hep-ph/9506369

A Review of Minimal Supersymmetric Electro Weak Theory

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Abstract

In this review article we study the Minimal Supersymmetric Electro-Weak theory. The Lagrangian is constructed step by step in great detail, both in the superfield and component field formalism — both on and off shell. Furthermore the Lagrangian is written in the more familiar four component formalism. Electro weak symmetry breaking is discussed, and the physical chargino- and neutralino states are introduced and discussed.

Contents

Preface

When I first started to work on supersymmetry, my interest fell upon the minimal supersymmetric electro-weak theory, or if you like, on supersymmetric quantum flavour dynamics (S-QFD). To my disappointment, as a person at that time with no background in supersymmetry, I was not able to find any good *detailed* review article on this subject.

In this report I try to present such a review article with the hope that it may be useful to others. The material is presented in great detail, and somebody may rightly say that the presentation is too comprehensive. For that reason most of the detailed calculations are reserved for the appendices. However, my personal motivation for including so much details was to easy the chance of following the calculations step by step for a person not familiar with the Minimal Supersymmetric Standard Model.

In this report I have mostly followed the notation used by the authors of ref. [31, 32, 33] and I give some useful formulae and comments about notation in appendix A. These references together with ref. [34] are also good introductions to the necessary background of supersymmetry needed for this report.

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I would like to take the opportunity to express my deep appreciation to Prof. Dr.tech. Haakon A. Olsen at the University of Trondheim, Norway. He has been very helpful, and I in particular thank him for great many stimulating and clarifying discussions and for fruitful suggestions during the course of this work.

Trondheim, April 1995

Ingve Simonsen

Chapter 1

Supersymmetric Extension of QFD.

We will now start the construction of a supersymmetric extension of QFD of leptons. In this chapter the Lagrangian, in the superfield formalism, will be derived.

However, before we do so, we will say a few words about possible extensions of the Standard Model (SM).

1.1 Possible Extensions of the Standard Model.

In a supersymmetric theory, any fermionic state has to be accompanied by a bosonic one, and vice versa. In the early days of SUSY, one had hoped that some of the states required by SUSY, could be identified with some of the known particle states. For instance one tried to identify the spin-0 fields associated with the neutrino- and the electron-fields, as the photon and Higgs-field respectively [1]. Unfortunately, this idea runs into difficulties. Firstly, if one of the spin-0 neutrino states is associated with the photon, what happens to the lepton-quark symmetry? Secondly, and more convincing, is the observation that the spin-0 states, associated with the leptons and quarks, carry lepton number and colour respectively. By demanding a theory with unbroken colour and electromagnetism, only the scalar neutrino can acquire a vacuum expectation value. This results in a theory with the unwanted possibility of lepton number violation. However, this scenario can not be completely ruled out [3], but no realistic model, with such properties, exists. Thus, in consequence, one is forced to introduce a complete Higgs (SUSY) multiplet in addition to the multiplets of leptons and quarks.

In the SM, it is sufficient with only one Higgs doublet (and its charge conjugated) in order to generate masses for the leptons and charge-$\frac{1}{3}$ and -$\frac{2}{3}$ quarks. In SUSY, however, one has to have at least two Higgs doublets if suitable mass terms shall be generated [2, 4, 5, 13]. The reason is rather technical and relies on the fact that SUSY do not allow for charge conjugation[1].

1.1.1 The Minimal Supersymmetric Standard Model.

The different supersymmetric extensions of the SM are naturally divided into two main classes. The first one, is the Minimal Supersymmetric Standard Model (MSSM) [4—22] containing the minimal number of fields and parameters required to construct a realistic model of leptons and quarks. The second class, goes under the name of Non-Minimal Supersymmetric Standard Models (NMSSM) [23]. Several such models can also be constructed, but they typically increase the number of parameters (and fields) without any corresponding increase in predictive power and physical motivation.

The MSSM has a high degree of predictivity, and within this model all masses and coupling constants of the Higgs boson sector, can be calculated at tree level.

Since the MSSM is the most attractive one from a practical point of view, and since no theoretical aspects (at present) seem to discredit it, we will be considering this model in the present work. It is also interesting to note that the MSSM has survived *all* the stringent phenomenological tests coming from resent LEP-experiments, and that in most of its parameter space the (relevant) MSSM predictions are impressively close to the SM values (calculated for a relative light SM Higgs) [24].

Model Ingredients.

In a more complete way, the central ingredients of the MSSM can be defined by the following points:

- The minimal gauge group: $SU(3) \times SU(2) \times U(1)$.
- The minimal particle content, holding three generations of leptons and quarks, twelve gauge bosons (defined in the usual way), two Higgs doublets and, of course, all these particles superpartners.
- SUSY breaking parametrized by *soft* breaking terms.
- An exact discrete R-parity.

The three first points need no further comments at this early stage. However, the same can not be said about the fourth point. If we construct a theory based on the three first points only, a theory possessing baryon- and lepton-number violation will emerge [25]. The terms responsible for this, give unacceptable physics (fast rates of nucleon decay). Thus, these

[1]Two Higgs doublets are also needed in order to avoid gauge anomalies originating from the spin-$\frac{1}{2}$ higgsinos.

terms must somehow be avoided, and it is believed that this can only be done in a satisfactory manner by introducing additional symmetries, e.g. gauge- or discrete-symmetries. The last possibility is used in the MSSM. Here an unbroken R-symmetry [1, 26, 27] with a corresponding R-parity, or equivalently matter-parity, is introduced in order to eliminate the offending terms. The R-parity of a state is related to its spin (S), baryon-number (B), and lepton-number (L) according to

$$R_p = (-1)^{2J+3B+L} . \tag{1.1}$$

Note that the assumption of baryon- and lepton-number conservation implies the conservation of R-parity.

Furthermore, an immediate consequence of the above expression is that all SM particles (including the Higgs bosons) are R-even, while their superpartners are R-odd. As a result the "new" supersymmetric particles can only be pair-produced, and any of their decay products have to contain an odd number of supersymmetric particles. This implies that the lightest supersymmetric particle (LSP) has to be stable, since it has no allowed decay channels.

1.2 The Lagrangian for Supersymmetric QFD.

In this section, we shall construct a (minimal) supersymmetric extension of QFD. We have chosen to work within the framework of the MSSM, and consider supersymmetric QFD to be a part of this more fundamental theory[2]. Thus the content of the Higgs-sector is defined to contain two Higgs-doublets, as we discussed in the previous section.

In order to construct the Lagrangian of supersymmetric QFD (S-QFD), we will assume that the theory can be viewed as a low-energy limit of a SUGRAV-theory. Thus the Lagrangian of S-QFD has to have the form

$$\mathcal{L}_{S-QFD} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}. \tag{1.2}$$

Here $\mathcal{L}_{SUSY}$ is a supersymmetric piece, while $\mathcal{L}_{Soft}$ explicitly breaks SUSY.

The ultimate aim of this section, will be to specify the different terms of $\mathcal{L}_{S-QFD}$. However before we do so, we have to define the different fields which are present in S-QFD.

The first version of the MSSM was constructed in the early eighties by the authors of refs. 28 and 29 and later discussed in refs. 13 and 35. They promoted all the lepton fields of the SM to chiral superfields, one for each generation. The same we will do, and denote these superfields by $\hat{l}(x,\theta,\bar{\theta})$ and $\hat{\nu}_l(x,\theta,\bar{\theta})$. Here the former contains the charged leptons (like

[2] An alternative contemplation could be to consider the MSSM for leptons only. Hence the $SU(3)$-gauge invariance becomes trivial as in the SM (of leptons), where all fields except the quark- and gluon-fields are $SU(3)$-singlets, and a non-trivial $SU(2)\times U(1)$ theory remains. This resulting theory may be considered, as is correct, to be a supersymmetric extension of QFD (or equivalently the Glasow-Weinberg-Salam theory).

the electron) and the latter the corresponding neutrinos. Here the generational indices have been suppressed[3].

It is useful, and we will henceforward use it, to assume, as for "ordinary" QFD, that the neutrinos are completely left-handed. Hence the left-handed lepton superfields (for each generation) can be arranged in an $SU(2)$-doublet and the right-handed in an $SU(2)$-singlet according to[4 5]

$$\hat{L}(x,\theta,\bar{\theta}) = \begin{pmatrix} \hat{\nu}_l(x,\theta,\bar{\theta}) \\ \hat{l}(x,\theta,\bar{\theta}) \end{pmatrix}_L, \tag{1.3}$$

$$\hat{R} = \hat{l}_R(x,\theta,\bar{\theta}). \tag{1.4}$$

From the previous section, we already know that the MSSM, and hence S-QFD, contains two doublets of (chiral) Higgs superfields, which we will defined as

$$\hat{H}_1(x,\theta,\bar{\theta}) = \begin{pmatrix} \hat{H}_1^1(x,\theta,\bar{\theta}) \\ \hat{H}_1^2(x,\theta,\bar{\theta}) \end{pmatrix}, \tag{1.5}$$

and

$$\hat{H}_2(x,\theta,\bar{\theta}) = \begin{pmatrix} \hat{H}_2^1(x,\theta,\bar{\theta}) \\ \hat{H}_2^2(x,\theta,\bar{\theta}) \end{pmatrix}. \tag{1.6}$$

Note that the upper index on these superfields, say $\hat{H}_1^2(x,\theta,\bar{\theta})$, is an $SU(2)$ index taking values in the set $\{1,2\}$. The same applies to $\hat{L}(x,\theta,\bar{\theta})$.

As for non-supersymmetric QFD, S-QFD possesses an $SU(2)\times U(1)$-gauge invariance. This means that the theory contains four different gauge vector superfields — $\hat{V}'(x,\theta,\bar{\theta})$ for the $U(1)$-gauge group and $\hat{V}^a(x,\theta,\bar{\theta})$ $(a=1,2,3)$ for $SU(2)$. As usual we will take the gauge vector superfields to be Lie algebra valued, i.e.

$$\hat{V}'(x,\theta,\bar{\theta}) = Y\hat{v}'(x,\theta,\bar{\theta}), \tag{1.7}$$

$$\hat{V}(x,\theta,\bar{\theta}) = T^a\hat{V}^a(x,\theta,\bar{\theta}), \qquad a=1,2,3. \tag{1.8}$$

Here Y and T^a are the generators of $U(1)$ and $SU(2)$ respectively.

In table 1.1 the above definitions, together with the quantum numbers, are summerized.

1.2.1 The Supersymmetric Term $\mathcal{L}_{SUSY}$.

The term $\mathcal{L}_{SUSY}$, is obtained by "supersymmetrizing" the Lagrangian of ordinary QFD. In this generalizing procedure, the Yang-Mills Lagrangian [31, 32, 33] is useful. However

[3]Summation over the generational indices will be understood everywhere, if nothing else is said to indicate otherwise.

[4]Here the subscripts L and R mean left- and right-handed respectively.

[5]From now on we will use hats ($\hat{}$) on the superfield quanteties of our S-QFD model.

Multiplet type	Superfields	Quantum Numbers	
		$SU(2)$	$U(1)$
Matter	$\hat{L}(x,\theta,\bar{\theta})$	doublet	-1
	$\hat{R}(x,\theta,\bar{\theta})$	singlet	2
	$\hat{H}_1(x,\theta,\bar{\theta})$	doublet	-1
	$\hat{H}_2(x,\theta,\bar{\theta})$	doublet	1
Gauge	$\hat{V}'(x,\theta,\bar{\theta})$	singlet	0
	$\hat{V}^a(x,\theta,\bar{\theta})$	triplet	0

Table 1.1: The notation and quantum numbers used for the superfields in S-QFD (for leptons). The index a labels $SU(2)$ triplets of gauge bosons. All the generational indices are suppressed.

the S-QFD Lagrangian becomes slightly more complicated due to the fact that we have a larger gauge group, a richer particle spectrum with both left- and right-handed states, and in addition a Higgs-sector as well to take into account.

With the identifications we made in the previous chapter for the kinetic terms of chiral- and vector-superfields, the S-QFD Lagrangian reads

$$\mathcal{L}_{SUSY} = \mathcal{L}_{Lepton} + \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs}, \tag{1.9}$$

where

$$\mathcal{L}_{Lepton} = \int d^4\theta \left[\hat{L}^\dagger e^{2g\hat{V}+g'\hat{V}'} \hat{L} + \hat{R}^\dagger e^{2g\hat{V}+g'\hat{V}'} \hat{R} \right], \tag{1.10}$$

$$\mathcal{L}_{Gauge} = \frac{1}{4} \int d^4\theta \left[W^{a\,\alpha} W^a_\alpha + W'^{\,\alpha} W'_\alpha \right] \delta^2(\bar{\theta}) + h.c., \tag{1.11}$$

and finally

$$\mathcal{L}_{Higgs} = \int d^4\theta \left[\hat{H}_1^\dagger e^{2g\hat{V}+g'\hat{V}'} \hat{H}_1 + \hat{H}_2^\dagger e^{2g\hat{V}+g'\hat{V}'} \hat{H}_2 + W\,\delta^2(\bar{\theta}) + \bar{W}\,\delta^2(\theta) \right]. \tag{1.12}$$

Here g and g' are the (gauge) coupling constants for $SU(2)$ and $U(1)$ respectively and W_α and W'_α are the $SU(2)$- and $U(1)$-fieldstrengths defined by

$$W_\alpha = -\frac{1}{8g} \bar{D}\bar{D} e^{-2g\hat{V}} D_\alpha e^{2g\hat{V}}, \tag{1.13}$$

$$W'_\alpha = -\frac{1}{4} DD\bar{D}_{\dot{\alpha}} \hat{V}'. \tag{1.14}$$

Furthermore, $W \equiv W[\hat{L}, \hat{R}, \hat{H}_1, \hat{H}_2]$ is the superpotential of the theory which we will discuss in a moment[6].

[6] We will not write the fieldstrengths without spinor indices so confusion between the symbols for the superpotential and the fieldstengths will arise.

The factors of 2 appearing in eqs. (1.10), (1.12) and (1.13) in connection with the $SU(2)$-coupling constant g, are inserted for convenience. With this choice the (non-SUSY) field-strength $V^a_{\mu\nu}$ contained in W_α correspondes to that of the SM.

The Superpotential.

In order to give a complete expression for $\mathcal{L}_{SUSY}$, the superpotential $W[\hat{L}, \hat{R}, \hat{H}_1, \hat{H}_2]$ has to be specified. The superpotential can at maximum be cubic in the superfields in order to guarantee a renormalizable theory.

In the MSSM the superpotential takes on the form

$$W = W_H + W_Y,$$

with the "Higgs-part" given by

$$W_H = \mu\, \varepsilon^{ij} \hat{H}_1^i \hat{H}_2^j,$$

and the corresponding "Yukawa-part" by[7]

$$W_Y[\hat{L}, \hat{R}, \hat{H}_1 \hat{H}_2] = \varepsilon^{ij} \left[f \hat{H}_1^i \hat{L}^j \hat{R} + f_1 \hat{H}_1^i \hat{Q}^j \hat{D} + f_2 \hat{H}_2^j \hat{Q}^i \hat{U} \right].$$

Here μ is a mass parameter and ε^{ij} is an anti-symmetric tensor defined by

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{1.15}$$

Furthermore, f, f_1 and f_2 are all (Yukawa) coupling constants containing one generational index which has been suppressed. It is often the case that only the largest Yukawa couplings (for the third generation) are of importance. However, we will not in particular take a stand on this point.

As alluded to earlier, we will not be concerned about the quark-sector of S-QFD. Hence the superpotential reduces to

$$\begin{aligned} W &= W_H + W_Y \\ &= \mu\, \varepsilon^{ij} \hat{H}_1^i \hat{H}_2^j + f\, \varepsilon^{ij} \hat{H}_1^i \hat{L}^j \hat{R}. \end{aligned} \tag{1.16}$$

The first term of the above superpotential needs some further comments. If this term is missing (i.e. $\mu = 0$), the theory has an additional Peccei-Quinn symmetry [36]. Under this symmetry the Higgs superfield $\hat{H}_1$ undergoes a phase transformation. In cases where the bosonic component of $\hat{H}_1^1$ gets a non-vanishing vacuum expectation value, this symmetry is spontaneously broken. The result of such a breaking is an experimentally unacceptable Weinberg-Wilczek axion [37]. Hence, $\mu \neq 0$ is required in order to get a physically acceptable theory.

[7]Here $\hat{Q}$ is a quark $SU(2)$-doublet while $\hat{U}$ and $\hat{D}$ are quark $SU(2)$-singlets.

1.2.2 The Soft SUSY-Breaking Term $\mathcal{L}_{Soft}$.

The most general soft SUSY breaking terms where described by Giraedello and Grisaru [30]. They found that the allowed terms can be categorized as follows; scalar mass tems, gaugino mass tems and finally trilinear scalar interaction terms. However, S-QFD, as the MSSM, has to possess R-invariance, as referred to in the previous section. This implicates that trilinear terms contained in $W|_{\theta=0}$, have to be disregarded (and we do it from now) since they are not R-invariant. The actual proof of this fact will be given in subsect. 1.3.3.

By adjusting the remaining allowed soft terms to our notation of S-QFD, one gets the following Lagrangian (appropriate to Fermi scale) in terms of superfields:

$$\mathcal{L}_{Soft} = \mathcal{L}_{SMT} + \mathcal{L}_{GMT}, \tag{1.17}$$

where the scalar mass term (SMT) piece reads

$$\begin{aligned}\mathcal{L}_{SMT} &= -\int d^4\theta \, \Big[M_L^2 \, \hat{L}^\dagger \hat{L} + m_R^2 \hat{R}^\dagger \hat{R} + m_1^2 \hat{H}_1^\dagger \hat{H}_1 \\ &\qquad + m_2^2 \hat{H}_2^\dagger \hat{H}_2 - m_3^2 \varepsilon^{ij} \left(\hat{H}_1^i \hat{H}_2^j + h.c. \right) \Big] \, \delta^4(\theta, \bar{\theta}),\end{aligned} \tag{1.18}$$

and the gauge mass term (GMT) is

$$\mathcal{L}_{GMT} = \frac{1}{2} \int d^4\theta \, \Big[\Big(M \, W^{a\,\alpha} W_\alpha^a + M' \, W'^{\,\alpha} W'_\alpha \Big) + h.c. \Big] \, \delta^4(\theta, \bar{\theta}). \tag{1.19}$$

Here

$$M_L^2 \, \hat{L}^\dagger \hat{L} = m_{\tilde{\nu}}^2 \, \hat{\nu}^\dagger \hat{\nu} + m_L^2 \, \hat{l}_L^\dagger \hat{l}_L,$$

while the (soft) mass-parameters M and M' are corresponding to the SU(2)- and U(1)-gauge group respectively. The factor of $\frac{1}{2}$ in front of $\mathcal{L}_{GMT}$ is inserted for later convenience.

Within the framework of MSSM, the different couplings and mass-terms, appearing in the above Lagrangian, are all undetermined both in origin and magnitude. However they are usually interpreted as remnants of a more fundamental spontaneously broken ($N = 1$) SUGRAV-theory. Keep in mind that at the Fermi scale, which we are working at, one deals with renormalized parameters which are connected to their values at the Planck scale via the renormalization group equations.

1.2.3 Conclusion.

To conclude this section, we collect our results for the Lagrangian $\mathcal{L}_{S-QFD}$, in terms of superfields, for later reference. It reads:

$$\begin{aligned}\mathcal{L}_{S-QFD} &= \mathcal{L}_{SUSY} + \mathcal{L}_{Soft} \\ &= \int d^4\theta \, \Big\{ \hat{L}^\dagger e^{2g\hat{V} + g'\hat{V}'} \hat{L} + \hat{R}^\dagger e^{2g\hat{V} + g'\hat{V}'} \hat{R}\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{4}\left[\left(W^{a\,\alpha}W^a_\alpha + W'^{\,\alpha}W'_\alpha\right)\delta^2(\bar\theta) + h.c.\right] \\
&+\hat H_1^\dagger e^{2g\hat V + g'\hat V'}\hat H_1 + \hat H_2^\dagger e^{2g\hat V + g'\hat V'}\hat H_2 \\
&+W\,\delta^2(\bar\theta) + \bar W\,\delta^2(\theta) \\
&-\Big[M_L^2\,\hat L^\dagger\hat L + m_R^2\,\hat R^\dagger\hat R + m_1^2\,\hat H_1^\dagger\hat H_1 \\
&\qquad + m_2^2\hat H_2^\dagger\hat H_2 - m_3^2\varepsilon^{ij}\left(\hat H_1^i\hat H_2^j + h.c.\right)\Big]\,\delta^4(\theta,\bar\theta) \\
&+\frac{1}{2}\left[\left(M\,W^{a\,\alpha}W^a_\alpha + M'\,W'^{\alpha}W'_\alpha\right) + h.c.\right]\delta^4(\theta,\bar\theta)\Big\}. \qquad (1.20)
\end{aligned}
$$

1.3 Invariances of the Lagrangian $\mathcal{L}_{S-QFD}$.

In this section, we will establish some of the symmetries of $\mathcal{L}_{S-QFD}$, and we start by demonstrating the SUSY invariance of $\mathcal{L}_{SUSY}$.

1.3.1 The SUSY Invariance of $\mathcal{L}_{SUSY}$.

It is well known that the highest (mass) dimensional component of any superfield combination is always supersymmetric (up to a total derivative) [31, 32, 33]. With this in mind, the SUSY-invariance of $\mathcal{L}_{SUSY}$ is easy to verify, due to its possible formulation in terms of superfields[8].

With eq. $(A.142)$ we have that a four dimensional integration with respect to Grassmann variables projects out the $\theta\theta\,\bar\theta\bar\theta$-component of the integrand. This is the highest, non-vanishing dimensional component possible, because of the anti-commuting property of the Grassmann variables. Hence, we may on this ground conclude that $\mathcal{L}_{Lepton}$ and the two first terms of $\mathcal{L}_{Higgs}$ are supersymmetric.

The highest component of a product of two or three left-handed (right-handed) chiral superfields is a $\theta\theta$-component ($\bar\theta\bar\theta$-component). Hence, since $\hat L$, $\hat R$, $\hat H_1$, $\hat H_2$ and the fieldstrenghts W_α and W'_α are all left-handed chiral superfields, while their hermitian conjugated are right-handed, $\mathcal{L}_{Gauge}$ and the remaining terms of $\mathcal{L}_{Higgs}$ are SUSY-invariant. Note that the two-dimensional delta functions over a Grassmann algebra, are inserted in order to adopt with the four-dimensional Grassmann integration.

Hence $\mathcal{L}_{SUSY}$ is proven to be SUSY-invariant.

