Čerenkov radiation by neutrinos in a supernova core

Subhendra Mohanty† and Manoj K. Samal#

Theory Group, Physical Research Laboratory,

Ahmedabad - 380 009, India

Abstract

Neutrinos with a magnetic dipole moment propagating in a medium with a velocity larger than the phase velocity of light emit photons by the Čerenkov process. The Čerenkov radiation is a helicity flip process via which a left-handed neutrino in a supernova core may change into a sterile right-handed one and free-stream out of the core. Assuming that the luminosity of the sterile right-handed neutrinos is less than 10^{53} ergs/sec gives an upper bound on the neutrino magnetic dipole moment $\mu_\nu < 0.5 \times 10^{-13} \mu_B$. This is two orders of magnitude more stringent than the previously established bounds on μ_ν from considerations of supernova cooling rate by right-handed neutrinos.

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†E-mail: *mohanty@prl.ernet.in* #E-mail: *mks@prl.ernet.in*

PRL-TH-95/10

June 1995

The observation of neutrinos [1] from the supernova SN1987A has provided a number of constraints on the properties of neutrinos [2–4]. Most of the mechanisms for the constraints on the mass and magnetic moment of the neutrinos depend upon the the helicity flip of a left-handed neutrino into a sterile right-handed one which can free-stream out of the core and hence deplete the energy of the supernova core within a timescale of ~ 1 sec. Since the observed time-scale of neutrino emission [1] is of the order of 1-10 secs, it is expected that the luminosity of the right-handed neutrinos is less than 10^{53} ergs/ sec, which is the total neutrino luminosity of the supernova. The mechanism for helicity flip caused by a neutrino magnetic moment that have been considered so far are (i) helicity flip in an external magnetic field of the neutron star in the supernova core [2] and (ii) helicity flip by scattering with charged fermions *i.e.* the proccesses $\nu_L e^- \rightarrow \nu_R e^-, \quad \nu_L p \rightarrow \nu_L p$ [3]. The proccess (i) leads to an upper bound $\mu_\nu < 10^{-14}\mu_B$ ($\mu_B = e/2m_e$, the Bohr magneton), but is unreliable since it relies on a high magnetic field ($\sim 10^{14}$ Gauss) in a supernova core which has not been observed. The scattering proccess (ii) leads to an upper bound $\mu_\nu < (0.2 - 0.8) \times 10^{-11}\mu_B$ [3].

In this letter we propose a third mechanism for the neutrino helicity flip which occurs via a Čerenkov radiation proccess in the medium of the supernova core. In the supernova core the refractive index of photons is determined by the electric permitivity of the e^-, p^+ plasma and the paramagnetic susceptibility of the degenerate neutron gas. We find that Čerenkov emission of a photon from a neutrino is allowed in the photon frequency range $\omega_p \lesssim \omega < 2E/(n+1)$, (where ω_p is the plasma frequency, E is the initial neutrino energy and n is the refractive index of the medium). Since the Čerenkov emission proccess is due to the magnetic dipole operator $\mu_\nu \sigma_{\mu\nu} k_\nu \epsilon_\mu$, it is a helicity flipping proccess $\nu_L \rightarrow \nu_R \gamma$. The helicity flipping is more efficient in the Čerenkov proccess because unlike the proccess (i) there is no dependence on external magnetic field and unlike (ii) it is a single vertex proccess, so the rate is larger than the scattering rate $\nu_L e^- \rightarrow \nu_R e^-$ by $\alpha_{em} e^{-\tilde{\mu}_e/T}$, where the exponetial factor is due to the Pauli blocking of the outgoing charged fermion. We compute the luminosity Q_{ν_R} of the right-handed neutrinos produced by the Čerenkov proccess. The constraint that

$Q_{\nu_R} < 10^{53}$ ergs/sec (the total observed luminosity) leads to the bound on the neutrino magnetic moment $\mu_\nu < 0.5 \times 10^{-13}\mu_B$. This is two order of magnitude improvement over the previously established bound [3] owing to to the fact that the Čerenkov radiation is a single vertex proccess unlike the scattering proccesses considered in ref [3].

