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The Hyperfine Spin Splittings In Heavy Quarkonia

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Abstract

The hyperfine spin splittings in heavy quarkonia are studied using the recently developed renormalization group improved spin-spin potential which is independent of the scale parameter μ. The calculated energy difference between the J/ψ and the η_c fits the experimental data well, while the predicted energy difference ΔM_p between the center of the gravity of $1^3P_{0,1,2}$ states and the 1^1P_1 state of charmonium has the correct sign but is somewhat larger than the experimental data. This is not surprising since there are several other contributions to ΔM_p, which we discuss, that are of comparable size ($\sim$ 1 MeV) that should be included, before precise agreement with the data can be expected. The mass differences of the $\psi' - \eta_c'$, $\Upsilon(1S) - \eta_b$, $\Upsilon(2S) - \eta_b'$, and $B_c^* - B_c$ are also predicted.

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I. INTRODUCTION

The hyperfine splittings in heavy quark-antiquark systems can provide information about strong interactions or Quantum Chromodynamics (QCD) at low energies. Since the motion of the heavy quark and antiquark in heavy quarkonia is nonrelativistic, their dynamics can be well described by nonrelativistic potential models. The hyperfine splittings arise from higher order relativistic corrections and can be calculated using perturbation theory, given the appropriate spin-dependent potential. Recently, significant progress has been made in the theoretical study of the spin dependent potential [1]. In this work the spin-dependent potentials were derived from QCD first principles using the Heavy Quark Effective Theory (HQET) [2]. The spin-dependent potential was separated into short distance parts involving Wilson coefficients and long distance parts which were expressed in terms of gauge invariant correlation functions of the color-electric and color-magnetic fields weighted by the Wilson loop path integral [3]. If the tree level values for the Wilson coefficients are used the potential reduces to Eichten's and Feinberg's result [3]. And using the one-loop values of the Wilson coefficients, also calculating the correlation functions to one-loop in perturbation theory, the spin-dependent potential at the one-loop level in perturbation QCD [4,5] is recovered. However, the leading logarithmic terms appearing in perturbative calculations were also summed up in Ref. [1] using the Renormalization Group Equation (RGE) to obtain a scale independent result. Therefore, the spin-dependent heavy quark-antiquark potential derived in Ref. [1] is scale-independent and thus improves upon and generalizes both Eichten's and Feinberg's result [3] and the one-loop perturbative result [4,5]. In addition, this improved result [1] satisfies all the general relations among the different parts of the spin-dependent potential [6]. In the following we use this improved, more general potential [1] to calculate the hyperfine splittings in the $c\bar{c}$, $c\bar{b}$, and $b\bar{b}$ systems. Specifically, we calculate the energy difference between the ${}^3S_1$ and the ${}^1S_0$ states and the difference ΔM_p between the center of the gravity of the ${}^3P_{0,1,2}$ states and the ${}^1P_1$ state.

First we note that the P-wave hyperfine splitting ΔM_p in charmonium has been exper-

imentally determined to be -0.9 ± 0.2 MeV [7], which is not only much smaller than the splittings caused by the spin-orbit and the tensor interactions, but also the S-wave hyperfine splittings, which typically are $50 - 100$ MeV. Naively, one might estimate the hyperfine splitting to be smaller than the spin-orbit and tensor splittings by the order of v^2, where v is the relative quark-antiquark velocity, or about $1/10$ in charmonium. This interesting point has been studied previously [8–13]. According to the Fermi-Breit formula, which follows from lowest order perturbation theory, the hyperfine spin splitting is proportional to the wavefunction at the origin, which vanishes for P-waves. However, one-loop corrections give logarithmic terms that are nonlocal and allow a non-zero contribution to the P-wave hyperfine splittings. Several previous calculations [8–11] of ΔM_p used only the one-loop perturbative spin-spin potential [4,5] and the results are remarkably close to the experimental value. This agreement with the experimental value of ΔM_p, taking into account only the one-loop contribution is surprising, since there are other contributions to ΔM_p of similar size; for example, nonlocal terms coming from higher orders. We will discuss such effects below, as it is instructive to see how these higher order contributions could affect the results.

