Reinterpretation of Thermal Dilepton Emission Rate by Spectral Functions[*]

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Abstract

We reinterpret the dilepton emission rate from a hadronic gas expected to be produced in heavy ion collisions in terms of the spectral functions available from the e^+e^- annihilation and the τ lepton decays experiments. We take into account all possible hadronic state especially the multi-pion contributions to the dilepton emission and the parity mixing phenomenon due to the soft final-state corrections. A new compilation of the experimental data for the spectral functions is presented.

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The lepton pair emissions are the ideal probes of the dense system of strongly interacting particles expected to be produced in ultrarelativistic heavy nucleus-nucleus collisions [1]. Once produced, they will decouple from the strongly interacting system without further interactions and thus carry the information on the dynamical properties of the hot nuclear system. The theoretical predictions on the emission rate, both from a quark-gluon plasma at high temperature (T) and from a hot hadron gas at relatively lower T, have been extensively discussed in literatures [2–6]. There already exist some experimental data from various CERN SPS experiments (for a recent review, see [7]), and more thorough investigations will be made at upcoming RHIC and LHC experiments. The quantitative interpretation of the data is beginning to be possible [8,9]

The main theoretical uncertainties in calculating the dilepton emissions rate come from the fact (apart from the crucial assumption that the system thermalizes in short time and stays in local equilibrium) that the system is strongly interacting and the perturbation theory fails. In a hot hadronic gas, though the effective degrees of freedom are mainly pions, pions interact strongly among themselves in many resonance channels, and effectively the resonances have to be taken into account. One obvious example is the pion electromagnetic form factor which exhibits the ρ-meson dominance. Near the resonances the naive pole approximations often violate the unitarity of the partial waves. There may also be some potential double counting problems associated with the emission rate at the location of the resonance peaks, as to whether or not the real resonance dilepton decays should be counted separately. Also, the multi-pion state emissions, e.g. $3\pi \to \ell^+\ell^-$ and $4\pi \to \ell^+\ell^-$ may not be so suppressed as they can proceed through the resonance poles, e.g. $3\pi \to \omega^*, \phi^* \to \ell^+\ell^-$ and $4\pi \to \pi a_1 \to \ell^+\ell^-$. Taking all these reactions into account by the kinetic theory is a difficult task though not impossible. One may always wonder the validity of a calculation and if the omission of certain reactions oversimplifies the life. More importantly, process involving hadrons in the final state such as $n\pi \to \ell^+\ell^- m\pi$ may have been entirely overlooked. Even in the phase of quark-gluon plasma where quarks and gluons are supposed to be weakly interacting, the high order corrections in α_s may not be so small.

The aim of this paper is to use and to develop the expression that allows the non-perturbative treatment of the strong interactions and avoids the detailed counting of the relevant reactions. Such an expression is known to exist [2,3,10] in the form of a low temperature expansion when T^2 is smaller compared to the dilepton invariant mass squared M^2. We shall reinterpret the dilepton emission rate in terms of the spectral functions of some hadronic currents that are known experimentally in the low energy e^+e^- annihilation process and τ lepton decays. While we shall comment on its application to the quark-gluon plasma, our approach is most relevant for the case of a hadronic gas. The recent development in calculating the correlation functions of vector currents in a low temperature pion gas using the PCAC and current algebra [11] enables us to obtain the important corrections to the rate with soft pions emitted together with the dilepton in the final state. Our parameterization clearly exhibits the mixing phenomenon of vector and axial vector currents at finite temperature and its relation to the dilepton production. We present a new compilation of the experimental data for the spectral functions in the energy range from 360 MeV to 2150 MeV, relevant to the dilepton yield from a hadron gas.

