisive. In nucleons-only matter, the pressure is significantly decreased by a kaon condensate, which lowers the maximum mass. An even larger reduction in the maximum mass occurs when hyperons are present in matter. The additional presence of condensed kaons in hyperonic matter induces relatively small changes in the EOS so that there is little influence of the condensate on the gross stellar properties.

III. COMPARISON WITH CHIRAL MODELS

In previous work [12], we employed the same baryon Lagrangian Eq. (5), but took the kaon kinetic energy and mass terms as well as the kaon-nucleon and kaon-hyperon interactions from the Kaplan–Nelson chiral Lagrangian [7]. We would like to compare this with the meson-exchange approach discussed in the preceding section. We choose to establish parameters that are in some sense compatible in the two cases via the optical potential. For the chiral model, the kaon Lagrangian in *nuclear* matter ($n_n = n_p$) takes the form

$$\mathcal{L}_K = \partial_\mu K^+ \partial^\mu K^- - m_K^2 K^+ K^- + \frac{3i}{8f_\pi^2} n (K^+ \partial_0 K^- - K^- \partial_0 K^+) + \frac{\Sigma^{KN}}{f_\pi^2} n^s K^+ K^- \ , \quad (21)$$

where the pion decay constant $f_\pi = 93$ MeV, $n = n_n + n_p$, $n^s = n_n^s + n_p^s$ and the kaon-nucleon sigma term $\Sigma^{KN} = -(a_1/2 + a_2 + 2a_3)m_s$, in terms of the standard parameters of the chiral Lagrangian [7]. It is straightforward [18] to show that the optical potential is

$$U_{opt}^{chK} \equiv S_{opt}^{chK} + V_{opt}^{chK} = -\frac{\Sigma^{KN} n^s}{2 m_K f_\pi^2} - \frac{3n}{8 f_\pi^2} \ . \quad (22)$$

The kaon-nucleon sigma term requires the parameters $a_1 m_s$, $a_2 m_s$ and $a_3 m_s$. The first two of these can be established from the hyperon-nucleon mass differences, but the third is related to the strangeness content of the proton, which is unknown. Taking the reasonable range of 0, 10 and 20% strangeness for the proton yields $\Sigma^{KN} = 167$, 344 and 520 MeV, respectively. This gives $S_{opt}^{chK} = -22, -45\ and\ -69$ MeV for the different choices. The value $V_{opt}^{chK} = -51$

MeV is given uniquely by the saturation density. Thus, the values of the optical potential are $U_{opt}^{chK} = -73,\ -96\ and\ -120$ MeV. Due to the fact that the vector part of the potential is only about $\frac{1}{3}$ of the value in Sec. II, the total optical potential is only about half of the value favored by Friedman et al. [27], although it is comparable to the value obtained in their $t_{eff}\rho$ approximation. It must, however, be borne in mind that there are uncertainties in their analysis and also in simply expropriating the real part of a complex potential as we have done. Further, our main interest here is in a comparison of the chiral model with the meson-exchange model.

We choose the coupling constants of the meson-exchange model such that the scalar and vector parts of the optical potential as given in Eq. (20) are equal to the corresponding chiral values in Eq. (22). This does not determine the kaon-rho coupling for which we take $x_{\rho K} = 1/3$. The values of the coupling constants thus determined are given in Table 6. Comparison with Table 3 shows that the ω coupling is substantially reduced, whereas the σ coupling is increased for the larger values of Σ^{KN}.

Before proceeding, a comparison of the expressions for the critical densities obtained in the meson-exchange and chiral models is useful. Both can be written in the form

$$\mu^2 + 2\mu\alpha - m_K^{*2} = 0\ . \tag{23}$$

For simplicity, we restrict ourselves to the case in which only the Λ and Σ^- hyperons are considered in addition to nucleons. In the chiral model, α and m_K^{*2} are

$$\begin{aligned} \alpha &= \frac{2n_p + n_n - n_{\Sigma^-}}{2f_\pi^2} \\ m_K^{*2} &= m_K^2 + \left[2a_1 n_p^s + (2a_2 + 4a_3)(n_p^s + n_n^s + n_{\Sigma^-}^s) + \left(\tfrac{5}{3}(a_1 + a_2) + 4a_3\right) n_\Lambda^s\right] \frac{m_s}{2f_\pi^2}\ . \end{aligned} \tag{24}$$

