New Physics with three-photon events at LEP

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Abstract

The effect of the most general $Z \to 3\gamma$ vertex in the reaction $e^+e^- \to 3\gamma$ is studied with a particular attention to LEP searches. We give exact analytical expressions including realistic cuts for the signal and present a detailed analysis based on a Monte Carlo that includes the effect of the irreducible 3γ QED cross section. As special applications we discuss the effect of heavy scalars, fermions and gauge bosons and comment on the "monopole" connection.

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1 Introduction

The second phase of LEP will soon be in operation. The achievements of the first phase have already been formidable as we have learnt a great deal from the precision measurements at the Z peak on the parameters of the Standard Model, $\mathcal{SM}$. Some of the future analyses will need to exploit the full statistics to be accumulated at the end of the LEP1 runs[†] in order to put stringent constraints on rare processes that are usually the hallmark of New Physics. One such process is the decay of the Z into three photons. Within the $\mathcal{SM}$ this process is so rare that even the once-discussed High-Luminosity option of LEP1 [2] would not be able to see it. Indeed, branching ratio for this process calculated within the $\mathcal{SM}$ is $Br_{sm}(Z \to 3\gamma) \simeq 5.4\ 10^{-10}$ [3]. This makes it an ideal candidate to search for contributions that are beyond the $\mathcal{SM}$. It is interesting to note that this kind of quartic vertex, $Z3\gamma$, is an example of an "anomalous" boson self-coupling which can be probed at LEP1 whereas the second phase of LEP will be mostly dedicated to the tri-linear $WW\gamma, WWZ$ couplings. The study of this coupling thus provides a nice bridge, when moving from LEP1 to LEP2, in the general topic of the *direct*[‡] investigations on the vector boson self-couplings. From the experimental point of view, another motivation for studying this decay is the observation that the description of this rare process is, as we shall see, identical to the description of the scattering of light by light parameterised by a 4-γ coupling. Although QED has been with us for a long time, this non-linear effect has not been directly experimentally investigated. As far as we are aware, one has only experimentally studied Delbrück scattering and photon splitting near a heavy nucleus at low energy [4]. One should therefore not miss any opportunity of investigating a very similar kind of physics, especially since the theoretical situation at LEP1 with the $Z \to 3\gamma$ is much cleaner. Thus one should take full advantage of the resonance enhancement at the Z peak which provides an alternative to study New Physics contributions to an Abelian non-linear effect.

Before the advent of LEP, it had been argued [5, 6] that this decay could check the hypothesis of a composite Z. The argument heavily borrows from the ρ system and vector meson dominance [7]. To explain the universality of the weak coupling in this picture, one adapts the vector dominance idea to the W triplet by implementing a $W^0-\gamma$ transition [8].

[†] By the end of 1994 each LEP collaboration has collected about 140 pb^{-1} ($\sim$ 5.5 $10^6 Z$) of data samples but only a fraction of these has been used in published analyses of rare processes. At the end of the LEP1 runs one hopes to accumulate about 200 pb^{-1} per experiment [1].

[‡] Of course, LEP1 is of an unsurpassable precision when it comes to the investigation of the vector bosons self-energies and hence on what might be considered as bi-linear anomalous couplings.

The universality of the QED electromagnetic coupling is thus transmitted to the W/Z-fermion coupling. The difference between the ρ system and the W is that the strength of this transition, directly related to the weak mixing angle s_W, is not small as would be expected for an electromagnetic transition but of order 1/2. Since this is large, one should expect that other electromagnetic transitions be enhanced [9]. More specifically, in this picture, one can build up on the "strength" of the $Z\gamma$ transition to predict a large decay of the Z into three photons. Of course, the Z can not decay into two on-shell photons (Yang's theorem [10]) and in any case the $Z2\gamma$ vertex violates $\mathcal{C}$ and $\mathcal{P}$ and thus could be forbidden. However, $Z \to 3\gamma$ is perfectly allowed. Arguing along these lines, a *non-relativistic* bound state calculation [5] of the $Z \to 3\gamma$ leads to a branching ratio of the order of 10^{-5}. This is in fact the present order of magnitude the LEP experiments [11, 12, 13, 14] have set, as a limit, on this decay.

Recently De Rújula [15] has suggested that this decay may be large if it is induced by "monopoles". This idea has already been put forth [16] a few years ago to motivate a non-negligible 4-γ coupling. The aim of this paper is, first, to give the most general framework for the study of the $Z \to 3\gamma$ where the above two examples (so-called "composite" Z and "monopoles") will be seen to be specific cases of the most general effective Lagrangian. We will then give a detailed analysis on the signature of the $Z \to 3\gamma$ decays. For the most general Lagrangian we give exact analytical formulae for various distributions including the effect of the correlation with the initial e^+e^-state. We also derive analytical formulae taking into account experimental cuts that can be used for a quick estimate of the acceptances of the 4 LEP detectors. Finally we conduct a detailed discussion based on a Monte Carlo analysis which includes not only the signal but also the irreducible $e^+e^- \to 3\gamma$ purely QED background as well as the effect of the interference. We show which distributions are most sensitive to the presence of the $Z \to 3\gamma$ coupling. Finally a short discussion about the effect of this coupling far away from the Z resonance is presented. In the Appendix we collect some simple and compact expressions for the standard and non-standard helicity amplitudes based on the technique of the inner spinor product that were implemented in our matrix element Monte Carlo generator.

2 Effective Lagrangians and Models for $Z \to 3\gamma$

Probably the oldest example of an effective Lagrangian is the celebrated Euler Lagrangian [17] that describes the self-interaction of photons. The first non-trivial interaction describes a 4-γ vertex in its leading part and accounts for the scattering of light-by-light.

The basic idea of the effective Lagrangian is to give a general description of a phenomenon even if one does not know its origin or the underlying theory behind it. All we need to construct the ensuing operators is the known symmetries at low energies which are, in this example, contained in the $U(1)$ gauge invariance. For the 4-γ vertex one is led to

$$\mathcal{L}^{4\gamma}_{eff.} = \frac{\alpha^2}{M^4}\left(\beta_2(F_{\mu\nu}F^{\mu\nu})^2 \;+\; \beta_3(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\right) \qquad \text{with} \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}. \tag{2.1}$$

We remark that even though one is dealing with an effective Lagrangian, and restricting oneself to the *leading* operators, the Lagrangian is very constrained since it contains only two operators. These are necessarily of dimension eight. The mass M is directly related to the scale of New Physics. Non-leading operators contain extra derivatives that give correction factors of order $(E_\gamma/M)^2$ to the dominant terms we are considering, and therefore their effect is very negligible. We can also argue that if these terms were to be taken into account then this would *also* mean that multi-photon amplitudes $6\gamma, ...$ etc would not be negligible and thus the New Physics would be more conspicuous.

Different models, or rather types of heavy particles, give definite predictions for $\beta_{2,3}$. For instance, considering the case of a massive particle (mass M) of unit charge that contributes to the one-loop 4-γ amplitude, we have in the case of fermions [17, 18]

$$\beta_2^F = \frac{4}{7}\beta_3^F = \frac{1}{90}. \tag{2.2}$$

For a charged scalar the coefficients β_i's are different with [18]

$$\beta_2^S = 7\beta_3^S = \frac{7}{16}\frac{1}{90}. \tag{2.3}$$

For completeness, we can also give the contribution of a heavy charged gauge vector boson. Here as the spin increases the values of the coefficients increase as well. This is not only due to the counting of degrees of freedom. Although the coefficients for the $\beta's$ in the case of scalars and fermions have been known for a long time [18], it is only in the last few years that the issue of the vector, more specifically the W-loop, has been investigated [19]. The $\beta's$ can be inferred from [19]:

$$\beta_2^V = \frac{29}{160} \qquad \beta_3^V = \frac{27}{160}. \tag{2.4}$$

It is instructive to realise that if instead of considering $\beta_{2,3}$ one takes the combination

$$\beta_\pm = \beta_2 \pm \beta_3 \tag{2.5}$$

the counting of the degrees of freedom, with the correct Bose-Fermi statistics factor, is reflected only in β_-:

$$\begin{aligned} \beta_+^S &= \frac{1}{180} \quad ; \quad \beta_-^S = \frac{1}{240} \\ \beta_+^F &= \frac{11}{360} \quad ; \quad \beta_-^F = -2\,\beta_-^S \\ \beta_+^V &= \frac{7}{20} \quad ; \quad \beta_-^V = 3\,\beta_-^S. \end{aligned} \tag{2.6}$$

We will have more to say about the extraction of these coefficients and about the so-called supersymmetric relations [20] between the β_- elsewhere [21].

The $Z \to 3\gamma$ vertex is constructed along the same mould. In fact, since we are replacing *only one* photon by a Z the general leading-order effective Lagrangian has exactly the same structure. Therefore, we choose to parameterise the $Z3\gamma$ anomalous coupling in the form

$$\begin{aligned} \mathcal{L}_{Z3\gamma} &= \frac{4\alpha^2}{M^4}\left(\tilde{\beta}_2(F_{\mu\nu}F^{\mu\nu})(F_{\mu\nu}Z^{\mu\nu}) \;+\; \tilde{\beta}_3(Z_{\mu\nu}\tilde{F}^{\mu\nu})(F_{\mu\nu}\tilde{F}^{\mu\nu})\right) \\ &\equiv \frac{4\alpha^2}{M^4}\left((\tilde{\beta}_2 - 2\tilde{\beta}_3)(F_{\mu\nu}F^{\mu\nu})(F_{\mu\nu}Z^{\mu\nu}) \;+\; 4\tilde{\beta}_3(Z_{\mu\nu}F^{\nu\lambda}F_{\lambda\rho}F^{\rho\mu})\right). \end{aligned} \tag{2.7}$$

The factor of 4 in the definition of the $Z3\gamma$ coupling compared to the case of the 4γ coupling is such that the two Lagrangians lead to the same strength, and expression in momentum space, for the $Z3\gamma$ and 4γ coupling modulo the overall change $\beta_i \to \tilde{\beta}_i$.

We refrain here from giving specific examples of the coefficient $\tilde{\beta}_{2,3}$ for the different spin species as, beside the electric charge of the particles, we need to know their hypercharge or $SU(2)_{weak}$ assignment. Nonetheless the relations between $\tilde{\beta}_2$ and $\tilde{\beta}_3$ are maintained in the same form as in the all 4-γ case (Eq. 2.2- 2.3), apart from the case of the spin-1, where there is a subtlety. For the latter one needs to know both the quantum number of the pure gauge part *and* the Goldstone part of the massive vector bosons. In practice the pure gauge part is by far dominant. Since these are rather technical observations and calculations we leave this discussion to a forthcoming paper [21].

Again it is worth working, instead of the two independent constants $\tilde{\beta}_2$ and $\tilde{\beta}_3$, with the combination

$$\tilde{\beta}_\pm = \tilde{\beta}_2 \pm \tilde{\beta}_3. \tag{2.8}$$

The reason is that $\tilde{\beta}_+$ and $\tilde{\beta}_-$ contribute to different helicity amplitudes and therefore they do not interfere. It is easy to see the reason why this is so. For simplicity it is best to stick with the 4-γ Lagrangian and rewrite it in terms of the self-dual, $F^{+}_{\mu\nu}$, and anti-self-dual, $F^{-}_{\mu\nu}$, fields which for photons correspond to *definite* helicity states, namely

$$F^{\mu\nu}_{\pm} = \frac{1}{2}(F^{\mu\nu} \pm i\, \tilde{F}^{\mu\nu}) \quad ; \quad F_{-} \cdot F_{+} = F^{\mu\nu}_{-}\, F_{+\,\mu\nu} = 0. \tag{2.9}$$

In this case one has

$$\mathcal{L}^{4\gamma}_{eff.} \equiv \frac{\alpha^2}{M^4}\left(2\beta_{+}\, F_{+}^{2}F_{-}^{2} \;+\; \beta_{-}\,\left((F_{+}^{2})^2 \;+\; (F_{-}^{2})^2\right)\right). \tag{2.10}$$

It is then obvious that the terms in β_{+} contribute to different helicity amplitudes than those with β_{-}: formally taking a configuration with only either a self-dual or an anti-self-dual (either $F^{+} \to 0$ or $F^{-} \to 0$) the β_{+} does not contribute. For the same reason, in the process $Z \to 3\gamma$ based on the identical Lagrangian, only $\tilde{\beta}_{-}$ contributes to the amplitude where all 3γ have the same helicity.