In the following we will use the general formula for the spin-spin part of the renormalization-group-improved spin-dependent potential that was derived in Ref. [1] to calculate the hyperfine spin splittings in the $c\bar{c}$, $b\bar{b}$, and $c\bar{b}$ systems. Since the spin-spin potential is a short distance feature, perturbation theory can reliably be used in the calculation. Our result for the $1^3S_1 - 1^1S_0$ splitting between the J/ψ and the η_c agrees well with the experimental value and our predictions for the mass differences $\psi' - \eta_c'$, $\Upsilon(1S) - \eta_b$, $\Upsilon(2S) - \eta_b'$, and $B_c^* - B_c$ are reasonable. However, the contribution to the P-wave energy difference, ΔM_p, between the center of the gravity of $1^3P_{0,1,2}$ states and the 1^1P_1 state, while having the correct sign, is somewhat larger than the experimental data. That is, when the contributions of the leading logarithmic terms are summed up and included, the agreement with that data is not as good as when only the one-loop perturbative spin-spin potential, in which the leading logarithmic contributions are not summed up and included is used.

We will discuss the implications of these results in greater detail below and point out that there are several other contributions to the rather small energy difference ΔM_p which estimates indicate are of the same order of magnitude as the spin-spin contribution. It therefore appears that the agreement of the one-loop perturbative result with the data is probably fortuitous.

The following Sec. II is devoted to the calculational methods. In Sec. III we present our numerical results and in Sec. VI we discuss these results and our conclusions.

II. CALCULATIONAL METHODS

To calculate the hyperfine splittings in the heavy quark-antiquark systems we will use the spin-spin part of the renormalization-group improved general formula for the spin-dependent potential [1] derived in the framework of HQET [2]. In the derivation the renormalized two-particle effective Lagrangian was first calculated to order $1/m^2$. Then, treating the terms of higher order in $1/m$ in the effective Lagrangian as perturbations, the four point Green's function on the Wilson loop [15] with the time interval T was calculated in the limit where $m \to \infty$ first followed by $T \to \infty$ [16,3]. In this limit, using standard perturbative methods, the large T behavior of the Green's function is of the form

$$I \propto e^{-T\epsilon(m,r)}. \tag{1}$$

From Eq. (1) $\epsilon(m,r)$, the potential energy between the quark and the antiquark, can be extracted. Expanding $\epsilon(m,r)$ in powers of $1/m$ each of the spin-dependent potentials can be factorized into a short distance part, involving Wilson coefficients, and a long distance part, which can be expressed in terms of correlation functions of the color-electric and color-magnetic fields weighted by the Wilson-loop integral. Using the notation of Ref. [1], the resulting spin-spin potential is

$$\Delta H_{ss}(m_1,m_2,r) = \frac{S_1 \cdot S_2}{3m_1m_2}\left[\, c_3(\mu,m_1)c_3(\mu,m_2)V_4(\mu,r) - 6N_c g_s^2(\mu)d(\mu)\delta^3(r)\,\right], \tag{2}$$

where m_1, m_2, and S_1, S_2 are the masses and the spins of the heavy quark and the antiquark, respectively, μ is the renormalization subtraction point, N_c is the number of colors, and $g_s(\mu)$ is the running coupling constant. The Wilson coefficients $c_3(\mu, m)$ and $d(\mu)$ were calculated in leading logarithmic approximation in Ref. [14] and Ref. [1], respectively, and are

$$c_3(\mu, m) = \left(\frac{\alpha_s(\mu)}{\alpha_s(m_1)}\right)^{-\frac{9}{25}}, \tag{3}$$

and

$$\begin{aligned} d(\mu) &= \frac{N_c^2 - 1}{8N_c^2} c_3(m_2, m_1)[1 - c_3^2(\mu, m_2)] \\ &= \frac{N_c^2 - 1}{8N_c^2} \left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)}\right)^{-\frac{9}{25}} \left[1 - \left(\frac{\alpha_s(\mu)}{\alpha_s(m_2)}\right)^{-\frac{18}{25}}\right]. \end{aligned} \tag{4}$$

In Eq. (2) $V_4(\mu, r)$ is the color magnetic-magnetic correlation function which can be expressed as

$$V_4(\mu, r) \equiv \lim_{T\to\infty} \int_{-T/2}^{T/2} dz \int_{-T/2}^{T/2} dz' \frac{g_s^2(\mu)}{T} \langle B^i(x_1, z) B^i(x_2, z')\rangle / \langle 1\rangle, \tag{5}$$

where $\langle \cdots \rangle$ is defined by

$$\langle \cdots \rangle \equiv \int [dA^\mu] Tr \left\{ P \left[\exp\left(ig \oint_{C(r,T)} dz_\mu A^\mu(z) \right) \cdots \right] \right\}_{x\in C} \exp(iS_{YM}(A)), \tag{6}$$

Here $C(r, T)$ represents the Wilson loop [15], P denotes the path ordering, and $r \equiv |x_1 - x_2|$.