In the meson-exchange model, we have

$$\begin{aligned} \alpha &= \left(G_{KN}^\omega - \tfrac{1}{2}G_{KN}^\rho\right) n_n + \left(G_{KN}^\omega + \tfrac{1}{2}G_{KN}^\rho\right) n_p + G_{K\Lambda}^\omega n_\Lambda + (G_{K\Lambda}^\omega - G_{K\Lambda}^\rho)\, n_{\Sigma^-} \\ m_K^{*2} &= m_K^2 + G_{KN}^\sigma m_K \left[bM(g_{\sigma N}\sigma)^2 + c(g_{\sigma N}\sigma)^3 - n_n^s - n_p^s\right] - G_{K\Lambda}^\sigma m_K (n_\Lambda^s + n_{\Sigma^-}^s)\ , \end{aligned} \tag{25}$$

where we have used the definitions of Eq. (18). Comparing these expressions for the two models we see that the weightings of the various densities are different. In addition, the "non-linear" b and c terms do not play a role in the chiral expressions. So already the threshold condition is different in the two approaches, even though they use the same underlying baryonic model and give the same optical potential in nuclear matter. Above threshold, θ enters in different ways in the two models and this will introduce additional differences.

The properties of neutron stars in the meson-exchange model are shown in Table 7 with and without kaons and hyperons; here, the parameters are chosen on the basis of the optical potentials, as discussed above, and we focus on the GM cases. These results can be directly compared with those of the chiral model in Table 8 (note the values listed here differ slightly from those of Ref. [12], where an equilibrium nuclear matter density of 0.16 fm^{-3} was employed). When kaons are excluded, the models are, of course, identical. In the case that hyperons are absent, the threshold for condensation, u_{crit}, is noticeably lower in the chiral model by 0.2–0.7 units of the density ratio. Nevertheless, the results for the maximum masses and central densities in the two models are, for the most part, similar. This would suggest that our procedure of adjusting the couplings via the optical model is reasonable.

Turning to the case where hyperons are present, the meson-exchange model only yields a condensate for the largest value of Σ^{KN}. This is the bottom row of Table 7. Only for parameters GM2 and GM3 is the critical density less than the central density; but, even in these cases, the kaon condensation produces only a minor modification of the stellar properties. For the chiral case in Table 8, kaon condensation occurs for Σ^{KN} values of 344 and 520 MeV. For the latter, the critical densities are much lower than for the meson exchange model. We recall from Fig. 2 that, for $g_{\omega K}/m_\omega \sim 0.7$, the critical density is very sensitive to the precise value of the σ coupling. Thus, it is to be expected that when hyperons are present, the question of kaon condensation is quite delicate and depends sensitively on the parameters employed, as well as the model chosen. Finally, as the notation in Table 8 indicates, the effective mass drops to zero in all the chiral cases; the problem is more severe here than in meson-exchange models, because there is an explicit negative contribution from the condensate to the baryon masses. Thus, we are unable to compare the stellar properties of the two models.

IV. CRITIQUE OF THE MODELS

In this section, we want to point out clearly that there are significant uncertainties and difficulties associated with these models. We first discuss the implications of uncertainties in the hyperon coupling constants, and then we delineate difficulties with effective masses.

A. Hyperon couplings

In the previous sections, we assumed that the couplings of the Σ and Ξ were equal to those of the Λ hyperon. Here, we relax this assumption and explore the sensitivity to unequal couplings of the different hyperons. Of the many possibilities, we pick three for study. These are listed in Table 9, in terms of the ratio to the nucleon couplings as defined in Sec. II.B.2. For the Λ, we use the values discussed previously. For the Σ, we use two sets of values which gave satisfactory fits to the Σ^- atom data in the work of Mareš et al., [25]. This was based on a mean field description of nuclear matter using the nucleon couplings of Horowitz and Serot [24], who did not include non-linear terms ($b = c = \zeta = 0$). The parameters are listed in Table 1 as HS81. Partly for consistency and partly because this model is often used as a baseline in the literature, we will adopt these parameters. (Qualitatively similar results are obtained for other values of the nucleon couplings, which yield more realistic values of the compression modulus.) Finally, we need the couplings of the Ξ. Since there is little information, we take the couplings to be equal to those of either the Λ or the Σ. Note that set 1 in Table 9 is close to the set that we have been using in the previous discussion.