For the discussion we will consider 5 typical one-parameter "models" of β:

- $\tilde{\beta}_{2,3}$: As stressed above, since it is only the combination $\tilde{\beta}_{\pm}$ that appears at the helicity amplitude level, it is not possible to disentangle a model with only $\tilde{\beta}_2$ from a model with only $\tilde{\beta}_3$ in three-photon production. The model with $\tilde{\beta}_2$ has, in an abuse of terms, been referred to as a "composite" Z. $\tilde{\beta}_2$ could, more appropriately, be associated with the effect of a heavy scalar having a two-γ and $Z\gamma$ coupling, while $\tilde{\beta}_3$ with that of a pseudo-scalar. Two of us [22] have devoted a preliminary analysis to the effect of the operator $\tilde{\beta}_2$ some time ago.

- a model with $\tilde{\beta}_{+}$ but $\tilde{\beta}_{-} = 0$

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- a model that we will refer to as the β_4 model with $\tilde{\beta}_{+} = -11/3\ \tilde{\beta}_{-}$. This is the same model as that considered in [15]. It corresponds to the effect of a heavy fermion which in [15] is considered to be a monopole. In [15] the exotic fermion is assumed to be a $SU(2)$ singlet. With this information it is straightforward to relate the 4γ and $Z3\gamma$ strengths as

$$\tilde{\beta}_{2,3} = \frac{s_W}{c_W}\beta^{F}_{2,3} \qquad (c_W^2 = 1 - s_W^2) \tag{2.11}$$

  where $\beta^{F}_{2,3}$ are given by Eq. 2.2. Taking the fermion to be a point-like monopole (free isolated magnetic charge) one is also led to assume that the strength of its coupling to the photon, g_m, is strong. In fact Dirac's quantification [23] condition relates this coupling to the electric coupling through

$$eg_m = 2\pi n \quad (n = \text{ integer}). \tag{2.12}$$

Therefore in our parameterisation one has to allow for the change $e \to g_m$ in Eq. 2.7. Although one may be tempted by giving a monopole motivation to this type of coupling, it remains that one still has two parameters: the mass of the monopole and the magnetic strength, g_m (or quantification number n) which are lumped in one effective parameter M/n, exactly as what one has with the effective Lagrangian (Eq. 2.7). The discriminating relation Eq. 2.11 indicates the spin-1/2 nature of the point-like monopole. Note, for the record, that the idea of a monopole to induce a large 4-γ coupling is not new; it has been exploited some time ago by Ginzburg and Panfil [16]. They also considered the case of a spin-0 monopole which again can be accommodated by the general effective Lagrangian through the discriminating relation $\tilde{\beta}_{2,3} = \frac{s_W}{c_W}\beta^S_{2,3}$ (and $e \to g_m$); however, from what we remarked earlier, for the same quantification condition the spin-1/2 monopole gives a larger contribution.

- In fact, by far, the most interesting scenario is that of the effect of a heavy spin-1 boson. This could be a model with $\tilde{\beta}_+ = 28\tilde{\beta}_-$. This corresponds to the effect of a very heavy vector boson whose pure gauge component and Goldstone component have the same quantum numbers. This may be considered as the vector boson version of the "monopole" considered by De Rújula [15]. In practice this model is very similar to a model with $\tilde{\beta}_- = 0$ in view of the much larger value of $\tilde{\beta}_+$, especially if one keeps in mind that the main contributions of these operators, as we shall see, are quadratic in the couplings. For a thorough discussion on these models and others we refer to [21].

In the monopole vein, one should not dismiss the eventuality that due to the necessarily very strong coupling of the monopoles, bound states may form with a mass that is much smaller than their combined constituent masses. One interesting possibility would be that the ground state is of spin-0 that decays predominantly into $\gamma\gamma$ and $Z\gamma$, therefore dynamically realising models with $\tilde{\beta}_{2,3}$. In the remaining of the paper we will steer away from considerations pertaining to the manifestation of the monopoles. This is not only because one would have to argue that their masses are, against what is generally but not universally *believed*, much below the GUT scale (in order to evade a Lilliputian unobservable effective $Z \to 3\gamma$ coupling), but also because one has to quantify their effect on the two-point functions that are extremely well measured at LEP1 and contrast the information with the one given by $Z \to 3\gamma$ [§]. We will stick to the general unbiased approach of studying the manifestation of a general $Z \to 3\gamma$ coupling referring to the models as defined through the relation between $\tilde{\beta}_+$ and $\tilde{\beta}_-$. We take the view that the "monopole" is just a possible paradigm for the existence of a not too small $Z \to 3\gamma$ coupling.

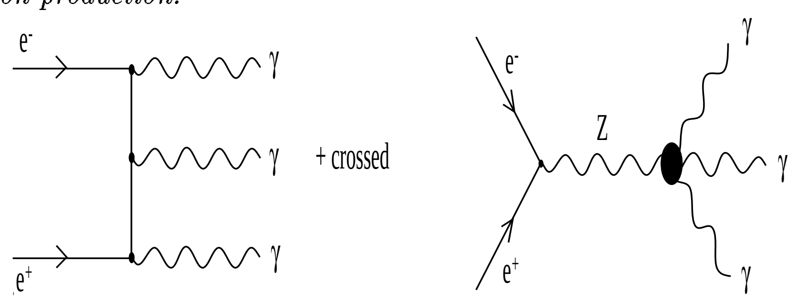
We also note for further discussion that the authors of reference [25] have also given a parameterisation in terms of two operators but have only studied the double integrated $Z \to 3\gamma$ partial width and the totally integrated width. To make contact with their parameters we note that their constants $G_{1,2}$ are related to ours as

$$G_1 = \frac{32\alpha^2}{M^4}(\tilde{\beta}_3 - \tilde{\beta}_2) = -\frac{32\alpha^2}{M^4}\tilde{\beta}_- \qquad G_2 = \frac{32\alpha^2}{M^4}\tilde{\beta}_3. \tag{2.13}$$

3 Analytical Formulae for $\Gamma(Z \to 3\gamma)$ and $e^+e^- \xrightarrow{Z} 3\gamma$

From the general Lagrangian, it is an easy task to compute the Z width into three photons as well as the correlation with the initial state through the cross section $e^+e^- \to 3\gamma$. We will start by providing analytical formulae for the partial width and will give compact analytical formulae including realistic experimental cuts, that could be considered as canonical cuts for this process. The latter can be used to quickly estimate the acceptances.

Figure 1: *The t-channel QED process and the s-channel anomalous $Z \to 3\gamma$ contribution to three-photon production.*



From the start, we would like to point out, before exhibiting any formula, that since the effective operator is of dimension 8, constructed out of field strengths, the s-channel

[§] Moreover, beside the controversial "lightness" ($\sim$ TeV) and point-like nature of the Dirac monopole, consistency of QED in the presence of a Dirac monopole has been questioned [24].

produced photons, see Fig. 1, will be hard photons that will not be collinear to the beam. This is in contrast to the t-channel QED background (see Fig. 1). This is a general characteristics of photons that probe New Physics (See [22]). In the case at hand, we should therefore expect that the distribution in the least energetic photon, for instance, will be more sensitive to the presence of the $Z \to 3\gamma$ vertex.

Specialising to the decay $Z \to 3\gamma$ and taking as variables the reduced photon energies $x_i = 2E_i/M_Z$, the double distribution in the energies writes:

$$
\begin{aligned}
\frac{1}{\Gamma}\frac{d\Gamma}{dx_1 dx_2} &= \frac{120}{3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2}\Big\{(\tilde{\beta}_+^2 + \tilde{\beta}_-^2)(1-x_1)(1-x_2)(1-x_3) \quad + \\
&\quad (\tilde{\beta}_+^2 + 2\tilde{\beta}_-^2)\left((1-x_1)^2(1-x_2)^2 + (1-x_1)^2(1-x_3)^2 + (1-x_2)^2(1-x_3)^2\right)\Big\} \\
&\equiv \frac{60}{3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2}\Big\{(\tilde{\beta}_+^2 + 2\tilde{\beta}_-^2)\left((1-x_1)^2 x_1^2 + (1-x_1)^2 x_2^2 + ((1-x_3)^2 x_3^2\right) \\
&- \quad 2\tilde{\beta}_-^2(1-x_1)(1-x_2)(1-x_3)\Big\}, \qquad (3.1)
\end{aligned}
$$

where the totally integrated partial width is calculated to be

$$
\Gamma = \frac{M_Z}{18}\frac{1}{\pi^3}\left(\alpha\frac{M_Z^2}{M^2}\right)^4\left(\frac{3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2}{120}\right). \qquad (3.2)
$$

We disagree with the expression given in [15] in the case of the fermion monopole, although we confirm the expression for the normalised differential cross section. We find a rate that is 12 times smaller than the one given in [15] ¶ . On the other hand, we confirm the direct one-loop calculation of the supersymmetric charged Higgses given in [26]. Needless to say that for this weakly interacting heavy particle the effect is hopelessly beyond observability. We also agree with the expression given in [25] in terms of $G_{1,2}$. We note, in connection with the monopole case, that had we considered a spin-1 rather than spin-1/2 exotic, the spin-1 gives a factor of about 120 enhancement (if one keeps the same other quantum numbers)!

¿From these expressions we see that present experimental limits [13, 12], $Br(Z \to 3\gamma) \sim 10^{-5}$ place a bound on the strength of the $Z3\gamma$ of the order of

$$
4\left(\frac{\alpha M_Z^2}{M^2}\right)^2 \tilde{\beta}_{2,3} \sim 0.4. \qquad (3.3)
$$

¶This would mean that the extracted L3 [13] limit on the mass of the monopole should be corrected by a factor $\simeq 2$. Moreover from the MC that we will provide it will be possible to implement the correct acceptance factor for this model.

Therefore, one can easily argue that a conventional weakly interacting particle ($\tilde{\beta} \sim 1$) can not be probed through the $Z \to 3\gamma$ decay. This is not to say that we should not look for these decays; this would be like arguing that lacking any model that gives large $WW\gamma/Z$ anomalous coupling one should not probe the $WW\gamma/Z$ couplings at LEP2!

The single-photon energy distribution is

$$\frac{1}{\Gamma}\frac{d\Gamma}{dx} = \frac{20}{3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2} x^3 \left((\tilde{\beta}_+^2 + \tilde{\beta}_-^2)(1-x) + \frac{\tilde{\beta}_+^2 + 2\tilde{\beta}_-^2}{5}(20 - 40x + 21x^2) \right). \tag{3.4}$$

It is more telling and appropriate to distinguish the photons by ordering them according to their energy; we will label them as $E_1 > E_2 > E_3$ (E_i is the energy). The energy distribution of the least energetic photon is

$$\begin{aligned}\frac{1}{\Gamma}\frac{d\Gamma}{dx_3} &= \frac{12}{3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2} \Big\{ x_3^3 \left[(\tilde{\beta}_+^2 + 2\tilde{\beta}_-^2)(1 + 3x_3(1-x_3) + 24(1-x_3)^2) - 5\tilde{\beta}_-^2(1-x_3) \right] \theta(\frac{1}{2} - x_3) \\ &+ (2 - 3x_3) \left[(\tilde{\beta}_+^2 + 2\tilde{\beta}_-^2)(1 + 3(1-x_3)(1 + (1-x_3)(4 - 23x_3 + 27x_3^2)) \right. \\ &- \left. 5\tilde{\beta}_-^2(1-x_3)(1 - 3(1-x_3)^2) \right] \theta(2/3 - x_3)\theta(x_3 - 1/2) \Big\}, \end{aligned} \tag{3.5}$$

where $\theta(x)$ is the step function.