As have been stated up to several time, $\mathcal{L}_{Soft}$ breaks SUSY. To see this, it is enough to note

[8]Later on, when the component-form of $\mathcal{L}_{SUSY}$ is obtained, we will also verify the SUSY invariance explicitly without any reference to the superfield formalism. As we will see then, this line of action is much more demanding then the approach made here.

that

$$\int d^4\theta \; \hat{S} \; \delta^4(\theta, \bar{\theta}) \;\; = \;\; \hat{S}\Big|_{\theta=\bar{\theta}=0} \tag{1.21}$$

is a (mass) dimensional zero term, with $\hat{S}$ being any superfield (or superfield combination). Then according to our earlier discussion $\mathcal{L}_{Soft}$ is notoriously not SUSY-invariant.

1.3.2 The Gauge Invariance of $\mathcal{L}_{S-QFD}$.

The gauge transformations on chiral- and vector-superfields are defined by

$$\left.\begin{array}{lcll} \Phi'(x,\theta,\bar{\theta}) & = & e^{-ig\Lambda(x,\theta,\bar{\theta})}\Phi(x,\theta,\bar{\theta}), & \bar{D}_{\dot{\alpha}}\Lambda = 0 \\ \Phi'^{\dagger}(x,\theta,\bar{\theta}) & = & \Phi^{\dagger}(x,\theta,\bar{\theta})e^{ig\Lambda^{\dagger}(x,\theta,\bar{\theta})} & D_{\alpha}\Lambda^{\dagger} = 0 \\ e^{gV'} & = & e^{-ig\Lambda^{\dagger}}\, e^{gV}\, e^{ig\Lambda} & \end{array}\right\}. \tag{1.22}$$

and that of the fieldstrength W_{α}^{a} by

$$W_{\alpha} \rightarrow W'_{\alpha} \;\; = \;\; e^{-ig\Lambda}\, W_{\alpha}\, e^{ig\Lambda}. \tag{1.23}$$

These transformations will be extensively used in this subsection.

We start by showing the $SU(2)$ invariance of the theory.

The $SU(2)$-Invariance.

Since $[\hat{V}, \hat{V}'] = [\hat{\Lambda}, \hat{V}'] = 0$, the term $\hat{L}^{\dagger}e^{2g\hat{V}+g'\hat{V}'}\hat{L}$ is shown to be $SU(2)$-gauge invariant as follows

$$\begin{array}{rcl} \hat{L}^{\dagger}e^{2g\hat{V}+g'\hat{V}'}\hat{L} = \hat{L}^{\dagger}e^{2g\hat{V}}e^{g'\hat{V}'}\hat{L} & \longrightarrow & \hat{L}^{\dagger}e^{2ig\hat{\Lambda}^{\dagger}}e^{-2ig\hat{\Lambda}^{\dagger}}e^{2g\hat{V}}e^{2ig\hat{\Lambda}}e^{g'\hat{V}'}e^{-2ig\hat{\Lambda}}\hat{L} \\ & = & \hat{L}^{\dagger}e^{2g\hat{V}+g'\hat{V}'}\hat{L}. \end{array} \tag{1.24}$$

The invariance of the corresponding kinetic terms of $\hat{R}$, $\hat{H}_1$ or $\hat{H}_2$ are shown in the same manner[9].

If we can show that $W^{\alpha a}W_{\alpha}^{a}$, $W'^{\alpha}W'_{\alpha}$, and the superpotential $W \equiv W[\hat{L}, \hat{R}, \hat{H}_1, \hat{H}_2]$ are gauge invariant, then we have established the SU(2)-invariance of $\mathcal{L}_{SUSY}$. This is so because the invariance of the other terms can be obtained by hermitian conjugation. From eq. (1.23) we have

$$\begin{array}{rcl} W^{\alpha\, a}W_{\alpha}^{a} = \dfrac{1}{k}\,Tr\,(W^{\alpha}W_{\alpha}) & \longrightarrow & \dfrac{1}{k}\,Tr\left(e^{-2ig\hat{\Lambda}}W^{\alpha}e^{2ig\hat{\Lambda}}e^{-2ig\hat{\Lambda}}W_{\alpha}e^{2ig\hat{\Lambda}}\right) \\ & = & \dfrac{1}{k}\,Tr\,(W^{\alpha}W_{\alpha}) \\ & = & W^{\alpha\, a}W_{\alpha}^{a} \end{array} \tag{1.25}$$

[9]Note that the invariance of the term containing $\hat{R}$ is trivial since $\hat{R}$ transform like a singlet under $SU(2)$.

Here we have used the cyclic property of the trace. The SU(2)-invariance of $W'^{\alpha}W'_{\alpha}$ is trivial since W'_{α} is a singlet under this group.

Now we shall demonstrate the invariance of the superpotential W, and we start by W_H,

$$\begin{aligned} W_H = \mu\varepsilon^{ij}\,\hat{H}_1^i\hat{H}_2^j \quad \longrightarrow & \quad \mu\varepsilon^{ij}\,\left[e^{-2ig\hat{\Lambda}}\hat{H}_1\right]^i\left[e^{-2ig\hat{\Lambda}}\hat{H}_2\right]^j, \quad i,j=1,2 \\ = & \quad \mu\varepsilon^{ij}\,\mathcal{U}^{ik}\mathcal{U}^{jl}\,\hat{H}_1^k\hat{H}_2^l, \qquad\qquad \mathcal{U}=e^{-2ig\hat{\Lambda}}. \end{aligned} \tag{1.26}$$

In order for W_H to be invariant we must have

$$\varepsilon^{kl} \;=\; \varepsilon^{ij}\,\mathcal{U}^{ik}\mathcal{U}^{jl}. \tag{1.27}$$

This relation is in fact satisfied as we now will show. The matrix $\mathcal{U}=e^{-2ig\hat{\Lambda}}$ is obviously a 2×2-matrix, and its determinant is

$$\det\mathcal{U} = e^{-2igTr\left(\hat{\Lambda}\right)} = 1, \tag{1.28}$$

since $Tr\left(\hat{\Lambda}\right)\equiv Tr\left(T^a\hat{\Lambda}^a\right)=0$. Hence $\mathcal{U}$ is an $SU(2)$-matrix. Then $\mathcal{U}$, as any $SU(2)$-matrix, can be written as

$$\mathcal{U} \;=\; \begin{pmatrix} \hat{A} & \hat{B} \\ -\hat{B}^{\dagger} & \hat{A}^{\dagger} \end{pmatrix}, \tag{1.29}$$

with

$$\hat{A}^{\dagger}\hat{A}+\hat{B}^{\dagger}\hat{B} \;=\; 1. \tag{1.30}$$

Here $\hat{A}$ and $\hat{B}$ are functionals of the chiral superfields $\hat{\Lambda}^a$. Their actual dependence on these superfields are of no importance to us, so we will not worry about them.

Hence

$$\begin{aligned} \varepsilon^{ij}\,\mathcal{U}^{ik}\mathcal{U}^{jl} &= \left[\mathcal{U}^T\varepsilon\,\mathcal{U}\right]^{kl} \\ &= \begin{pmatrix} 0 & \hat{A}^{\dagger}\hat{A}+\hat{B}^{\dagger}\hat{B} \\ -\left(\hat{A}^{\dagger}\hat{A}+\hat{B}^{\dagger}\hat{B}\right) & 0 \end{pmatrix}^{kl} \\ &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{kl} \\ &= \varepsilon^{kl}, \end{aligned} \tag{1.31}$$

and W_H is (gauge) invariant under $SU(2)$.

The invariance of W_Y is showed as above since $\hat{H}_1$ and $\hat{L}$ are both doublets under SU(2), while $\hat{R}$ is a singlet under this group. Thus the superpotential $W=W_H+W_Y$ is $SU(2)$-gauge invariant.

We now would like to draw the attention towards the SUSY-breaking term $\mathcal{L}_{Soft}$. Because of the particular form of $\mathcal{L}_{GMT}$ (cf. eqs. (1.19)) the only invariance which has not been checked yet, is that of $\mathcal{L}_{SMT}$. Since

$$\hat{L}^\dagger \hat{L} \quad \longrightarrow \quad \hat{L}^\dagger e^{2ig\hat{\Lambda}} e^{-2ig\hat{\Lambda}} \hat{L} = \hat{L}^\dagger \hat{L}, \tag{1.32}$$

is invariant, and the same applies for the corresponding terms of $\hat{R}$, $\hat{H}_1$ and $\hat{H}_2$, we may conclue that $\mathcal{L}_{SMT}$, and thus $\mathcal{L}_{Soft}$, are SU(2)-invariant[10].

Thus the total Lagrangian $\mathcal{L}_{S-QFD}$ is SU(2)-gauge invariant as it should.

The $U(1)$-Invariance.

Many of the invariances showed above easily generalize to U(1) with the substitutions $2g \rightarrow g'$, $T^a\hat{\Lambda}^a \rightarrow Y\hat{\lambda}' = \hat{\Lambda}'$. This applies to all terms containing only vector superfields, and terms built out of vector superfields and only one type of chiral superfields[11].

The remaining U(1)-invariance to check, is that of terms holding two, or more, types of chiral superfields. Such terms are only contained in the superpotential W, and the invariance is proved as follows

$$\begin{aligned} W_H = \mu\varepsilon^{ij}\ \hat{H}_1^i \hat{H}_2^j \quad &\longrightarrow \quad \mu\varepsilon^{ij}\ e^{-ig'\left(Y_{H_1}+Y_{H_2}\right)\hat{\lambda}'}\ \hat{H}_1^i \hat{H}_2^j \\ &= \quad W_H \end{aligned} \tag{1.33}$$

and

$$\begin{aligned} W_Y = f\varepsilon^{ij}\ \hat{H}_1^i \hat{L}^j \hat{R} \quad &\longrightarrow \quad f\varepsilon^{ij}\ e^{-ig'\left(Y_{H_1}+Y_L+Y_R\right)\hat{\lambda}'} \hat{H}_1^i \hat{L}^j \hat{R} \\ &= \quad W_Y \end{aligned} \tag{1.34}$$

since $Y_{H_1} + Y_{H_2} = 0$ and $Y_{H_1} + Y_L + Y_R = 0$ according to table 1.1. Hence the theory is U(1)-invariant as well.

This completes the proof of the full SU(2)×U(1) gauge invariance of the theory.

1.3.3 The R-Invariance.

The definition of R-symmetry, generated by the operator R, was introduced by the authors of refs. 1 and 27. It acts on left-handed chiral superfields $\Phi(x,\theta,\bar{\theta})$, and its (right-handed) hermitian conjugated, as follows

$$R\Phi(x,\theta,\bar{\theta}) \quad = \quad e^{2in_\Phi\alpha}\Phi(x, e^{-i\alpha}\theta, e^{i\alpha}\bar{\theta}) \tag{1.35}$$

$$R\Phi^\dagger(x,\theta,\bar{\theta}) \quad = \quad e^{-2in_\Phi\alpha}\Phi^\dagger(x, e^{-i\alpha}\theta, e^{i\alpha}\bar{\theta}), \tag{1.36}$$

[10] Note that the last two terms of $\mathcal{L}_{SMT}$ are invariant for the same reason as for instance W_H.

[11] The different types in our model are $\hat{L}$, $\hat{R}$, $\hat{H}_1$ and $\hat{H}_2$.

and on vector multiplets according to

$$RV(x,\theta,\bar\theta) \;=\; V(x,e^{-i\alpha}\theta,e^{i\alpha}\bar\theta). \tag{1.37}$$

Here α is a continuous real parameter, while n_Φ is called the *R-character* of the chiral superfield $\Phi(x,\theta,\bar\theta)$.

In terms of component fields, the above transformations read for the chiral multiplet

$$\left.\begin{array}{lcr} A & \longrightarrow & e^{2in_\Phi\alpha}A \\ \psi & \longrightarrow & e^{2i\left(n_\Phi-\frac{1}{2}\right)\alpha}\psi \\ F & \longrightarrow & e^{2i(n_\Phi-1)\alpha}F \end{array}\right\}, \tag{1.38}$$

and for the vector multiplet

$$\left.\begin{array}{lcr} f & \longrightarrow & f \\ \psi & \longrightarrow & e^{-i\alpha}\psi \\ m & \longrightarrow & e^{-2i\alpha}m \\ V_\mu & \longrightarrow & V_\mu \\ \lambda & \longrightarrow & e^{i\alpha}\lambda \\ d & \longrightarrow & d \end{array}\right\}. \tag{1.39}$$

Here the transformations for the remaining components are given by hermitian conjugation.

For products of left-handed chiral superfields we have [38]

$$R\prod_a \Phi_a(x,\theta,\bar\theta) \;=\; e^{2i\sum_a n_a\alpha}\prod_a \Phi(x,e^{-i\alpha}\theta,e^{i\alpha}\bar\theta), \tag{1.40}$$

and the following general superfield terms are all R-invariant:

$$\int d^4\theta\;\Phi^\dagger(x,\theta,\bar\theta)\Phi(x,\theta,\bar\theta), \tag{1.41}$$

$$\int d^4\theta\;\Phi^\dagger(x,\theta,\bar\theta)e^{V(x,\theta,\bar\theta)}\Phi(x,\theta,\bar\theta), \tag{1.42}$$

$$\int d^4\theta\;\prod_a \Phi_a(x,\theta,\bar\theta)\,\delta^2(\bar\theta), \qquad \text{if } \sum_a n_a = 1, \tag{1.43}$$

$$\int d^4\theta\;\prod_a \Phi_a(x,\theta,\bar\theta)\,\delta^4(\theta,\bar\theta), \qquad \text{if } \sum_a n_a = 0, \tag{1.44}$$

Now returning to S-QFD, we have at once, from the above results, that $\mathcal{L}_{S-QFD}$ is R-invariant if and only if

$$n_1+n_2 \;=\; 1, \tag{1.45}$$

$$n_1+n_L+n_R \;=\; 1. \tag{1.46}$$

Here we have used obvious notation, and we have chosen to give the superfields arranged in doublets, the same R-character for convenience. Since the R-characters of the superfields in question are somewhat ambiguous, we will in addition take up the convention of all n's being positive. With the choices made in table 1.2, $\mathcal{L}_{S-QFD}$ is R-invariant, as it should.

Before we close this chapter, we will make one concluding remark. From eqs. (1.43) and (1.44) it is obvious that both $\int d^4\theta\ W\,\delta^2(\theta)$ (from $\mathcal{L}_{SUSY}$) and the soft term $\int d^4\theta\ W\,\delta^4(\theta,\bar{\theta})$ can not be R-invariant at the same time. On the other hand, R-invariance alone does not favour one from the other. However, the unbroken S-QFD theory, described by $\mathcal{L}_{SUSY}$, must have appropriate Yukawa-terms. This implies that $\int d^4\theta\ W\,\delta^2(\theta)$ must be included in $\mathcal{L}_{SUSY}$, while the soft term $\int d^4\theta\ W\,\delta^4(\theta,\bar{\theta})$ has to be excluded from $\mathcal{L}_{Soft}$ (due to R-invariance) as mentioned earlier in this chapter.

Superfields	**R**-character
$\hat{L}(x,\theta,\bar{\theta})$	1/4
$\hat{R}(x,\theta,\bar{\theta})$	1/4
$\hat{H}_1(x,\theta,\bar{\theta})$	1/2
$\hat{H}_2(x,\theta,\bar{\theta})$	1/2

Table 1.2: The R-character of the different chiral superfields of S-QFD.

Chapter 2

Component Field Expansion of $\mathcal{L}_{S-QFD}$.

In this chapter, the component expansion of the full Lagrangian $\mathcal{L}_{S-QFD}$ will be developed, even if it is $\mathcal{L}_{SUSY}$ which will be our main consurn. Much of the explicit calculations are pretty lengthy and are performed in the appendices. The time-consuming possedure of explicitely proving the SUSY-invariance of $\mathcal{L}_{SUSY}$ will also be given in this chapter. Finally we will transform the full Lagrangian into four-component notation.

2.1 Component Expansion of $\mathcal{L}_{SUSY}$.

Before we go into the component expansion of $\mathcal{L}_{SUSY}$, the component form of the different superfields of the model have to be given. This we will do now.

In the previous chapter, we arranged for one of the lepton superfields to be an SU(2)-doublet ($\hat{L}$) and the other an singlet ($\hat{R}$). These chiral superfields will be given the following component expansions[1]

$$
\begin{aligned}
\hat{L}(x,\theta,\bar{\theta}) &= \begin{pmatrix} \hat{\nu}_l(x,\theta,\bar{\theta}) \\ \hat{l}(x,\theta,\bar{\theta}) \end{pmatrix}_L \\
&= \tilde{L}(x) + i\,\theta\sigma^\mu\bar{\theta}\,\partial_\mu\tilde{L}(x) - \frac{1}{4}\,\theta\theta\,\bar{\theta}\bar{\theta}\,\partial^\mu\partial_\mu\tilde{L}(x) \\
&\quad + \sqrt{2}\,\theta L^{(2)}(x) + \frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\sigma}^\mu\partial_\mu L^{(2)}(x) + \theta\theta\,F_L(x), \qquad (2.1) \\
\hat{R}(x,\theta,\bar{\theta}) &= \hat{l}_R(x) \\
&= \tilde{R}(x) + i\,\theta\sigma^\mu\bar{\theta}\,\partial_\mu\tilde{R}(x) - \frac{1}{4}\,\theta\theta\,\bar{\theta}\bar{\theta}\,\partial^\mu\partial_\mu\tilde{R}(x)
\end{aligned}
$$

[1]These component expansions, and coming, would been simpler in the (y,θ)-basis. However, this basis will not often be used, so we have decided to work in the $(x,\theta,\bar{\theta})$-basis form the very beginning.

$$+\sqrt{2}\,\theta R^{(2)}(x)+\frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\sigma}^{\mu}\partial_{\mu}R^{(2)}(x)+\theta\theta\;F_R(x). \tag{2.2}$$

Field name	Symbol	Spin	Charge
Leptons	$L^{(2)\,1}$	1/2	0
	$L^{(2)\,2}$	1/2	−1
	$R^{(2)}$	1/2	1
Sleptons	$\tilde{L}^1$	0	0
	$\tilde{L}^2$	0	−1
	$\tilde{R}$	0	1
Higgs bosons	H_1^1	0	0
	H_2^1	0	−1
	H_1^2	0	1
	H_2^2	0	0
Higgsinos	$\psi_{H_1}^1$	1/2	0
	$\psi_{H_1}^2$	1/2	−1
	$\psi_{H_2}^1$	1/2	1
	$\psi_{H_2}^2$	1/2	0
Gauge bosons	V_μ^a	1	–
	V'_μ	1	–
Gauginos	λ^a	1/2	–
	λ'	1/2	–

Table 2.1: A summary of the SM-fields and their superpartners present in the S-QFD model. The quantum numbers of the various fields are also summarized. All fermion fields are given in terms of two-component (Weyl) spinors.

Here the component fields are defined by

$$\tilde{L}(x)=\begin{pmatrix}\tilde{\nu}_l(x)\\ \tilde{l}_L(x)\end{pmatrix}\qquad L^{(2)}(x)=\begin{pmatrix}\nu_l^{(2)}(x)\\ l^{(2)}(x)\end{pmatrix}_L\qquad F_L(x)=\begin{pmatrix}f^{\nu}(x)\\ f_L^l(x)\end{pmatrix}, \tag{2.3}$$

and[2]

$$\tilde{R}(x)=\tilde{l}_R^{\dagger}(x)\qquad R^{(2)}(x)=l_R^{(2)}(x)\qquad F_R(x)=f_R^l(x). \tag{2.4}$$

[2]The relation $\tilde{R}=\tilde{l}_R^{\dagger}$ (with a dagger on only one side) may seem a little bit strange at first sight. It is introduced for convenience, and in particular to let $\tilde{L}^{\dagger}$ and $\tilde{R}^{\dagger}$ both create negatively charged sleptons. If we have identified $\tilde{R}=\tilde{l}_R$, then $\tilde{R}^{\dagger}$ would have created positively charged sleptons[39].

Field name	Symbol	Spin	Charge
Auxiliary Lepton Fields	f^ν	0	0
	f^l_L	0	-1
	f^l_R	0	1
Auxiliary Higgs Fields	f^1_1	0	0
	f^2_1	0	-1
	f^1_2	0	1
	f^2_2	0	0
Auxiliary Gauge Fields	D^a	1	–
	D'	1	–

Table 2.2: A summary of the auxiliary fields of the S-QFD model and their quantum numbers.

In the same way, we have for the two Higgs (doublet) superfields

$$\begin{aligned} \hat{H}_1(x,\theta,\bar{\theta}) &= \begin{pmatrix} \hat{H}^1_1(x,\theta,\bar{\theta}) \\ \hat{H}^2_1(x,\theta,\bar{\theta}) \end{pmatrix} \\ &= H_1(x) + i\,\theta\sigma^\mu\bar{\theta}\,\partial_\mu H_1(x) - \frac{1}{4}\,\theta\theta\,\bar{\theta}\bar{\theta}\,\partial^\mu\partial_\mu H_1(x) \\ &\quad + \sqrt{2}\,\theta\tilde{H}^{(2)}_1(x) + \frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\sigma}^\mu\partial_\mu\tilde{H}^{(2)}_1(x) + \theta\theta\,F_1(x), \end{aligned} \tag{2.5}$$

$$\begin{aligned} \hat{H}_2(x,\theta,\bar{\theta}) &= \begin{pmatrix} \hat{H}^1_2(x,\theta,\bar{\theta}) \\ \hat{H}^2_2(x,\theta,\bar{\theta}) \end{pmatrix} \\ &= H_2(x) + i\,\theta\sigma^\mu\bar{\theta}\,\partial_\mu H_2(x) - \frac{1}{4}\,\theta\theta\,\bar{\theta}\bar{\theta}\,\partial^\mu\partial_\mu H_2(x) \\ &\quad + \sqrt{2}\,\theta\tilde{H}^{(2)}_2(x) + \frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\sigma}^\mu\partial_\mu\tilde{H}^{(2)}_2(x) + \theta\theta\,F_2(x), \end{aligned} \tag{2.6}$$

where the component fields read

$$H_1(x) = \begin{pmatrix} H^1_1(x) \\ H^2_1(x) \end{pmatrix} \qquad \tilde{H}^{(2)}_1(x) = \begin{pmatrix} \psi^1_{H_1}(x) \\ \psi^2_{H_1}(x) \end{pmatrix} \qquad F_1(x) = \begin{pmatrix} f^1_1(x) \\ f^2_1(x) \end{pmatrix}, \tag{2.7}$$

and

$$H_2(x) = \begin{pmatrix} H^1_2(x) \\ H^2_2(x) \end{pmatrix} \qquad \tilde{H}^{(2)}_2(x) = \begin{pmatrix} \psi^1_{H_2}(x) \\ \psi^2_{H_2}(x) \end{pmatrix} \qquad F_2(x) = \begin{pmatrix} f^1_2(x) \\ f^2_2(x) \end{pmatrix}. \tag{2.8}$$

Note that *all* the F-fields are auxiliary fields, which later on, when constructing the on-shell Lagrangian, will be removed through the Euler-Lagrange equations.