We emphasize that this is a general result for the hyperfine part of the spin-dependent potential to order $1/m^2$. It absorbs the short distance contributions to the potential into the coefficients $c_3(\mu, m)$ and $d(\mu)$ while the long distance contributions to the potential are contained in the correlation function $V_4(\mu, r)$. Moreover, the result is independent of the factorization scale since the μ-dependence in the coefficients cancels the μ-dependence in the correlation function. The first term in the bracket in Eq. (2) is a nonlocal term while the second term is a local one which is generated by mixing with the first (nonlocal) term under renormalization. We note that if the coefficients are evaluated at tree level; i.e., $c_3(\mu, m) = 1$ and $d(\mu) = 0$, the potential reduces to the Eichten-Feinberg result [3]. And if these coefficients are expanded to order $\alpha_s(\mu)$ and the correlation function is also evaluated

only to one-loop, the logarithmic terms in Eq. (2) then reduce to the one-loop spin-spin potential [4,5]. Therefore, this renormalization-group improved potential, Eq. (2), extends both Eichten's and Feinberg's result [3] as well as the one-loop perturbative potential [4,5], containing each of these results as special cases.

In a heavy quarkonium state the typical momentum transfer is of order $m\,v$, where v is the relative velocity of the heavy quark and the antiquark, and the typical size is of order $1/(mv)$. In such a low momentum region the correlation function $V_4(\mu, r)$ could in principle have nonperturbative contributions and should, therefore, be calculated using nonperturbative methods. However, since confinement in QCD is color-electrical, it is reasonable to expect the color-magnetic field to be predominately a short distance effect. Thus the color magnetic-magnetic correlation function $V_4(\mu, r)$ should fall off quite fast when the distance r becomes large. This is confirmed both by lattice calculations [18] and by the experimental fact that ΔM_p is empirically very small. If $V_4(\mu, r)$ had a significant long distance component ΔM_p would be considerably larger, contrary to the data. We therefore can safely assume that the potential $V_4(\mu, r)$ is a short distance effect which can be calculated using perturbative QCD. Its perturbative expression can be obtained from the following arguments: As mentioned above, the result for $\Delta H_{ss}(r)$ in Eq. (2) is μ-independent since the μ-dependence in the coefficients cancels the μ-dependence in the correlation function. However, to explicitly demonstrate this cancellation to all orders one must calculate the correlation function to all orders, which is impossible to do directly. Fortunately, using the RGE, this can be done in the leading logarithmic approximation. In momentum space the Fourier transformation of $V_4(\mu, r)$, denoted by $\widetilde{V}_4(\mu, q)$, is dimensionless and is only a function of the two variables μ and q. It must, therefore, be a function of $\ln(q^2/\mu^2)$ and these logarithms can be summarized using the RGE in the effective theory. Alternately, there is another simple approach: If we choose $\mu = q$ all of these logarithmic terms vanish and only the tree level term $\dfrac{N_c^2 - 1}{N_c} g_s^2(\mu)|_{\mu=q}$ remains in $\widetilde{V}_4(\mu, q)$ in the leading logarithmic approximation. Then all the nonlocal logarithmic terms having been absorbed into the coefficients $c_3(q, m)$ and

$d(q)$. Consequently, we find the hyperfine part of the spin-dependent potential in momentum space in the leading logarithmic approximation to be

$$\Delta\widetilde{H}_{ss}(m_1,m_2,q) = \frac{S_1\cdot S_2}{3m_1m_2}g_s^2(q)\left[\frac{N_c^2-1}{N_c}c_3(q,m_1)c_3(q,m_2) - 6N_c d(q)\right], \tag{7}$$

where $c_3(q,m)$ and $d(q)$ are given by Eqs. (3) and (4). This final formula, Eq. (7), for the hyperfine spin-dependent potential, which we will use in our calculations of the hyperfine spin-splittings in the $c\bar{c}$, $c\bar{b}$, and $b\bar{b}$ systems, improves upon the one-loop perturbative calculation in two important respects: (i) it is independent of μ and (ii) it includes the higher order logarithmic terms.