The average energy of the least energetic photon, E_3, is

$$< E_3 > = \frac{M_Z}{42} \frac{1}{3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2} \left(\frac{751}{27}\tilde{\beta}_+^2 + \frac{367}{8}\tilde{\beta}_-^2 \right). \tag{3.6}$$

This shows that independently of $\tilde{\beta}_-, \tilde{\beta}_+$ the average energy of the photon is almost the same for the two independent operators, *i.e.*, $0.221 M_Z$ for $\tilde{\beta}_+$ and 0.218 for $\tilde{\beta}_-$. As expected these are hard photons as compared to the expected average energy of the softest photon one gets for a typical QED 3γ event. Therefore, it is relatively easy to disentangle the QED and $Z \to 3\gamma$ events; however it is not easy to distinguish between different types of New Physics. This will be made clearer when we show the results of the full simulations including cuts. It is also difficult to differentiate between the models on the basis of the average energy of the most energetic photon and the "medium" one.

It is relatively straightforward to implement the cuts within the event plane as done in the LEP experiments. Requiring the separation angle between any two photons (θ_{ij}) to be above a certain value and requiring a cut on the energy of the photons, such that

$$\theta_{ij} > 2\sqrt{\mathrm{Arcsin}\,\delta} \quad ; \quad x_i = \frac{2E_i}{M_Z} > \varepsilon, \tag{3.7}$$

we calculate the acceptance to be

$$\frac{\Gamma_{cut}}{\Gamma} - 1 \simeq -\frac{1}{3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2} \left((\tilde{\beta}_+^2 + 2\tilde{\beta}_-^2)\, \frac{18\delta}{7} + (\tilde{\beta}_+^2 + \tilde{\beta}_-^2)\, \frac{9\delta^2}{7} + (4\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2)\, \frac{2\delta^3}{7} \right.$$
$$\left. + (5\tilde{\beta}_+^2 + 9\tilde{\beta}_-^2)\, 15\varepsilon^4 \right). \tag{3.8}$$

We refrain from giving the exact expression, not only because it is lengthy and uninspiring but also because the typical values of the cuts applied by the LEP experiments are such that the above approximate expression is more than enough. Note for instance that the cut on the energy is of order ε^4 reflecting once again the preferentially energetic photon characteristics of the New Physics and the quartic dependence of the differential width on the energy. This is the reason we have kept terms up to fourth order in the cut-offs. At this order there is "no-mixing" between the cuts on the opening angles and the energy. With $E_i > M_Z/20$ and $\theta_{ij} > 20^0$ (See OPAL [14]), one does not lose more than a few percent of the signal. The message then is that one can afford applying harder cuts than the above without much affecting the signal; even with cuts that are twice as large as the typical LEP experiments cuts our approximate formula, Eq. 3.8, is excellent. The main purpose of the "mild" LEP1 experimental cuts is to bring about a decent and clean 3γ sample (which in the absence of any new physics is purely QED). With higher statistics we could afford cutting harder. It is worth emphasising that the acceptance is sensibly the same for different manifestations of New Physics, *i.e.*, whether one takes $\tilde{\beta}_+$ or $\tilde{\beta}_-$. Again it has to do with the fact that the dependence on the photon energies for different choices of $\tilde{\beta}$ are not terribly different.

Introducing the correlations with the initial state, we have been able to obtain some exact results including cuts. For the sake of generality we also give the cross section for polarised electron beams.

For the 3γ final state we have that

$$\sigma(e^+ e^-_{L,R} \xrightarrow{Z} 3\gamma) = 2\, \frac{g_{L,R}^2}{g_L^2 + g_R^2} \sigma_{unp.} \tag{3.9}$$

where $\sigma_{unp.}$ is the unpolarised cross section and

$$g_L = -\frac{1}{2} + s_W^2 \qquad ; \qquad g_R = s_W^2. \tag{3.10}$$

We introduce, for a centre-of-mass energy, $\sqrt{s}$, the Z propagator via

$$D_Z(s) = \frac{(s - M_Z^2) - i\Gamma_Z M_Z}{(s - M_Z^2)^2 + (\Gamma_Z M_Z)^2} \tag{3.11}$$

where we have chosen to keep an energy-independent width. In terms of the physical width of the Z into electrons, Γ_Z^{ee}, and with $dLips$ being the invariant 3γ phase space, the differential cross section writes as

$$\begin{aligned}
&\frac{1}{90\pi}\frac{d\sigma(e^+e^- \xrightarrow{Z} 3\gamma)}{(16\pi)^2 dLips} = 12\pi\frac{\Gamma_Z^{ee}\Gamma_Z^{3\gamma}}{M_Z^2}\left(\frac{s}{M_Z^2}\right)^4 |D_Z(s)|^2 \Bigg\{ \\
&\frac{\tilde\beta_+^2+\tilde\beta_-^2}{3\tilde\beta_+^2+5\tilde\beta_-^2}\Bigg[2(1-x_1)(1-x_2)(1-x_3) - \Bigg(x_1^2c_1^2\,(1-x_1-\frac{x_2x_3}{2}) \\
&\qquad + \quad x_2^2c_2^2\,(1-x_2-\frac{x_1x_3}{2}) + x_3^2c_3^2\,(1-x_3-\frac{x_1x_2}{2})\Bigg)\Bigg] \\
&+ \quad \frac{\tilde\beta_+^2+2\tilde\beta_-^2}{2(3\tilde\beta_+^2+5\tilde\beta_-^2)}\Big[\Big((1-x_1)^2(1-x_2)^2+(1-x_1)^2(1-x_3)^2+(1-x_2)^2(1-x_3)^2 \\
&\qquad - 3(1-x_1)(1-x_2)(1-x_3)\Big) \\
&+ \quad \frac{1}{2}\Big(x_1^2c_1^2\,(1-2x_1+x_2^2+x_3^2)+x_2^2c_2^2\,(1-2x_2+x_1^2+x_3^2)+x_3^2c_3^2\,(1-2x_3+x_1^2+x_2^2)\Big)\Big]\Bigg\}.
\end{aligned} \tag{3.12}$$

Here c_i is the cosine of the angle between photon i and the beam, $c_i = \cos(e^-\gamma_i)$ and $x_i = E_i/\sqrt{s}$.

From the above expression we can look at the orientation of the photons relative to the beam. For instance, the distribution in the angle ψ between the event plane and the beam axis, with $z = \sin(\psi)$ $(0 < \psi < \pi/2)$, is given by

$$\frac{1}{\sigma}\frac{d\sigma}{dz} = \frac{27}{8}\frac{\tilde\beta_+^2+2\tilde\beta_-^2}{3\tilde\beta_+^2+5\tilde\beta_-^2}\left\{1-\frac{z^2}{9}\,\frac{3\tilde\beta_+^2+14\tilde\beta_-^2}{\tilde\beta_+^2+2\tilde\beta_-^2}\right\}. \tag{3.13}$$

When specialising to the "fermion-monopole" case, we disagree with [15]. In principle it is this distribution that can disentangle between different $\tilde\beta$. The average value of z may be easily computed

$$< z > = \frac{3}{32}\frac{15\tilde\beta_+^2+22\tilde\beta_-^2}{3\tilde\beta_+^2+5\tilde\beta_-^2}. \tag{3.14}$$

The totally integrated cross section at a given energy writes very simply in terms of the width measured at the Z resonance:

$$\sigma^{NP}(e^+e^-\to 3\gamma) = \frac{12\pi}{M_Z^2}\,\Gamma_Z^{e^+e^-}\;\Gamma_Z^{3\gamma}\;\frac{1}{s}\left(\frac{s}{M_Z^2}\right)^4 |sD_Z(s)|^2. \tag{3.15}$$

This shows that, as expected, the cross section grows like s^3. Hence, at some energy much above the Z peak this growth factor can give an enhancement factor of the order of that given by the resonance (see below).

Of course at the Z peak one has the usual Breit-Wigner formula:

$$\sigma^{NP}(e^+e^- \to 3\gamma) = \frac{12\pi}{M_Z^2}\frac{\Gamma_{Z\to e^+e^-}\Gamma_{Z\to 3\gamma}}{\Gamma_Z^{tot}}. \tag{3.16}$$

It is also possible to give an exact expression for the s-channel cross section including realistic cuts. Besides the cuts on the photon energies and photon-photon separations (see Eq. 3.7), we can include a cut on the angle, ψ, between the event plane and the beam to avoid forward-backward events and reduce the purely QED contribution such that $\sin(\psi) > \sin(\psi_0) = z_0$. The exact analytical formula is lengthy and not very telling. With the typical values of the experimental cuts as applied by the LEP collaborations the exact formula is well approximated by:

$$\frac{\sigma_{cuts}}{\sigma} - 1 \simeq \frac{1}{3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2}\left\{15(5\tilde{\beta}_+^2 + 9\tilde{\beta}_-^2)\varepsilon^4 - 18(\tilde{\beta}_+^2 + 2\tilde{\beta}_-^2)(\frac{1}{7}\delta + \frac{3}{16}z_0)\right\} \tag{3.17}$$

4 Monte Carlo Analysis

4.1 Behaviour of the cross section and effect of the interference

The above expression can prove handy for a quick estimate of the efficiencies on the detection and for the measurement of the $Z \to 3\gamma$ partial width. However, to conduct a detailed analysis it is essential to consider the QED 3γ background as well as the effect of the interference and be able to apply any cut that is needed for a particular detector. To include more sophisticated cuts a MC program is necessary. We have written a fast MC which includes both the pure QED contribution with the 3 *observable* photons as well as the anomalous terms. The program is based on a calculation of the matrix elements. This allows us to study the effect of the interference between the standard part and the non-QED part. The very compact expressions are derived through the use of the spinor inner product [27, 28, 29, 30, 31] and can be found in the Appendix. By implementing the same cuts that we implemented in the analytical formulae we find excellent agreement between the analytical results and the output of the MC for all the parts contributing to the cross section. We will explicitly show that the interference term can always be dropped, hence simplifying the experimental analysis even when one is far away from the peak.

Of course at the Z peak, there is no interference since the tree-level QED amplitude is purely real whereas the anomalous is purely imaginary. We find that if the search at the

Z peak reveals that the anomalous part is smaller than the QED part (which is the case) this implies values for the couplings such that even at LEP2 the interference is negligible. Moreover, it is only the term in $\tilde{\beta}_+$ that contributes to the interference, the reason being that the tree-level cross section for the production of 3 photons with the same helicity is known to be exactly zero [27]‖. Since, as remarked in the first section, the classification into $\tilde{\beta}_+$ and $\tilde{\beta}_-$ refers to two classes of helicities for the photons, only $\tilde{\beta}_+$ interferes. Details are found in the Appendix.