In the lowest order in the electromagnetic interactions, the emission rate can be expressed non-perturbatively in terms of the correlation functions of the hadronic electromagnetic current. When the lepton mass is ignored, the rate reads [2,3,10]

$$\frac{dR}{d^4q} = \frac{4\alpha^2}{3(2\pi)^3}\frac{1}{q^4}(q^\mu q^\nu - q^2 g^{\mu\nu})W_{\mu\nu}(q) \tag{1}$$

where

$$W_{\mu\nu}(q) = \int d^4x e^{-iqx}\langle\langle J_\mu^{em}(x)J_\nu^{\dagger em}(0)\rangle\rangle . \tag{2}$$

q is the time-like 4-momentum of the lepton pair, J_μ^{em} is the electromagnetic current for hadrons and $\langle\langle\cdots\rangle\rangle$ stands for the thermal average at a give temperature T. Note that $W_{\mu\nu}(q)$ is the thermal structure function, and there is no time ordering in the product of $J_\mu^{em}(x)$ and $J_\nu^{\dagger em}(0)$, which is crucial in relating $W_{\mu\nu}$ directly to the spectral functions. (1) is shown to agree with the relativistic kinetic theory reaction by reaction if one inserts the complete set $\sum_F |F\rangle\langle F| = 1$ for each component $|I\rangle$ of the thermal density matrix

$$W_{\mu\nu}(q) = \sum_{F}\sum_{I}\int d^x e^{-iqx}\langle I|J_\mu^{em}(x)|F\rangle\langle F|J_\nu^{\dagger em}(0)|I\rangle\frac{e^{-\beta E_I}}{\mathcal{Z}} \ , \tag{3}$$

where $\mathcal{Z}$ is the canonical partition function of the system. (3) describes the all possible transitions $I \to F\ell^+\ell^-$. Summing up the disconnected parts of the matrix elements, one recovers the appropriate Fermi or Bose distribution functions $f(E_i) = (e^{\beta E_i} \pm 1)^{-1}$ for each initial-state particle and the suppression or enhancement factor $(1 \mp f(E_i))$ for each final-state particle. $W_{\mu\nu}(q)$ has no non-trivial zero temperature limit. To get the leading temperature dependence, one alternatively sums up all initial states $|I\rangle$'s using the completeness $\sum_I |I\rangle\langle I|$ and the energy conservation $E_I = E_F + q^0$ [10]

$$\begin{aligned} W_{\mu\nu}(q) &= e^{-\beta q^0}\sum_{F}\int d^4 x e^{iqx}\langle F|J_\nu^{\dagger em}(x)J_\mu^{em}(0)|F\rangle\frac{e^{-\beta E_F}}{\mathcal{Z}} \\ &= e^{-\beta q^0}\int d^4 x e^{iqx}\langle\langle J_\nu^{\dagger em}(x)J_\mu^{em}(0)\rangle\rangle \equiv e^{-\beta q^0}\rho_{\mu\nu}(q) \ . \end{aligned} \tag{4}$$

$\rho_{\mu\nu}(q)$ now has a non-trivial limit as $T \to 0$. Though $|F\rangle$'s and $|I\rangle$'s seem to switch their positions in (3) and (4), the same kinetic theory result would be obtained if one uses the identity $(1 \mp f(E))e^{-\beta E} = f(E)$.

We would like to compute $\rho_{\mu\nu}(q)$ for a low temperature hadronic gas, not reaction by reaction as one does in the kinetic theory, but to obtain a low T expansion, in which each term is the summation of all possible reactions. The leading term is independent of temperature, which is obtained by taking the limit $T \to 0$ where only the vacuum state in $|F\rangle$'s contributes to the connected part, namely

$$\rho_{\mu\nu}(q) \xrightarrow{T\to 0} \rho_{\mu\nu}^{em}(q) \equiv \int d^4 x e^{iqx}\langle 0|J_\nu^{\dagger em}(x)J_\mu^{em}(0)|0\rangle \ . \tag{5}$$