The amplitude for the Čerenkov radiation proccess $\nu_L(p) \to \nu_R(p')\gamma(k)$ is given by

$$\mathcal{M} = \frac{\mu_\nu}{n}\overline{u}(p', s')\sigma^{\mu\nu}k_\nu u(p,s)\epsilon_\mu(k,\lambda), \tag{1}$$

where μ_ν is the magnetic dipole moment of neutrino and n is the refractive index of the medium. The transition rate of the Čerenkov proccess is

$$\Gamma = \frac{1}{2E}\int \frac{d^3p'}{(2\pi)^3 2E'}\frac{d^3k}{(2\pi)^3 2\omega}(2\pi)^4\,\delta^{(4)}(p - p' - k)|\mathcal{M}|^2, \tag{2}$$

where $p = (E, p)$, $p' = (E', p')$ and $k = (\omega, k)$ are the four momenta of the incoming neutrino, outgoing neutrino and the emitted photon respectively. Using the identity

$$\int \frac{d^3p'}{2E'} = \int d^4p'\ \theta(E')\,\delta(p'^2 - m_\nu^2),$$

and integrating over the δ function in (2) we obtain

$$\Gamma = \frac{1}{16\pi}\int \frac{k^2 dk}{E^2\omega^2 n} d(\cos\theta)\,\delta(\frac{2\omega E - k^2}{2|k||p|} - \cos\theta)|\mathcal{M}|^2, \tag{3}$$

where $\cos\theta$ is the angle between the emitted photon and the incoming neutrino.

In a medium with the refractive index $n(= |k|/\omega)$, the δ function in (3) constrains $\cos\theta$ to have the value

$$\cos\theta = \frac{1}{nv}[1 + \frac{(n^2-1)\omega}{2E}], \tag{4}$$

where $v = |p|/E$ is the particle velocity and $k^2 = -(n^2-1)\omega^2$. It is clear that the kinematically allowed region for the Čerenkov proccess is where $|\cos\theta| \leq 1$. This leads to an upper limit on the range of the allowed photon frequency

$$\omega < \frac{2E}{n+1}. \tag{5}$$

where we have taken $|p| = E(1 - m^2/E^2)^{1/2} \simeq E$ since we are dealing with extremely relativistic neutrinos with $m^2/E^2 < 10^{-6}$. The refractive index of photons in a medium is $n^2 = \epsilon\mu$, where ϵ and μ are the electric permitivity and magnetic permiability of the medium. For the case of supernova the medium is a plasma consisting of degenerate electrons, protons and neutrons at temperature $T \approx 60$ MeV [5]. The permitivity ϵ is given by

$$\epsilon = (1 - \frac{\omega_p^2}{\omega^2}), \tag{6}$$

where ω_p is the plasma frequency that is determined by the chemical potential of the electrons $\tilde{\mu}_e \approx 280$ MeV and $\omega_p = (4\alpha/3\pi)^{1/2}\tilde{\mu}_e \approx 15$ MeV [6]. The neutrons do not contribute to ϵ, but they do contribute towards μ due to the paramagnetic susceptibility of the neutron gas which can be treated as a degenerate Fermi gas of magnetic dipoles with para-magnetic susceptibility χ given by [7]

$$\mu = 1 + 4\pi\chi,$$

with χ equals to

$$\chi = \frac{(2m_n)^{3/2}{\mu_n}^2\tilde{\mu}_n^{1/2}}{2\pi^2}, \tag{7}$$

where m_n, $\mu_n(= -1.91e/2m_p)$ and $\tilde{\mu}_n$ being the mass, magnetic moment and chemical potential of neutron respectively. In the supernova core the neutron density is $\rho_n \sim 4 \times 10^{14}$ gm/ cm^3 and $\tilde{\mu}_n \sim 400$ MeV [5]. The paramagnetic susceptibility is $\chi = 0.084$ and the magnetic permiability $\mu = 1 + 4\pi\chi = 2.055$. Thus the refractive index of the medium becomes

$$n^2 = 1 + 4\pi\chi - \frac{\omega_p^2}{\omega^2}. \tag{8}$$

For $\chi = 0.084$, we have $n = 1.433$ at $\omega >> \omega_p$. The refractive index is positive definite and the Čerenkov condition (5) is satisfied in the frequency range

$$0.70\,\omega_p \leq \omega \leq 0.82\,E, \tag{9}$$

which is the parameter space of the allowed Čerenkov radiation frequency.