It is always possible to give a parameterisation for the cross section as a function of the cms energy, including any cuts. A very simple scaling law parametrisation is obtained if the angular cuts are the same at all energies and if the cut on the photon energies are implemented through a fixed ratio with the total cms energy. In an obvious notation

$$\begin{aligned} \sigma(e^+e^- \to 3\gamma) &= \sigma_0 \frac{1}{\tilde{s}} + \frac{4\alpha^2}{m^4}\,\tilde{\beta}_+\,\sigma_i\,\frac{(\tilde{s}-1)\tilde{s}^2}{\left(\frac{\tilde{s}-1}{\gamma}\right)^2+1} \\ &+ (\frac{4\alpha^2}{m^4})^2\,(\tilde{\beta}_+^2\sigma_+ + \tilde{\beta}_-^2\sigma_-)\frac{\tilde{s}^5}{\left(\frac{\tilde{s}-1}{\gamma}\right)^2+1}, \end{aligned} \tag{4.1}$$

$$\text{with} \quad \tilde{s} = \frac{s}{M_Z^2} \quad ; \quad \gamma = \frac{\Gamma_Z}{M_Z} \quad ; \quad m = \frac{M}{M_Z}. \tag{4.2}$$

For instance, taking the OPAL cuts [14]

$$|\cos\theta_{e\gamma}| < 0.9 \quad ; \quad \theta_{\gamma\gamma} > 20^0 \quad ; \quad E_\gamma > \frac{\sqrt{s}}{20} \tag{4.3}$$

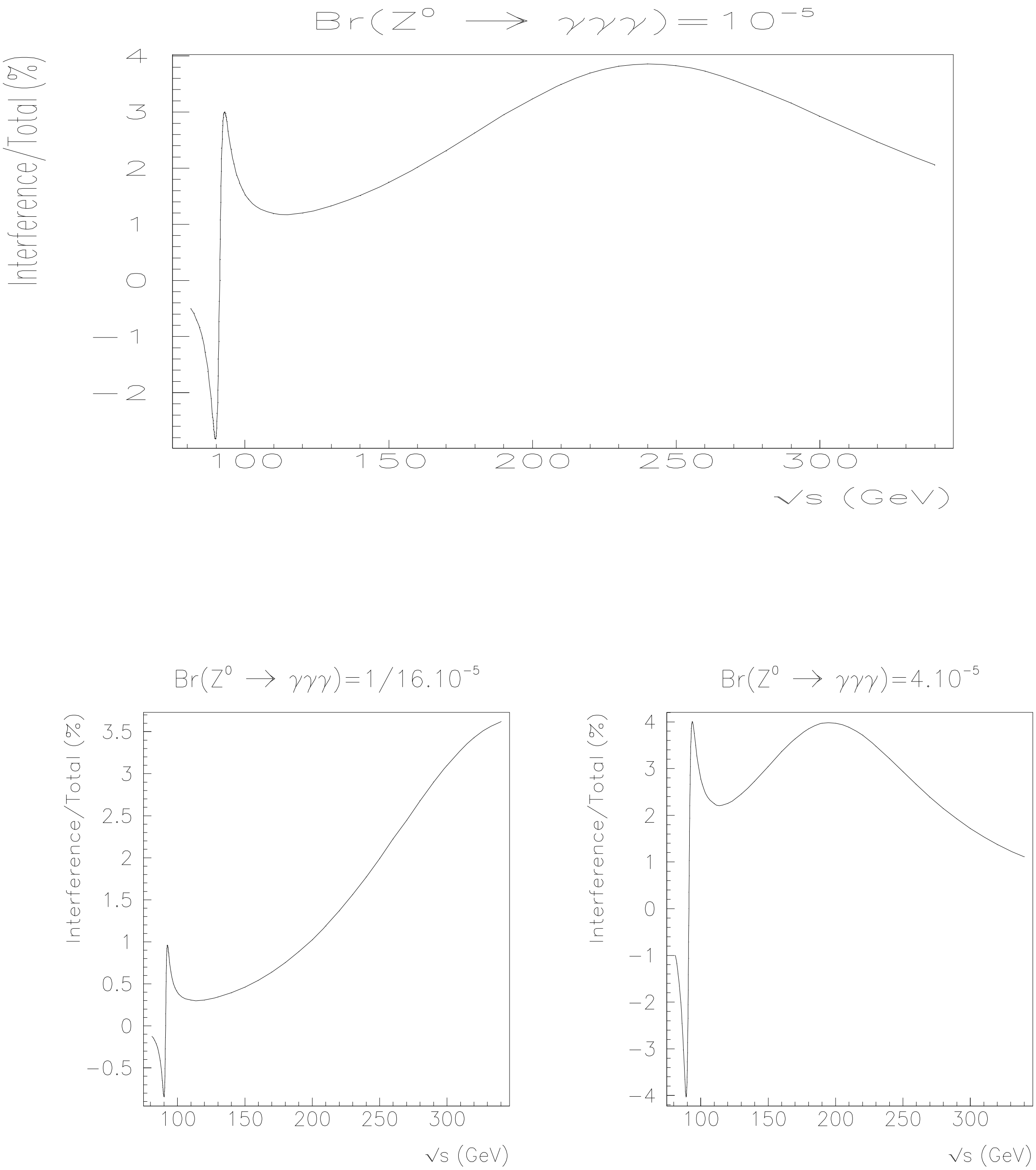
the values of the cross section $\sigma_{0,i,+,-}$ calculated with $M_Z = 91.187$ GeV, $\Gamma_Z = 2.490$ GeV, $\sin^2\theta_W = s_W^2 = 0.232$, are

$$\begin{aligned} \sigma_0 &= 0.705 \text{ pb} \\ \sigma_i &= 0.082 \text{ pb} \\ \sigma_+ &= 4.181 \text{ pb} \\ \sigma_- &= 6.420 \text{ pb}. \end{aligned} \tag{4.4}$$

Armed with the MC, simulating an experimental environment, we can first exactly quantify the effect of the interference. We only consider models with $\tilde{\beta}_+$ since there is no interference with $\tilde{\beta}_-$. As a first illustration we take a value of $\tilde{\beta}_+/M^4$ which corresponds

‖This also holds for same-helicity multi-photon final state processes [29].

Figure 2: *Relative effect of the interference for various values of $\tilde{\beta}_+$. Positive $\tilde{\beta}_+$ were taken. We have preferred, instead of quoting the values of the $\tilde{\beta}_+$, to give the associated 3γ width as calculated from Eq.3.2.*



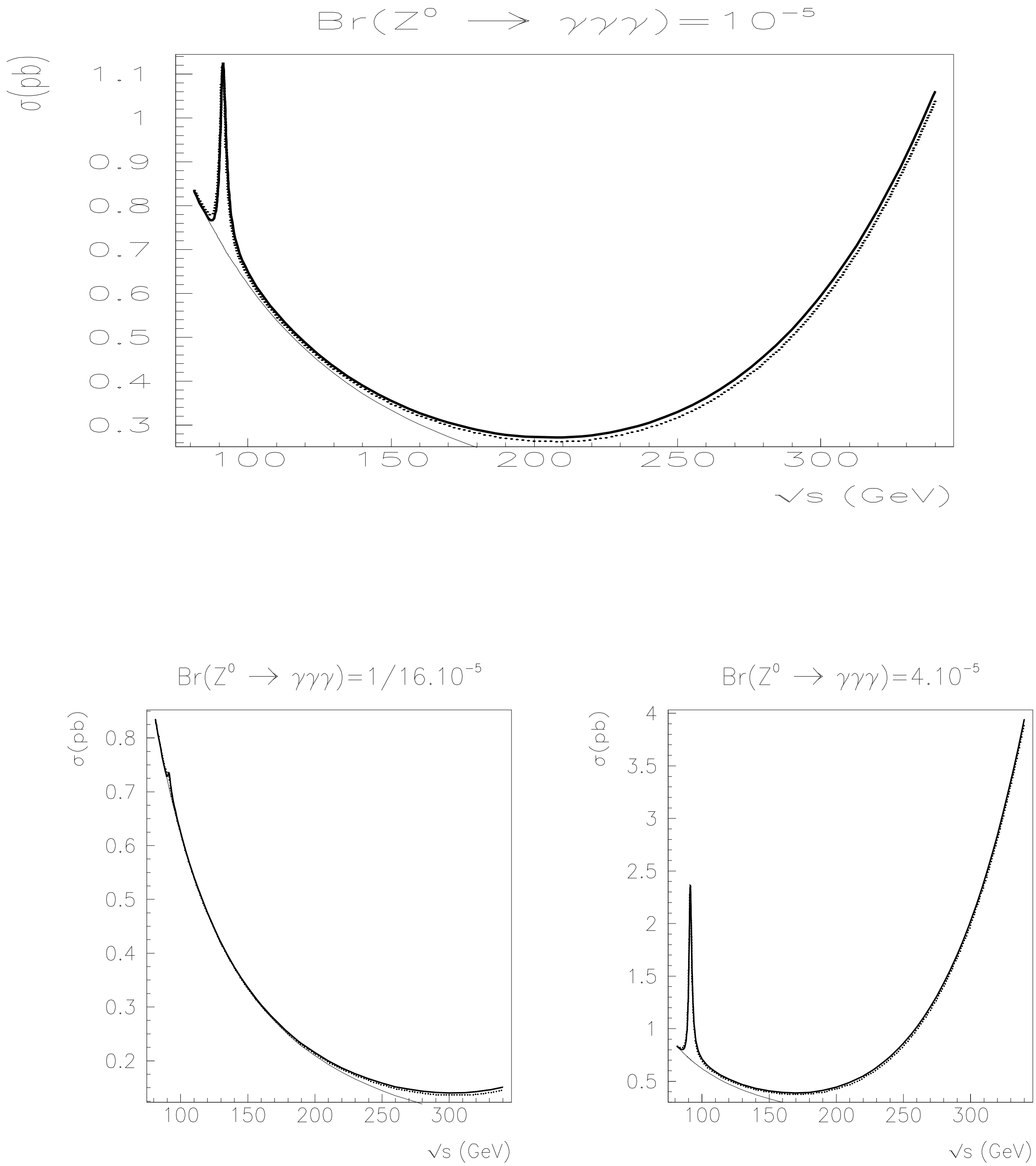
to the best limit set on the branching ratio for $Z \to 3\gamma$ [13]: $Br(Z \to 3\gamma) \sim 10^{-5}$. We clearly see (Fig. 2) that around the energies scanned by LEP1, the effect is negligible. The relative error introduced by neglecting the interference is below 3% all the way up to LEP2 energies. Even for an energy around 300 GeV the effect does not exceed 3%. We have also looked at the effect of the interference for a value of $\tilde{\beta}_+$ which is 4 times smaller (that would give a branching fraction 16 times smaller than the best published limit and which could be considered as the ultimate limit from LEP1). The effect up to LEP2 energies is below the 1% level. We have also considered values that are 20 times smaller; however, foreseeing the accumulated luminosity at LEP1 it would not be possible to detect the effect of an anomalous $Z \to 3\gamma$ at LEP. Nonetheless, even for such values the effect of the interference is too small even at LEP2 (and beyond) energies. Just to make the point we also show that there is negligible effect even when taking larger values of the coupling (Figs 2).

Figures 3 show the merit of sitting at the peak contrasted to the improvement one gains by moving to higher (than LEP2) energies, taking for $\tilde{\beta}$ typical values that one may hope to measure at LEP1. We see that just above the Z peak the combined cross section (QED+anomalous) decreases (due to the Breit-Wigner factor and the $1/s$ drop of the QED cross section) and that it starts bending over and increasing only, unfortunately, around the highest LEP2 energies. One conclusion, to which we will come back later, is that these figures convey the message that if there is no sign of New Physics affecting the $Z \to 3\gamma$ at LEP1 even the highest energies foreseen for LEP2 will not be enough to probe this vertex better, even with the higher nominal luminosity of LEP2 (500 pb^{-1}).