In Fig. 5, the upper, center and lower panels refer to hyperon coupling sets 1, 2 and

3, respectively. The upper panel is similar to results already discussed; note that kaons do not condense up to the maximum density displayed, $u = 4.5$. In discussing the other cases, we first mention the seeming paradox that increasing the coupling constants of a hyperon species delays its appearance to a higher density. The explanation [5,6] is that the threshold equation receives contributions from the σ, ω and ρ mesons, the net result being positive due to the ω. Thus, if all the couplings are scaled up, the positive contribution becomes larger, and the appearance of the particle is delayed to a higher density. With this in mind, consider the center panel of Fig. 5 which corresponds to set 2 of Table 9. The Σ couplings are larger than set 1 (upper panel), so the Σ^- no longer appears, thus allowing the chemical potential μ to continue rising (cf. Figs. 3 and 4). This allows the Ξ^- to appear at $u = 2.2$, essentially substituting for the Σ^-. Of course, were we to reduce the Ξ couplings on the grounds that this hyperon contains two strange quarks, the Ξ^- would appear at an even lower density. Turning to the lower panel of Fig. 5, we recall that this corresponds to set 3 of Table 9, for which both the Σ and Ξ couplings are increased. Neither of them now appear, and since the chemical potential, μ, continues to increase with density, it becomes favorable for kaons to condense at $u = 3.6$; the fraction Y_{K^-}, however, remains rather small.

Clearly, the lesson to be drawn from this is that the thresholds for the strange particles, hyperons and kaons, are sensitive to coupling constants which are poorly known. Thus, while strangeness plays a significant role in determining the constitution and physical properties of a neutron star, the detailed behavior cannot be tied down at the present time.

B. Effective Masses

We have several times alluded to effective masses going to zero, and we wish to clarify the situation here; for clarity kaons will be excluded from the initial discussion. The situation is best illustrated by reference to Fig. 6. Here, we display for all the parameter sets we have discussed (see Table 1) the effective mass ratio, M_n^*/M, for the neutron, since this is the first particle to show pathological behavior. The top panel is for the case in which only nucleons are allowed, and we see that there is no pathological behavior with the effective mass going smoothly to zero with increasing density in all cases. Indeed, in pure neutron matter, with degeneracy $\gamma = 2$, or in nuclear matter, with $\gamma = 4$, at high density the effective mass has the limiting form [19]

$$M_n^* \to M \left[1 + \frac{g_{\sigma N}^2}{m_\sigma^2} \frac{\gamma k_{Fn}^2}{4\pi^2} \right]^{-1} , \tag{26}$$

when the non-linear couplings are neglected, $b = c = 0$. By contrast, when hyperons are allowed with the couplings of Table 2, the middle panel of Fig. 6 shows that in most cases the neutron effective mass becomes zero. Even for the GM2 and 3 cases, this will happen if one goes beyond the density range plotted. The density at which M_n^* becomes zero is clearly correlated with the effective nucleon mass in equilibrium nuclear matter. Values ~ 0.6, as favored by nuclei [24], cause this to happen at $u \sim 4$, while for $M_n^*/M \sim 0.8$, it is postponed to $u > 10$. The B91 model behaves differently, with the neutron mass becoming zero at $u \sim 7.5$, even though the effective mass in nuclear matter is 0.6. This behavior of the effective mass turning negative has been noted earlier by Lévai et al. [28] in a mixture of nucleons and Δ baryons at finite temperature and chemical potential.

The problem of effective masses turning negative is generic to multi-component systems in which the constituents have dissimilar masses and different couplings to the σ field. This can be seen clearly if one considers the σ field equation (11) (again for simplicity, choosing $b = c = 0$). At very high densities, one can take the limit $k_{FB}/M_B^* \gg 1$ for all baryons present. Defining

$$G_B^\sigma = g_{\sigma B} g_{\sigma N}/m_\sigma^2 \,, \qquad x_B = g_{\sigma B}/g_{\sigma N} \qquad and \qquad y_B = M_B/M \,, \tag{27}$$

the σ field equation can be rewritten as

$$\frac{g_{\sigma N}\sigma}{M} = \frac{\sum_B G_B^\sigma y_B k_{FB}^2}{\sum_B G_B^\sigma x_B k_{FB}^2} \left(\frac{1}{1 + \frac{2\pi^2}{\left(\sum_B G_B^\sigma x_B k_{FB}^2\right)}} \right) . \tag{28}$$