When all possible initial states $|I\rangle$'s are saturated between the two currents, (5) corresponds to the "hard" particle contributions illustrated in [2] where "hard" means "not necessarily soft". As we are interested in the kinematical region $T^2 \ll q^2$, any non-vacuum states $|F\rangle$'s are necessarily soft since they are kinematically limited to $E \leq T$ by the presence of a thermal distribution function. The low but finite temperature corrections to the leading term $\rho_{\mu\nu}^{em}(q)$ arise from the soft final-state particle emissions and the soft initial-state particles,

both are limited to $E \leq T$. In a hadronic gas, the soft modes are the pions. One may use the soft pion theorem based on the PCAC to calculate the soft pion corrections. Unlike the case in the quark-gluon plasma where there is a contribution of order T^2, the insertion of the soft pions (the initial states) in the intermediate states of the correlator does not pick up a contribution of order T^2. In fact, the extra powers of soft thermal pion momentum leads to a suppression factor of order T^4/M^4 in the chiral limit, as shown by Leutwyler and Smilga in [12] and Eletsky in [11]. This is the consequence of the low temperature theorem [12] (the finite temperature version of the Adler theorem) that the chiral symmetry protects all of the masses from picking up a contribution of order T^2. We shall ignore the effects of the initial-state soft corrections with the understanding that they start at $O(T^4/M^4)$.

On the other hand, the corrections from the soft final-state thermal pions, as shown by Dey, Eletsky and Ioffe [11], leads to the mixing of vector and axial vector correlators. The essence is to approximate the matrix elements containing soft pions with their threshold value at zero pion momentum, which is completely determined by the current algebra or the symmetry of the system [13]. For example,

$$
\begin{aligned}
\langle n\pi^+ | J_\nu^{\dagger em}(x) J_\mu^{em}(0) | n\pi^+ \rangle &= -\frac{1}{F_\pi^2} \langle (n-1)\pi^+ | J_\nu^{\dagger em}(x) V_\mu^3(0) - A_\nu^-(x) A_\mu^+(0) \\
&\quad - A_\nu^+(x) A_\mu^-(0) + V_\nu^3 J_\mu^{em}(0) | (n-1)\pi^+ \rangle \\
&= \frac{2}{F_\pi^2} (-1)^n 3^{n-1} \langle 0 | V_\nu^3(x) V_\mu^3(0) - A_\nu^3(x) A_\mu^3(0) | 0 \rangle ,
\end{aligned}
$$

(6)

where V_μ^3 and A_μ^3 are the third (neutral) component of the vector- and axial-vector- isovector currents, $F_\pi = 93$ MeV. By doing so, we may compute all reactions with soft pions in the final state $I \to \ell^+ \ell^- n\pi^\pm$. It is worth pointing out that although J_μ^{em} contains all flavors (u, d, s, c, b, t), the soft pion emission selects only two flavors (u, d) since V_μ^3 and A_μ^3 contains no strange or higher flavor contents. Also, when only u, d flavors are effective at lower energy range, $J_\mu^{em} \equiv V_\mu^3 + B_\mu/2$ where B_μ is the baryonic current which contains both isovector $(I = 1)$ and isosinglet $(I = 0)$ parts, while V_μ^3 and A_μ^3 are purely isovector. When $n \geq 2$, (6) combining with the integration of the Bose distribution functions gives the corrections of order $(T^2/6F_\pi^2)^2$ or higher. However, the multi-pions in the final state can interact with

themselves and cause further phase shifts. Inclusion of the $\pi\pi$ rescattering effects gives a correct T^4 term [11]. We neglect the non-zero pion momentum that causes a correction suppressed by M^2 and summarize the corrections to T^4 in the spectral tensor

$$\rho_{\mu\nu}(q) = \rho_{\mu\nu}^{em}(q) - (\epsilon - \frac{\epsilon^2}{2})(\rho_{\mu\nu}^{V}(q) - \rho_{\mu\nu}^{A}(q)) , \tag{7}$$

where $\epsilon = T^2/6F_\pi^2$. $\rho_{\mu\nu}^{V}(q)$ and $\rho_{\mu\nu}^{A}(q)$ are the correlators defined in (5) with J_μ^{em} replaced with V_μ^3 and A_μ^3 respectively. Clearly, the dilepton production from a thermal source comes not only from the vector channel but also from the axial vector channel since they can interchange their parity identities through the interaction with a thermal background. The soft corrections tend to suppress the contributions from the vector channel and to increase the rate by adding to it the axial channel contributions.