4.2 Distributions

The selection of the optimal cuts that should be applied in order to bring out a $Z \to 3\gamma$ effect should be guided by the knowledge of the various distributions in energies and angles. These are best given by the MC. We have looked at a few distributions applying on both the QED and anomalous parts the same cuts as defined in the previous subsection, see Eq. 4.3. We have chosen to show separately the *normalised* (to one) cross section for the anomalous and the QED; the effect of the interference is not taken into account. However we have seen that the interference can be safely neglected; moreover, in terms of the parameter $\tilde{\beta}_-$, this interference is not present, as it is not present at all on the Z peak for all operators. Beside the variables we discussed earlier we also looked at the distribution in the acollinearity in the transverse plane defined by the most energetic

Figure 3: *Energy dependence of the cross section for various values of $\tilde{\beta}_{+}$. The full thick curve is the total contribution including the interference while the dotted curve is with no interference. Also shown, separately, the irreducible QED cross section.*



photon (1) and the "medium" photon (2). This is defined from the photon momenta transverse to the beam, $\vec{k}^T_{1,2}$ as

$$\chi = \text{Arcos}\left(\frac{\vec{k}^T_1.\vec{k}^T_2}{|\vec{k}^T_1|\,|\vec{k}^T_2|}\right). \tag{4.5}$$

For all the distributions we show the effect of $\tilde{\beta}_-$ and $\tilde{\beta}_+$ and one of the other "models" defined in the second section in order to see how these models may be disentangled, in principle. For the single distribution variables we also show the average value of the corresponding variable including the cuts for the QED as well as the anomalous.

We start with the energy distributions. As argued above there is a clear-cut distinction between the QED and the anomalous contribution that is best understood for the least energetic photon. In QED $e^+e^- \to 3\gamma$ is built up from $e^+e^- \to 2\gamma$ where the "added" photon can be considered to be a bremstrahl. One should, therefore, expect that among the three photons, one is much less energetic than the other two (that will tend to take the beam energy). In the case of the $Z \to 3\gamma$, all three photons have on average an energy of the same order, thus the energy is shared somehow equally between them. This is the reason one of the most salient differences appears in the distribution of the least energetic γ. This is well rendered by Fig. 4 where two very distinctive spectra stand out. This distribution is thus a powerful tool for separating the QED from the anomalous. Note, however, that this distribution is almost insensitive to the model of New Physics contributing to the s-channel $Z \to 3\gamma$. Therefore based on this remark one could devise a "model-independent" optimised cut to unravel the anomalous contribution**. A similar discrimination between the anomalous and the QED occurs in the case of the medium-energy photon without "resolving" the models. The distribution in the hardest photon shows a less dramatic difference (see Figs. 5). Other distributions in the event plane variables relate to the opening angles between any two photons. Although these distributions are directly related to the energies ($\sin^2(\theta_{ij}/2) = (x_i + x_j - 1)/x_i x_j$), besides bringing out the markedly different structure of the QED and the anomalous, one finds that given enough non-QED events a certain discrimination may be perceptible, especially in the distribution of the angle between the two softest, Figs. 6.

Another characteristic of QED events, besides the softness of one of the photons, is the collinearity of the photon with the particle that emits that photon. This is not expected for the $Z \to 3\gamma$ where the photon does not connect to the e^+e^- line. Thus another

**The "knee" that shows up at $M_Z/4$ is purely kinematical and has to do with the ordering of the photons. This effect is also manifest in the analytical formula Eq. 3.5 through the occurrence of the step function at $x_3 = 1/2$.

Figure 4: *Normalised distribution in the energy of the softest photon for the case of QED as well as for the models defined in the text. Cuts as defined by Eq. 4.3 have been applied. The arrows point to the average values of the energy of the softest photon for the models considered.*

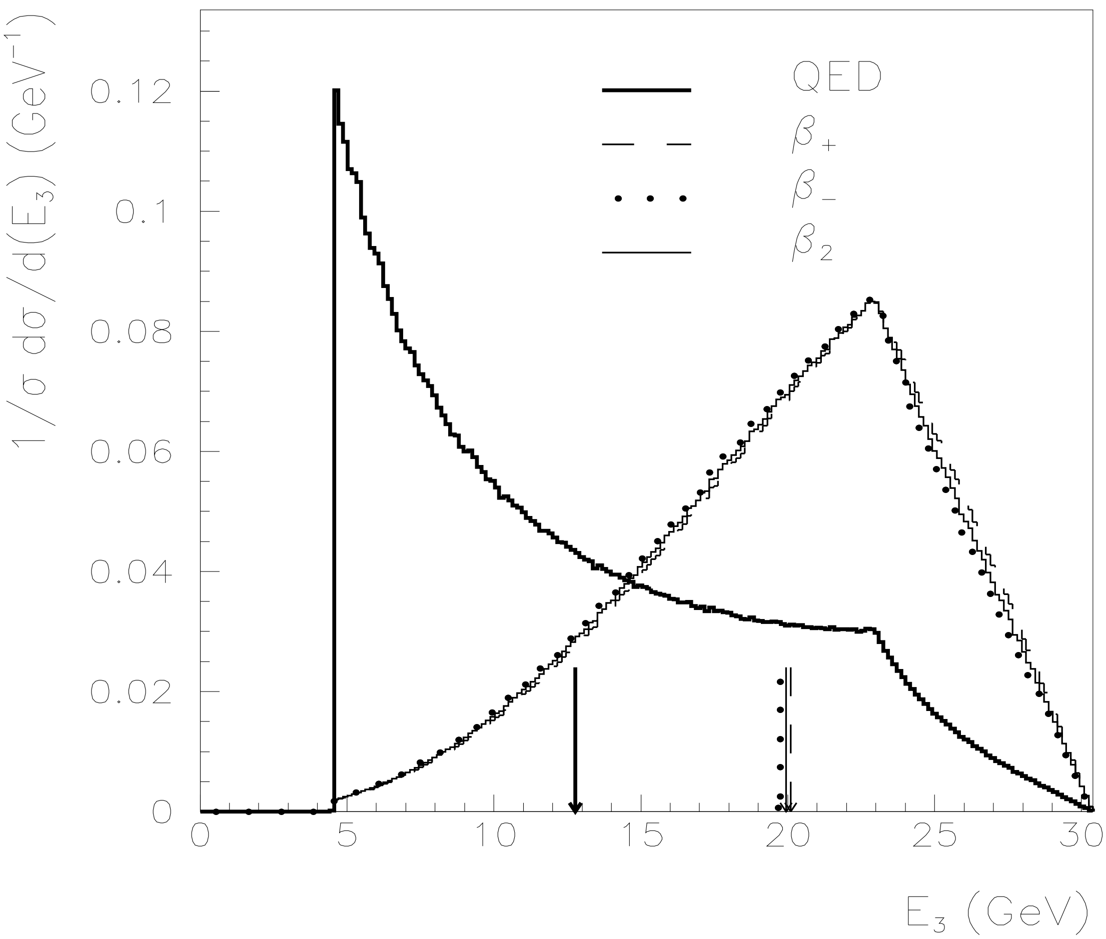
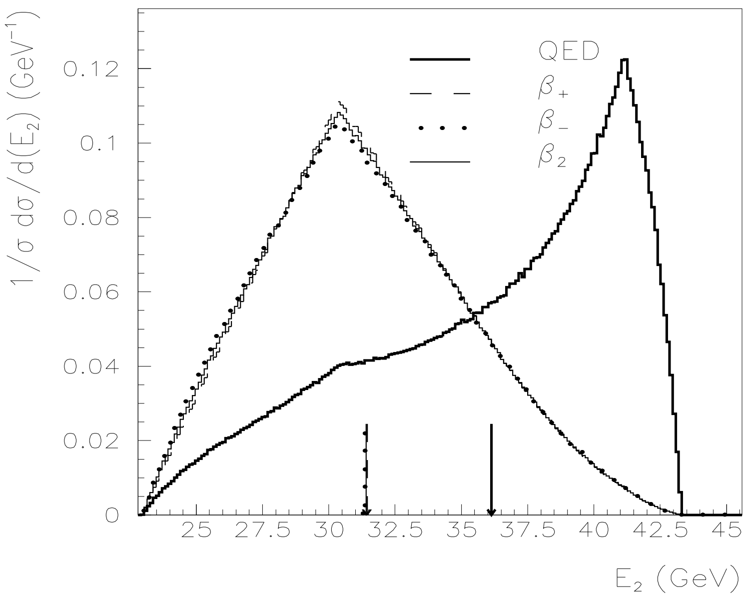


Figure 5: *As in the previous figure for the medium photon, E_2 and the hardest, E_1.*



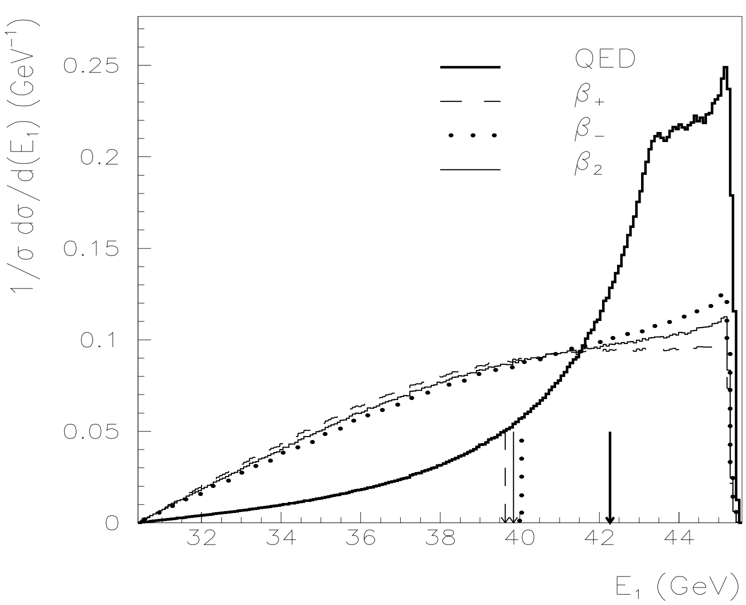
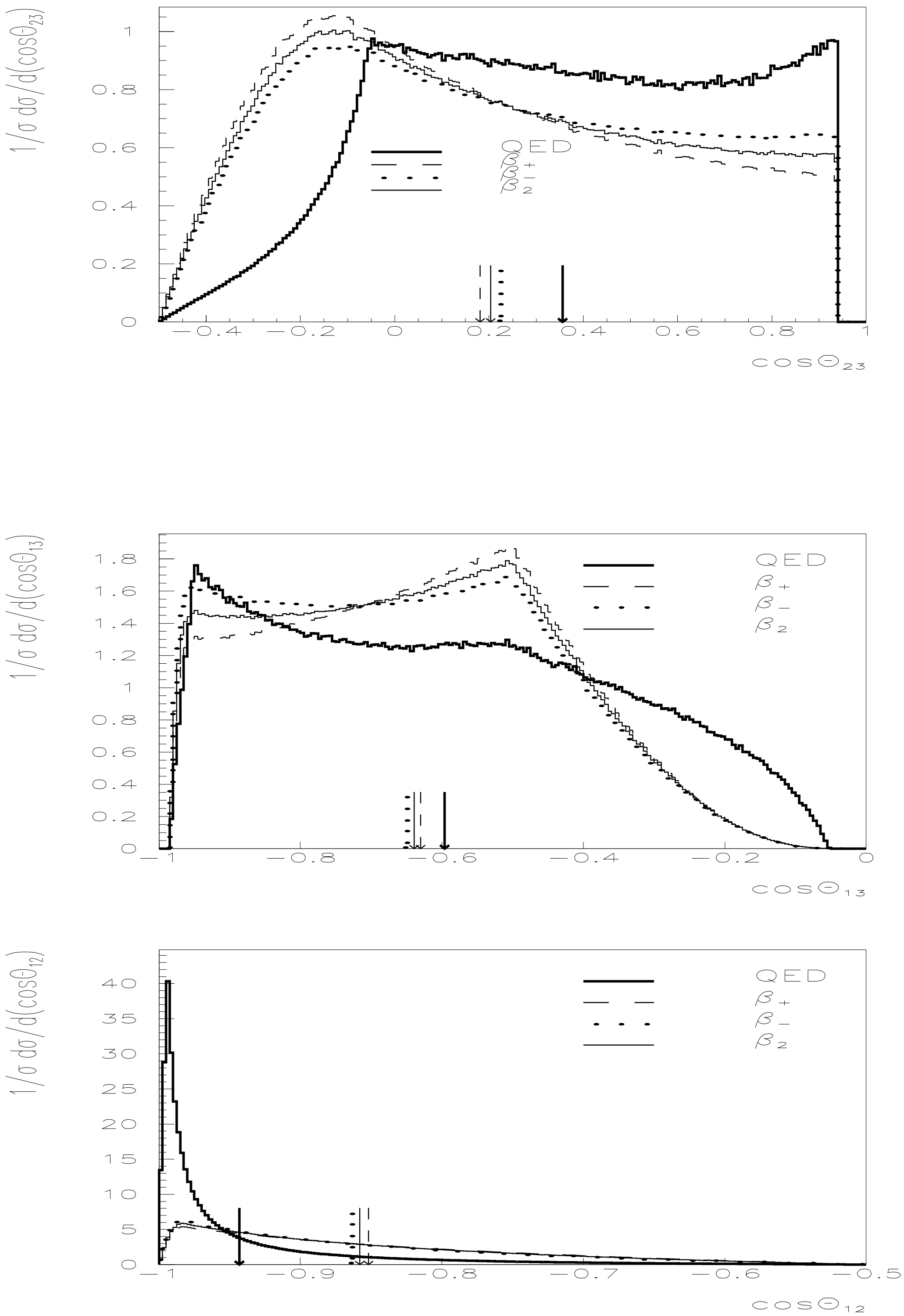