Evaluating from (1) we have

$$|\mathcal{M}|^2 = \frac{\mu_\nu^2}{n^2}[-16(p.k)^2 + 12(p.k)k^2 + 16m_\nu^2 k^2]. \tag{10}$$

Substituting the above expression in (3) and performing the integral over δ function and using (4) for $\cos\theta$, we have the expression for transition rate for the Čerenkov process [8,9]

$$\Gamma = \frac{\mu_\nu^2}{16\pi E^2}\int_{\omega_1}^{\omega_2} d\omega[2(n^2-1)^2\omega^4 - 16m_\nu^2(n^2-1)\omega^2], \tag{11}$$

where the limits of the integral are from (9), $\omega_1 = 0.7\omega_p$ and $\omega_2 = 0.8E$. The Čerenkov proccess $\nu_L(p) \rightarrow \nu_R(p')\gamma(k)$ changes the ν_L's to sterile ν_R's which can free stream out of the supernova core. Neglecting the second term (since $m_\nu \sim 1$ eV and $\omega \sim 60$ MeV) in the expression for Γ, the luminosity of the sterile ν_R's is

$$Q_{\nu_R} = \frac{V\mu_\nu^2}{8\pi E^2}\int_0^\infty dE[f_\nu(E) - f_{\overline{\nu}}(E)]E^2\int_{\omega_1}^{\omega_2} d\omega(E-\omega)(n^2-1)^2\omega^4, \tag{12}$$

where $f_\nu(E) = [e^{(E-\tilde{\mu}_\nu)/T}+1]^{-1}$ and $f_{\overline{\nu}}(E) = [e^{(E+\tilde{\mu}_\nu)/T}+1]^{-1}$ are the statistical distribution function of the ν_L and $\overline{\nu_L}$ in the supernova. Performing the integrals over E, the luminosity of right handed neutrinos is

$$Q_{\nu_R} = V\mu_\nu^2(n^2-1)^2(0.002)[\frac{\tilde{\mu}_\nu^7}{7} + \pi^2T^2\tilde{\mu}_\nu^5 + \frac{7}{3}\pi^4T^4\tilde{\mu}_\nu^3)]. \tag{13}$$

We take the volume $V \approx 4\times 10^{18}$ cm^3, $\tilde{\mu_\nu} \approx 160$ MeV, $T = 60$ MeV [3,5] for the supernova core parameters within 1 second after collapse. The luminosity turns out to be

$$Q_{\nu_R} = 0.164\times 10^{53} \quad \mu_\nu^2 \quad (GeV^4), \tag{14}$$

in terms of the magnetic moment μ_ν. Assuming that the entire energy of the core collapse is not carried out by the right handed sterile neutrinos, *i.e.* $Q_{\nu_R} < 10^{53} ergs/sec$, we have from (13) the upper bound on the neutrino magnetic dipole moment

$$\mu_\nu < 0.5\times 10^{-13}\mu_B.$$

This is two orders of magnitude better than the previously established [3] upper bound from the ν_R luminosity of supernova. The proccess for generating ν_R in the supernova core considered in ref. [3] is via the helicity flip scattering $\nu_L e^- \rightarrow e^- \nu_R$ and $\nu_L p^- \rightarrow p^- \nu_R$ etc. This proccess has an extra electromagnetic vertex and a Pauli blocking factor for the outgoing charged fermion compared to the proccess that we have considered. That accounts for the more stringent bound we established compared to ref. [3].

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