To first order perturbation theory in ΔH_{ss} the energy shift caused by $\Delta H_{ss}(r)$ is

$$\Delta E = \int d^3r \Psi^*_{l,l_z}(r)\Delta H_{ss}(r)\Psi_{l,l_z}(r) \tag{8}$$

where $\Psi_{l,l_z}(r)$ is the nonrelativistic wavefunction of the bound state with total angular momentum l and z-component l_z. For simplicity we suppress spin and color indices and retain only the space-dependent indices. Separating the radial part, $u(r)$, we write $\Psi_{l,l_z}(r)$ as

$$\Psi_{l,l_z}(r) = u(r)\, Y_{l,l_z}(\theta,\phi), \tag{9}$$

where $Y_{l,m}(\theta,\phi)$ are the standard spherical harmonics. Rotational invariance implies that ΔE is independent of l_z. Averaging over l_z and using properties of the spherical harmonics, ΔE can be expressed as

$$\Delta E = \int \frac{d^3r}{4\pi}|u(r)|^2\Delta H_{ss}(r). \tag{10}$$

Taking the Fourier transform, in momentum space ΔE is given by

$$\Delta E = \int \frac{d^3q}{(2\pi)^3}\xi(q)\Delta\widetilde{H}_{ss}(q), \tag{11}$$

where

$$\xi(q) = \int \frac{d^3r}{4\pi}e^{iq\cdot r}|u(r)|^2 = \frac{1}{q}\int dr\, r\sin qr\,|u(r)|^2. \tag{12}$$

Finally, doing the angular integration we have

$$\Delta E = \frac{1}{2\pi^2} \int dq\, q^2 \xi(q) \Delta \widetilde{H}_{ss}(q), \tag{13}$$

which we will use to numerically calculate the hyperfine splittings. Of course, the radial wave function $u(r)$ must first be obtained by numerically solving the Schrödinger equation with some chosen potential and then $\xi(q)$ [Eq. (12)] can easily be calculated by using the Fast Fourier Transformation program.

III. NUMERICAL RESULTS

Using Eqs. (7), (12), and (13) the hyperfine spin splittings for both the S-wave and the P-wave states were numerically calculated. The radial wavefunction was obtained by numerically solving the Schrödinger equation. For comparison, we used three popular potential models. One was the Cornell model [19] in which the potential has the form,

$$V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}, \tag{14}$$

with

$$\begin{aligned} m_c &= 1.84 \;\; GeV \;\; , \; m_b = 5.18 \;\; GeV, \\ \kappa &= 0.52 \;\; GeV \;\; , \; a = 2.34 \;\; GeV. \end{aligned} \tag{15}$$

The second one was the logarithmic potential [20] given by

$$V(r) = -0.6635\ GeV + (0.733\ GeV) \log(r \cdot 1GeV), \tag{16}$$

with

$$m_c = 1.5 \;\; GeV \;\; , \; m_b = 4.906 \;\; GeV. \tag{17}$$

The third one was the improved QCD-motivated potential [8] with the form

$$V(r) = \frac{r}{a^2} - \frac{16\pi}{25} \frac{1}{r f(r)} \left[1 + \frac{2\gamma_E + \frac{53}{75}}{f(r)} - \frac{462}{625} \frac{\ln f(r)}{f(r)} \right], \tag{18}$$

where $f(r)$ was given by

$$f(r) = 2\ln\left(\frac{\Lambda_{\overline{MS}} + (\Lambda_{II} - \Lambda_{\overline{MS}})\exp\left[-\left[15\,(0.75\Lambda_{II} - \Lambda_{\overline{MS}})\,r\right]^2\right]}{\Lambda_{II}\Lambda_{\overline{MS}}\,r} + C\right) \tag{19}$$

with

$$\begin{aligned} m_c &= 1.478 \;\; GeV \;\; , \; m_b = 4.878 \;\; GeV, \\ \Lambda_{II} &= 0.72 \;\; GeV, \quad a = 2.59 \;\; GeV^{-1}, \\ C &= 4.62. \end{aligned} \tag{20}$$