Here hats (ˆ), as in the previous chapter, indicate superfields while tildes (˜) denote supersymmetric partners of the SM particles. The subscripts L and R on fermionic-fields, mean as usual, left- and right-handed fields[3], while the superscript "(2)" means that we are dealing with two-component (Weyl) spinors. The same goes for the SU(2) components of the higgsino doublets $\tilde{H}_1^{(2)}$ and $\tilde{H}_1^{(2)}$, even if the above mentioned superscript is missing on the ψ's.

The (minimal) S-QFD model also contains vector multiplets. As a matter of convenience, we choose to work in the WZ-gauge. In this gauge the component expansions of the SU(2)- and U(1)-gauge superfields $\hat{V} = T^a \hat{V}^a$ and $\hat{V}' = Y \hat{v}'$, are given by

$$\hat{V}^a(x,\theta,\bar{\theta}) \;=\; -\,\theta\sigma^\mu\bar{\theta}\; V_\mu^a(x) + i\;\theta\theta\;\bar{\theta}\bar{\lambda}^a(x) - i\;\bar{\theta}\bar{\theta}\;\theta\lambda^a(x) + \frac{1}{2}\;\theta\theta\,\bar{\theta}\bar{\theta}\; D^a(x), \qquad (2.9)$$

and

$$\hat{v}'(x,\theta,\bar{\theta}) \;=\; -\,\theta\sigma^\mu\bar{\theta}\; V'_\mu(x) + i\;\theta\theta\;\bar{\theta}\bar{\lambda}'(x) - i\;\bar{\theta}\bar{\theta}\;\theta\lambda'(x) + \frac{1}{2}\;\theta\theta\,\bar{\theta}\bar{\theta}\; D'(x). \qquad (2.10)$$

Here $\lambda^a(x)$ and $\lambda'(x)$ are the two-component (Weyl) gaugino fields, the superpartners of the (SM) gauge bosons, and the D-fields are auxiliary fields.

With the above definitions, the Lagrangian $\mathcal{L}_{SUSY}$ can be expanded in terms of component fields. In appendix B, this calculation is performed in detail, and the result is according to eq. $(B.61)$

$$\begin{aligned}
\mathcal{L}_{SUSY} \;=\; & \left(D^\mu \tilde{L}\right)^\dagger \left(D_\mu \tilde{L}\right) + \left(D^\mu \tilde{R}\right)^\dagger \left(D_\mu \tilde{R}\right) - i\;\bar{L}^{(2)}\bar{\sigma}^\mu D_\mu L^{(2)} - i\;\bar{R}^{(2)}\bar{\sigma}^\mu D_\mu R^{(2)} \\
& + \tilde{L}^\dagger \left(gT^aD^a - \frac{1}{2}g'D'\right)\tilde{L} + \tilde{R}^\dagger g'D'\tilde{R} \\
& + \sqrt{2}i\;\tilde{L}^\dagger\left(gT^a\lambda^a - \frac{1}{2}g'\lambda'\right)L^{(2)} - \sqrt{2}i\;\bar{L}^{(2)}\left(gT^a\bar{\lambda}^a - \frac{1}{2}g'\bar{\lambda}'\right)\tilde{L} \\
& + \sqrt{2}i\;\tilde{R}^\dagger g'\lambda' R^{(2)} - \sqrt{2}i\;\bar{R}^{(2)}g'\bar{\lambda}'\tilde{R} \\
& + F_L^\dagger F_L + F_R^\dagger F_R \\
& - i\;\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - i\;\bar{\lambda}'\bar{\sigma}^\mu D_\mu\lambda' \\
& - \frac{1}{4}\;\left(\;V^{a\;\mu\nu}V^a_{\mu\nu} + V'^{\mu\nu}V'_{\mu\nu}\;\right) + \frac{1}{2}\;\left(\;D^aD^a + D'D'\;\right) \\
& + \left(D^\mu H_1\right)^\dagger\left(D_\mu H_1\right) + \left(D^\mu H_2\right)^\dagger\left(D_\mu H_2\right) \\
& - i\;\bar{\tilde{H}}_1^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_1^{(2)} - i\;\bar{\tilde{H}}_2^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_2^{(2)} \\
& + H_1^\dagger\left(gT^aD^a - \frac{1}{2}g'D'\right)H_1 + H_2^\dagger\left(gT^aD^a + \frac{1}{2}g'D'\right)H_2 \\
& + \sqrt{2}i\;H_1^\dagger\left(gT^a\lambda^a - \frac{1}{2}g'\lambda'\right)\tilde{H}_1^{(2)} - \sqrt{2}i\;\bar{\tilde{H}}_1^{(2)}\left(gT^a\bar{\lambda}^a - \frac{1}{2}g'\bar{\lambda}'\right)H_1 \\
& + \sqrt{2}i\;H_2^\dagger\left(gT^a\lambda^a + \frac{1}{2}g'\lambda'\right)\tilde{H}_2^{(2)} - \sqrt{2}i\;\bar{\tilde{H}}_2^{(2)}\left(gT^a\bar{\lambda}^a + \frac{1}{2}g'\bar{\lambda}'\right)H_2
\end{aligned}$$

[3]When those subscripts occur on bosonic-fields, say on $\tilde{L}_L$, it only denotes a particular field and has nothing to do with left-and right-handed fields (which are not defined for bosonic-fields).

$$
\begin{aligned}
&+ F_1^\dagger F_1 + F_2^\dagger F_2 \\
&+ \mu\varepsilon^{ij} \Big[H_1^i F_2^j + H_1^{i\,\dagger} F_2^{j\,\dagger} + F_1^i H_2^j + F_1^{i\,\dagger} H_2^{j\,\dagger} - \tilde{H}_1^{(2)\,i} \tilde{H}_2^{(2)\,j} - \bar{\tilde{H}}_1{}^{(2)\,i} \bar{\tilde{H}}_2{}^{(2)\,j} \Big] \\
&+ f\varepsilon^{ij} \Big[F_1^i \tilde{L}^j \tilde{R} + F_1^{i\,\dagger} \tilde{L}^{j\,\dagger} \tilde{R}^\dagger + H_1^i F_L^j \tilde{R} + H_1^{i\,\dagger} F_L^{j\,\dagger} \tilde{R}^\dagger \\
&\qquad + H_1^i \tilde{L}^j F_R + H_1^{i\,\dagger} \tilde{L}^{j\,\dagger} F_R^\dagger - \tilde{H}_1^{(2)\,i} L^{(2)\,j} \tilde{R} - \bar{\tilde{H}}_1{}^{(2)\,i} \bar{L}^{(2)\,j} \tilde{R}^\dagger \\
&\qquad - H_1^i L^{(2)\,j} R^{(2)} - H_1^{i\,\dagger} \bar{L}^{(2)\,j} \bar{R}^{(2)} - R^{(2)} \tilde{H}_1^{(2)\,i} \tilde{L}^j - \bar{R}^{(2)} \bar{\tilde{H}}_1{}^{(2)\,i} \tilde{L}^{j\,\dagger} \Big] \\
&+ t.d. \qquad\qquad (2.11)
\end{aligned}
$$

Here t.d. means a total derivative and D_μ is the standard $SU(2) \times U(1)$-covariant derivative defined by

$$D_\mu = \partial_\mu + igT^a V_\mu^a + ig' \frac{Y}{2} V'_\mu, \qquad a = 1, 2, 3. \qquad (2.12)$$

Note that when D_μ operates on e.g. the gauginos λ^a and λ', which lay in the adjoint representation of SU(2) and U(1) respectively, i.e.

$$
\begin{aligned}
\left[T^c_{adj} \right]^{ab} &= -if^{cab}, \\
Y_{adj} &= 0,
\end{aligned}
$$

we have (cf. eq. $(B.23)$)

$$D_\mu \lambda^a = \partial_\mu \lambda^a - g f^{abc} V_\mu^b \lambda^c, \qquad (2.13)$$

$$D_\mu \lambda' = \partial_\mu \lambda'. \qquad (2.14)$$

The various fields of the Lagrangian (2.11) are summarized in tables 2.1 and 2.2. Note that this Lagrangian contains auxiliary fields, i.e. F- and D-fields, and thus is off-shell.

2.2 Elimination of the Auxiliary Fields.

The aim of this section will be to construct the on-shell Lagrangian, i.e. to eliminate the different auxiliary fields given in table 2.2. When we do so, we will see that mass terms for Higgs-bosons and different interaction terms between Higgses, Leptons and Sleptons, without any Lepton-Slepton interaction, will appear.

If we pick all the terms from the off-shell Lagrangian (2.11) containing Lepton-, Higgs- and Gauge-auxiliary fields (F- and D-fields) we get

$$\mathcal{L}_{Aux} = \mathcal{L}_{Aux-F} + \mathcal{L}_{Aux-D}, \qquad (2.15)$$

with

$$\begin{aligned}\mathcal{L}_{Aux-F} &= F_L^\dagger F_L + F_R^\dagger F_R + F_1^\dagger F_1 + F_2^\dagger F_2 \\ &+ \mu\, \varepsilon^{ij} \left[H_1^i F_2^j + H_1^{i\,\dagger} F_2^{j\,\dagger} + F_1^i H_2^j + F_1^{i\,\dagger} H_2^{j\,\dagger} \right] \\ &+ f\, \varepsilon^{ij} \left[F_1^i \tilde{L}^j \tilde{R} + F_1^{i\,\dagger} \tilde{L}^{j\,\dagger} \tilde{R}^\dagger + H_1^i F_L^j \tilde{R} + H_1^{i\,\dagger} F_L^{j\,\dagger} \tilde{R}^\dagger \right. \\ &\qquad \left. + H_1^i \tilde{L}^j F_R + H_1^{i\,\dagger} \tilde{L}^{j\,\dagger} F_R^\dagger \right], \end{aligned} \tag{2.16}$$

and

$$\begin{aligned}\mathcal{L}_{Aux-D} &= \frac{1}{2}\,(\, D^a D^a + D' D' \,) \\ &+ \tilde{L}^\dagger \left(g T^a D^a - \frac{1}{2} g' D' \right) \tilde{L} + \tilde{R}^\dagger g' D' \tilde{R} \\ &+ H_1^\dagger \left(g T^a D^a - \frac{1}{2} g' D' \right) H_1 + H_2^\dagger \left(g T^a D^a + \frac{1}{2} g' D' \right) H_2. \end{aligned} \tag{2.17}$$

We will now show that these fields can be eliminated through the Euler-Lagrange equations [41]

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0,$$

where ϕ is *any* (also hermitian conjugated) Minkowski field. Formally auxiliary fields are defined as fields having no kinetic terms. Thus, this definition immediately yields that the Euler-Lagrange equations for auxiliary fields simplify to $\frac{\partial \mathcal{L}}{\partial \phi} = 0$.

Applying these simplified equations to various auxiliary F-fields yields the following relations

$$F_L^{j\,\dagger} = -f\, \varepsilon^{ij} H_1^i \tilde{R}, \tag{2.18}$$

$$F_R^\dagger = -f\, \varepsilon^{ij} H_1^i \tilde{L}^j, \tag{2.19}$$

$$F_1^{i\,\dagger} = -\mu\, \varepsilon^{ij} H_2^j - f\, \varepsilon^{ij} \tilde{L}^j \tilde{R}, \tag{2.20}$$

$$F_2^{j\,\dagger} = -\mu\, \varepsilon^{ij} H_1^i. \tag{2.21}$$

Expressions for, say F_L^j and so on, are given by hermitian conjugation of the above relations. Substituting these expressions for the F-fields into eq. (2.16) yields according to eq. ($D.7$)

$$\begin{aligned}\mathcal{L}_{Aux-F} &= -\mu^2\, H_1^\dagger H_1 - \mu^2\, H_2^\dagger H_2 - \mu f \left[H_2^\dagger \tilde{L}\, \tilde{R} + \tilde{L}^\dagger H_2\, \tilde{R}^\dagger \right] \\ &- f^2 \left[\tilde{L}^\dagger \tilde{L}\, \tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 \left(\tilde{L}^\dagger \tilde{L} + \tilde{R}^\dagger \tilde{R} \right) - H_1^\dagger \tilde{L} \left(H_1^\dagger \tilde{L} \right)^\dagger \right]. \end{aligned} \tag{2.22}$$

Note that mass terms for the Higgs bosons and Higgs-Lepton and Lepton-Lepton interactions have now been generated as we clamed at the beginning of this section.

The same program for the D-fields gives

$$D^a = -g \left[\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2 \right], \tag{2.23}$$

$$D' = \frac{g'}{2}\, \tilde{L}^\dagger \tilde{L} - g'\, \tilde{R}^\dagger \tilde{R} + \frac{g'}{2}\, H_1^\dagger H_1 - \frac{g'}{2}\, H_2^\dagger H_2, \tag{2.24}$$

and according to eq. $(D.9)$ this means for $\mathcal{L}_{Aux-D}$

$$\begin{aligned}\mathcal{L}_{Aux-D} &= -\frac{g^2}{2}\left(\tilde{L}^\dagger T^a\tilde{L}+H_1^\dagger T^a H_1+H_2^\dagger T^a H_2\right)\left(\tilde{L}^\dagger T^a\tilde{L}+H_1^\dagger T^a H_1+H_2^\dagger T^a H_2\right)\\ &\quad -\frac{g'^2}{8}\left(\tilde{L}^\dagger\tilde{L}-2\tilde{R}^\dagger\tilde{R}+H_1^\dagger H_1-H_2^\dagger H_2\right)^2. \end{aligned}\tag{2.25}$$

Now Higgs-Higgs, Higgs-Slepton and Slepton-Slepton interactions have come into play.

By substituting the expression for $\mathcal{L}_{Aux}$ back into $\mathcal{L}_{SUSY}$, the on-shell Lagrangian is obtained. According to eq. $(D.11)$ the result is

$$\begin{aligned}\mathcal{L}_{SUSY} &= \left(D^\mu\tilde{L}\right)^\dagger\left(D_\mu\tilde{L}\right)+\left(D^\mu\tilde{R}\right)^\dagger\left(D_\mu\tilde{R}\right)-i\,\bar{L}^{(2)}\bar{\sigma}^\mu D_\mu L^{(2)}-i\,\bar{R}^{(2)}\bar{\sigma}^\mu D_\mu R^{(2)}\\
&\quad+\sqrt{2}i\,\tilde{L}^\dagger\left(gT^a\lambda^a-\frac{1}{2}g'\lambda'\right)L^{(2)}-\sqrt{2}i\,\bar{L}^{(2)}\left(gT^a\bar{\lambda}^a-\frac{1}{2}g'\bar{\lambda}'\right)\tilde{L}\\
&\quad+\sqrt{2}i\,\tilde{R}^\dagger g'\lambda' R^{(2)}-\sqrt{2}i\,\bar{R}^{(2)}g'\bar{\lambda}'\tilde{R}\\
&\quad-i\,\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a-i\,\bar{\lambda}'\bar{\sigma}^\mu D_\mu\lambda'-\frac{1}{4}\left(V^{a\,\mu\nu}V^a_{\mu\nu}+V'^{\mu\nu}V'_{\mu\nu}\right)\\
&\quad+\left(D^\mu H_1\right)^\dagger\left(D_\mu H_1\right)+\left(D^\mu H_2\right)^\dagger\left(D_\mu H_2\right)\\
&\quad-i\,\bar{\tilde{H}}_1^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_1^{(2)}-i\,\bar{\tilde{H}}_2^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_2^{(2)}\\
&\quad+\sqrt{2}i\,H_1^\dagger\left(gT^a\lambda^a-\frac{1}{2}g'\lambda'\right)\tilde{H}_1^{(2)}-\sqrt{2}i\,\bar{\tilde{H}}_1^{(2)}\left(gT^a\bar{\lambda}^a-\frac{1}{2}g'\bar{\lambda}'\right)H_1\\
&\quad+\sqrt{2}i\,H_2^\dagger\left(gT^a\lambda^a+\frac{1}{2}g'\lambda'\right)\tilde{H}_2^{(2)}-\sqrt{2}i\,\bar{\tilde{H}}_2^{(2)}\left(gT^a\bar{\lambda}^a+\frac{1}{2}g'\bar{\lambda}'\right)H_2\\
&\quad-\varepsilon^{ij}\left[\mu\left(\tilde{H}_1^{(2)\,i}\tilde{H}_2^{(2)\,j}+\bar{\tilde{H}}_1^{(2)\,i}\bar{\tilde{H}}_2^{(2)\,j}\right)+f\left(\tilde{H}_1^{(2)\,i}L^{(2)\,j}\tilde{R}+\bar{\tilde{H}}_1^{(2)\,i}\bar{L}^{(2)\,j}\tilde{R}^\dagger\right)\right.\\
&\qquad\left.+f\left(H_1^iL^{(2)\,j}R^{(2)}+H_1^{i\,\dagger}\bar{L}^{(2)\,j}\bar{R}^{(2)}+R^{(2)}\tilde{H}_1^{(2)\,i}\tilde{L}^j+\bar{R}^{(2)}\bar{\tilde{H}}_1^{(2)\,i}\tilde{L}^{j\,\dagger}\right)\right]\\
&\quad-\mu^2\,H_1^\dagger H_1-\mu^2\,H_2^\dagger H_2-\mu f\left[H_2^\dagger\tilde{L}\,\tilde{R}+\tilde{L}^\dagger H_2\,\tilde{R}^\dagger\right]\\
&\quad-f^2\left[\tilde{L}^\dagger\tilde{L}\,\tilde{R}^\dagger\tilde{R}+H_1^\dagger H_1\left(\tilde{L}^\dagger\tilde{L}+\tilde{R}^\dagger\tilde{R}\right)-H_1^\dagger\tilde{L}\left(H_1^\dagger\tilde{L}\right)^\dagger\right]\\
&\quad-\frac{g^2}{2}\left(\tilde{L}^\dagger T^a\tilde{L}+H_1^\dagger T^a H_1+H_2^\dagger T^a H_2\right)\left(\tilde{L}^\dagger T^a\tilde{L}+H_1^\dagger T^a H_1+H_2^\dagger T^a H_2\right)\\
&\quad-\frac{g'^2}{8}\left(\tilde{L}^\dagger\tilde{L}-2\tilde{R}^\dagger\tilde{R}+H_1^\dagger H_1-H_2^\dagger H_2\right)^2+t.d.\end{aligned}\tag{2.26}$$

This concludes this section.

2.3 Introducing the Photon-, W- and Z-Gauge Boson Fields.

In order for our model to be realistic, the sector of the theory containing the SM-particles has to coincide with non-supersymmetric QFD. In particular this means that the photon and heavy W- and Z-bosons have to be present. However, "generation" of heavy gauge bosons requires some sort of gauge symmetry breaking as in the SM, and this will be discussed in detail in the next chapter. Nevertheless, it is practical at this stage to introduce the W- and Z-gauge fields even if they before gauge symmetry breaking are massless.

In analogy with the Standard Model we define

$$A_\mu(x) = \cos\theta_{\rm W}\, V'_\mu(x) + \sin\theta_{\rm W}\, V^3_\mu(x), \tag{2.27}$$

$$Z_\mu(x) = -\sin\theta_{\rm W}\, V'_\mu(x) + \cos\theta_{\rm W}\, V^3_\mu(x), \tag{2.28}$$

$$W^\pm_\mu(x) = \frac{V^1_\mu(x) \mp i V^2_\mu(x)}{\sqrt{2}}, \tag{2.29}$$

and for the corresponding spin-1/2 gauginos

$$\lambda_A(x) = \cos\theta_{\rm W}\, \lambda'(x) + \sin\theta_{\rm W}\, \lambda^3(x), \tag{2.30}$$

$$\lambda_Z(x) = -\sin\theta_{\rm W}\, \lambda'(x) + \cos\theta_{\rm W}\, \lambda^3(x), \tag{2.31}$$

$$\lambda^\pm(x) = \frac{\lambda^1(x) \mp i\lambda^2(x)}{\sqrt{2}}. \tag{2.32}$$

With these definitions the $SU(2)\times U(1)$-covariant derivative becomes (cf. eq. (C.15))

$$\begin{aligned} D_\mu &= \partial_\mu + igT^a V^a_\mu + ig'\frac{Y}{2}V'_\mu \\ &= \partial_\mu + \frac{ig}{\sqrt{2}}T^+ W^+_\mu + \frac{ig}{\sqrt{2}}T^- W^-_\mu + ieQA_\mu + \frac{ig}{\cos\theta_{\rm W}}\left[T^3 - Q\sin^2\theta_{\rm W}\right] Z_\mu, \end{aligned} \tag{2.33}$$

where the charge operator Q (with eigenvalues in units of the elementary charge "e") is

$$Q = T^3 + \frac{Y}{2}, \tag{2.34}$$

and

$$T^\pm = T^1 \pm iT^2. \tag{2.35}$$

It is important to note that **Q** and the T's are assumed to operate on the same field as D_μ. For instance, if D_μ operates on an SU(2)-doublet, $T^a = \sigma^a/2$, and D_μ is a 2×2-matrix, while for an SU(2)-singlet $T^a = 0$, and D_μ is no matrix at all.

In terms of the new fields (2.27)–(2.32), the Lagrangian $\mathcal{L}_{SUSY}$, in two-component notation, can be obtained from appendix C by substituting for the various terms of eq. (2.26) rewritten in this appendix.