Figure 6: *As in Fig. 4 but in the opening angles between the photons. "3" is the softest and "1" the hardest.*



clear signature is the distribution in the angle between any photon and the beam. The QED events, as Figs. 7 show, are clearly peaked in the forward-backward direction even after the cuts, whereas the anomalous events are rather central. Note should however be made that the softest non-QED photon has also a slight tendency of preferring to be in the forward-backward region. In this respect it is really the most energetic photon distribution that shows the most marked difference between the QED and the non-QED events. Once again, one sees that all models for Z-initiated 3γ events give a sensibly similar distribution and thus it is highly unlikely that one can resolve any difference between the models from the distributions that we have seen up to now. The same can be said about the distribution in the acollinearity, once the cuts are applied.

On the other hand, the angular distribution of the event plane, or equivalently the normal to this plane, with respect to the beam axis, does show some interesting structure that could be the best way of discriminating between the different models. This is clearly seen in Fig. 8 , where we see that not only the average values are well separated but that there is a distinct lifting of the "degeneracy" between the models, especially for values of the cosine of the angle between the plane and the beam axis above 0.6.

This lifting of the degeneracy with the devising of more optimal cuts to bring out the new physics and attempt to look at its origin can be made much clearer by studying distributions in a combination of appropriate variables in the form of scatter plots, for instance. In the scatter plot of the energy of the two softest photons, the pure QED and the s-channel 3γ are confined to two opposite corners (see Fig. 9). Another obvious discrimination is to examine the scatter plot involving the least energetic photon and the angle of the most energetic photon with the beam. The scatter plots involving the angle of the event plane with the beam show what could, in principle, be the best strategy to differentiate between various models. We illustrate this in Fig. 10 by taking as a second variable either the energy in the softest or the hardest photon.

5 Conclusions and Discussion

We have given a detailed analysis of the most general manifestation of New Physics in the coupling of $Z \to 3\gamma$. The analytical formulae should help in quickly estimating the acceptances and providing a limit on the operators from the ongoing LEP1 searches. We have indicated how to optimise the searches (and the limits) and in the eventuality of a signal how to disentangle various effects. Our study also shows that for searches around LEP1 energies and with current limits on the anomalous $Z \to 3\gamma$ couplings the

Figure 7: *As in Fig. 4 but in the cosine of the angle between the photons and the beam.*

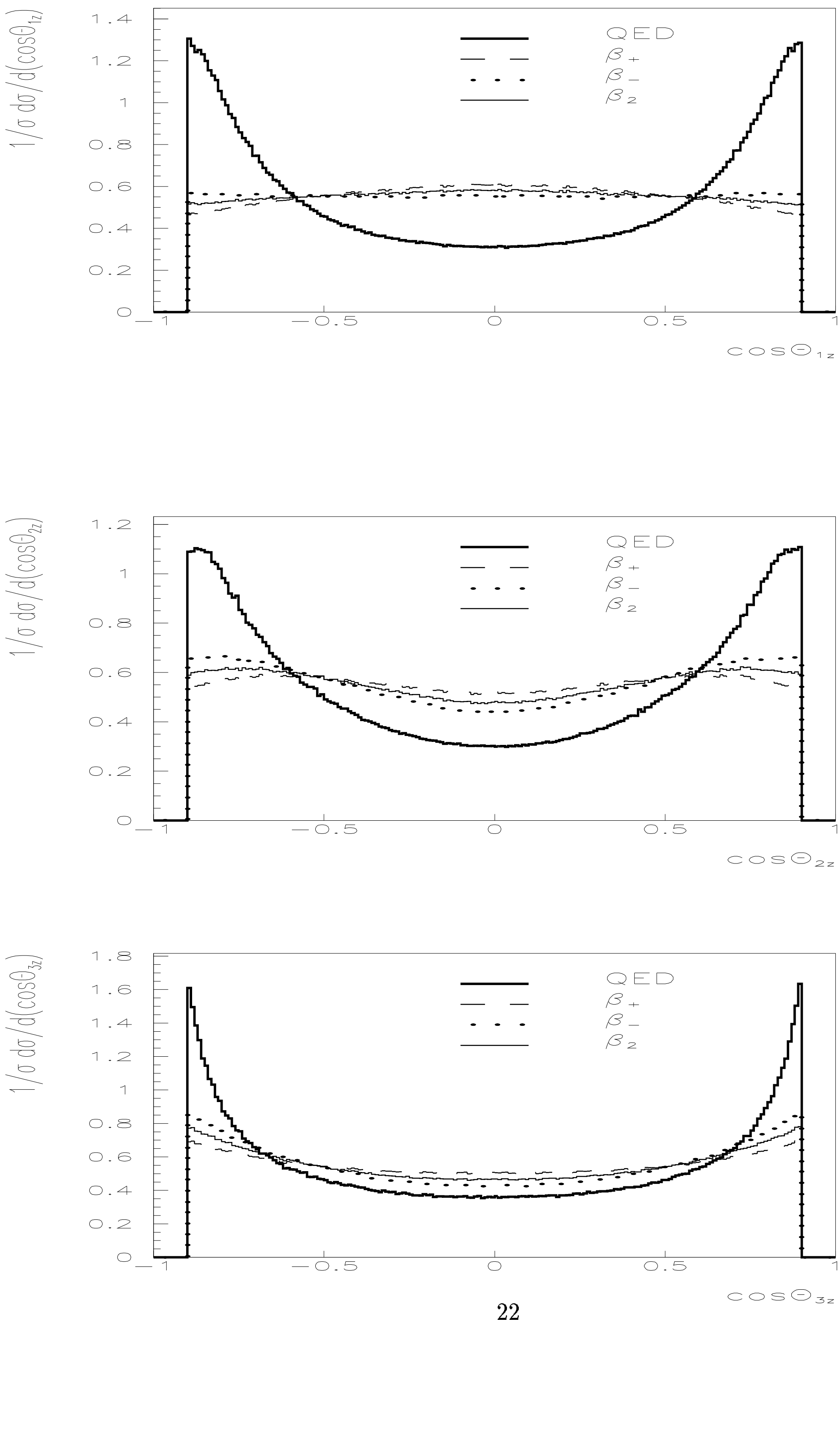


Figure 8: *Distribution in the cosine of the angle between the beam and the normal to the event plane.*

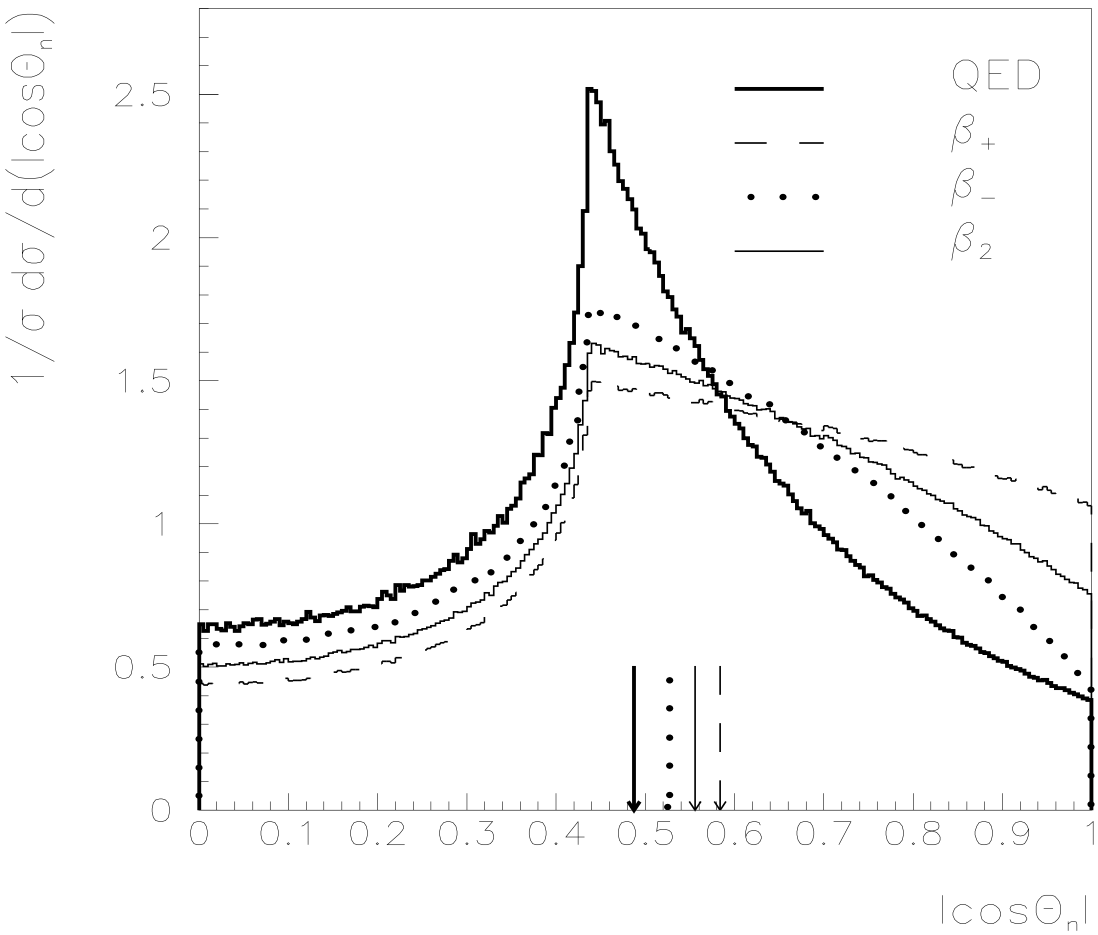


Figure 9: *Normalised scatter plots that show the difference between the QED distributions and those due to the independent β_+ and β_- couplings. The first is in the energy of the softest photon (E_3) and the cosine of the angle between the beam and the hardest photon ($\cos\theta_{1z}$). The second set of scatter plots is in the energies of the softest (E_3) and medium photon (E_2).*