To proceed with the calculation we also required an expression for the running coupling constant $\alpha_s(q)$. The familiar RGE, one-loop result is

$$\alpha_s(q) = \frac{4\pi}{b_0 \ln \dfrac{q^2}{\Lambda^2_{\overline{MS}}}}, \tag{21}$$

where $b_0 = 11N_c - 2N_f$ and N_f is the number of quark flavors. It is clear from Eq. (21) that $\alpha_s(q)$ contains a Landau singularity in the nonperturbative region when $q^2 = \Lambda^2_{\overline{MS}}$ and becomes negative for $q^2 < \Lambda^2_{\overline{MS}}$. To avoid the resulting numerical ambiguities we first moved this singularity to $q^2 = 0$ and used a modified form of $\alpha_s(q)$ in the actual numerical calculations; namely,

$$\alpha_s(q) = \frac{4\pi}{b_0 \ln\left(\dfrac{q^2}{\Lambda^2_{\overline{MS}}} + 1\right)}. \tag{22}$$

In the next section we shall discuss alternative approaches, the sensitivity of the results, and their implications. The value of $\Lambda_{\overline{MS}}$ was taken to be 200 MeV and 250 MeV in the numerical calculations, which is within the experimental range $\Lambda_{\overline{MS}} = 195 + 65 - 50$ MeV [21]. Our numerical results for the three potentials are presented in Tables I, II, III, respectively. For comparison we have also included the results for the $2S$ and $2P$ states. The main features of these results can be summarized as follows:

- The results are μ-independent, as they must be.

- The calculated energy difference between the J/Ψ and the η_c mesons is quite close to the experimental value for all three potentials.

- For each of these three potentials we predict the energy difference between Ψ' and η_c' to lie within the range $55 - 80$ MeV.

- For the $b\bar{b}$ system there are significant discrepancies between the Cornell model, with the parameters given by Eq. (15), and the other two models for the S-states. Since the Cornell model, with these parameters, does not predict the $b\bar{b}$ spectrum very well, the results calculated in the other two models are probably better predictions for the energy difference between the $\Upsilon(1S)$ and the η_b ($35-50$ MeV) and between the $\Upsilon(2S)$ and the η_b' (20 MeV).

- The predicted energy difference between B_c^* and B_c meson is in the range $40-70$ MeV from all three of these models, which is consistent with previous results [13].

- The calculated value of $\Delta M_p \equiv E(1^3P_J) - E(1^1P_1)$ for the charmonium $1P$ states is in the range of -4 to -6 MeV, which has the same sign but is several times larger than the experimental value of -0.9 ± 0.2 MeV [7]. This is not surprising since there are several other contributions to ΔM_p which estimates indicate are comparable in magnitude to the contribution coming from the hyperfine spin-spin interaction, H_{ss}. In fact, it is surprising that the prediction from only the one-loop spin-dependent potential is quite close to the experimental data. We discuss these other contributions in the next section.

IV. DISCUSSION AND CONCLUSIONS

We have calculated the hyperfine spin splittings in the $c\bar{c}$, $b\bar{b}$, and $b\bar{c}$ system using the RGE improved perturbative spin-spin potential [1]. The results for the hyperfine splittings of the S-wave states agree with the $J/\Psi - \eta_c$ measured splitting [21] and the prediction for splitting $\Upsilon - \eta_b$ is reasonable. However, the contribution to $\Delta M_p \equiv E(^3P_J) - E(^1P_1)$ for the

charmonium P-wave states is somewhat larger than the experimental data [7], although it agrees in sign. That is, after summing up the leading logarithmic terms and including them in the perturbation calculations, the agreement with the data is not as good as the one-loop calculations [8–11]. In order to illustrate this clearly, we can expand $\alpha_s(q)$ in terms of $\alpha_s(\mu)$ and truncate it at some finite order. In our final formula, Eq. (7), we used the expansion for $\alpha_s(q)$,

$$\alpha_s(q) = \frac{\alpha_s(\mu)}{1 - \frac{b_0}{4\pi}\alpha_s(\mu)\ln\frac{\mu^2}{q^2}} = \alpha_s(\mu)\left[\, 1 + \sum_{m=1}^{n}\left(\frac{b_0}{4\pi}\alpha_s(\mu)\ln\frac{\mu^2}{q^2}\right)^m \,\right], \tag{23}$$