Nevertheless, the result reads

$$\mathcal{L}_{SUSY} = \left(D^\mu\tilde{L}\right)^\dagger\left(D_\mu\tilde{L}\right) + \left(D^\mu\tilde{R}\right)^\dagger\left(D_\mu\tilde{R}\right) - i\,\bar{L}^{(2)}\bar{\sigma}^\mu D_\mu L^{(2)} - i\,\bar{R}^{(2)}\bar{\sigma}^\mu D_\mu R^{(2)}$$

$$
\begin{aligned}
&+ ig\left(\tilde{L}^{\dagger}T^{+}L^{(2)}\lambda^{+} - \bar{\lambda}^{+}\bar{L}^{(2)}T^{-}\tilde{L}\right) + ig\left(\tilde{L}^{\dagger}T^{-}L^{(2)}\lambda^{-} - \bar{\lambda}^{-}\bar{L}^{(2)}T^{+}\tilde{L}\right) \\
&+ \sqrt{2}ie\,Q_i\left(\tilde{L}^{\dagger\,i}L^{(2)\,i}\lambda_A - \bar{\lambda}_A\bar{L}^{(2)\,i}\tilde{L}^i\right) \\
&+ \frac{\sqrt{2}ig}{\cos\theta_{\rm W}}\left(\mathcal{T}_i^3 - Q_i\sin^2\theta_{\rm W}\right)\left[\tilde{L}^{\dagger\,i}L^{(2)\,i}\lambda_Z - \bar{\lambda}_Z\bar{L}^{(2)\,i}\tilde{L}^i\right] \\
&+ \sqrt{2}ie\left(\tilde{R}^{\dagger}\;R^{(2)}\lambda_A - \bar{\lambda}_A\bar{R}^{(2)}\;\tilde{R}\right) - \sqrt{2}ig\frac{\sin^2\theta_{\rm W}}{\cos\theta_{\rm W}}\left(\tilde{R}^{\dagger}\;R^{(2)}\lambda_Z - \bar{\lambda}_Z\bar{R}^{(2)}\;\tilde{R}\right) \\
&- i\,\bar{\lambda}^{+}\bar{\sigma}^{\mu}\partial_{\mu}\lambda^{+} - i\,\bar{\lambda}^{-}\bar{\sigma}^{\mu}\partial_{\mu}\lambda^{-} - i\,\bar{\lambda}_A\bar{\sigma}^{\mu}\partial_{\mu}\lambda_A - i\,\bar{\lambda}_Z\bar{\sigma}^{\mu}\partial_{\mu}\lambda_Z \\
&+ g\cos\theta_{\rm W}\left[\left(\bar{\lambda}_Z\bar{\sigma}^{\mu}\lambda^{-} - \bar{\lambda}^{+}\bar{\sigma}^{\mu}\lambda_Z\right)W_{\mu}^{+} - \left(\bar{\lambda}_Z\bar{\sigma}^{\mu}\lambda^{+} - \bar{\lambda}^{-}\bar{\sigma}^{\mu}\lambda_Z\right)W_{\mu}^{-}\right. \\
&\qquad\qquad \left. + \left(\bar{\lambda}^{+}\bar{\sigma}^{\mu}\lambda^{+} - \bar{\lambda}^{-}\bar{\sigma}^{\mu}\lambda^{-}\right)Z_{\mu}\right] \\
&+ e\left[\left(\bar{\lambda}_A\bar{\sigma}^{\mu}\lambda^{-} - \bar{\lambda}^{+}\bar{\sigma}^{\mu}\lambda_A\right)W_{\mu}^{+} - \left(\bar{\lambda}_A\bar{\sigma}^{\mu}\lambda^{+} - \bar{\lambda}^{-}\bar{\sigma}^{\mu}\lambda_A\right)W_{\mu}^{-}\right. \\
&\qquad \left. + \left(\bar{\lambda}^{+}\bar{\sigma}^{\mu}\lambda^{+} - \bar{\lambda}^{-}\bar{\sigma}^{\mu}\lambda^{-}\right)A_{\mu}\right] \\
&- \frac{1}{4}\;\left(\mathcal{W}^{+\,\mu\nu}\mathcal{W}_{\mu\nu}^{-} + \mathcal{W}^{-\,\mu\nu}\mathcal{W}_{\mu\nu}^{+} + \mathcal{A}^{\mu\nu}\mathcal{A}_{\mu\nu} + \mathcal{Z}^{\mu\nu}\mathcal{Z}_{\mu\nu}\right) \\
&+ \left(D^{\mu}H_1\right)^{\dagger}\left(D_{\mu}H_1\right) + \left(D^{\mu}H_2\right)^{\dagger}\left(D_{\mu}H_2\right) \\
&- i\;\bar{\tilde{H}}_1^{(2)}\bar{\sigma}^{\mu}D_{\mu}\tilde{H}_1^{(2)} - i\;\bar{\tilde{H}}_2^{(2)}\bar{\sigma}^{\mu}D_{\mu}\tilde{H}_2^{(2)} \\
&+ ig\left(H_1^{\dagger}T^{+}\tilde{H}_1^{(2)}\lambda^{+} - \bar{\lambda}^{+}\bar{\tilde{H}}_1^{(2)}T^{-}H_1\right) + ig\left(H_1^{\dagger}T^{-}\tilde{H}_1^{(2)}\lambda^{-} - \bar{\lambda}^{-}\bar{\tilde{H}}_1^{(2)}T^{+}H_1\right) \\
&+ \sqrt{2}ie\,Q_i\left(H_1^{\dagger\,i}\tilde{H}_1^{(2)\,i}\lambda_A - \bar{\lambda}_A\bar{\tilde{H}}_1^{(2)\,i}H_1^i\right) \\
&+ \frac{\sqrt{2}ig}{\cos\theta_{\rm W}}\left(\mathcal{T}_i^3 - Q_i\sin^2\theta_{\rm W}\right)\left[H_1^{\dagger\,i}\tilde{H}_1^{(2)\,i}\lambda_Z - \bar{\lambda}_Z\bar{\tilde{H}}_1^{(2)\,i}H_1^i\right] \\
&+ ig\left(H_2^{\dagger}T^{+}\tilde{H}_2^{(2)}\lambda^{+} - \bar{\lambda}^{+}\bar{\tilde{H}}_2^{(2)}T^{-}H_2\right) + ig\left(H_2^{\dagger}T^{-}\tilde{H}_2^{(2)}\lambda^{-} - \bar{\lambda}^{-}\bar{\tilde{H}}_2^{(2)}T^{+}H_2\right) \\
&+ \sqrt{2}ie\,Q_i\left(H_2^{\dagger\,i}\tilde{H}_2^{(2)\,i}\lambda_A - \bar{\lambda}_A\bar{\tilde{H}}_2^{(2)\,i}H_2^i\right) \\
&+ \frac{\sqrt{2}ig}{\cos\theta_{\rm W}}\left(\mathcal{T}_i^3 - Q_i\sin^2\theta_{\rm W}\right)\left[H_2^{\dagger\,i}\tilde{H}_2^{(2)\,i}\lambda_Z - \bar{\lambda}_Z\bar{\tilde{H}}_2^{(2)\,i}H_2^i\right] \\
&- \varepsilon^{ij}\;\left[\;\mu\left(\;\tilde{H}_1^{(2)\,i}\tilde{H}_2^{(2)\,j} + \bar{\tilde{H}}_1^{(2)\,i}\bar{\tilde{H}}_2^{(2)\,j}\;\right) + f\left(\;\tilde{H}_1^{(2)\,i}L^{(2)\,j}\tilde{R} + \bar{\tilde{H}}_1^{\;(2)\,i}\bar{L}^{(2)\,j}\tilde{R}^{\dagger}\;\right)\right. \\
&\qquad\qquad \left. +f\left(H_1^iL^{(2)\,j}R^{(2)} + H_1^{i\,\dagger}\bar{L}^{(2)\,j}\bar{R}^{(2)} + R^{(2)}\tilde{H}_1^{(2)\,i}\tilde{L}^j + \bar{R}^{(2)}\bar{\tilde{H}}_1^{\;(2)\,i}\tilde{L}^{j\,\dagger}\right)\right] \\
&- \mu^2\,H_1^{\dagger}H_1 - \mu^2\,H_2^{\dagger}H_2 - \mu f\left[\,H_2^{\dagger}\tilde{L}\,\tilde{R} + \tilde{L}^{\dagger}H_2\,\tilde{R}^{\dagger}\,\right] \\
&- f^2\left[\,\tilde{L}^{\dagger}\tilde{L}\;\tilde{R}^{\dagger}\tilde{R} + H_1^{\dagger}H_1\left(\tilde{L}^{\dagger}\tilde{L} + \tilde{R}^{\dagger}\tilde{R}\right) - H_1^{\dagger}\tilde{L}\left(H_1^{\dagger}\tilde{L}\right)^{\dagger}\,\right] \\
&- \frac{g^2}{2}\left(\,\tilde{L}^{\dagger}T^a\tilde{L} + H_1^{\dagger}T^aH_1 + H_2^{\dagger}T^aH_2\,\right)\left(\,\tilde{L}^{\dagger}T^a\tilde{L} + H_1^{\dagger}T^aH_1 + H_2^{\dagger}T^aH_2\,\right) \\
&- \frac{g'^2}{8}\left(\,\tilde{L}^{\dagger}\tilde{L} - 2\tilde{R}^{\dagger}\tilde{R} + H_1^{\dagger}H_1 - H_2^{\dagger}H_2\,\right)^2 + t.d. \qquad (2.36)
\end{aligned}
$$

Here, cf. eqs. $(C.34)$, $(C.34)$, $(C.36)$ and $(C.37)$,

$$\begin{aligned}
\mathcal{A}_{\mu\nu} &= \cos\theta_{\rm W}\, V'_{\mu\nu} + \sin\theta_{\rm W}\, V^3_{\mu\nu} \\
&= A_{\mu\nu} + ie\left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right), && (2.37) \\
\mathcal{Z}_{\mu\nu} &= -\sin\theta_{\rm W}\, V'_{\mu\nu} + \cos\theta_{\rm W}\, V^3_{\mu\nu} \\
&= Z_{\mu\nu} + ig\cos\theta_{\rm W}\left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right), && (2.38) \\
\mathcal{W}^+_{\mu\nu} &= \frac{V^1_{\mu\nu} - iV^2_{\mu\nu}}{\sqrt{2}} \\
&= W^+_{\mu\nu} + ie\left(A_\mu W^+_\nu - W^+_\mu A_\nu\right) + ig\cos\theta_{\rm W}\left(Z_\mu W^+_\nu - W^+_\mu Z_\nu\right), && (2.39) \\
\mathcal{W}^-_{\mu\nu} &= \frac{V^1_{\mu\nu} + iV^2_{\mu\nu}}{\sqrt{2}} \\
&= W^-_{\mu\nu} - ie\left(A_\mu W^-_\nu - W^-_\mu A_\nu\right) - ig\cos\theta_{\rm W}\left(Z_\mu W^-_\mu - W^-_\mu Z_\nu\right), && (2.40)
\end{aligned}$$

and $A_{\mu\nu}$, $Z_{\mu\nu}$ and $W^\pm_{\mu\nu}$ are the usual fieldstrengths given by

$$\begin{aligned}
A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, && (2.41) \\
Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu, && (2.42) \\
W^\pm_{\mu\nu} &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu. && (2.43)
\end{aligned}$$

Note that the "scripted kinetic terms" are defined in complete analogy with eqs. (2.27)–(2.29) and that they also contain interaction terms for the gauge bosons.

2.4 Introducing Four-Component Spinors.

In order to make use of the Lagrangian (2.36) in field theoretical calculations, it is practical to express it in terms of four-component spinors. This will be done in this section.

The interactions of the gauge-fermions of eq. (2.36) suggest that we introduce the Majorana spinors

$$\tilde{A}(x) = \begin{pmatrix} -i\lambda_A(x) \\ i\bar{\lambda}_A(x) \end{pmatrix}, \qquad (2.44)$$

$$\tilde{Z}(x) = \begin{pmatrix} -i\lambda_Z(x) \\ i\bar{\lambda}_Z(x) \end{pmatrix}, \qquad (2.45)$$

and the Dirac spinors

$$\tilde{W}(x) = \begin{pmatrix} -i\lambda^+(x) \\ i\bar{\lambda}^-(x) \end{pmatrix}, \qquad (2.46)$$

$$\tilde{W}^c(x) = \begin{pmatrix} -i\lambda^-(x) \\ i\bar{\lambda}^+(x) \end{pmatrix}. \qquad (2.47)$$

Here the Photino $\tilde{A}(x)$ and the Zino $\tilde{Z}(x)$ are neutral fields, while the Wino-field describes charged ($\pm e$) Winos. The state $\tilde{W}^c$ is the charge conjugated of the Wino-state $\tilde{W}$ (cf. eq. (A.91)).

In sect. 2.1 we saw that the Higgs-sector contains two charged and neutral states (cf. table 2.1). Hence we introduce the weak interacting neutral Majorana Higgsino states

$$\tilde{H}_1 = \begin{pmatrix} \psi^1_{H_1} \\ \bar{\psi}^1_{H1} \end{pmatrix}, \tag{2.48}$$

$$\tilde{H}_2 = \begin{pmatrix} \psi^2_{H_2} \\ \bar{\psi}^2_{H2} \end{pmatrix}, \tag{2.49}$$

and the charged Dirac Higgsino states

$$\tilde{H} = \begin{pmatrix} \psi^1_{H_2} \\ \bar{\psi}^2_{H_1} \end{pmatrix}, \tag{2.50}$$

$$\tilde{H}^c = \begin{pmatrix} \psi^2_{H_1} \\ \bar{\psi}^1_{H_2} \end{pmatrix}. \tag{2.51}$$

The (four-component) leptons are as usual Dirac spinors, and they have according to subsect. A.5.2, the form

$$l = \begin{pmatrix} l^{(2)}_L \\ \bar{l}^{(2)}_R \end{pmatrix}. \tag{2.52}$$

By working in the Weyl basis for the γ-matrices (cf. eqs. (A.48) and (A.49)), we demonstrate in great detail in appendix C, eq. (C.51), that the four component version of the two-component Lagrangian (2.26) (or equivalently (2.36)) is

$$\begin{aligned}
\mathcal{L}_{SUSY} = {} & \left(D^\mu \tilde{L}\right)^\dagger \left(D_\mu \tilde{L}\right) + \left(D^\mu \tilde{R}\right)^\dagger \left(D_\mu \tilde{R}\right) - i\, \bar{L}\gamma^\mu D_\mu L - i\, \bar{R}\gamma^\mu D_\mu R \\
& - g\left[\left\{\bar{L}^1 \tilde{W}\, \tilde{L}^2 + \bar{L}^2 \tilde{W}^c\, \tilde{L}^1\right\} + h.c.\right] + \sqrt{2}e\left[\left\{\bar{L}^2 \tilde{A}\, \tilde{L}^2 - \bar{\tilde{A}} R\, \tilde{R}\right\} + h.c.\right] \\
& - \frac{\sqrt{2}g}{\cos\theta_{\rm W}}\left[\left\{\left(\mathcal{T}^3_i - Q_i \sin^2\theta_{\rm W}\right) \bar{L}^i \tilde{Z}\, \tilde{L}^i - \sin^2\theta_{\rm W}\, \bar{\tilde{Z}} R\, \tilde{R}\right\} + h.c.\right] \\
& - i\, \bar{\tilde{W}}\gamma^\mu \partial_\mu \tilde{W} - \frac{i}{2}\, \bar{\tilde{A}}\gamma^\mu \partial_\mu \tilde{A} - \frac{i}{2}\, \bar{\tilde{Z}}\gamma^\mu \partial_\mu \tilde{Z} \\
& - g\cos\theta_{\rm W}\left[\, \bar{\tilde{Z}}\gamma^\mu \tilde{W}\, W^-_\mu + \bar{\tilde{W}}\gamma^\mu \tilde{Z}\, W^+_\mu - \bar{\tilde{W}}\gamma^\mu \tilde{W}\, Z_\mu\right] \\
& - e\left[\, \bar{\tilde{A}}\gamma^\mu \tilde{W}\, W^-_\mu + \bar{\tilde{W}}\gamma^\mu \tilde{A}\, W^+_\mu - \bar{\tilde{W}}\gamma^\mu \tilde{W}\, A_\mu\right] \\
& - \frac{1}{4}\mathcal{W}^{+\,\mu\nu}\mathcal{W}^-_{\mu\nu} - \frac{1}{4}\mathcal{W}^{-\,\mu\nu}\mathcal{W}^+_{\mu\nu} - \frac{1}{4}\mathcal{Z}^{\mu\nu}\mathcal{Z}_{\mu\nu} - \frac{1}{4}\mathcal{A}^{\mu\nu}\mathcal{A}_{\mu\nu} \\
& + \left(D^\mu H_1\right)^\dagger \left(D_\mu H_1\right) - \mu^2 H_1^\dagger H_1 + \left(D^\mu H_2\right)^\dagger \left(D_\mu H_2\right) - \mu^2 H_2^\dagger H_2 \\
& - \bar{\tilde{H}}\left(i\gamma^\mu \partial_\mu - \mu\right)\tilde{H} - \frac{i}{2}\, \bar{\tilde{H}}_1 \gamma^\mu \partial_\mu \tilde{H}_1 - \frac{i}{2}\, \bar{\tilde{H}}_2 \gamma^\mu \partial_\mu \tilde{H}_2 - \frac{\mu}{2}\bar{\tilde{H}}_1 \tilde{H}_2 - \frac{\mu}{2}\bar{\tilde{H}}_2 \tilde{H}_1
\end{aligned}$$

$$
\begin{aligned}
& -\frac{g}{\sqrt{2}}\Big[\,\Big(\bar{\tilde{H}}\gamma^\mu P_R \tilde{H}_1-\bar{\tilde{H}}\gamma^\mu P_L \tilde{H}_2\Big)\,W_\mu^+ + h.c.\Big] + e\;\bar{\tilde{H}}\gamma^\mu \tilde{H}\;A_\mu \\
& +\frac{g}{2\cos\theta_{\rm W}}\Big[\,\Big(1-2\sin^2\theta_{\rm W}\Big)\,\bar{\tilde{H}}\gamma^\mu\tilde{H}-\frac{1}{2}\Big(\bar{\tilde{H}}_1\gamma^\mu\gamma_5\tilde{H}_1-\bar{\tilde{H}}_2\gamma^\mu\gamma_5\tilde{H}_2\Big)\,\Big]\,Z_\mu \\
& -g\,\Big[\Big(\bar{\tilde{W}}P_R\tilde{H}\;H_1^1+\bar{\tilde{H}}P_R\tilde{W}\;H_2^2+\bar{\tilde{H}}_1P_R\tilde{W}\;H_1^2+\bar{\tilde{W}}P_R\tilde{H}_2\;H_2^1\Big)+h.c.\Big] \\
& +\sqrt{2}e\,\Big[\,\Big(\bar{\tilde{A}}P_R\tilde{H}\;H_1^2-\bar{\tilde{H}}P_R\tilde{A}\;H_2^1\Big)+h.c.\Big] \\
& -\frac{g}{\sqrt{2}\cos\theta_{\rm W}}\,\Big[\Big\{\bar{\tilde{Z}}P_R\tilde{H}_1\;H_1^1-\bar{\tilde{H}}_2P_R\tilde{Z}\;H_2^2 \\
& \qquad\qquad -\Big(1-2\sin^2\theta_{\rm W}\Big)\Big(\bar{\tilde{Z}}P_R\tilde{H}\;H_1^2-\bar{\tilde{H}}P_R\tilde{Z}\;H_2^1\Big)\Big\}+h.c.\Big] \\
& +f\Big[\Big\{\bar{\tilde{H}}L^1\,\tilde{R}-\bar{\tilde{H}}_1L^2\,\tilde{R}+\bar{R}L^1\,H_1^2-\bar{R}L^2\,H_1^1+\bar{R}\tilde{H}^c\,\tilde{L}^1-\bar{R}\tilde{H}_1\,\tilde{L}^2\Big\}+h.c.\Big] \\
& -\mu f\Big[H_2^\dagger\tilde{L}\,\tilde{R}+h.c.\Big]-f^2\Big[\tilde{L}^\dagger\tilde{L}\,\tilde{R}^\dagger\tilde{R}+H_1^\dagger H_1\Big(\tilde{L}^\dagger\tilde{L}+\tilde{R}^\dagger\tilde{R}\Big)-H_1^\dagger\tilde{L}\Big(H_1^\dagger\tilde{L}\Big)^\dagger\Big] \\
& -\frac{g^2}{2}\Big(\,\tilde{L}^\dagger T^a\tilde{L}+H_1^\dagger T^aH_1+H_2^\dagger T^aH_2\,\Big)\Big(\,\tilde{L}^\dagger T^a\tilde{L}+H_1^\dagger T^aH_1+H_2^\dagger T^aH_2\,\Big) \\
& -\frac{g^2\tan^2\theta_{\rm W}}{8}\Big(\,\tilde{L}^\dagger\tilde{L}-2\tilde{R}^\dagger\tilde{R}+H_1^\dagger H_1-H_2^\dagger H_2\,\Big)^2+t.d. \qquad (2.53)
\end{aligned}
$$

Here P_L and P_R are the left- and right-handed projection operators given by eqs. (A.80) and (A.81).

This concludes this section, and after the long discussion of the Lagrangian $\mathcal{L}_{SUSY}$ we will finally draw our attention towards the soft-breaking piece $\mathcal{L}_{Soft}$.

2.5 Component Field Expansion of $\mathcal{L}_{Soft}$.

From chapter 1, eq. (1.17), we recall that

$$
\mathcal{L}_{Soft} \;=\; \mathcal{L}_{SMT}+\mathcal{L}_{GMT}, \qquad (2.54)
$$

with

$$
\begin{aligned}
\mathcal{L}_{SMT} \;=\; & -\int d^4\theta\;\Big[M_L^2\;\hat{L}^\dagger\hat{L}+m_R^2\hat{R}^\dagger\hat{R}+m_1^2\hat{H}_1^\dagger\hat{H}_1 \\
& \qquad +m_2^2\hat{H}_2^\dagger\hat{H}_2-m_3^2\varepsilon^{ij}\Big(\hat{H}_1^i\hat{H}_2^j+h.c.\Big)\Big]\,\delta^4(\theta,\bar{\theta}), \qquad (2.55)
\end{aligned}
$$

and

$$
\mathcal{L}_{GMT} \;=\; \frac{1}{2}\int d^4\theta\;\Big[\Big(\,M\;W^{\alpha\,a}W_\alpha^a+M'\;W'^{\,\alpha}W'_\alpha\,\Big)+h.c.\Big]\,\delta^4(\theta,\bar{\theta}). \qquad (2.56)
$$

Now the component expansion of $\mathcal{L}_{Soft}$ will be calculated, and we start with $\mathcal{L}_{SMT}$. With the component expansions of $\hat{L}$, $\hat{R}$, $\hat{H}_1$ and $\hat{H}_2$ from sect. 2.1, we have (cf. [31, 32, 33])

$$
\begin{aligned}
\mathcal{L}_{SMT} \;=\; & -M_L^2\;\tilde{L}^\dagger\tilde{L}-m_R^2\tilde{R}^\dagger\tilde{R}-m_1^2H_1^\dagger H_1-m_2^2H_2^\dagger H_2 \\
& +m_3^2\varepsilon^{ij}\Big(H_1^iH_2^j+h.c.\Big)\,, \qquad (2.57)
\end{aligned}
$$

and correspondingly for $\mathcal{L}_{GMT}$

$$\mathcal{L}_{GMT} = -\frac{1}{2}M\left(\lambda^a\lambda^a+\bar{\lambda}^a\bar{\lambda}^a\right)-\frac{1}{2}M'\left(\lambda'\lambda'+\bar{\lambda}'\bar{\lambda}'\right). \tag{2.58}$$

Here $M_L^2\,\tilde{L}^\dagger\tilde{L}$ is defined in analogy with the corresponding superfield definition, i.e. $M_L^2\,\tilde{L}^\dagger\tilde{L} = m_{\tilde{\nu}}^2\,\tilde{\nu}^\dagger\tilde{\nu}+m_L^2\,\tilde{l}_L^\dagger\tilde{l}_L$.