For sufficiently high densities, the term in the bracket will approach unity. Since $M_n^*/M = 1 - g_{\sigma N}\sigma/M$, the neutron effective mass will eventually become negative if the term in front of the bracket is greater than unity. This is the case for the middle panel of Fig. 6, since $y_B \geq 1$ and $x_B \leq 1$. (Note that instead of taking the nucleon coupling and mass as a reference, one could have taken any one of the baryon couplings and masses.) Thus, one of the baryon effective masses will go negative at high densities unless all the baryon couplings are chosen to fulfill

$$x_B = y_B\,, \qquad or \qquad g_{\sigma B}/g_{\sigma N} = M_B/M\,. \tag{29}$$

This prescription yields the following ratios for the hyperon to nucleon coupling constants:

$$x_{\sigma\Lambda} = 1.190\,, \qquad x_{\sigma\Sigma} = 1.272 \qquad and \qquad x_{\sigma\Xi} = 1.405\,. \tag{30}$$

The bottom panel of Fig. 6 shows that, with these values of x, the effective neutron mass now goes to zero only at infinite density, as in nucleons-only matter. Similar behavior is obtained for the other baryons. It must be emphasized that while the choice of couplings in Eq. (30) leads to physically sensible effective masses, it fails to reproduce the Λ binding in Eq. (17), unless a significantly larger value for the ω coupling is used.

We would like to briefly assess the implications of Eq. (30) for the composition of neutron stars. For present purposes, we will use the x values of Eq. (30) for all the meson-hyperon couplings. Results are shown in Fig. 7 for the parameter sets H300 and GM2; the latter can be compared with Figs. 3 and 4. The upper panels (without kaons) show that hyperons appear, but that the fractions Y of the various species are small and tend to drop with increasing density. The matter is dominantly neutron matter. This is reflected in the maximum mass for the GM2 case, which is $2.04 M_\odot$, essentially the same as the np case of Table 5. If one allows for the presence of kaons, with the parameters of Table 3, the lower panels of Fig. 7 show they appear at $u \sim 3$ and quickly balance the number of protons, $Y_{K^-} \approx Y_p$, and this is close to the neutron fraction, Y_n, at high density. Thus, the matter is dominantly npK matter, and again this is reflected in the maximum mass for the GM2 case, which is $1.86 M_\odot$ (cf. Table 5). In a nutshell, with these values of x, hyperons play only a very minor role.

The behavior of the effective masses, namely that they vanish at a finite baryon density, may indicate that the mean field model is being pushed to the limits of its applicability. Inclusion of quantum loop corrections and the effects of correlations may well alter this behavior. Vanishing effective masses naturally arise in models with spontaneously broken chiral symmetry, where the vacuum masses are generated by a non-zero value for a scalar field. (The Nambu–Jona-Lasinio model, with four-Fermi interactions of the type $(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\boldsymbol{\tau}\psi)^2$, is an example.) These models also suggest that at a finite density chiral symmetry is restored, i.e. the effective masses become zero, even for a single fermion species. In the Walecka type model we are considering this occurs at infinite density for nucleons of zero strangeness, but at a finite density when fermions of different strangeness enter. Whether this may be interpreted as *strangeness-induced chiral restoration* in this approach depends on (i) whether the effective mass can be viewed as an order parameter, and (ii) also on whether the scalar terms of the Walecka Lagrangian can be shown to arise from chiral Lagrangians. While arguments to support such an interpretation may be adduced, further work is clearly necessary.

V. CONCLUSIONS

We have examined the presence of strangeness in the form of hyperons and/or a kaon condensate in neutron star matter. Calculations were performed in the framework of a relativistic mean field theory in which baryon-baryon and kaon-baryon interactions are generated by the exchange of σ, ω and ρ mesons. Our results allow for comparisons with the results obtained from the chiral Kaplan-Nelson model, where the kaons and baryons interact directly.

The qualitative results of kaon condensation in the meson-exchange model are similar to those of the chiral model when the magnitudes of the kaon-baryon interactions in the two models are required to be compatible with the kaon optical potential in nuclear matter. Specifically, we find that when matter contains nucleons and leptons only, kaons condense around $3-4$ times the nuclear matter saturation density, as found earlier using the chiral model. The effects of kaon condensation include a softening of the equation of state, which leads to a reduction in the maximum mass and to an increased proton concentration, which implies a rapid cooling of the star's core through direct Urca (beta decay) processes.