and truncated at several choices of n. Specifically, we repeated the numerical calculations for the improved QCD motivated potential [8] for $n = 1$, 2 and 4, choosing the scale μ, now to be $\mu = 1.5$ GeV, 4.0 GeV, and 2.5 GeV for the $c\bar{c}$, $b\bar{b}$, $b\bar{c}$ systems, respectively. The numerical results are presented in Tables IV, V, and VI corresponding to $n = 1$, 2, and 4, respectively. For comparison, we also presented the results obtained using the complete one-loop hyperfine potential [4,5] in Table VII. Comparing Table IV and Table VII we see that ΔM_P for $n = 1$ is quite close to the complete one-loop result. However, from Table V and Table VI we see that the predicted values of ΔM_p are about 60%-80% and 150%-200% larger than when terms up to order 2 and order 4 are kept in the expansion of $\alpha_s(q)$, Eq. (23). We note that we also repeated these calculations for the logarithmic potential [20] and the Cornell potential [19]. All three potentials predicted similar values for ΔM_p. This clearly indicates that the nonlocal logarithmic terms from high loop perturbative calculations are quite important. In fact, even using the RGE to sum up these logarithmic terms does not allow one to understand the experimental value of ΔM_p, indicating that the success of the one-loop calculations [8–11] was probably fortuitous. In fact, there are several additional contributions that are possibly comparable in magnitude. These include the following:

The contributions of the spin-orbit and and tensor potentials in the second order of perturbation theory: These contributions to ΔM_p only cancel to first order in perturbation theory. However, according to the power counting rules introduced in Ref. [22], the spin-orbit and tensor potential potentials shift the energies of the P-wave states by an amount

of order mv^4 in first order, which indeed cancel in ΔM_p, but they do make a contribution to ΔM_p of order mv^6 in the second order of perturbation theory. This estimate is several MeV for the P-wave charmonium states, and therefore should not be neglected.

Higher dimensional operators: Unlike the dimension-six operators, these give non-zero contributions to ΔM_p even at tree level. Compared to the one-loop contribution, these are suppressed by v^2 but enhanced by α_s^{-1} and $v^2/\alpha_s \sim 1$ in charmonium.

The color-octet S-wave component in P-wave quarkonia states [22]: This component of the wavefunction receives a tree-level contribution from the local term $\delta^3(r)$ in the spin-spin potential. This contribution too could be of order $v^2/\alpha_s \sim 1$ compared to what has been calculated.

Next-to-leading order perturbative contributions from the two-loop potential: These are suppressed by order α_s, but since α_s is not a very small quantity in charmonium, one cannot dismiss the possibility that this contribution could be significant.

Before comparing with the experimental value of ΔM_p in charmonium, which is only about 1 MeV, all the above contribution should be included since they are possibly comparable in magnitude. In the $b\bar{b}$ case these effects are less important and one can expect the perturbative calculations the $b\bar{b}$ system to be more reliable, although ΔM_p is smaller, also.

Finally, to explore the sensitivity of our results to the location of the Landau singularity in $\alpha_s(q)$ we replaced the expression, Eq. (20), by

$$\alpha_s(q) = \frac{4\pi}{b_0 \ln\left(\frac{q^2}{\Lambda^2_{\overline{MS}}} + \lambda^2\right)} \,. \tag{24}$$

and varied λ^2. The results for the S- wave hyperfine splitting were not sensitive to λ^2 and only for large λ^2 did ΔM_p significantly decrease. To fit ΔM_p to the measured value required λ^2 quite large, about 16, clearly out of the perturbative region.

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Table I. The hyperfine spin splittings in MeV predicted by Eq (7) with Cornell potential [19]

	$c\bar{c}$		$b\bar{b}$		$b\bar{c}$	
$\Lambda_{\overline{MS}}$ (MeV)	200	250	200	250	200	250
$E(1^3S_1) - E(1^1S_0)$	117.1	128.3	97.7	104.3	67.0	71.4
$E(2^3S_1) - E(2^1S_0)$	75.3	82.5	39.6	42.2	37.7	40.3
$E(1^3P_J) - E(1^1P_1)$	-4.8	-7.0	-3.0	-4.1	-4.0	-5.6
$E(2^3P_J) - E(2^1P_1)$	-3.6	-5.2	-2.1	-2.9	-3.0	-4.2

Table II. The hyperfine spin splittings in MeV predicted by Eq (7) with Logarithmic potential [20]