Since eq. (2.58) contains two-component Weyl-spinors, we will, as in the previous section, introduce four-component notation.

Hence, with eq. (2.46) we have

$$\begin{aligned}-\frac{1}{2}M\left(\lambda^1\lambda^1+\bar{\lambda}^1\bar{\lambda}^1\right)-\frac{1}{2}M\left(\lambda^2\lambda^2+\bar{\lambda}^2\bar{\lambda}^2\right)&\\ = -M\left(\lambda^-\lambda^++\bar{\lambda}^-\bar{\lambda}^+\right)&\\ = M_{\tilde{W}}\,\bar{\tilde{W}}\tilde{W},&\end{aligned} \tag{2.59}$$

where $M_{\tilde{W}} = M$. Similarly, with eqs. (2.44) and (2.45)

$$\begin{aligned}&-\frac{1}{2}M\left(\lambda^3\lambda^3+\bar{\lambda}^3\bar{\lambda}^3\right)-\frac{1}{2}M'\left(\lambda'\lambda'+\bar{\lambda}'\bar{\lambda}'\right)\\ &= -\frac{1}{2}\left(M\sin^2\theta_{\mathrm{W}}+M'\cos^2\theta_{\mathrm{W}}\right)\left(\lambda_A\lambda_A+\bar{\lambda}_A\bar{\lambda}_A\right)\\ &\quad-\frac{1}{2}\left(M\cos^2\theta_{\mathrm{W}}+M'\sin^2\theta_{\mathrm{W}}\right)\left(\lambda_Z\lambda_Z+\bar{\lambda}_Z\bar{\lambda}_Z\right)\\ &\quad-\frac{1}{2}\left(M-M'\right)\sin 2\theta_{\mathrm{W}}\ \left(\lambda_A\lambda_Z+\bar{\lambda}_A\bar{\lambda}_Z\right)\\ &= \frac{1}{2}\left(M\sin^2\theta_{\mathrm{W}}+M'\cos^2\theta_{\mathrm{W}}\right)\bar{\tilde{A}}\tilde{A}+\frac{1}{2}\left(M\cos^2\theta_{\mathrm{W}}+M'\sin^2\theta_{\mathrm{W}}\right)\bar{\tilde{Z}}\tilde{Z}\\ &\quad+\frac{1}{2}\left(M-M'\right)\sin 2\theta_{\mathrm{W}}\ \bar{\tilde{A}}\tilde{Z}\\ &= \frac{1}{2}\,M_{\tilde{A}}\,\bar{\tilde{A}}\tilde{A}+\frac{1}{2}\,M_{\tilde{Z}}\,\bar{\tilde{Z}}\tilde{Z}+\frac{1}{2}\left(M_{\tilde{Z}}-M_{\tilde{A}}\right)\tan 2\theta_{\mathrm{W}}\,\bar{\tilde{A}}\tilde{Z},\end{aligned} \tag{2.60}$$

where we have introduced the notation

$$M_{\tilde{A}} = M'\cos^2\theta_{\mathrm{W}}+M\sin^2\theta_{\mathrm{W}}, \tag{2.61}$$

$$M_{\tilde{Z}} = M'\sin^2\theta_{\mathrm{W}}+M\cos^2\theta_{\mathrm{W}}. \tag{2.62}$$

Thus eq. (2.58) reads

$$\mathcal{L}_{GMT} = M_{\tilde{W}}\,\bar{\tilde{W}}\tilde{W}+\frac{1}{2}\,M_{\tilde{A}}\,\bar{\tilde{A}}\tilde{A}+\frac{1}{2}\,M_{\tilde{Z}}\,\bar{\tilde{Z}}\tilde{Z}+\frac{1}{2}\left(M_{\tilde{Z}}-M_{\tilde{A}}\right)\tan 2\theta_{\mathrm{W}}\,\bar{\tilde{A}}\tilde{Z}, \tag{2.63}$$

and this section is concluded.

2.6 Conclusion — The Full Four-Component Lagrangian $\mathcal{L}_{S-QFD}$.

With the results from eqs. (2.53) and (2.63) we may conclude for $\mathcal{L}_{S-QFD} = \mathcal{L}_{SUSY} + \mathcal{L}_{Soft}$

$$
\begin{aligned}
\mathcal{L}_{S-QFD} \;=\; & \left(D^\mu \tilde{L}\right)^\dagger \left(D_\mu \tilde{L}\right) - M_L^2\, \tilde{L}^\dagger \tilde{L} + \left(D^\mu \tilde{R}\right)^\dagger \left(D_\mu \tilde{R}\right) - m_R^2 \tilde{R}^\dagger \tilde{R} \\
& - i\, \bar{L}\gamma^\mu D_\mu L - i\, \bar{R}\gamma^\mu D_\mu R \\
& - g\left[\left\{\bar{L}^1 \tilde{W}\ \tilde{L}^2 + \bar{L}^2 \tilde{W}^c\ \tilde{L}^1\right\} + h.c.\right] + \sqrt{2}e\left[\left\{\bar{L}^2 \tilde{A}\ \tilde{L}^2 - \bar{\tilde{A}} R\ \tilde{R}\right\} + h.c.\right] \\
& - \frac{\sqrt{2}g}{\cos\theta_{\mathrm{W}}}\left[\left\{\left(\mathcal{T}_i^3 - Q_i \sin^2\theta_{\mathrm{W}}\right)\bar{L}^i \tilde{Z}\ \tilde{L}^i - \sin^2\theta_{\mathrm{W}}\, \bar{\tilde{Z}} R\ \tilde{R}\right\} + h.c.\right] \\
& - \bar{\tilde{W}}\left(i\gamma^\mu\partial_\mu - M_{\tilde{W}}\right)\tilde{W} - \frac{1}{2}\bar{\tilde{A}}\left(i\gamma^\mu\partial_\mu - M_{\tilde{A}}\right)\tilde{A} - \frac{1}{2}\bar{\tilde{Z}}\left(i\gamma^\mu\partial_\mu - M_{\tilde{Z}}\right)\tilde{Z} \\
& + \frac{1}{2}\left(M_{\tilde{Z}} - M_{\tilde{A}}\right)\tan 2\theta_{\mathrm{W}}\, \bar{\tilde{A}}\tilde{Z} \\
& - g\cos\theta_{\mathrm{W}}\left[\ \bar{\tilde{Z}}\gamma^\mu\tilde{W}\ W_\mu^- + \bar{\tilde{W}}\gamma^\mu\tilde{Z}\ W_\mu^+ - \bar{\tilde{W}}\gamma^\mu\tilde{W}\ Z_\mu\right] \\
& - e\left[\ \bar{\tilde{A}}\gamma^\mu\tilde{W}\ W_\mu^- + \bar{\tilde{W}}\gamma^\mu\tilde{A}\ W_\mu^+ - \bar{\tilde{W}}\gamma^\mu\tilde{W}\ A_\mu\right] \\
& - \frac{1}{4}\mathcal{W}^{+\,\mu\nu}\mathcal{W}_{\mu\nu}^- - \frac{1}{4}\mathcal{W}^{-\,\mu\nu}\mathcal{W}_{\mu\nu}^+ - \frac{1}{4}\mathcal{Z}^{\mu\nu}\mathcal{Z}_{\mu\nu} - \frac{1}{4}\mathcal{A}^{\mu\nu}\mathcal{A}_{\mu\nu} \\
& + \left(D^\mu H_1\right)^\dagger\left(D_\mu H_1\right) - \left(m_1^2 + \mu^2\right) H_1^\dagger H_1 \\
& + \left(D^\mu H_2\right)^\dagger\left(D_\mu H_2\right) - \left(m_2^2 + \mu^2\right) H_2^\dagger H_2 + m_3^2\varepsilon^{ij}\left(H_1^i H_2^j + h.c.\right) \\
& - \bar{\tilde{H}}\left(i\gamma^\mu\partial_\mu - \mu\right)\tilde{H} - \frac{i}{2}\,\bar{\tilde{H}}_1\gamma^\mu\partial_\mu\tilde{H}_1 - \frac{i}{2}\,\bar{\tilde{H}}_2\gamma^\mu\partial_\mu\tilde{H}_2 - \frac{\mu}{2}\bar{\tilde{H}}_1\tilde{H}_2 - \frac{\mu}{2}\bar{\tilde{H}}_2\tilde{H}_1 \\
& - \frac{g}{\sqrt{2}}\left[\ \left(\bar{\tilde{H}}\gamma^\mu P_R\tilde{H}_1 - \bar{\tilde{H}}\gamma^\mu P_L\tilde{H}_2\right) W_\mu^+ + h.c.\right] + e\ \bar{\tilde{H}}\gamma^\mu\tilde{H}\ A_\mu \\
& + \frac{g}{2\cos\theta_{\mathrm{W}}}\left[\ \left(1 - 2\sin^2\theta_{\mathrm{W}}\right)\bar{\tilde{H}}\gamma^\mu\tilde{H} - \frac{1}{2}\left(\bar{\tilde{H}}_1\gamma^\mu\gamma_5\tilde{H}_1 - \bar{\tilde{H}}_2\gamma^\mu\gamma_5\tilde{H}_2\right)\ \right] Z_\mu \\
& - g\left[\left(\bar{\tilde{W}} P_R\tilde{H}\ H_1^1 + \bar{\tilde{H}} P_R\tilde{W}\ H_2^2 + \bar{\tilde{H}}_1 P_R\tilde{W}\ H_1^2 + \bar{\tilde{W}} P_R\tilde{H}_2\ H_2^1\right) + h.c.\right] \\
& + \sqrt{2}e\left[\ \left(\bar{\tilde{A}} P_R\tilde{H}\ H_1^2 - \bar{\tilde{H}} P_R\tilde{A}\ H_2^1\right) + h.c.\right] \\
& - \frac{g}{\sqrt{2}\cos\theta_{\mathrm{W}}}\left[\left\{\bar{\tilde{Z}} P_R\tilde{H}_1\ H_1^1 - \bar{\tilde{H}}_2 P_R\tilde{Z}\ H_2^2\right.\right. \\
& \qquad\qquad \left.\left. - \left(1 - 2\sin^2\theta_{\mathrm{W}}\right)\left(\bar{\tilde{Z}} P_R\tilde{H}\ H_1^2 - \bar{\tilde{H}} P_R\tilde{Z}\ H_2^1\right)\right\} + h.c.\right] \\
& + f\left[\left\{\bar{\tilde{H}} L^1\ \tilde{R} - \bar{\tilde{H}}_1 L^2\ \tilde{R} + \bar{R} L^1\ H_1^2 - \bar{R} L^2\ H_1^1 + \bar{R}\tilde{H}^c\ \tilde{L}^1 - \bar{R}\tilde{H}_1\ \tilde{L}^2\right\} + h.c.\right] \\
& - \mu f\left[H_2^\dagger\tilde{L}\ \tilde{R} + h.c.\right] - f^2\left[\tilde{L}^\dagger\tilde{L}\ \tilde{R}^\dagger\tilde{R} + H_1^\dagger H_1\left(\tilde{L}^\dagger\tilde{L} + \tilde{R}^\dagger\tilde{R}\right) - H_1^\dagger\tilde{L}\left(H_1^\dagger\tilde{L}\right)^\dagger\right] \\
& - \frac{g^2}{2}\left(\tilde{L}^\dagger T^a\tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right)\left(\tilde{L}^\dagger T^a\tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right)
\end{aligned}
$$

$$-\frac{g^2\tan^2\theta_{\rm W}}{8}\left(\tilde{L}^\dagger\tilde{L}-2\tilde{R}^\dagger\tilde{R}+H_1^\dagger H_1-H_2^\dagger H_2\right)^2+t.d. \tag{2.64}$$

This Lagrangian is the final result for our S-QFD theory, but before we close this chapter we will make several observations about this Lagrangian.

Firstly, it contains the correct kinetic terms for the bosons (sleptons, photons, Z-bosons, higgs bosons ...) and fermions (leptons, photinos, zinos, winos, ...) of the theory.

Secondly, it holds the well known SM-interaction terms for the SM-particles, and in addition interaction terms between SM- and SUSY-particles and SUSY-particles alone. Note the rich number of different interactions, both cubic and quadratic, that are possible in this theory.

Thirdly, we observe that for the wino- and charged higgsino-fields, their charge conjugated fields also appear in the Lagrangian. Such a situation is unknown from the SM. In part 2 we will see that this has the strange consequence that the theory will contain fermion-number violating vertices and propagators.

After these remarks we close this chapter.

Chapter 3

Symmetry Breaking and Physical Fields.

In this chapter the breaking of electroweak gauge symmetry and the introduction of physical states will be demonstrated.

As alluded to earlier, the breaking of gauge symmetry is in the MSSM directly connected to the breaking of supersymmetry. In fact this breaking — called radiative breaking — is an effect of radiative corrections to the soft mass-parameters as we now will discuss in detail.

3.1 Radiative $SU(2) \times U(1)$ Breaking.

Our model has the attractive virtue of allowing for the possibility of a phenomenologically acceptable radiative breaking of the electroweak gauge symmetry [4–22, 43–46]. This is obtained through a generalization of the original Coleman-Weinberg mechanism [48]. Radiative breaking also has the advantage, when combined with some additional plausible assumptions, of being very powerful since it excludes large regions of parameterspace as we will see. This takes part in increasing the predictiveness of the model. Now we will work out the Coleman-Weinberg scheme [48] for our supersymmetric field theory.

In SUSY-theories, one has two kinds of potentials — superpotentials and scalar potentials. Superpotentials have been discussed earlier in this thesis, so in consequence we now consider the scalar potential, which has its analogy in the SM.

Contributions to the MSSM scalar potential, V_{MSSM}, arise from three sources — the auxiliary F- and D-fields and the soft terms. We write

$$V_{MSSM} = V_D + V_F + V_{Soft}, \tag{3.1}$$

where[1] [2]

$$
\begin{aligned}
V_D &= -\mathcal{L}_{Aux-D} \\
&= \frac{g^2}{2}\left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right)\left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right) \\
&\quad + \frac{g'^2}{8}\left(\tilde{L}^\dagger \tilde{L} - 2\tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 - H_2^\dagger H_2\right)^2, \qquad (3.2) \\
V_F &= -\mathcal{L}_{Aux-F} \\
&= \mu^2\, H_1^\dagger H_1 + \mu^2\, H_2^\dagger H_2 + \mu f\left[\, H_2^\dagger \tilde{L}\, \tilde{R} + \tilde{L}^\dagger H_2\, \tilde{R}^\dagger \right] \\
&\quad + f^2\left[\tilde{L}^\dagger \tilde{L}\, \tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1\left(\tilde{L}^\dagger \tilde{L} + \tilde{R}^\dagger \tilde{R}\right) - H_1^\dagger \tilde{L}\left(H_1^\dagger \tilde{L}\right)^\dagger\right], \qquad (3.3)
\end{aligned}
$$

and

$$
\begin{aligned}
V_{Soft} &= -\mathcal{L}_{SMT} \\
&= M_L^2\, L^\dagger L + m_R^2 R^\dagger R + m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 \\
&\qquad - m_3^2 \varepsilon^{ij}\left(H_1^i H_2^j + h.c.\right). \qquad (3.4)
\end{aligned}
$$

Now we leave this general scalar potential, and instead consider the pure scalar Higgs potential because it is this potential which is of interest in the discussion of gauge symmetry breaking.

3.1.1 The Scalar Higgs Potential.

Thus, for the pure Higgs sector of the theory, the (tree-level) scalar Higgs potential $V \equiv V_{Higgs}$ reads[3] according to eqs. (3.1)–(3.4)

$$
\begin{aligned}
V &= \left(m_1^2 + \mu^2\right) H_1^\dagger H_1 + \left(m_2^2 + \mu^2\right) H_2^\dagger H_2 - m_3^2\, \varepsilon^{ij}\left(H_1^i H_2^j + h.c.\right) \\
&\quad + \frac{g^2}{2}\left(H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right)\left(H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right) \\
&\quad + \frac{g'^2}{8}\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2. \qquad (3.5)
\end{aligned}
$$

However, in appendix E this potential is rewritten for later convenience, and the result is (cf. eq. $(E.2)$)

$$
\begin{aligned}
V &= m_1^2\, H_1^\dagger H_1 + m_2^2\, H_2^\dagger H_2 - m_3^2\, \varepsilon^{ij}\left(H_1^i H_2^j + h.c.\right) \\
&\quad + \frac{1}{8}\left(g^2 + g'^2\right)\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \frac{g^2}{2}\left|H_1^\dagger H_2\right|^2. \qquad (3.6)
\end{aligned}
$$

[1] Generally can $D' \to D' + \xi$, where ξ is a Fayet-Iliopoulos term [42], but we will henceforth assume that this term is neglectable.

[2] Note that it is $-V_{MSSM}$ which appears in the Lagrangian.

[3] This potential is a special case of the general two-Higgs doublet potential [49, 50].

Here we have taken advantage of the arbitrary nature of the soft mass-parameters m_1^2 and m_2^2, and absorbed μ^2 into these, i.e.

$$\begin{aligned} m_1^2 + \mu^2 &\longrightarrow m_1^2, \\ m_2^2 + \mu^2 &\longrightarrow m_2^2. \end{aligned}$$

Without loss of generality, we may choose the phases of the (scalar) Higgs fields in such a way that all mass parameters m_i^2 $(i = 1, 2, 3)$ are real and that the vacuum expectation values (v.e.v.'s) of the Higgs fields are non-negative. As in the Standard Model (SM), the $SU(2) \times U(1)$ gauge symmetry has to be broken down to $U(1)_{EM}$. This means that electromagnetism is unbroken and hence the charged components of the Higgs-doublets can not develop non-vanishing v.e.v.'s. Hence

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \tag{3.7}$$

$$\langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \tag{3.8}$$

and the potential becomes at the vacuum

$$V = m_1^2 v_1^2 + m_2^2 v_2^2 - 2m_3^2 v_1 v_2 + \frac{1}{8}\left(g^2 + g'^2\right)\left[v_1^2 - v_2^2\right]^2. \tag{3.9}$$

For this potential to be bound from below, e.g. in the direction $v_1 = v_2$, one has to be careful and demand

$$\mathcal{B} \equiv m_1^2 + m_2^2 - 2m_3^2 \geq 0. \tag{3.10}$$

This relation will hereafter be referred to as the stability condition.

From the SM Higgs-mechanism, it is a well-known fact that when the Higgs v.e.v. is non-vanishing this signals breaking of the $SU(2) \times U(1)$-symmetry because origo is "unstable". This situation applies equivalently well to the two Higgs doublet model [49, 50]. However, what demands do we have to make in order to obtain non-vanishing v.e.v.'s? As long as V_{min} is non-negative, the minimum ($V_{min} = 0$) lies at the origo, i.e. at $v_1 = v_2 = 0$, and the gauge symmetry is unbroken. Thus V_{min} has to be negative to obtain breaking of gauge symmetry.

Now we will derive a condition on the mass parameters for this to happen. Rewriting eq. (3.9) yields

$$V = \mathrm{v}^T \mathcal{M}^2 \mathrm{v} + \frac{1}{8}\left(g^2 + g'^2\right)\left[v_1^2 - v_2^2\right]^2, \tag{3.11}$$

where

$$\begin{aligned} \mathrm{v} &= \begin{pmatrix} v_1 \\ -v_2 \end{pmatrix}, \\ \mathcal{M}^2 &= \begin{pmatrix} m_1^2 & m_3^2 \\ m_3^2 & m_2^2 \end{pmatrix}. \end{aligned}$$

Since $\mathcal{M}^2$ is a symmetric matrix, the following is true for the quadratic form $\mathrm{v}^T \mathcal{M}^2 \mathrm{v}$ [51]

$$\lambda_- \left|\mathrm{v}\right|^2 \;\; \leq \;\; \mathrm{v}^T \mathcal{M}^2 \mathrm{v} \;\; \leq \;\; \lambda_+ \left|\mathrm{v}\right|^2 . \tag{3.12}$$

Here $\lambda_\pm$ are the eigenvalues of $\mathcal{M}^2$ given by

$$\lambda_\pm \;\; = \;\; \frac{1}{2}\left(m_1^2 + m_2^2 \pm \sqrt{\left(m_1^2 + m_2^2\right)^2 - 4\left(m_1^2 m_2^2 - m_3^4\right)} \right), \tag{3.13}$$

and the norm $|\mathrm{v}|$ is taken relative to the inner product space $\mathbb{R}^2$.

Since the last term of eq. (3.11) is non-negative, the quadratic form $\mathrm{v}^T \mathcal{M}^2 \mathrm{v}$ has to be at its minimum value in order to get a minimum of V, i.e.

$$\mathrm{v}^T \mathcal{M}^2 \mathrm{v} \;\; = \;\; \lambda_- \left|\mathrm{v}\right|^2 .$$

Hence in order for $V_{min} < 0$ we must have $\lambda_- < 0$, something which implies

$$\det \mathcal{M}^2 = m_1^2 m_2^2 - m_3^4 < 0. \tag{3.14}$$

Thus if eq. (3.14), in addition to the stability condition (3.10), are satisfied, this signals $SU(2) \times U(1)$-gauge symmetry breaking. Later on this will be demonstrated explicitly.

$m_1^2 + m_2^2 - 2\left|m_3^2\right|$ $\qquad$ $m_1^2 m_2^2 - m_3^4$

12(130.00,49.00)(130.00,47.00)(130.00,46.00) 12(130.00,44.00)(130.00,42.00)(130.00,41.00) 12(130.00,39.00)(

Figure 3.1: The figure shows $\mathcal{B}$ and det $\mathcal{M}$ as functions of scale Q. The various sectors where $SU(2) \times U(1)$ is broken in a satisfactory/unsatisfactory manner are also indicated.