Within the meson-exchange model, we also investigated the presence of hyperons in matter and its influence on kaon condensation. Due to limited guidance about the couplings of the strange particles, be they kaons or hyperons, the densities at which they appear in dense matter are uncertain. The importance of individual hyperon species likewise remains unclear. For example, with the choice of couplings made in Sec. II (suggested by the Λ binding in nuclei), hyperons play a dominant role, while kaons, which appear at a higher density, are of lesser importance. However, these roles may be altered by other suitable choices of the coupling constants. If, in addition to the Λ couplings implied by hypernuclei, the couplings implied by Σ^- atoms are employed, then, depending upon the couplings of the Ξ, it is possible that the K^- is the first negatively charged hadron to appear in matter. This highlights the importance of further work in this area using inputs from hypernuclear physics and advances in both theory and techniques for calculating the energy of interacting systems.

In addition, we have pointed out that in mean field theories, effective masses will become zero and negative, unless a particular choice is made for the couplings of the hyperons to the σ field; this choice is, however, not supported by data on hypernuclei. Even if this problem does not occur within the range of densities considered, an instability in the basic theory gives one cause for concern in assessing the results obtained in these and other calculations. Thus, there is need to devise alternative means to treat a system of many baryons in the presence of scalar interactions.

Despite these caveats, an overall theme does emerge. Namely, that the presence of strangeness in dense matter necessarily implies that the equation of state is softened and the maximum mass is reduced. As noted in Ref. [1], protoneutron stars with strangeness-bearing components have larger maximum masses than catalyzed neutron stars, in contrast to the case of nucleons-only stars. This leads to metastable protoneutron stars that could collapse to black holes during their deleptonization era. In addition, since strangeness is accompanied by negative charge, the proton fraction in processed stars will be large enough for the direct Urca process to operate. This leads to more rapid cooling than the "standard" modified Urca process, which requires a spectator nucleon. The rate of cooling can be determined from the inferred surface temperatures of neutron stars [4], but further work is needed before a definitive answer can be given.

We refer the interested reader to a related paper by Schaffner and Mishustin [29], which became available as this work was completed.

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TABLE 1. Coupling constants fitted to a binding energy of -16.3 MeV at an equilibrium density of $n_0 = 0.153\ fm^{-3}$ in nuclear matter with a compression modulus K and effective mass M^*. The symmetry energy coefficient is 32.5 MeV. The equilibrium scalar and vector fields are also listed. Models termed 'H' are from Ref. [22], models termed 'GM' are from Ref. [23]. For the model termed 'B91' from Ref. [20] and the model termed 'HS81' from Ref. [24], the binding energy is -15.75 MeV at $n_0 = 0.1484\ fm^{-3}$ and the symmetry energy coefficient is 35 MeV.

	M_N^*/M_N	K (MeV)	$\frac{g_{\sigma N}}{m_\sigma}$ (fm)	$\frac{g_{\omega N}}{m_\omega}$ (fm)	$\frac{g_{\rho N}}{m_\rho}$ (fm)	b	c	S (MeV)	V (MeV)
H300	0.65	300	3.655	2.932	2.035	0.002319	-0.002129	329	260
H200	0.65	200	3.758	2.932	2.035	0.003845	-0.005035	"	"
GM1	0.7	300	3.434	2.674	2.100	0.002947	-0.001070	282	216
GM2	0.78	300	3.025	2.195	2.189	0.003478	0.01328	207	146
GM3	0.78	240	3.151	2.195	2.189	0.008659	-0.002421	"	"
B91	0.6	250	4.068	3.392	2.173	0.0014028	0.0001193	376	304
HS81	0.541	545	3.974	3.477	2.069	0.0	0.0	431	354

TABLE 2. Ratios of hyperon-meson to nucleon-meson coupling constants.

	M_N^*/M_N	$x_{\sigma H}$	$x_{\omega H}$	$x_{\rho H}$
H200,300	0.65	0.6	0.652	0.6
GM1	0.7	0.6	0.653	0.6
GM2,3	0.78	0.6	0.659	0.6
B91	0.6	0.6	0.649	0.6
HS81	0.541	0.6	0.651	0.6

TABLE 3. Kaon-baryon coupling constants that reproduce the phase shift data.