	$c\bar{c}$		$b\bar{b}$		$b\bar{c}$	
$\Lambda_{\overline{MS}}$ (MeV)	200	250	200	250	200	250
$E(1^3S_1) - E(1^1S_0)$	106.1	117.1	36.0	38.2	40.6	42.6
$E(2^3S_1) - E(2^1S_0)$	54.2	59.7	18.7	19.8	21.2	22.3
$E(1^3P_J) - E(1^1P_1)$	-5.4	-7.8	-3.4	-4.5	-4.5	-6.4
$E(2^3P_J) - E(2^1P_1)$	-2.6	-3.8	-2.1	-2.8	-2.8	-4.0

Table III. The hyperfine spin splittings in MeV predicted by Eq (7) with Improved-QCD motivated potential [8]

	$c\bar{c}$		$b\bar{b}$		$b\bar{c}$	
$\Lambda_{\overline{MS}}$ (MeV)	200	250	200	250	200	250
$E(1^3S_1) - E(1^1S_0)$	107.9	119.1	44.6	47.6	43.4	45.7
$E(2^3S_1) - E(2^1S_0)$	68.5	75.6	20.9	22.4	25.2	26.7
$E(1^3P_J) - E(1^1P_1)$	-4.6	-6.7	-2.7	-3.7	-3.7	-5.3
$E(2^3P_J) - E(2^1P_1)$	-3.4	-5.0	-1.8	-2.5	-2.6	-3.8

Table IV. The hyperfine spin splittings in MeV predicted by Eq (7) with $n = 1$ for the Improved-QCD motivated potential [8]

	$c\bar{c}$		$b\bar{b}$		$b\bar{c}$	
$\Lambda_{\overline{MS}}$ (MeV)	200	250	200	250	200	250
$E(1^3S_1) - E(1^1S_0)$	111.8	125.4	45.7	49.2	45.2	48.6
$E(2^3S_1) - E(2^1S_0)$	71.3	80.1	21.5	23.2	26.3	28.4
$E(1^3P_J) - E(1^1P_1)$	-1.5	-1.9	-1.0	-1.2	-1.2	-1.5
$E(2^3P_J) - E(2^1P_1)$	-1.0	-1.3	-0.7	-0.8	-0.9	-1.2

Table V. The hyperfine spin splittings in MeV predicted by Eq (7) with $n = 2$ for the Improved-QCD motivated potential [8]

	$c\bar{c}$		$b\bar{b}$		$b\bar{c}$	
$\Lambda_{\overline{MS}}$ (MeV)	200	250	200	250	200	250
$E(1^3S_1) - E(1^1S_0)$	110.5	123.4	45.2	48.5	44.7	47.8
$E(2^3S_1) - E(2^1S_0)$	70.6	79.1	21.3	23.0	26.1	28.2
$E(1^3P_J) - E(1^1P_1)$	-2.5	-3.4	-1.6	-2.0	-2.0	-2.7
$E(2^3P_J) - E(2^1P_1)$	-1.6	-2.2	-1.0	-1.3	-1.4	-1.8

Table VI. The hyperfine spin splittings in MeV predicted by Eq (7) with $n = 4$ for the Improved-QCD motivated potential [8]

	$c\bar{c}$		$b\bar{b}$		$b\bar{c}$	
$\Lambda_{\overline{MS}}$ (MeV)	200	250	200	250	200	250
$E(1^3S_1) - E(1^1S_0)$	108.8	120.4	44.8	47.9	43.9	46.5
$E(2^3S_1) - E(2^1S_0)$	69.3	76.8	21.0	22.7	25.6	27.4
$E(1^3P_J) - E(1^1P_1)$	-4.0	-6.2	-2.3	-3.0	-3.1	-4.3
$E(2^3P_J) - E(2^1P_1)$	-2.8	-4.3	-1.4	-1.9	-2.1	-2.9

Table VII. The hyperfine spin splittings in MeV predicted by the complete one-loop spin-spin potential [4,5] with Improved-QCD motivated potential [8]

	$c\bar{c}$		$b\bar{b}$		$b\bar{c}$	
$\Lambda_{\overline{MS}}$ (MeV)	200	250	200	250	200	250
$E(1^3S_1) - E(1^1S_0)$	127.7	145.7	50.1	54.3	39.2	41.5
$E(2^3S_1) - E(2^1S_0)$	81.5	93.3	23.4	25.6	22.6	23.9
$E(1^3P_J) - E(1^1P_1)$	-1.2	-1.5	-0.6	-0.7	-0.6	-0.7
$E(2^3P_J) - E(2^1P_1)$	-1.1	-1.3	-0.5	-0.6	-0.5	-0.6