Since J_μ^{em} and V_μ^3 are conserved currents (neglecting the isospin breaking), $\rho_{\mu\nu}^{em}(q)$ and $\rho_{\mu\nu}^{V}(q)$ have only transverse (spin one) part. A_μ^3 receives both spin one and spin zero (pions) contributions. However, the longitudinal part ($\propto q^\mu q^\nu$) does not contribute when contracted with the lepton pair tensor. Define the spectral functions $\rho^{(i)}(s)$ $(i = em, V, A)$ [13]

$$\rho_{\mu\nu}^{(i)}(q) = 2\pi(q_\mu q_\nu - q^2 g_{\mu\nu})\theta(q^0)\rho^{(i)}(s) , \tag{8}$$

where $s = q^2$, one obtains the dilepton emission rate in terms of three spectral functions

$$\frac{dR}{dq^4} = \frac{4\alpha^2}{(2\pi)^2}e^{-\beta q^0}\left\{\rho^{em}(s) - (\epsilon - \frac{\epsilon^2}{2})[\rho^V(s) - \rho^A(s)]\right\} , \tag{9}$$

or if the transverse mass and the rapidity are integrated over

$$\frac{dR}{dM^2} = \frac{4\alpha^2}{2\pi}MTK_1(M/T)\left\{\rho^{em}(M) - (\epsilon - \frac{\epsilon^2}{2})[\rho^V(M) - \rho^A(M)]\right\} . \tag{10}$$

As we have noted earlier, because the spectral tensor does not involve the time ordering of the currents, it is directly represented by the spectral functions on the physical axis $q^2 > 0$. In the case of time ordering correlator, the spectral function is (twice) the imaginary part of the correlator. To get the real part, one has to perform the dispersion integral of the spectral function. Clearly, the usual pole-plus-continuum approximation widely used in the

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Weinberg's sum rules [13] would not do since it is precisely the energy dependence of the spectral densities that determine the shape of the dilepton spectrum.

Other than a theoretical estimate, a more sound procedure is to use experimental date which have been accumulated over years and measured accurately enough to truly represent these spectral functions. Obviously, $\rho^{em}(s)$ can be extracted from the total hadronic cross section (the R value) of e^+e^- annihilations (note that $\sigma(had \rightarrow e^+e^-) = 4\sigma(e^+e^- \rightarrow had)$ because of the helicity summation)

$$\rho^{em}(s) = \frac{s\sigma(e^+e^- \rightarrow had)}{16\pi^3\alpha^2} = \frac{R(s)}{12\pi^2} \ . \tag{11}$$

To get $\rho^V(s)$, we have to first restrict the open flavors to u, d. This can be done effectively by selecting the events in which only pions are produced. We also need to eliminate the $I = 0$ events in e^+e^- annihilations. Since the $I = 0$ states will have the G parity $G = C(-1)^I = -1$, which can only decay into an odd number of pions (a pion has $G = -1$). So by selecting the events consisting only of even number of pions, we have for ρ^V [14,15]

$$\rho^V(s) = \frac{s}{16\pi^3\alpha^2} \sum_{n=1} \sigma(e^+e^- \rightarrow 2n\pi) \ . \tag{12}$$

ρ^A can be extracted from the differential probabilities of the τ lepton decays into odd number of pions $\tau \rightarrow \nu_\tau(2n + 1)\pi$ [16]

$$\rho^A(s) = \frac{8\pi m_\tau^3}{G_F^2 \cos^2\theta_c (m_\tau^2 + 2s)(m_\tau^2 - s)^2} \sum_{n=1} \frac{d\Gamma(\tau \rightarrow \nu_\tau(2n + 1)\pi)}{ds} \ , \tag{13}$$

where s is understood to be the invariant mass squared of $(2n + 1)\pi$'s, G_F is the Fermi constant and θ_c is the Cabibbo angle.