	$\frac{g_{\sigma K}}{m_\sigma}$ (fm)	$\frac{g_{\omega K}}{m_\omega}$ (fm)	$\frac{g_{\rho K}}{m_\rho}$ (fm)
H300	0.669	1.699	0.639
H200	0.650	1.699	0.639
GM1	0.712	1.863	0.619
GM2	0.808	2.269	0.594
GM3	0.776	2.269	0.594
B91	0.601	1.468	0.599
HS81	0.615	1.433	0.629

TABLE 4. Scalar and vector contributions and the kaon optical potential in equilibrium nuclear matter using the kaon-baryon couplings from the phase shifts.

	$-S^K_{opt}$ (MeV)	$-V^K_{opt}$ (MeV)	$-U^K_{opt}$ (MeV)
H300	30	150	180
H200	28	150	178
GM1	29	151	180
GM2	28	151	179
GM3	25	151	176
B91	28	132	159
HS81	33	146	179

TABLE 5. Gravitational mass and central density of the maximum mass neutron stars for matter with and without kaon condensates. The critical density ratio for condensation is given in the middle column. The symbol np denotes matter containing nucleons and leptons, and npH denotes matter containing nucleons, hyperons and leptons. The kaon coupling constants are taken from Table 3. The symbol * marks models for which the nucleon effective mass drops to zero before reaching the central stellar density. The symbol ** indicates that for this choice of constants no condensation takes place up to the maximum density considered ($u = 10$).

		without kaons			with kaons	
		$\frac{M_{max}}{M_\odot}$	u_{cent}	u_{crit}	$\frac{M_{max}}{M_\odot}$	u_{cent}
np	H300	2.529	5.13	2.44	2.395	5.56
	H200	2.508	5.32	2.40	2.378	5.81
	GM1	2.346	5.70	2.49	2.185	6.46
	GM2	2.064	6.58	2.60	1.854	8.37
	GM3	2.005	7.14	2.59	1.809	9.18
	B91	2.097	5.80	3.19	2.014	5.59
	HS81	2.954	3.85	3.35	2.886	3.86
npH	H300	—	*	**	—	—
	H200	—	*	**	—	—
	GM1	1.776	6.53	**	—	—
	GM2	1.655	6.96	3.37	1.645	7.36
	GM3	1.544	7.98	3.20	1.536	8.46
	B91	1.463	6.18	**	—	—
	HS81	—	*	**	—	—

TABLE 6. Kaon-baryon coupling constants determined from the optical potential in chiral models. For ρ-meson exchange, we take the ratio $x_{\rho K} = g_{\rho K}/g_{\rho N}$ to be $\frac{1}{3}$.

	$\frac{g_{\sigma K}}{m_\sigma}$ (fm)			$\frac{g_{\omega K}}{m_\omega}$ (fm)	$\frac{g_{\rho K}}{m_\rho}$ (fm)
Σ^{KN} (MeV)	167	344	520		
GM1	0.537	1.106	1.672	0.631	0.700
GM2	0.650	1.340	2.025	0.769	0.730
GM3	0.678	1.396	2.110	0.769	0.730

TABLE 7. Gravitational mass and central density of the maximum mass neutron stars for matter with and without kaon condensates in the meson-exchange model. The critical density ratio for condensation is also listed. Symbols are: np for nucleons-only matter, npK for nucleons-only matter with a kaon condensate; npH and npHK denote matter which also includes hyperons. The kaon coupling constants are from Table 6. The symbol ** indicates that no condensation takes place for this choice of coupling constants.

		GM1			GM2			GM3		
	Σ^{KN}	$\frac{M_{max}}{M_{\odot}}$	u_{cent}	u_{crit}	$\frac{M_{max}}{M_{\odot}}$	u_{cent}	u_{crit}	$\frac{M_{max}}{M_{\odot}}$	u_{cent}	u_{crit}
np		2.346	5.70	—	2.064	6.58	—	2.005	7.14	—
	167	2.339	5.62	4.74	2.038	6.22	4.90	1.950	6.66	4.75
npK	344	2.288	5.56	3.74	1.956	5.97	3.95	1.831	7.82	3.75
	520	2.177	6.35	2.94	1.818	8.85	3.17	1.770	9.25	2.99
npH		1.776	6.53	—	1.655	6.96	—	1.554	7.98	—
	167	—	—	**	—	—	**	—	—	**
npHK	344	—	—	**	—	—	**	—	—	**
	520	—	—	7.72	1.646	6.89	5.10	1.516	8.35	3.93

TABLE 8. Same as Table 7, but for the chiral model. The symbol * indicates that the effective mass drops to zero below the expected central stellar density. The critical density for condensation is marked with a ** if the nucleon effective mass drops to zero prior to condensation.