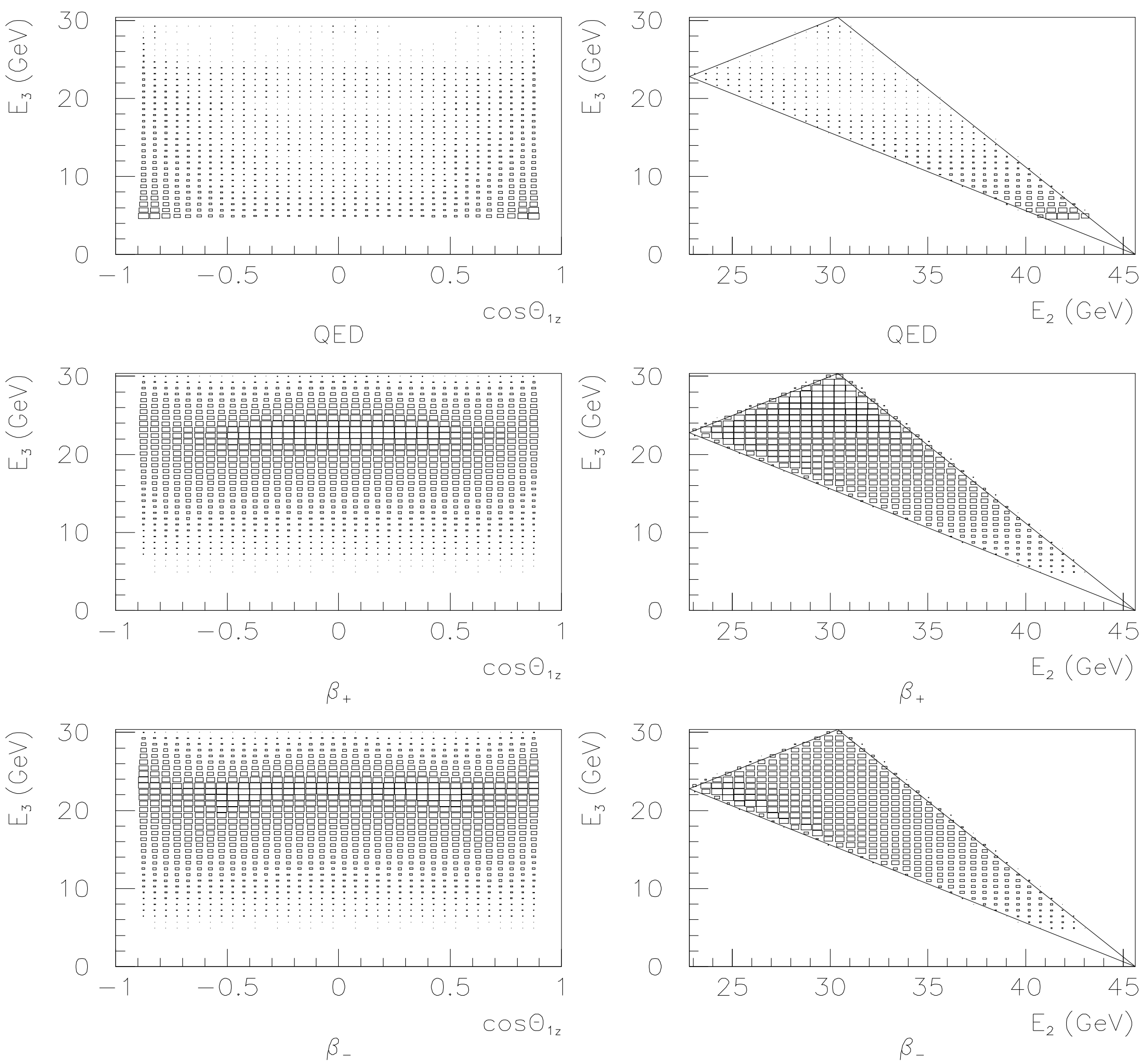
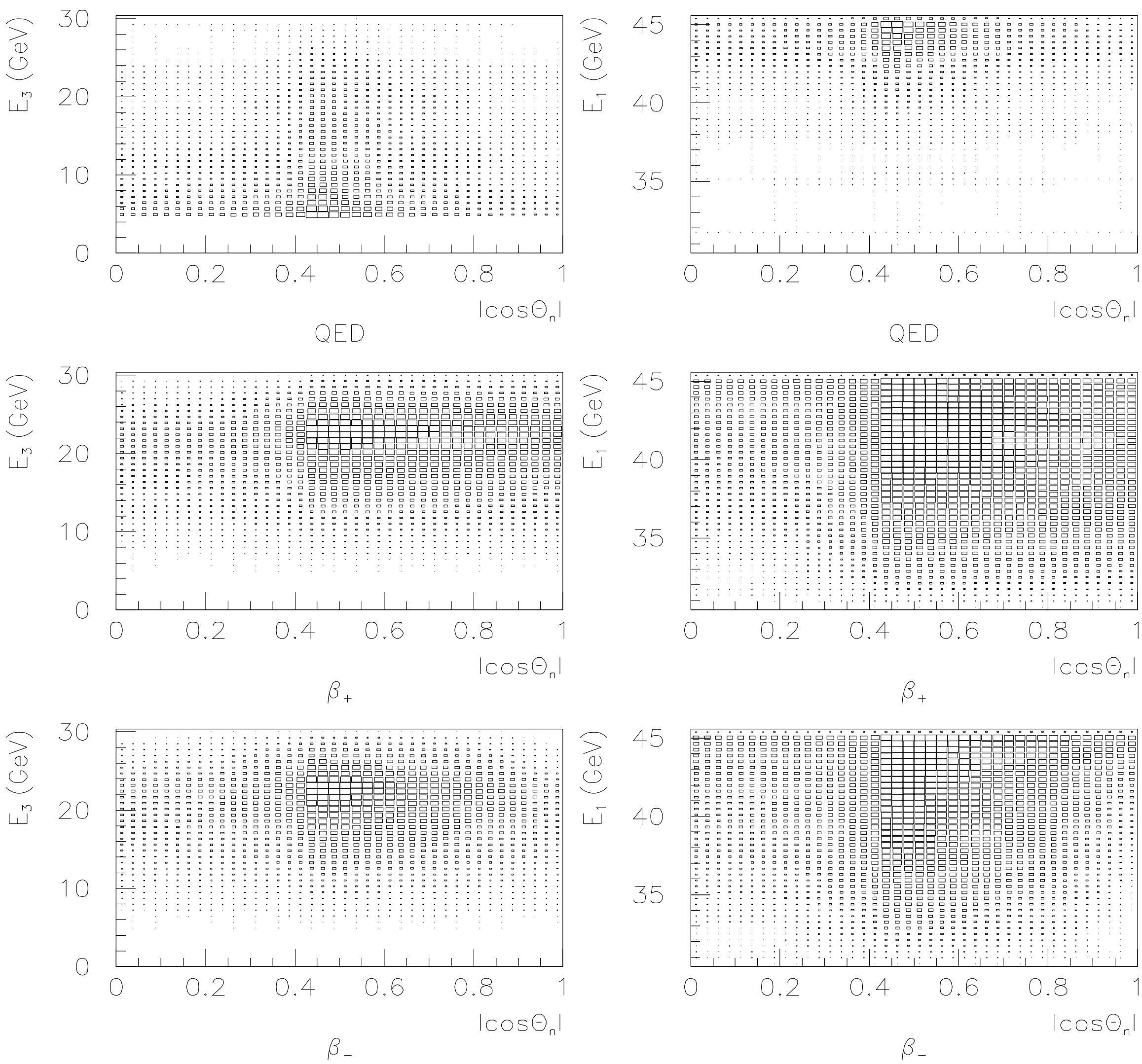


Figure 10: *As in the previous scatter plots but as variables: the cosine of the angle between the normal to the event plane and the beam, and the energy of the softest or the hardest photon.*



effect of the interference between New Physics and the irreducible QED 3γ background is abysmally insignificant and can thus be neglected, although it is very straightforward to include.

Let us now ascertain the limits one would derive on the parameters of the effective Lagrangian at the end of the LEP1 run. We will assume that each experiment has collected a total luminosity $\int\mathcal{L}$ =200 pb^{-1} and further assume a 100% efficiency on the detection of the 3γ once the experimental cuts defined by Eq. 4.3 have been applied. Let us stress that since all models, *i.e.*, both $\tilde{\beta}_+$ and $\tilde{\beta}_-$ give sensibly the same distributions and that since it will not be possible to reconstruct the helicities of the energetic photons it is extremely difficult to distinguish between the different $\tilde{\beta}$'s especially if the "anomalous" signal is weak. Therefore, the limits will have to be extracted from the total cross section after cuts have been applied. It ensues that we will only have access to the combination of $\tilde{\beta}_+$ and $\tilde{\beta}_-$ $(\tilde{\beta}_+^2\sigma_+ + \tilde{\beta}_-^2\sigma_-)$ where $\sigma_\pm$ are defined in Eq. 4.1 and depend (mildly) on the cuts applied. Without the cuts one has the combination that appears in the width, $(3\tilde{\beta}_+^2 + 5\tilde{\beta}_-^2)$. With a healthy statistics and a good signal it could be possible by judiciously varying the cuts to "reconstruct" the correct combination of $\tilde{\beta}_-$ and $\tilde{\beta}_+$. In what follows we will restrict the discussion to one parameter. Requiring a 5σ deviation, with only the statistical error taken into account, one finds for $\tilde{\beta}_-$ for instance

$$\frac{4\alpha^2}{m^4}|\tilde{\beta}_-| < 0.215 \qquad (0.256 \ \text{for} \int\mathcal{L} = 100 \ \text{pb}^{-1}). \tag{5.1}$$

The increase in luminosity does not, unfortunately, tremendously improve the limits, as the effect of the New Physics is quadratic in the couplings and therefore the sensitivity scales only as $\mathcal{L}^{1/4}$. This remains true, as we have seen, even when one moves away from the Z peak. Thus, one may wonder, since the QED cross section decreases with energy as $1/s$ while the anomalous grows as s^3, whether one could set a better limit at LEP2. We have already answered by the negative. We show in Fig. 11 how the significances are changed as the energy is increased while keeping the same cuts and normalising to a common integrated luminosity of $\mathcal{L}$ =100 pb^{-1}[††]. As the figures demonstrate, for various values of β_+ (in the range set by LEP1), the statistical significances are never better at LEP2 than what they are at LEP1. One will take full advantage of the energy increase of the anomalous cross section, bettering the resonance enhancement, only for energies around 300 GeV. Taking again the operator defined by $\tilde{\beta}_-$, and with LEP2 meaning

[††] Of course, as the energy increases one expects the luminosity to improve as well, however we stress again that since the effect of the interference is marginal, the significance can be easily calculated through the scaling law $\mathcal{L}^{1/4}$.