When condition (3.14) is satisfied, the neutral components of H_1 and H_2 start to develop non-vanishing v.e.v.'s ($v_1, v_2 \neq 0$). Now we will derive some useful relations, and an expression for the potential at its minimum. At $V_{\rm min}$, the potential has to fulfil the equations $\frac{\partial V_{min}}{\partial v_1} = \frac{\partial V_{min}}{\partial v_2} = 0$ and $\frac{\partial^2 V_{min}}{\partial v_1 \partial v_2} > 0$. This yields the following relations

$$m_1^2 v_1 - m_3^2 v_2 + \frac{1}{4}\left(g^2 + g'^2\right)\left[v_1^2 - v_2^2\right] v_1 \;\; = \;\; 0, \tag{3.15}$$

$$m_2^2 v_2 - m_3^2 v_1 - \frac{1}{4}\left(g^2 + g'^2\right)\left[v_1^2 - v_2^2\right] v_2 \;\; = \;\; 0, \tag{3.16}$$

$$-2 m_3^2 - \left(g^2 + g'^2\right) v_1 v_2 \;\; > \;\; 0. \tag{3.17}$$

By multiplying eqs. (3.15) and (3.16) by v_1^{-1} and v_2^{-1} respectively, and then adding and subtracting the resulting equations, we obtain

$$
\begin{aligned}
m_1^2 + m_2^2 &= m_3^2 \left(\tan\beta + \cot\beta\right), & (3.18)\\
v_1^2 - v_2^2 &= \frac{-2}{g^2 + g'^2}\left[m_1^2 - m_2^2 - \left(m_1^2 + m_2^2\right)\frac{\tan\beta - \cot\beta}{\tan\beta + \cot\beta}\right] \\
&= \frac{-2}{g^2 + g'^2}\left[m_1^2 - m_2^2 + \left(m_1^2 + m_2^2\right)\cos 2\beta\right], & (3.19)
\end{aligned}
$$

where

$$
\tan\beta = \frac{v_2}{v_1}. \qquad (3.20)
$$

Here the angle β is a new parameter of the model and since $v_1, v_2 \geq 0$ we have

$$
0 \leq \beta \leq \frac{\pi}{2}. \qquad (3.21)
$$

With eqs. (3.15), (3.16) and (3.19) the minimum of the potential can be written as

$$
V_{min} = \frac{-1}{2(g^2 + g'^2)}\left[\left(m_1^2 - m_2^2\right) + \left(m_1^2 + m_2^2\right)\cos 2\beta\right]^2, \qquad (3.22)
$$

All the parameters of the model have a functional dependence on the renormalization point[4] Q. This in particular applies to the mass parameters m_i^2 ($i = 1, 2, 3$) and thus to det $\mathcal{M}^2(Q)$. To proceed, one has to take the complicated (coupled) renormalization group equations (RGE's) into account. This we will not do here, but only refer the interested reader to the literature [47]. The rest of the discussion of this section will be kept on a qualitative level.

At the Planck scale, M_{Pl}, condition (3.14) is not fulfilled, and hence the critical scale reads det $\mathcal{M}^2(Q_c) = 0$, where $Q_c < M_{Pl}$. Below Q_c, non-vanishing Higgs v.e.v.'s start to develop, signalling $SU(2)\times U(1)$-breaking as discussed earlier, but only as long as $\mathcal{B}(Q) \geq 0$. However, for some particular scale $Q_s < Q_c$, $\mathcal{B}(Q_s) < 0$ is driven negative and for $Q < Q_s$ one is in an instability region where $SU(2) \times U(1)$ is broken in an unsatisfactory manner. Our picture is recapitulated in figure 3.1 for various scales Q.

Note that in the supersymmetric limit, where *all* (soft) mass parameters of $\mathcal{L}_{Soft}$ are set equal to zero, det $\mathcal{M}^2 = 0$ and no electroweak breaking is possible in view of condition (3.14). So, in our model the gauge symmetry breaking is connected to the breaking of supersymmetry, as we already have noted several times[5].

[4]This Q-dependence may for instance come from the renormalization-group-improved tree-level potential which incorporates the large logarithmic corrections proportional to $\alpha \log\left(M_{GUT}/Q\right)$. Here M_{GUT} is a grand unification scale.

[5]It is possible to construct non-minimal models [23] where the Higgs-sector is enlarged by an $SU(2) \times U(1)$-gauge singlet and where the gauge-symmetry and SUSY can be broken separately.

Before we close this section, we will make one final comment. Instead of our naive use of the tree-level scalar potential (3.6), we should have used the full one-loop corrected effective potential

$$V_1(Q) \;=\; V(Q) + \Delta V(Q).$$

Here ΔV is the one-loop radiative correction to the scalar potential and in the leading logarithm approximation it reads

$$\Delta V(Q) \;\sim\; m_t^4 \log\left(\frac{M_{GUT}}{Q}\right)^2,$$

where m_t is the top-quark mass and M_{GUT} some super-high unification scale. By choosing a low renormalization scale, one gets substantial contributions from ΔV. Until recently, it was believed that the large logarithmic terms could be reabsorbed into the soft parameters[6] of $V(Q)$, and in consequence, ΔV only contained small logarithmic corrections. However, this only applies to so-called field independent radiative corrections. For the field dependent corrections we still can get substantial contributions as explained e.g. in ref. 52.

Even though the scalar potential can receive large corrections from ΔV, the use of the tree-level potential $V(Q)$ is adequate for our discussion. Furthermore, it simplifies the discussion enormously.

3.2 The Physical Higgs Boson Spectrum.

In the previous section we derived the condition for electroweak symmetry breaking. Henceforth we will assume that these conditions, i.e. eqs. (3.10) and (3.14), are fulfilled, and show that this implies the correct symmetry breaking pattern.

In the SM one starts by expanding around the Higgs v.e.v.'s and identify the new state as the physical state. However, by performing the same scheme for the MSSM, these new weak interacting eigenstates do not represent physical (mass) eigenstates, as we will see. So, before we proceed, we will work out the physical Higgs boson states.

The physical eigenstates are obtained by diagonalizing the Higgs boson mass-square matrix. This is most easily done in a real basis where

$$H_1 \;=\; \begin{pmatrix} h_1 + ih_2 \\ h_3 + ih_4 \end{pmatrix}, \tag{3.23}$$

$$H_2 \;=\; \begin{pmatrix} h_5 + ih_6 \\ h_7 + ih_8 \end{pmatrix}. \tag{3.24}$$

[6] Recall that the soft parameters in our theory are arbitrary.

In this basis the scalar (Higgs) potential (3.6) reads

$$\begin{aligned} V(h_i) &= m_1^2 \sum_{i=1}^{4} h_i^2 + m_2^2 \sum_{i=5}^{8} h_i^2 - 2m_3^2 \left(h_1 h_7 + h_4 h_6 - h_3 h_5 - h_2 h_8\right) \\ &+ \frac{1}{8}\left(g^2 + g'^2\right)\left[\sum_{i=1}^{4} h_i^2 - \sum_{i=5}^{8} h_i^2\right]^2 \\ &+ \frac{g^2}{2}\left(h_1 h_5 + h_2 h_6 + h_3 h_7 + h_4 h_8\right)^2 \\ &+ \frac{g^2}{2}\left(h_1 h_6 + h_3 h_8 - h_2 h_5 - h_4 h_7\right)^2 . \end{aligned} \tag{3.25}$$

From this potential it is apparent that the Higgs field basis that we are working in can not be a physical basis since it contains off-diagonal mass terms. Thus we are forced to transform to a mass-eigenstate basis, and the method which we will apply, is described in detail in ref. 49 for a general two doublet model.

The physical Higgs boson states are obtained by diagonalizing the Higgs boson mass-square matrix[7] given by [49]

$$M_{ij}^2 = \frac{1}{2} \left.\frac{\partial^2 V}{\partial h_i\, \partial h_j}\right|_{\min} . \tag{3.26}$$

Here the term "min" means setting $\langle h_1 \rangle = v_1$, $\langle h_7 \rangle = v_2$ and $\langle h_i \rangle = 0$ for all other i's. Note from eq. (3.25) that the "mixed" second order partial derivatives of $V(h_i)$ are continuous and thus equal (i.e. $\frac{\partial^2 V}{\partial h_i \partial h_j} = \frac{\partial^2 V}{\partial h_j \partial h_i}$), implying a symmetric mass-matrix, i.e. $M_{ij}^2 = M_{ji}^2$.

Now, the different parts of the Higgs sector will be analyzed in detail, and this will be the aim of the next three subsections.

3.2.1 The Charged Higgs Sector; Indices 3, 4, 5 and 6.

With eqs. (3.25) and (3.26) the Higgs boson mass-square matrix is easily calculated. Observe that the real and imaginary sector decouple i.e.

$$M_{56}^2 = M_{54}^2 = M_{36}^2 = M_{34}^2 = 0.$$

The remaining mass-square matrix components read

$$\begin{aligned} M_{55}^2 &= m_2^2 - \frac{1}{4}\left(g^2 + g'^2\right)\left(v_1^2 - v_2^2\right) + \frac{1}{2} g^2 v_1^2 \\ &= \frac{1}{2}\left(g^2 + \frac{2m_3^2}{v_1 v_2}\right) v_1^2 , \end{aligned}$$

[7] The factor of $\frac{1}{2}$ in front of definition (3.26) stems from the normalization of the scalar fields in eqs. (3.23) and (3.24).

$$
\begin{aligned}
M_{53}^2 &= m_3^2 + \frac{1}{2} g^2 v_1 v_2 \\
&= \frac{1}{2}\left(g^2 + \frac{2m_3^2}{v_1 v_2}\right) v_1 v_2, \\
M_{33}^2 &= m_1^2 + \frac{1}{4}\left(g^2 + g'^2\right)\left(v_1^2 - v_2^2\right) + \frac{1}{2} g^2 v_2^2 \\
&= \frac{1}{2}\left(g^2 + \frac{2m_3^2}{v_1 v_2}\right) v_2^2,
\end{aligned}
$$

and

$$
\begin{aligned}
M_{66}^2 &= M_{55}^2, \\
M_{44}^2 &= M_{33}^2, \\
M_{64}^2 &= -M_{53}^2. \qquad (3.27)
\end{aligned}
$$

Here eqs. (3.15) and (3.16) have been taken advantage of in eliminating the mass parameters m_1^2 and m_2^2. Hence in the basis's (h_5, h_3) and $(-h_6, h_4)$, the charged Higgs mass-square matrix reads[8]

$$
M_{\pm}^2 = \frac{1}{2}\left(g^2 + \frac{2m_3^2}{v_1 v_2}\right)\begin{pmatrix} v_1^2 & v_1 v_2 \\ v_1 v_2 & v_2^2 \end{pmatrix}. \qquad (3.28)
$$

To obtain the physical charged Higgs states and their masses, one has to orthogonal diagonalize [9] the matrix $M_{\pm}^2$ since physical states always are orthogonal to each other. Note that $M_{\pm}^2$ always will be orthogonal diagonalizable because it is symmetric [51].

By calculating the eigenvalues and the corresponding set of orthonormal eigenvectors, the charged mass matrix $M_{\pm}^2$ can be written in the form $(tan\beta = v_2/v_1)$

$$
M_{\pm}^2 = \begin{pmatrix} -\sin\beta & \cos\beta \\ \cos\beta & \sin\beta \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & m_{H\pm}^2 \end{pmatrix}\begin{pmatrix} -\sin\beta & \cos\beta \\ \cos\beta & \sin\beta \end{pmatrix}, \qquad (3.29)
$$

where

$$
m_{H\pm}^2 = \frac{1}{2}\left(g^2 + \frac{2m_3^2}{v_1 v_2}\right)\left(v_1^2 + v_2^2\right), \qquad (3.30)
$$

is the mass-square of the physical charged Higgs-bosons. Note that by this diagonalization procedure, two massless and two massive states have appeared. The mass-zero states will be associated with Goldstone bosons, as we will see in a moment.

[8] The particular sign of the basis $(-h_6, h_4)$, owing to the appearance of the sign in eq. (3.27), is chosen such that the two mass matrices coincide with each other.

[9] Recall that an orthogonal diagonalizable $n\times n$-matrix A always can be written in the form $A = PDP^{-1}$, where D is given by $D = \text{diag}\begin{pmatrix} \lambda_1 & \lambda_2 & \ldots & \lambda_n \end{pmatrix}$, and P is the orthogonal matrix containing the eigenvectors in the following way $P = \begin{pmatrix} v_1 & v_2 & \ldots & v_n \end{pmatrix}$. Here (λ_i, v_i) are corresponding sets of eigenvalues and eigenvectors.

After completing the diagonalizing procedure, the mass terms for the charged Higgs-bosons in the Lagrangian can be written in the following way

$$\begin{aligned}
&\begin{pmatrix} h_5 & h_3 \end{pmatrix} M_{\pm}^2 \begin{pmatrix} h_5 \\ h_3 \end{pmatrix} + \begin{pmatrix} -h_6 & h_4 \end{pmatrix} M_{\pm}^2 \begin{pmatrix} -h_6 \\ h_4 \end{pmatrix} \\
&\quad = \begin{pmatrix} h_5 + ih_6 & h_3 - ih_4 \end{pmatrix} M_{\pm}^2 \begin{pmatrix} h_5 - ih_6 \\ h_3 + ih_4 \end{pmatrix} \\
&\quad = \begin{pmatrix} H_2^1 & H_1^{2\,\dagger} \end{pmatrix} M_{\pm}^2 \begin{pmatrix} H_2^{1\,\dagger} \\ H_1^2 \end{pmatrix} \\
&\quad = \begin{pmatrix} -H_2^1 \sin\beta + H_1^{2\,\dagger} \cos\beta \\ H_2^1 \cos\beta + H_1^{2\,\dagger} \sin\beta \end{pmatrix}^T \begin{pmatrix} 0 & 0 \\ 0 & m_{H\pm}^2 \end{pmatrix} \begin{pmatrix} -H_2^{1\,\dagger} \sin\beta + H_1^2 \cos\beta \\ H_2^{1\,\dagger} \cos\beta + H_1^2 \sin\beta \end{pmatrix} \\
&\quad = \begin{pmatrix} G^+ & H^+ \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & m_{H\pm}^2 \end{pmatrix} \begin{pmatrix} G^- \\ H^- \end{pmatrix}.
\end{aligned}$$

Here

$$G^- = H_1^2 \cos\beta - H_2^{1\,\dagger} \sin\beta, \tag{3.31}$$

$$H^- = H_1^2 \sin\beta + H_2^{1\,\dagger} \cos\beta, \tag{3.32}$$

and

$$G^+ = \left(G^-\right)^\dagger,$$
$$H^+ = \left(H^-\right)^\dagger,$$

where $G^\pm$ are the charged Goldstone bosons while $H^\pm$ are the charged Higgs bosons.

This completes this subsection.

3.2.2 The Neutral Higgs Sector; Indices 2 and 8.

In the previous subsection we saw that the charged Higgs sector decouples into a real and an imaginary part. This is also the case for the neutral Higgs sector as the reader may easily verify by showing that $M_{ij}^2 = 0$ for $i = 1, 7$ and $j = 2, 8$. This owing to the fact that our theory is CP-invariant. We start the discussion with the imaginary (CP-odd) sector, and consider the real (CP-even) part in the next subsection[10].

Proceeding as in the previous subsection the mass-square matrix becomes

$$\frac{m_3^2}{v_1 v_2} \begin{pmatrix} v_1^2 & v_1 v_2 \\ v_1 v_2 & v_2^2 \end{pmatrix}$$

[10]The various CP-assignments can be obtained by e.g. studying the interactions of (neutral) Higgs- and gauge-bosons.

in the basis (h_8, h_2). By diagonalizing this matrix, which is identical to that for the charged sector, the physical mass eigenstates are obtained as follows

$$
\begin{aligned}
&\frac{m_3^2}{v_1 v_2}\begin{pmatrix} h_8 & h_2 \end{pmatrix}\begin{pmatrix} v_1^2 & v_1 v_2 \\ v_1 v_2 & v_2^2 \end{pmatrix}\begin{pmatrix} h_8 \\ h_2 \end{pmatrix} \\
&\quad = \begin{pmatrix} -h_8 \sin\beta + h_2 \cos\beta \\ h_8 \cos\beta + h_2 \sin\beta \end{pmatrix}^T \begin{pmatrix} 0 & 0 \\ 0 & m_{H_3^0}^2 \end{pmatrix}\begin{pmatrix} -h_8 \sin\beta + h_2 \cos\beta \\ h_8 \cos\beta + h_2 \sin\beta \end{pmatrix} \\
&\quad = \begin{pmatrix} \frac{G^0}{\sqrt{2}} & \frac{H_3^0}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & m_{H_3^0}^2 \end{pmatrix}\begin{pmatrix} \frac{G^0}{\sqrt{2}} \\ \frac{H_3^0}{\sqrt{2}} \end{pmatrix}.
\end{aligned}
$$

Here

$$
\begin{aligned}
G^0 &= \sqrt{2}\,(\, h_2 \cos\beta - h_8 \sin\beta \,) \\
&= \sqrt{2}\left(\mathrm{Im}\, H_1^1 \cos\beta - \mathrm{Im}\, H_2^2 \sin\beta\right), \qquad (3.33)
\end{aligned}
$$

$$
\begin{aligned}
H_3^0 &= \sqrt{2}\,(\, h_2 \sin\beta + h_8 \cos\beta \,) \\
&= \sqrt{2}\left(\mathrm{Im}\, H_1^1 \sin\beta + \mathrm{Im}\, H_2^2 \cos\beta\right), \qquad (3.34)
\end{aligned}
$$

where G^0 is a Goldstone boson (in this case neutral), and H_3^0 is a neutral Higgs boson. The mass of the Higgs boson is[11]

$$
\begin{aligned}
m_{H_3^0}^2 &= \frac{m_3^2}{v_1 v_2}\left(v_1^2 + v_2^2\right) \\
&= m_{H^\pm}^2 - m_{\mathrm{W}}^2. \qquad (3.35)
\end{aligned}
$$

The factors of $\sqrt{2}$ are inserted in order for these fields to have the conventional kinetic energy terms.

3.2.3 The Neutral Higgs Sector; Indices 1 and 7.

After completing the diagonalizing of the neutral imaginary sector, we will now consider the corresponding real sector. For this sector the mass-square matrix reads

$$
M_0^2 = \begin{pmatrix} A & B \\ B & C \end{pmatrix}
$$

relative to the basis (h_1, h_7). Here we have introduced the abbreviations

$$
\begin{aligned}
A &= \frac{1}{2}\left(g^2 + g'^2\right) v_1^2 + m_3^2 \frac{v_2}{v_1}, \\
B &= -\frac{1}{2}\left(g^2 + g'^2\right) v_1 v_2 - m_3^2, \\
C &= \frac{1}{2}\left(g^2 + g'^2\right) v_2^2 + m_3^2 \frac{v_1}{v_2},
\end{aligned}
$$

[11] In sect. 3.3 we will show that the W- and Z-mass are respectively given by $m_{\mathrm{W}}^2 = \frac{1}{2} g^2 \left(v_1^2 + v_2^2\right)$ and $m_{\mathrm{Z}}^2 = \frac{1}{2}\left(g^2 + g'^2\right)\left(v_1^2 + v_2^2\right)$.

and we notice that $A, C \geq 0$ and $B \leq 0$. Also here eqs. (3.15) and (3.16) have been used to eliminating the mass parameters m_1^2 and m_2^2.

The orthogonal diagonalization scheme for this sector is not as straightforward as above. Accordingly some more details will be given. The eigenvalues of M_0^2 read[11]

$$
\begin{aligned}
m^2_{H_1^0, H_2^0} &= \frac{1}{2}\left[A + C \pm \sqrt{(A-C)^2 + 4B^2} \right] \\
&= \frac{1}{2}\left[m^2_{H_3^0} + m_Z^2 \pm \sqrt{\left(m^2_{H_3^0} + m_Z^2\right)^2 - 4 m_Z^2 m^2_{H_3^0} \cos^2 2\beta} \right], \qquad (3.36)
\end{aligned}
$$

where the positive (negative) sign is associated with $m^2_{H_1^0}$ ($m^2_{H_2^0}$). The corresponding eigenvectors are[12]

$$
v_{1,2} = N_{1,2} \begin{pmatrix} 1 \\ \frac{-(A-C) \pm \sqrt{(A-C)^2 + 4B^2}}{2B} \end{pmatrix}. \qquad (3.37)
$$

Here $N_{1,2}$ are normalization constants.

As will become clear soon, it is useful to introduce the mixing angel α (not to be confused with the fine structure constant) defined by

$$
\begin{aligned}
\sin 2\alpha &= \frac{2B}{\sqrt{(A-C)^2 + 4B^2}} \\
&= -\sin 2\beta \left(\frac{m^2_{H_1^0} + m^2_{H_2^0}}{m^2_{H_1^0} - m^2_{H_2^0}} \right) \\
\cos 2\alpha &= \frac{A-C}{\sqrt{(A-C)^2 + 4B^2}} \\
&= -\cos 2\beta \left(\frac{m^2_{H_3^0} - m_Z^2}{m^2_{H_1^0} - m^2_{H_2^0}} \right).
\end{aligned}
$$

From the mathematical identities $\sin 2\alpha = 2 \sin\alpha \cos\alpha$ and $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$, one easily obtains the second order equation

$$
x^2 + 2\cot(2\alpha)\, x - 1 = 0,
$$

where $x = \tan\alpha$. This equation generally has two distinct solutions. However, earlier we have chosen $v_1, v_2 \geq 0$ or equivalently $0 \leq \beta \leq \frac{\pi}{2}$, something which according to ref. 53 implies that $-\frac{\pi}{2} \leq \alpha \leq 0$. With this constraint in mind, one can uniquely solve for x, and the result is (remember that $B \leq 0$)

$$
\tan\alpha = \frac{-(A-C) + \sqrt{(A-C)^2 + 4B^2}}{2B}, \qquad (3.38)
$$

[12] Here v_1 and v_2 correspond to the eigenvalues $m^2_{H_1^0}$ and $m^2_{H_2^0}$ respectively.

and by inversion (and some algebra)

$$\cot\alpha \;=\; \frac{(A-C)+\sqrt{(A-C)^2+4B^2}}{2B}. \tag{3.39}$$

By comparing eqs. (3.38) and (3.39) with eq. (3.37), we see that the second component of v_1 (v_2) can up to a sign be identified with $\tan\alpha$ ($\cot\alpha$). The mixing angle, α, was defined in order to obtain this.