We present a new compilation of the data for those spectral functions in the energy range fro 360 MeV and 2150 MeV where the data are rich. The total hadronic cross section of e^+e^- annihilation above $\sqrt{s} = 1420$ MeV is measured by Adone $\gamma\gamma2$ and MEA [17], below which the ORSAY data exist [18] but are fairly sparse, especially at ρ, ω, ϕ peaks. We fill in more data from OLYA and CMD $e^+e^- \rightarrow 2\pi$ measurement [19], Orsay DM1 $e^+e^- \rightarrow 3\pi$ measurement [20], and VEPP-2M $e^+e^- \rightarrow K^+K^-, K_S K_L$ [21] in the resonance

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region. To extract ρ^V, we consider, in addition to $e^+e^- \to 2\pi$, the multi-pion reactions $e^+e^- \to 2\pi^+2\pi^-$, $\pi^+\pi^-2\pi^0$, $2\pi^+2\pi^-2\pi^0$, $3\pi^+3\pi^-$ which are available from Adone [22]. For ρ^A, we assume the equality of $\Gamma(\tau \to \nu_\tau\pi^+2\pi^-)$ and $\Gamma(\tau \to \nu_\tau\pi^-2\pi^0)$ which is expected if the 3π state is dominated by the $a_1(1230)$ resonance, and use the recent ARGUS data [23]. Clearly, ρ^A thus obtained is limited to $\sqrt{s} < m_\tau \sim 1.7$ GeV. However, the rapid fall off of ρ^A beyond m_{a_1} indicates a rather negligible contribution at a larger $\sqrt{s}$.

We plot our compilation of the numerical values of the three spectral functions in Fig. 1 for the convenience of further phenomenological studies. Data with poor statistics are removed by hand. The dilepton emission rate at a typical temperature $T = 160$ MeV is plotted in Fig. 2 for the invariant mass ranging from 360 MeV to 2150 MeV. The dotted line is the leading contribution without the soft final-state emissions. An enhancement located around $m_{a_1} \simeq 1230$ MeV is visible due to ρ^A.

We have reinterpreted the dilepton emission rate in terms of spectral functions available directly from the data. The leading contribution is related to the total inclusive hadronic cross section of e^+e^- annihilation, which sums up all possible combinations of mesons and resonances such as $P+P$, $V+V$, $V+P \to \ell^+\ell^-$ recently considered by Gale and Lichard [5], and three- and four- body combinations, and many more. It is free of theoretical uncertainties in calculating these non-perturbative processes and avoids the possible double counting of the light mesons and their resonances. As emphasized by McLerran and Toimela [2], the leading term by no means contains only the hard particle contributions. It is merely not necessarily soft which includes the soft particle contributions as well. The parity mixing phenomenon, also considered in [6,9,24] is explicit in our reinterpretation by the presence of the axial spectral density ρ^A, which arises from the soft final-state emission of pions. Our formalism is applicable to a hadronic gas as long as $T^2 \ll M^2$, $T^2 \ll 6F_\pi^2$ and $T < T_c$. It may be possible to extend the approach to the case of quark-gluon plasma for the high invariant mass region $M > 2$ GeV, where $\sigma(e^+e^- \to had) \simeq \sigma(e^+e^- \to q\bar{q})$ and α_s corrections to all orders would be automatically included . When $M < 2$ GeV, the parton degree of freedom are not apparent in $\sigma(e^+e^- \to had)$. The existence of the parton degrees freedom

at $T > T_c$ even for low invariant mass parton pair is a fundamental assumption. To calculate the actual dilepton multiplicity in order to comapare it with the experimental data, one has to integrate the rate over the space-time history of the thermal source, which involes more assumptions and approximations, and will be reported elsewhere.

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Figure Captions

Fig. 1 Spectral functions $\rho^{em}(s)$, $\rho^V(s)$, and $\rho^A(s)$ from e^+e^- annihilation and τ decays. Note that $\rho^{em}(s)$ extends beyond the vertical scale at the ω and ϕ resonance peaks.

Fig. 2 Dilepton emission rate from a $T = 160$ MeV thermal source. Dotted line (error bars not shown) represents the leading temperature contribution without the soft pion emissions. The error bars are calculated assuming the statistical independence of different experiment sets.