	Σ^{KN}	GM1			GM2			GM3		
		$\frac{M_{max}}{M_{\odot}}$	u_{cent}	u_{crit}	$\frac{M_{max}}{M_{\odot}}$	u_{cent}	u_{crit}	$\frac{M_{max}}{M_{\odot}}$	u_{cent}	u_{crit}
np		2.346	5.70	—	2.064	6.58	—	2.005	7.14	—
	167	2.334	5.60	4.54	1.990	5.96	4.33	1.911	6.67	4.35
npK	344	2.270	5.66	3.48	1.796	8.68	3.28	1.783	9.22	3.29
	520	2.182	6.45	2.73	1.769	10.39	2.60	1.777	10.30	2.61
npH		1.776	6.53	—	1.655	6.96	—	1.554	7.98	—
	167	—	—	**	—	*	9.39	—	*	9.90
npHK	344	—	*	5.87	—	*	4.39	—	*	4.41
	520	—	*	3.33	—	*	2.86	—	*	2.86

TABLE 9. Ratios of hyperon-meson to nucleon-meson coupling constants, $x_{iH} = g_{iH}/g_{iN}$ where $i = \sigma,\ \omega$ or ρ and H is a hyperon species.

Case	$x_{\sigma\Lambda}$	$x_{\omega\Lambda}$	$x_{\rho\Lambda}$	$x_{\sigma\Sigma}$	$x_{\omega\Sigma}$	$x_{\rho\Sigma}$	$x_{\sigma\Xi}$	$x_{\omega\Xi}$	$x_{\rho\Xi}$
1	0.60	0.65	0.60	0.54	0.67	0.67	0.60	0.65	0.60
2	0.60	0.65	0.60	0.77	1.00	0.67	0.60	0.65	0.60
3	0.60	0.65	0.60	0.77	1.00	0.67	0.77	1.00	0.67

FIGURE CAPTIONS

Fig. 1. Contours of the critical density ratio $u_{crit} = n_{crit}/n_0$ for kaon condensation in matter containing nucleons and leptons as a function of the kaon-meson coupling constants $g_{\omega K}/m_\omega$ and $g_{\sigma K}/m_\sigma$. For the kaon-rho coupling the ratio $x_{\rho K} = g_{\rho K}/g_{\rho n}$ was taken to be $\frac{1}{3}$. Panels (1)–(3) show results for models GM1-GM3.

Fig. 2. Same as Fig.1, but in matter containing nucleons, hyperons and leptons. In region (A), kaons are present above the critical density up to the highest densities considered ($8n_0$ for panel (1) and $10n_0$ for panels (2) and (3)). In region (B), kaons appear at the critical density but disappear again at a higher density. In this region, for each choice of couplings the figure yields two densities, the lower one corresponding to the critical density for kaon condensation and the higher one to the highest density where kaons will still be present. In region (C), kaons do not condense.

Fig. 3. Matter containing nucleons and leptons with parameter set GM2. Solid (dashed) lines show quantities in matter with (without) kaons, as a function of the baryon density ratio $u = n/n_0$. Panel (1): Particle fractions $Y_i = n_i/n$. Panel (2): Kaon energies $\omega^\pm$ and effective mass m_K^*, meson field strengths and electron chemical potential μ. Panel (3): Kaon condensate amplitude, θ and strangeness/baryon, $|S|/B$. Panel (4): Pressure and energy density.

Fig. 4. As for Fig. 3, but for matter containing nucleons, hyperons and leptons.

Fig. 5. Particle fractions for model HS81 with different choices of Σ and Ξ coupling constants. Panels (1), (2) and (3) correspond to parameter sets 1, 2 and 3 of Table 9, respectively.

Fig. 6. Neutron effective mass ratios, M_n^*/M, in nucleons-only matter (panel 1), in matter containing hyperons with the parameters of Table 2 (panel 2) and in matter containing hyperons with the parameters of Eq. (30) (panel 3). The labels on the curves indicate the equation of state employed (see Table 1).

Fig. 7. Particle fractions in the case where the hyperon couplings to all mesons are chosen according to Eq. (30). The left panels show results for the HS300 parameter set and the right panels for the GM2 set. In the upper (lower) panels kaons are excluded from (included in) the calculations.