Thus we choose $N_1 = \cos\alpha$ and $N_2 = -\sin\alpha$ in order to obtain an orthonormal eigenvector set, and the mass-square matrix of the real neutral sector takes on the form

$$M_0^2 \;=\; \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} m^2_{H_1^0} & 0 \\ 0 & m^2_{H_2^0} \end{pmatrix} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}^{-1}. \tag{3.40}$$

The corresponding mass terms of the Lagrangian now become

$$\begin{aligned} &\begin{pmatrix} h_1 & h_7 \end{pmatrix} M_0^2 \begin{pmatrix} h_1 \\ h_7 \end{pmatrix} \\ &= \begin{pmatrix} h_1\cos\alpha + h_7\sin\alpha \\ -h_1\sin\alpha + h_7\cos\alpha \end{pmatrix}^T \begin{pmatrix} m^2_{H_1^0} & 0 \\ 0 & m^2_{H_2^0} \end{pmatrix} \begin{pmatrix} h_1\cos\alpha + h_7\sin\alpha \\ -h_1\sin\alpha + h_7\cos\alpha \end{pmatrix}. \end{aligned} \tag{3.41}$$

When we now proceed by identifying the physical Higgs states H_1^0 and H_2^0, we have to be careful. The reason is that these states, as any physical states, have to have zero vacuum expectation values. Hence we make the following identifications

$$\begin{aligned} \frac{H_1^0}{\sqrt{2}} + v_1\cos\alpha + v_2\sin\alpha &= h_1\cos\alpha + h_7\sin\alpha, \\ \frac{H_2^0}{\sqrt{2}} - v_1\sin\alpha + v_2\cos\alpha &= -h_1\sin\alpha + h_7\cos\alpha, \end{aligned}$$

or equivalently

$$H_1^0 \;=\; \sqrt{2}\left[\left(\mathrm{Re}\,H_1^1 - v_1\right)\cos\alpha + \left(\mathrm{Re}\,H_2^2 - v_2\right)\sin\alpha\right], \tag{3.42}$$

$$H_2^0 \;=\; \sqrt{2}\left[-\left(\mathrm{Re}\,H_1^1 - v_1\right)\sin\alpha + \left(\mathrm{Re}\,H_2^2 - v_2\right)\cos\alpha\right]. \tag{3.43}$$

This concludes this section.

3.2.4 Conclusion and Comments.

In the three previous subsections the physical content of the Higgs sector of the MSSM was obtained. It is the charged Higgs bosons ($H^\pm$), the neutral Higgs bosons[13] (H_i^0, $i = 1, 2, 3$) and finally the charged ($G^\pm$) and neutral Goldstone bosons (G^0).

[13]Some authors use the notation H^0, h^0 and A^0 instead of our H_1^0, H_2^0 and H_3^0.

The new fields in terms of the "old" are given in eqs. (3.31), (3.32), (3.33), (3.34), (3.42) and (3.43). However, in order to give the Lagrangian in terms of the physical fields, we have to invert the above relations. The results, obtained by straightforward calculations, are

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}\,[\, H_1^0 \cos\alpha - H_2^0 \sin\alpha + iH_3^0 \sin\beta + iG^0 \cos\beta \,] \\ H^- \sin\beta + G^- \cos\beta \end{pmatrix}, \tag{3.44}$$

$$H_2 = \begin{pmatrix} H^+ \cos\beta - G^+ \sin\beta \\ v_2 + \frac{1}{\sqrt{2}}\,[\, H_1^0 \sin\alpha + H_2^0 \cos\alpha + iH_3^0 \cos\beta - iG^0 \sin\beta \,] \end{pmatrix}. \tag{3.45}$$

By inserting these expressions into the Lagrangian (2.64) the interactions (and Feynman rules) of the physical Higgs bosons can be obtained.

From the formulae for the Higgs-masses obtained earlier, eqs. (3.28), (3.35) and (3.36), it is interesting to note that in the limit $m_{H_3^0} \to \infty$ (fixed $\tan\beta$), $H^\pm$, H_1^0 (and H_3^0) decouple from the theory, and thus the Higgs-sector contains only H_2^0. In this limit, it is possible to show that H_2^0 is identical to the Higgs of the (minimal) Standard Model.

It should be noticed that the Higgs-masses obtained in the previous subsections are tree-level formulae. They fulfil the following relations

$$\begin{aligned} & & m_{H^\pm} &\geq M_{\mathrm{W}}, \\ m_{H_2^0} &\leq & m_{\mathrm{Z}} &\leq m_{H_1^0}, \\ & & m_{H_3^0} &\geq m_{H_2^0}. \end{aligned}$$

Since $m_{H_2^0} \leq m_{\mathrm{Z}}$ (at tree-level) it is believed, due to the interaction picture of H_2^0, that H_2^0 could be produced and hopefully detected at LEP. No Higgs has ever been seen and this may seem like a problem. Thus it came like a relief to many physicists when it recently was reported [52] (see subsect. 3.1.1) that the MSSM Higgses could get radiative corrections as large as $\mathcal{O}(100)$ GeV. This at once may push the mass of H_2^0 far above that of the Z-boson (and outside the LEP 1 discovery range). These large radiative corrections also have the implications [54], due to the unsuccessful Higgs searches at LEP 1, that[14]

$$\tan\beta \geq 1, \tag{3.46}$$

in the context of the MSSM.

3.3 The W-, Z- and Lepton Mass.

In this section we will give an illustrative demonstration (and a control for the sceptic one) of the fact that our gauge symmetry breaking scheme is capable of "producing" masses of the W- and Z-bosons and the (charged) leptons.

As for the SM case, we will make use of the gauge freedom of the theory and transform to the unitary gauge. This consists of setting the Goldstone fields of eqs. (3.44) and (3.45) to zero, but it will have no practical implication for our discussion.

[14] It is usual to let $\tan\beta$ varies in the range $1 \leq \tan\beta \leq 50$.

3.3.1 The W- and Z-Mass.

From the Lagrangian (2.64) (after symmetry breaking) we pick the following terms

$$
\begin{aligned}
&(D^\mu \mathrm{v}_1)^\dagger (D_\mu \mathrm{v}_1) + (D^\mu \mathrm{v}_2)^\dagger (D_\mu \mathrm{v}_2) \\
&\quad = \begin{pmatrix} \frac{ig}{2\cos\theta_\mathrm{W}}\, v_1 Z^\mu \\ \frac{ig}{\sqrt{2}}\, v_1 W^{-\,\mu} \end{pmatrix}^\dagger \begin{pmatrix} \frac{ig}{2\cos\theta_\mathrm{W}}\, v_1 Z_\mu \\ \frac{ig}{\sqrt{2}}\, v_1 W^-_\mu \end{pmatrix} \\
&\qquad + \begin{pmatrix} \frac{ig}{\sqrt{2}}\, v_2 W^{+\,\mu} \\ -\frac{ig}{2\cos\theta_\mathrm{W}}\, v_2 Z^\mu \end{pmatrix}^\dagger \begin{pmatrix} \frac{ig}{\sqrt{2}}\, v_2 W^+_\mu \\ -\frac{ig}{2\cos\theta_\mathrm{W}}\, v_2 Z_\mu \end{pmatrix} \\
&\quad = \frac{g^2}{4\cos^2\theta_\mathrm{W}} \left(v_1^2 + v_2^2\right) Z^\mu Z_\mu \\
&\qquad + \frac{g^2}{4}\left(v_1^2 + v_2^2\right) W^{+\,\mu} W^-_\mu + \frac{g^2}{4}\left(v_1^2 + v_2^2\right) W^{-\,\mu} W^+_\mu .
\end{aligned}
$$

where $\mathrm{v}_1 = \begin{pmatrix} v_1 & 0 \end{pmatrix}^T$ and $\mathrm{v}_2 = \begin{pmatrix} 0 & v_2 \end{pmatrix}^T$.

Hence the Z- and W-mass can be identified as

$$
m_\mathrm{W}^2 = \frac{g^2}{2}\left(v_1^2 + v_2^2\right), \tag{3.47}
$$

$$
m_\mathrm{Z}^2 = \frac{1}{2}\frac{g^2}{\cos^2\theta_\mathrm{W}}\left(v_1^2 + v_2^2\right) = \frac{1}{2}\left(g^2 + g'^2\right)\left(v_1^2 + v_2^2\right), \tag{3.48}
$$

which is consistent with the results from the SM.

Note that with the above results $v_1^2 + v_2^2$ is fixed by the W-mass.

3.3.2 The Lepton Mass.

Now the lepton mass will be paid attention. The piece $f\varepsilon^{ij}\,\bar{R}L^i H_1^j + h.c.$, stemming from the Yukawa piece of the superpotential, gives raise to the lepton mass as we now will see.

With H_1 given by eq. (3.44), $f\varepsilon^{ij}\,\bar{R}L^i H_1^j + h.c.$ contains the following terms

$$
\begin{aligned}
-f\,\bar{R}L^2 v_1 + h.c. &= -f v_1 \left(\bar{l}_R l_L + \bar{l}_L l_R\right) \\
&= -f v_1 \left(\bar{l}\,P_L\,l + \bar{l}\,P_R\,l\right) \\
&= -f v_1\,\bar{l}l.
\end{aligned}
$$

Hence, we can make the identification

$$
m_l = f v_1,
$$

and as in the SM, we notice that the lepton mass is undetermined by the theory.

For later use we observe that the Yukawa coupling f can be written as

$$f = \frac{m_l}{v_1} = \frac{g m_l}{\sqrt{2} m_{\rm W} \cos\beta}. \tag{3.49}$$

3.4 The Physical Slepton States.

The Lagrangian (2.64) contains off-diagonal mass terms for the sleptons in the basis $(\tilde{l}_L, \tilde{l}_R)$. So, also here we have to perform a diagonalizing procedure to obtain the physical mass eigenstates, and hence we have

$$\begin{aligned}\mathcal{L}^{mass}_{slept.} &= -\mu f v_2\, \tilde{l}_L^\dagger \tilde{l}_R - \mu f v_2\, \tilde{l}_R^\dagger \tilde{l}_L - f^2 v_1^2 \left(\tilde{l}_L^\dagger \tilde{l}_L + \tilde{l}_R^\dagger \tilde{l}_R\right) \\ &\quad - m_L^2\, \tilde{l}_L^\dagger \tilde{l}_L - m_R^2\, \tilde{l}_R^\dagger \tilde{l}_R \\ &= -\begin{pmatrix} \tilde{l}_L^\dagger & \tilde{l}_R^\dagger \end{pmatrix} \begin{pmatrix} m_L^2 + f^2 v_1^2 & \mu f v_2 \\ \mu f v_2 & m_R^2 + f^2 v_1^2 \end{pmatrix} \begin{pmatrix} \tilde{l}_L \\ \tilde{l}_R \end{pmatrix}.\end{aligned}$$

By diagonalizing, one obtains the mass eigenstates (in the usual way)

$$\begin{aligned}\tilde{l}_1 &= \tilde{l}_L \cos\theta + \tilde{l}_R \sin\theta, \\ \tilde{l}_2 &= \tilde{l}_L \sin\theta - \tilde{l}_R \cos\theta,\end{aligned}$$

with[15]

$$\tan 2\theta = \frac{2\mu f v_2}{(m_L^2 - m_R^2)} = \frac{2\mu m_l \tan\beta}{(m_L^2 - m_R^2)},$$

and masses respectively given by

$$\begin{aligned}M^2_{\tilde{l}_1,\tilde{l}_2} &= f^2 v_1^2 + \frac{1}{2}\left[\left(m_L^2 + m_R^2\right) \pm \sqrt{\left(m_L^2 - m_R^2\right)^2 + 4\mu^2 f^2 v_2^2}\,\right] \\ &= m_l^2 + \frac{1}{2}\left[\left(m_L^2 + m_R^2\right) \pm \sqrt{\left(m_L^2 - m_R^2\right)^2 + 4\mu^2 m_l^2 \tan^2\beta}\,\right].\end{aligned} \tag{3.50}$$

Unfortunately, there do not exist much information about the parameters contained in the slepton mass matrix. All the same, we will assume maximal mixing, i.e. $\theta = \pi/4$ or

$$m_L^2 = m_R^2 = \tilde{m}^2. \tag{3.51}$$

A motivation for this choice can be taken from supersymmetric QED where this choice is made in order to keep parity unbroken.

[15]Notice from eqs. (3.20) and (3.49) that $f v_2 = f v_1 \frac{v_2}{v_1} = m_l \tan\beta$.

Hence

$$\tilde{l}_1 = \frac{\tilde{l}_L + \tilde{l}_R}{\sqrt{2}}, \tag{3.52}$$

$$\tilde{l}_2 = \frac{\tilde{l}_L - \tilde{l}_R}{\sqrt{2}}, \tag{3.53}$$

and

$$M^2_{\tilde{l}_1, \tilde{l}_2} = \tilde{m}^2 + m_l^2 \pm |\mu| \, m_l \tan\beta. \tag{3.54}$$

This concludes this section.

3.5 Chargino and Neutralino Mixing.

The gaugino-higgsino sector of the theory also contains off-diagonal mass terms, as easily seen from the Lagrangian (2.64). To obtain mass-eigenstates the now familiar diagonalization procedure has to be performed, and the resulting mass-eigenstates are called charginos, $\tilde{\chi}^{\pm}$, and neutralinos, $\tilde{\chi}^0$. The discussion of these states will be the aim of the present section.

3.5.1 Chargino Mixing.

Charginos $\tilde{\chi}_i^+$ $(i = 1, 2)$, which arise due to mixing of Winos, $\tilde{W}^{\pm}$, and charged Higgsinos, $\tilde{H}^{\pm}$, are four component Dirac spinors. Since there in principle are two independent mixings, i.e. $(\tilde{W}^-, \tilde{H}^-)$ and $(\tilde{W}^+, \tilde{H}^+)$, we will need two unitary matrices in order to diagonalize the resulting mass-matrix [55].

From the Lagrangian (2.64) we pick the terms

$$\begin{aligned} \mathcal{L}^{mass}_{\tilde{\chi}^{\pm}} &= -gv_1 \, \bar{\tilde{W}} P_R \tilde{H} - gv_1 \, \bar{\tilde{H}} P_L \tilde{W} - gv_2 \, \bar{\tilde{H}} P_R \tilde{W} - gv_2 \, \bar{\tilde{W}} P_L \tilde{H} \\ &\quad + \mu \, \bar{\tilde{H}} \tilde{H} + M_{\tilde{W}} \, \bar{\tilde{W}} \tilde{W}, \end{aligned}$$

which in two-component form reads

$$\mathcal{L}^{mass}_{\tilde{\chi}^{\pm}} = ig \left[v_1 \psi^2_{H_1} \lambda^+ + v_2 \lambda^- \psi^1_{H_2} \right] + \mu \, \psi^2_{H_1} \psi^1_{H_2} - M \, \lambda^- \lambda^+ + h.c. \tag{3.55}$$

By introducing the notation

$$\psi^+ = \begin{pmatrix} -i\lambda^+ \\ \psi^1_{H_2} \end{pmatrix}, \qquad \psi^- = \begin{pmatrix} -i\lambda^- \\ \psi^2_{H_1} \end{pmatrix},$$

and

$$\Psi^{\pm} = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix},$$

eq. (3.55) takes on the form

$$\mathcal{L}^{mass}_{\tilde{\chi}^\pm} = \frac{1}{2}\left(\Psi^\pm\right)^T Y^\pm \Psi^\pm + h.c.$$

Here

$$Y^\pm = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}, \tag{3.56}$$

with

$$X = \begin{pmatrix} M & -\sqrt{2}m_{\rm W}\sin\beta \\ -\sqrt{2}m_{\rm W}\cos\beta & \mu \end{pmatrix}. \tag{3.57}$$

Now, two-component mass-eigenstates can be defined by $(i, j = 1, 2)$

$$\chi^+_i = V_{ij}\psi^+_j, \tag{3.58}$$

$$\chi^-_i = U_{ij}\psi^-_j, \tag{3.59}$$

where U and V are unitary matrices, chosen in such a way that

$$U^* X V^\dagger = M^\pm_D. \tag{3.60}$$

Here $M^\pm_D$ is the chargino mass-matrix. Since we have assumed CP-invariance of our theory, this in particular holds for the chargino sector. Thus the chargino-masses will be real and non-negative[16]. Furthermore, the two-component spinors of eqs. (3.58) and (3.59) can be arranged in (four-component) Dirac-spinors as follows[17]:

$$\tilde{\chi}_i = \begin{pmatrix} \chi^+_i \\ \bar{\chi}^-_i \end{pmatrix}, \qquad i = 1, 2. \tag{3.61}$$

The Lagrangian (2.64) is given in terms of the non mass-eigenstates $\tilde{W}$ and $\tilde{H}$, because it leads to simpler expressions for the interaction terms. In converting to the (physical)

[16]It is possible to show that the masses read

$$M^2_{\tilde{\chi}_1} = A + \sqrt{B},$$
$$M^2_{\tilde{\chi}_2} = A - \sqrt{B},$$

where

$$A = \frac{1}{2}\left(M^2 + \mu^2\right) + m^2_{\rm W},$$
$$B = \frac{1}{4}\left(M^2 - \mu^2\right)^2 + m^4_{\rm W}\cos^2(2\beta) + m^2_{\rm W}\left(M^2 + \mu^2 + 2\mu M \sin(2\beta)\right).$$

[17]In what follows, we will use the abbreviation $\tilde{\chi} \equiv \tilde{\chi}^+$. Hence $\tilde{\chi}^c \equiv (\tilde{\chi}^+)^c = \tilde{\chi}^-$ is a negatively charged chargino.

charginos, the following relations are useful

$$P_L \tilde{W} = P_L V_{i1}^* \tilde{\chi}_i, \tag{3.62}$$
$$P_R \tilde{W} = P_R U_{i1} \tilde{\chi}_i, \tag{3.63}$$
$$P_L \tilde{H} = P_L V_{i2}^* \tilde{\chi}_i, \tag{3.64}$$
$$P_R \tilde{H} = P_R U_{i2} \tilde{\chi}_i. \tag{3.65}$$

Here, repeated indices are summed from 1 to 2, and, as usual, P_L and P_R are (projection) operators projecting out the top two and bottom two components of a Dirac-spinor. These relations are easy to prove with eqs. (3.58), (3.59), (3.61) and the unitarity of the matrices U and V. We will now demonstrate it for eq. (3.62).

Proof : With eq. (2.46), the left-hand side of eq. (3.62) reads

$$P_L \tilde{W} = P_L \begin{pmatrix} -i\lambda^+ \\ i\bar{\lambda}^- \end{pmatrix} = \begin{pmatrix} -i\lambda^+ \\ 0 \end{pmatrix}. \tag{3.66}$$

By pre-multiplying eq. (3.58) with V_{ik}^* $(i,k=1,2)$, and using the unitarity of V, we obtain

$$V_{ik}^* \chi_i^+ = V_{ik}^* V_{ij} \psi_j^+ = \delta_{kj} \psi_j^+ = \psi_k^+.$$

Hence

$$\begin{pmatrix} -i\lambda^+ \\ 0 \end{pmatrix} = \begin{pmatrix} \psi_1^+ \\ 0 \end{pmatrix} = V_{i1}^* \begin{pmatrix} \chi_i^+ \\ 0 \end{pmatrix} = P_L V_{i1}^* \tilde{\chi}_i,$$

and by comparing this result with eq. (3.66) the proof of eq. (3.62) is competed.

In the same way the following relations for the charge-conjugated fields are obtained

$$P_L \tilde{W}^c = P_L U_{i1}^* \tilde{\chi}_i^c, \tag{3.67}$$
$$P_R \tilde{W}^c = P_R V_{i1} \tilde{\chi}_i^c, \tag{3.68}$$
$$P_L \tilde{H}^c = P_L U_{i2}^* \tilde{\chi}_i^c, \tag{3.69}$$
$$P_R \tilde{H}^c = P_R V_{i2} \tilde{\chi}_i^c. \tag{3.70}$$

Observe that by hermitian conjugation, two corresponding sets of equations, like for instance $\bar{\tilde{W}} P_R = V_{i1} \bar{\tilde{\chi}}_i \, P_R$, can be obtained.

By this observation we conclude this subsection, and instead consider neutralino mixing.

3.5.2 Neutralino Mixing.

Neutralinos ($\tilde{\chi}^0_i, i = 1, \dots, 4$) are Majorana-spinors arising due to mixing of photino, zino and neutral higgsinos.

The appropriate mass-terms are

$$\mathcal{L}^{mass}_{\tilde{\chi}^0} = -\frac{g}{\sqrt{2}\cos\theta_W}\left[\left\{v_1\,\bar{\tilde{Z}}P_R\tilde{H}_1 - v_2\,\bar{\tilde{H}}_2P_R\tilde{Z}\right\} + h.c.\right] - \frac{\mu}{2}\,\bar{\tilde{H}}_1\tilde{H}_2 - \frac{\mu}{2}\,\bar{\tilde{H}}_2\tilde{H}_1$$
$$+\frac{1}{2}M_{\tilde{A}}\,\bar{\tilde{A}}\tilde{A} + \frac{1}{2}M_{\tilde{Z}}\,\bar{\tilde{Z}}\tilde{Z} + \frac{1}{2}\left(M_{\tilde{Z}} - M_{\tilde{A}}\right)\tan 2\theta_W\,\bar{\tilde{A}}\tilde{Z},$$

and in two-component form they read

$$\mathcal{L}^{mass}_{\tilde{\chi}^0} = \frac{ig}{\sqrt{2}\cos\theta_W}\left\{v_1\,\lambda_Z\psi^1_{H_1} - v_2\,\lambda_Z\psi^2_{H_2}\right\} - \mu\,\psi^1_{H_1}\psi^2_{H_2}$$
$$-\frac{1}{2}M_{\tilde{A}}\,\lambda_A\lambda_A - \frac{1}{2}M_{\tilde{Z}}\,\lambda_Z\lambda_Z - \frac{1}{2}\left(M_{\tilde{Z}} - M_{\tilde{A}}\right)\tan 2\theta_W\,\lambda_A\lambda_Z. \tag{3.71}$$

In the basis

$$\psi^0 = \begin{pmatrix} -i\lambda_A & -i\lambda_Z & \psi^1_{H_1} & \psi^2_{H_2}\end{pmatrix}^T, \tag{3.72}$$

eq. (3.71) can be written in the form

$$\mathcal{L}^{mass}_{\tilde{\chi}^0} = \frac{1}{2}\left(\psi^0\right)^T Y^0\psi^0 + h.c., \tag{3.73}$$

where Y^0 reads

$$Y^0 = \begin{pmatrix} M_{\tilde{A}} & \frac{1}{2}\left(M_{\tilde{Z}} - M_{\tilde{A}}\right)\tan 2\theta_W & 0 & 0 \\ \frac{1}{2}\left(M_{\tilde{Z}} - M_{\tilde{A}}\right)\tan 2\theta_W & M_{\tilde{Z}} & -m_Z\cos\beta & m_Z\sin\beta \\ 0 & -m_Z\cos\beta & 0 & -\mu \\ 0 & m_Z\sin\beta & -\mu & 0 \end{pmatrix}. \tag{3.74}$$

Note that Y^0 is symmetric, something which has to do with the Majorana nature of the neutralinos. In consequence, only one unitary matrix N is required in order to diagonalize Y^0:

$$N^* Y^0 N^\dagger = M^0_D. \tag{3.75}$$

Here M^0_D is the diagonal neutralino mass matrix[18].