$\sqrt{s} = 190$ GeV, $\mathcal{L}$ =500 pb^{-1} a limit based on a 5σ (statistical only) deviation gives

$$\frac{4\alpha^2}{m^4}|\tilde{\beta}_-| < 0.365, \tag{5.2}$$

i.e., worse than at LEP1. However, at a Next Linear Collider (NLC) operating at 350 GeV with $\mathcal{L}$ =10 fb^{-1} we find that the LEP1 limit is improved by as much as an order of magnitude:

$$\frac{4\alpha^2}{m^4}|\tilde{\beta}_-| < 0.025. \tag{5.3}$$

This said, it is not excluded that the strength of the 4-γ vertex is much larger than that of the $Z \to 3\gamma$ vertex, in which case 3γ production, proceeding through photon exchange, will be much better studied than at LEP1, since it does not receive any resonant enhancement. Therefore we should urge the LEP2 collaborations to scrutinise 3γ production. The topology of these 3γ events is *exactly* the same as those originating from the $e^+e^- \to Z \to 3\gamma$ that we have studied here; in particular, all the normalised distributions are the same. To adapt what we have investigated in this paper to the effect of a 4-γ vertex in 3-γ production one simply makes the replacement $\tilde{\beta}_\pm \to \beta_\pm$ together with the change of the Zee vertex into the electromagnetic one and the replacement of the Z propagator. For instance assuming that $\tilde{\beta}_\pm = s_W/c_W \beta_\pm$ as given by a model with a strongly interacting SU(2) singlet fermion, then the 4-γ strength is larger than that of the $Z \to 3\gamma$. For this particular case the combined effect of the photon and the Z is accounted for by the factor

$$\frac{g_L^2 \left(1 - \frac{c_W^2}{g_L}(1 - \frac{M_Z^2}{s})\right)^2 + g_R^2 \left(1 - \frac{c_W^2}{s_W^2}(1 - \frac{M_Z^2}{s})\right)^2}{g_L^2 + g_R^2} \tag{5.4}$$

that multiplies the results of taking into account the Z-channel only. At 190 GeV this enhances the Z cross section by almost seven fold. At LEP2, there is also the possibility of checking for the 4-γ vertex through 2γ production in the (peripheral) $\gamma\gamma$ processes. A dedicated investigation of this coupling for LEP2 and beyond is under investigation [32].

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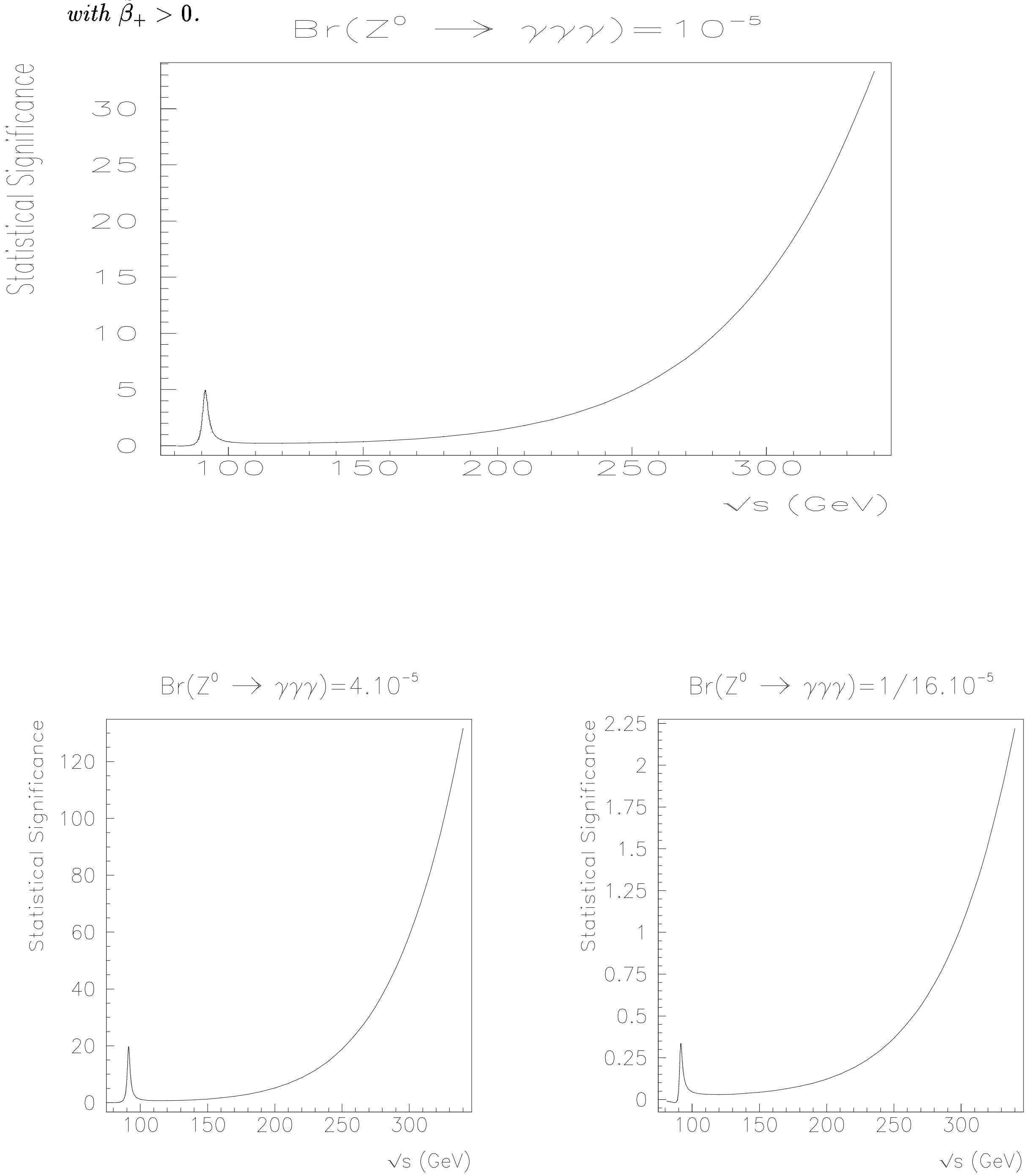
Figure 11: *Statistical significances as a function of the cms energy assuming a common effective luminosity $L = 100\ pb^{-1}$. Three typical values of the branching ratio are assumed with $\tilde{\beta}_+ > 0$.*



Appendix:

A Helicity amplitudes with the spinor inner product technique.

With the cuts that we had to implement, the helicity amplitudes could be evaluated by neglecting the electron mass. There is a variety of methods for the evaluation of the helicity amplitudes [27, 28]. The most powerful are those that have been developed for the calculation of multiparticle processes especially in massless QCD [30, 31, 28] and are thus well adapted for the process at hand. The amplitudes are written in terms of Spinor Inner Products. We have performed all the helicity amplitudes calculation with the help of FORM. We have also checked our results against known results for the QED part and against the usual trace technique summation for the matrix element squared. For the helicity technique we have made full use of gauge invariance by choosing the most appropriate choice of gauge for the polarisation vectors to render the expressions as simple as possible. This is a huge gain on the usual squaring technique. The helicity amplitudes can all be expressed though the inner product $S(p,q)$, where p,q are (light-like) momenta. Following the handy notation of [31], the massless spinors are written as

$$\begin{aligned} |p\pm > &\equiv u\left(p,\pm\frac{1}{2}\right) = v(p,\mp\frac{1}{2}) \\ < p\pm | &\equiv \bar{u}\left(p,\pm\frac{1}{2}\right) = \bar{v}(p,\mp\frac{1}{2}) \end{aligned} \tag{A.1}$$

then

$$\begin{aligned} S(p,q) &= < p - | q+ > = < q + | p- >^* = -S(q,p) \quad ; \\ |S(q,p)|^2 &= 2p\cdot q \quad ; \quad (p^2 = q^2 = 0). \end{aligned} \tag{A.2}$$

S is antisymmetric, unlike in [30].

We decompose the amplitudes into the standard QED part and the two non-interfering $\tilde{\beta}_\pm$ as

$$\mathcal{A} = -(8\pi\alpha)^{3/2}\left\{\mathcal{A}^{QED} - \frac{\alpha}{\pi M^4}\sqrt{\frac{\alpha(s)}{\alpha}}\frac{1}{s_W c_W}D_Z(s)\left(\tilde{\beta}_+\mathcal{B}^+ + \tilde{\beta}_-\mathcal{B}^-\right)\right\}. \tag{A.3}$$

The helicity components are labelled as $\mathcal{A}_{\sigma_1,\sigma_2;\lambda_1,\lambda_2,\lambda_3}$, where the first labels refer to the helicity of the e^- and e^+. Chirality ($m_e = 0$) only allows the helicity amplitudes $\mathcal{A}_{\sigma,-\sigma;\lambda_1,\lambda_2,\lambda_3}$. One then only needs the expressions for $\mathcal{A}_{+,-;\pm,\pm,\pm}, \mathcal{A}_{+,-;+,-,-}$ and

$\mathcal{A}_{+,-;++-}$. For QED we recover the known result that the amplitude with the like-sign photon helicities vanishes:

$$\mathcal{A}^{QED}_{+,-;\pm,\pm,\pm} = 0 \tag{A.4}$$

while the two others are simply given by

$$\begin{aligned}
\mathcal{A}^{QED}_{+-;--+} &= -\frac{S(p_1,p_2)S(p_1,k_3)^2}{S(p_1,k_1)S(p_1,k_2)S(p_2,k_1)S(p_2,k_2)} \\
\mathcal{A}^{QED}_{+-;++-} &= \left(\frac{S(p_1,p_2)S(p_2,k_3)^2}{S(p_1,k_1)S(p_1,k_2)S(p_2,k_1)S(p_2,k_2)}\right)^*
\end{aligned} \tag{A.5}$$

For the anomalous part, $\mathcal{B}^-$ only contributes to the like-sign photon helicity amplitude whereas $\mathcal{B}^+$ only for mixed helicity states:

$$\mathcal{B}^+_{+-;+++} = \mathcal{B}^+_{+-;---} = \mathcal{B}^-_{+-;++-} = \mathcal{B}^-_{+-;+--} = 0 \tag{A.6}$$

and

$$\begin{aligned}
\mathcal{B}^+_{+-;--+} &= g_R \;\; S(p_1,p_2)^*(S(k_3,p_1)S(k_1,k_2)^*)^2 \\
\mathcal{B}^+_{+-;++-} &= -g_R \;\; S(p_1,p_2)(S(k_3,p_2)^*S(k_1,k_2))^2
\end{aligned} \tag{A.7}$$

$$\begin{aligned}
\mathcal{B}^-_{+-;+++} &= g_R \;\; S(p_1,p_2)^* \;\; \Big(S(k_2,k_3)^2 S(k_1,p_1)^2 \\
&\qquad + S(k_1,k_3)^2 S(k_2,p_1)^2 + S(k_1,k_2)^2 S(k_3,p_1)^2\Big) \\
\mathcal{B}^-_{+-;---} &= -g_R \;\; S(p_1,p_2) \;\; \Big(S(k_2,k_3)^2 S(k_1,p_2)^2 \\
&\qquad + S(k_1,k_3)^2 S(k_2,p_2)^2 + S(k_1,k_2)^2 S(k_3,p_2)^2\Big)^* .
\end{aligned} \tag{A.8}$$

All other amplitudes are found by permutation in the photons and parity conjugation that reverses all signs of the helicity. For the Z mediated amplitudes this conjugation should also be accompanied by allowing for $g_R \to g_L$.

The numerical generation of $S(p,q)$ is done through the definition

$$S(p,q) = \frac{(p_1 + i p_2)}{\sqrt{(p_0+p_3)}}\sqrt{(q_0+q_3)} - \frac{(q_1 + i q_2)}{\sqrt{(q_0+q_3)}}\sqrt{(p_0+p_3)} \quad ; \quad p = (p_0,p_1,p_2,p_3). \tag{A.9}$$

Where to find the generator?

All the cross sections and distributions found in this paper have been obtained by numerical integration of helicity-amplitude-based matrix elements. This was necessary in order to show that the interference between the QED background and the anomalous signal is negligible for energies up to LEP2 and for reasonable values of the couplings. However, from an experimentalist point of view, this is not exactly what is needed. To conduct a detailed experimental analysis, it is more important to have an event generator for the QED background and a second one for the signal. The former can already be found on the market[33] and we now provide a new generator for the anomalous signal. It can be found at lapphp0.in2p3.fr/pub/preprints-theorie/ee3gammagenerator.uu using anonymous ftp. The uu-encoded file contains five files of which only one is important. The four others are there only to run a small demonstration program to see if it runs well on your system.

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