As in the previous subsection we define two-component mass-eigenstates by

$$\chi^0_i = N_{ij}\psi^0_j, \qquad i,j = 1,\dots,4, \tag{3.76}$$

[18]Also here the matrix N may be chosen in such a way that the elements of M^0_D are real and non-negative.

but in this case we arrange them in (four-component) Majorana spinors defined by

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \bar{\chi}_i^0 \end{pmatrix}, \qquad i = 1, \ldots, 4. \tag{3.77}$$

The relations corresponding to eqs. (3.62)–(3.65) read

$$P_L \tilde{A} = P_L \, N_{i1}^* \tilde{\chi}_i^0, \tag{3.78}$$

$$P_R \tilde{A} = P_R \, N_{i1} \tilde{\chi}_i^0, \tag{3.79}$$

$$P_L \tilde{Z} = P_L \, N_{i2}^* \tilde{\chi}_i^0, \tag{3.80}$$

$$P_R \tilde{Z} = P_R \, N_{i2} \tilde{\chi}_i^0 \tag{3.81}$$

$$P_L \tilde{H}_j = P_L \, N_{i,j+2}^* \tilde{\chi}_i^0, \qquad j = 1, 2, \tag{3.82}$$

$$P_R \tilde{H}_j = P_R \, N_{i,j+2} \tilde{\chi}_i^0, \tag{3.83}$$

and they are obtained in the same fashion. Here repeated indices are assumed to be summed from 1 to 4.

3.6 Concluding Remarks.

In the previous chapter the full four-component Lagrangian for our supersymmetric electroweak theory was established. Furthermore, we in this chapter introduced the physical states and described the gauge symmetry breaking which gives masses to the gauge bosons and the charged leptons.

With these elements at hand, one can in principle calculate any process contained within this minimal electro-weak theory.

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Appendix A

Notation and Conventions.

A.1 Relativistic Notation.

In this report we will adopt standard relativistic units, i.e.

$$\hbar = c = 1. \tag{A.1}$$

A general contravariant and covariant four-vector will be denoted by

$$\left.\begin{array}{rcccl} A^\mu & = & (A^0; A^1, A^2, A^3) & = & (A^0; \mathbf{A}) \\ A_\mu & = & (A_0; A_1, A_2, A_3) & = & (A^0; -\mathbf{A}) \end{array}\right\}. \tag{A.2}$$

The compact "Feynman slash" notation

$$\not{A} = \gamma^\mu A_\mu, \tag{A.3}$$

will be used. The metric tensor, $g^{\mu\nu}$, which connects A^μ and A_μ, is defined by

$$g^{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1). \tag{A.4}$$

Moreover, we will use the (relativistic) summation convention which states that repeated Greek indices, $\mu, \nu, \rho, \sigma, \tau$, are summed from 0 to 3 and latin indices run from 1 to 3 unless specifically indicated to the contrary.

The Minkowski product (the four-product) will be denoted by AB and defined as

$$AB \equiv A^\mu B_\mu = A^0 B^0 - \mathbf{A}\mathbf{B} \tag{A.5}$$

Practical notation for the four-gradients, ∂^μ and ∂_μ, will be used

$$\partial^\mu \equiv \frac{\partial}{\partial x_\mu} = (\frac{\partial}{\partial t}; -\nabla), \tag{A.6}$$

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = (\frac{\partial}{\partial t}; \nabla). \tag{A.7}$$

The totally antisymmetric Levi-Civita tensors in three and four dimensions are respectively defined by

$$\varepsilon_{ijk} = \begin{cases} +1 & , \text{ for even permutations of } 123 \\ -1 & , \text{ for odd permutations} \\ 0 & , \text{ otherwise,} \end{cases} \tag{A.8}$$

$$\varepsilon_{\mu\nu\rho\sigma} = \begin{cases} +1 & , \text{ for even permutations of } 0123 \\ -1 & , \text{ for odd permutations} \\ 0 & , \text{ otherwise,} \end{cases} \tag{A.9}$$

where

$$\varepsilon_{ijk} = \varepsilon^{ijk} , \tag{A.10}$$

$$\varepsilon_{\mu\nu\rho\sigma} = -\varepsilon^{\mu\nu\rho\sigma} . \tag{A.11}$$

A.2 Pauli Matrices.

The well known Pauli matrices are defined by

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.12}$$

and satisfy the commutator relation

$$[\sigma^i, \sigma^j] = 2i\varepsilon^{ijk}\sigma^k, \qquad i, j, k = 1, 2, 3.$$

From this definition it is evident that

$$(\sigma^i)^\dagger = \sigma^i, \qquad i = 1, 2, 3, \tag{A.13}$$

$$(\sigma^i)^2 = 1, \tag{A.14}$$

$$Tr(\sigma^i) = 0. \tag{A.15}$$

For later use, we also introduce[1]

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{A.16}$$

and a useful arrangement of these matrices is

$$\sigma^\mu = (\sigma^0 \,;\, \boldsymbol{\sigma}) = (\sigma^0 \,;\, \sigma^1, \sigma^2, \sigma^3).$$

[1]Note that different signs are used in the literature for the definition of this quantity.

The index structure of the σ-matrices is given by

$$\sigma^\mu \quad = \quad [\sigma^\mu_{\alpha\dot\alpha}]. \tag{A.17}$$

We now introduce some "Pauli related" matrices defined by

$$\bar\sigma^{\mu\,\dot\alpha\alpha} \equiv \sigma^{\mu\,\alpha\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta}\varepsilon^{\alpha\beta}\sigma^\mu_{\beta\dot\beta}, \tag{A.18}$$

where the "metrics" ε and $\bar\varepsilon$ have been used. By direct computations one can establish the following relations

$$\begin{aligned} \bar\sigma^0 &= \sigma^0 && \text{(A.19)} \\ \bar\sigma^i &= -\sigma^i, \qquad i = 1,2,3. && \text{(A.20)} \end{aligned}$$

Moreover, the following relations are true

$$\begin{aligned} \sigma^\mu_{\alpha\dot\alpha}\bar\sigma^{\dot\beta\beta}_\mu &= 2\,\delta_\alpha^{\;\beta}\delta_{\dot\alpha}^{\;\dot\beta} && \text{(A.21)} \\ Tr(\sigma^\mu\bar\sigma^\nu) &= 2g^{\mu\nu} && \text{(A.22)} \\ (\sigma^\mu\bar\sigma^\nu + \sigma^\nu\bar\sigma^\mu)_\alpha^{\;\beta} &= 2\,g^{\mu\nu}\delta_\alpha^{\;\beta} && \text{(A.23)} \\ (\bar\sigma^\mu\sigma^\nu + \bar\sigma^\nu\sigma^\mu)^{\dot\alpha}_{\;\dot\beta} &= 2\,g^{\mu\nu}\delta^{\dot\alpha}_{\;\dot\beta} && \text{(A.24)} \\ (\sigma^\mu\bar\sigma^\nu\sigma^\rho + \sigma^\rho\bar\sigma^\nu\sigma^\mu) &= 2\,(g^{\mu\nu}\sigma^\rho + g^{\nu\rho}\sigma^\mu - g^{\mu\rho}\sigma^\nu) && \text{(A.25)} \\ (\bar\sigma^\mu\sigma^\nu\bar\sigma^\rho + \bar\sigma^\rho\sigma^\nu\bar\sigma^\mu) &= 2\,(g^{\mu\nu}\bar\sigma^\rho + g^{\nu\rho}\bar\sigma^\mu - g^{\mu\rho}\bar\sigma^\nu) && \text{(A.26)} \\ Tr(\sigma^\mu\bar\sigma^\nu\sigma^\rho\bar\sigma^\sigma) &= 2\,(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma} - i\varepsilon^{\mu\nu\rho\sigma}). && \text{(A.27)} \end{aligned}$$

Most of the above relations are easily proved by direct computations. Besides, Müller-Kirsten and Wiedemann [33, subsec. 1.3.5], have proved most of them, and in particular eq. $(A.27)$ which is the most difficult one.

Anti-symmetric matrices $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ are defined by

$$\begin{aligned} \sigma^{\mu\nu} &= \frac{i}{4}\,(\sigma^\mu\bar\sigma^\nu - \sigma^\nu\bar\sigma^\mu), && \text{(A.28)} \\ \bar\sigma^{\mu\nu} &= \frac{i}{4}\,(\bar\sigma^\mu\sigma^\nu - \bar\sigma^\nu\sigma^\mu). && \text{(A.29)} \end{aligned}$$

By utilizing the index structure of the σ-matrices, it is easily seen that $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ must have the index structure $\sigma^{\mu\nu} = [(\sigma^{\mu\nu})_\alpha^{\;\beta}]$ and $\bar\sigma^{\mu\nu} = [(\bar\sigma^{\mu\nu})^{\dot\beta}_{\;\dot\alpha}]$. In fact are $\sigma^{\mu\nu}$ and $\bar\sigma^{\mu\nu}$ the generators of $SL(2,C)$ in the spinor representations $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$ respectively. The proofs together with the establishment of the below formulae can be found in ref. 33:

$$\begin{aligned} \sigma^{\mu\nu\,\dagger} &= \bar\sigma^{\mu\nu}, && \text{(A.30)} \\ \sigma^{\mu\nu} &= \frac{1}{2i}\varepsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}, && \text{(A.31)} \end{aligned}$$

$$
\begin{aligned}
\bar{\sigma}^{\mu\nu} &= -\frac{1}{2i}\varepsilon^{\mu\nu\rho\sigma}\bar{\sigma}_{\rho\sigma}, && \text{(A.32)}\\
Tr(\sigma^{\mu\nu}) &= Tr(\bar{\sigma}^{\mu\nu}) \;=\; 0 && \text{(A.33)}\\
Tr(\sigma^{\mu\nu}\sigma^{\rho\sigma}) &= \frac{1}{2}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) + \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}, && \text{(A.34)}\\
Tr(\bar{\sigma}^{\mu\nu}\bar{\sigma}^{\rho\sigma}) &= \frac{1}{2}\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right) - \frac{i}{2}\varepsilon^{\mu\nu\rho\sigma}. && \text{(A.35)}
\end{aligned}
$$

A.3 Dirac Matrices.

The Dirac γ-matices are defined by the anticommutation (Clifford) relations

$$
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{A.36}
$$

From the four γ-matrices above, it is possible to define a "fifth γ-matrix" by

$$
\gamma_5 \equiv \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{A.37}
$$

It possesses the following properties which follows easily from the definitions $(A.36)$ and $(A.37)$

$$
\begin{aligned}
\{\gamma^5, \gamma^\mu\} &= 0, && \text{(A.38)}\\
(\gamma^5)^2 &= 1. && \text{(A.39)}
\end{aligned}
$$

We will now state three explicit representations of the γ-matrices, namely the so-called Dirac representation, the Majorana representation, and finally the Chiral representation.

A.3.1 Representations

The lowest non-trivial representation of these matrices is of dimension four. and we will concentrate on this represntation. From now on, we will assume that a four dimensional representation is used.

The Dirac Representation or Canonical Basis.

In this particular representation the γ-matrices read

$$
\begin{aligned}
\gamma^0 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, && \text{(A.40)}\\
\gamma^i &= \begin{pmatrix} 0 & \sigma^i \\ \bar{\sigma}^i & 0 \end{pmatrix}, \qquad i = 1, 2, 3, && \text{(A.41)}
\end{aligned}
$$

$$\gamma^5 = \begin{pmatrix} 0 & \sigma^0 \\ \bar{\sigma}^0 & 0 \end{pmatrix}, \tag{A.42}$$

where 1 denotes the 2×2 identity matrix and σ^μ and $\bar{\sigma}^\mu$ are the Pauli matrices defined in the previous section.

The Majorana Representation.

In this representation all γ-matricrs are pure imaginary and have the explicit form:

$$\gamma^0 = \begin{pmatrix} 0 & \sigma^2 \\ -\bar{\sigma}^2 & 0 \end{pmatrix}, \tag{A.43}$$

$$\gamma^1 = \begin{pmatrix} i\sigma^3 & 0 \\ 0 & i\sigma^3 \end{pmatrix}, \tag{A.44}$$

$$\gamma^2 = \begin{pmatrix} 0 & -\sigma^2 \\ -\bar{\sigma}^2 & 0 \end{pmatrix}, \tag{A.45}$$

$$\gamma^3 = \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, \tag{A.46}$$

and finally

$$\gamma^5 = \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}. \tag{A.47}$$

The Chiral representation or Weyl Basis.

This basis is of particular interest to persons doing SUSY. In this representation the γ-matrices take on the explicite form

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \tag{A.48}$$

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{A.49}$$

A.4 Spinor Relations.

In two-component notation we have the anti-symmetric ε-metric. The tensor oby the following relations, which are proven by stright forward calculations

$$\varepsilon^{\alpha\beta}\, \varepsilon_{\gamma\delta} = \delta^\alpha{}_\delta\, \delta^\beta{}_\gamma - \delta^\alpha{}_\gamma\, \delta^\beta{}_\delta, \tag{A.50}$$

$$\varepsilon_{\dot\alpha\dot\beta}\,\varepsilon^{\dot\gamma\dot\delta} = \delta_{\dot\alpha}^{\ \dot\delta}\,\delta_{\dot\beta}^{\ \dot\gamma} - \delta_{\dot\alpha}^{\ \dot\gamma}\,\delta_{\dot\beta}^{\ \dot\delta}, \tag{A.51}$$

$$\varepsilon^{\alpha\beta}\,\varepsilon_{\beta\gamma} = \delta^{\alpha}_{\ \gamma}, \tag{A.52}$$

$$\varepsilon_{\dot\alpha\dot\beta}\,\varepsilon^{\dot\beta\dot\gamma} = \delta_{\dot\alpha}^{\ \dot\gamma}. \tag{A.53}$$

We start by postulating that the spinor components are Grassmann numbers, i.e.

$$\left.\begin{array}{l}\{\psi_\alpha,\psi_\beta\} = \{\psi^\alpha,\psi^\beta\} = \{\psi_\alpha,\psi^\beta\} = 0\\ \{\bar\chi_{\dot\alpha},\bar\chi_{\dot\beta}\} = \{\bar\chi^{\dot\alpha},\bar\chi^{\dot\beta}\} = \{\bar\chi_{\dot\alpha},\bar\chi^{\dot\beta}\} = 0\end{array}\right\}, \tag{A.54}$$

and also anti-commute with other Grassmann numbers (e.g. fermion fields, spinor charges etc.).

With this postulate an expression like $\psi^\alpha\chi_\alpha = \psi_2\chi_1 - \psi_1\chi_2$ do not vanish[2], and in particular

$$\left.\begin{array}{rcl}\psi^\alpha\chi_\alpha &=& -\chi_\alpha\psi^\alpha\\ \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} &=& -\bar\chi^{\dot\alpha}\bar\psi_{\dot\alpha}\end{array}\right\}. \tag{A.55}$$

Because of the signs in eq. $(A.55)$, it is not well-defined what we mean by $\psi\chi$ or $\bar\psi\bar\chi$. To tackle this problem, we introduce the *summation convention* that states that suppressed undotted spinor indices are summed from upper left to downer right, while suppressed dotted indices are summed from lower left to upper right. In particular this means, for instance, that

$$\psi\chi \equiv \psi^\alpha\chi_\alpha \tag{A.56}$$

$$\bar\psi\bar\chi \equiv \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} \tag{A.57}$$

$$\psi\sigma^\mu\bar\chi \equiv \psi^\alpha\sigma^\mu_{\alpha\dot\alpha}\bar\chi^{\dot\alpha} \tag{A.58}$$

$$etc.$$

We are now in position to establish some useful relations involving spinors which will frequently be use in calculations.

Let ψ, θ and χ be two-comonent (Weyl) spinors. Then the following relations hold:

$$\psi\chi = \chi\psi, \tag{A.59}$$

$$\bar\psi\bar\chi = \bar\chi\bar\psi, \tag{A.60}$$

$$(\psi\chi)^\dagger = \bar\chi\bar\psi, \tag{A.61}$$

$$\psi\sigma^\mu\bar\chi = -\bar\chi\bar\sigma^\mu\psi, \tag{A.62}$$

$$(\psi\sigma^\mu\bar\chi)^\dagger = \chi\sigma^\mu\bar\psi, \tag{A.63}$$

$$\psi\sigma^{\mu\nu}\chi = -\chi\sigma^{\mu\nu}\psi, \tag{A.64}$$

$$\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\,\theta\sigma^\nu\bar\theta = \bar\theta\bar\theta\left[\,\frac{1}{2}\delta_\alpha^{\ \beta}g^{\mu\nu} - i\,\left(\sigma^{\mu\nu}\right)_\alpha^{\ \beta}\right]\theta_\beta, \tag{A.65}$$

$$\psi\sigma^\mu\bar\sigma^\nu\chi = \psi\left[\,-2i\,\sigma^{\mu\nu} + g^{\mu\nu}\,\right]\chi, \tag{A.66}$$

[2]This observation can be taken as a motivation of the above postulate.

$$\bar{\psi}\bar{\sigma}^{\mu}\sigma^{\nu}\bar{\chi} = \bar{\psi}\,[\,-2i\,\bar{\sigma}^{\mu\nu} + g^{\mu\nu}\,]\,\bar{\chi}, \tag{A.67}$$

$$\theta^{\alpha}\,\theta^{\beta} = -\frac{1}{2}\,\varepsilon^{\alpha\beta}\,\theta\theta, \tag{A.68}$$

$$\theta_{\alpha}\theta_{\beta} = \frac{1}{2}\,\varepsilon_{\alpha\beta}\,\theta\theta, \tag{A.69}$$

$$\bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \frac{1}{2}\,\varepsilon^{\dot{\alpha}\dot{\beta}}\,\bar{\theta}\bar{\theta}, \tag{A.70}$$

$$\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\,\varepsilon_{\dot{\alpha}\dot{\beta}}\,\bar{\theta}\bar{\theta}. \tag{A.71}$$

The final results are only stated here. Most of the explicite proofs are given in detail in in ref. 33.

A.4.1 Fierz Rearrangemant Formulae.

Some other relations have proven useful. They go under the name of Fierz Rearrangement formulae and read:

$$\theta\psi\ \ \theta\chi = -\frac{1}{2}\,\theta\theta\ \ \psi\chi, \tag{A.72}$$

$$\bar{\theta}\bar{\psi}\ \ \bar{\theta}\bar{\chi} = -\frac{1}{2}\,\bar{\theta}\bar{\theta}\ \ \bar{\psi}\bar{\chi}, \tag{A.73}$$

$$\theta\psi\ \ \bar{\chi}_{\dot{\alpha}} = \frac{1}{2}\,\theta\sigma^{\mu}\bar{\chi}\ \ \psi^{\alpha}\sigma_{\mu\,\alpha\dot{\alpha}}, \tag{A.74}$$

$$\bar{\theta}\bar{\psi}\ \ \chi^{\alpha} = \frac{1}{2}\,\bar{\theta}\bar{\sigma}^{\mu}\chi\ \bar{\psi}_{\dot{\alpha}}\bar{\sigma}_{\mu}^{\dot{\alpha}\alpha}, \tag{A.75}$$

$$\psi_1\sigma^{\mu}\bar{\chi}_1\ \psi_2\sigma^{\nu}\bar{\chi}_2 = \frac{1}{2}g^{\mu\nu}\ \ \psi_1\psi_2\ \bar{\chi}_1\bar{\chi}_2, \tag{A.76}$$

$$\theta\chi\ \theta\sigma^{\mu}\bar{\psi} = -\frac{1}{2}\,\theta\theta\ \chi\sigma^{\mu}\bar{\psi}, \tag{A.77}$$

$$\bar{\theta}\bar{\chi}\ \bar{\theta}\bar{\sigma}^{\mu}\psi = -\frac{1}{2}\,\bar{\theta}\bar{\theta}\ \bar{\chi}\bar{\sigma}^{\mu}\psi, \tag{A.78}$$

$$\psi\sigma^{\mu}\bar{\sigma}^{\nu}\chi = \chi\sigma^{\nu}\bar{\sigma}^{\mu}\psi. \tag{A.79}$$

Neither these formulae we will prove explicitly. The proofs can be found from the same source as above.

A.5 Four Component notation.

A.5.1 The Projections Operators.

We start by defining the projection operators, well known from SM,

$$P_L = \frac{1}{2}\,(1-\gamma_5)\,, \tag{A.80}$$

$$P_R = \frac{1}{2}(1+\gamma_5). \tag{A.81}$$

With the properties of the γ-matrices from sect. A.3, it is straightforward to establish the relations

$$\begin{aligned}
P_L + P_R &= 1, && \text{(A.82)}\\
P_L P_L &= P_L, && \text{(A.83)}\\
P_L P_R &= P_R P_L = 0, && \text{(A.84)}\\
P_L^\dagger &= P_L, && \text{(A.85)}\\
P_L \gamma^\mu &= \gamma^\mu P_R, && \text{(A.86)}
\end{aligned}$$

and corresponding equations for P_R.

A.5.2 Connection Between the Two- and Four-Component Spinors.

Let us introduce the two two-component Weyl spinors ξ_α and $\bar{\eta}^{\dot{\alpha}}$

$$\begin{aligned}
\xi_\alpha &\in F,\\
\bar{\eta}^{\dot{\alpha}} &\in \dot{F}^*.
\end{aligned}$$

The vector-spaces F and $\dot{F}^*$ are inequivalent representation spaces of SL(2,C). Now we construct the direct sum space

$$D = F \oplus \dot{F}^*. \tag{A.87}$$

This space is a four-dimensional representation space of SL(2,C). The elements of D, are just the well-known four-component Dirac-spinors.

Thus a Dirac-spinor, Ψ, can be constructed from these Weyl spinors according to

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}. \tag{A.88}$$

Strictly speaking this is a Dirac-spinor in the Weyl-representation. Thus we see that if we work in the Weyl representation (subsect. A.3.1) we have a direct relation between two- and four-component spinors. Throughout this subsection we will thus assume the Weyl-representation.

A Majorana spinor, λ, is a (four-component) Dirac-spinor with the additional condition

$$\lambda = \lambda^c = C\bar{\lambda}^T. \tag{A.89}$$

Here C is the charge conjugation matrix[3] while $\bar{\lambda}$ means, as usual, the the Dirac adjoint spinor $\bar{\lambda} = \lambda^\dagger \gamma_0$ (independent of represnetation). In the Dirac-represntation C reads $C_D = i\gamma^2\gamma^0$, and in the Weyl-representation (with the correct index structure) [33, p. 135]

$$C_{\rm W} = \begin{pmatrix} (i\sigma^2\bar{\sigma}^0)_\alpha{}^\beta & 0 \\ 0 & (i\bar{\sigma}^2\sigma^0)^{\dot{\alpha}}{}_{\dot{\beta}} \end{pmatrix}. \tag{A.90}$$

Thus it is possible to show that [33, p. 140]

$$\Psi^c = C\bar{\Psi}^T = \begin{pmatrix} \eta_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}, \tag{A.91}$$

i.e. the charge conjugation (in the Weyl representation) flips ξ and η.

Hence, we may conclude that a Majorana-spinor,λ, defined in eq. $(A.89)$, can be written

$$\lambda = \begin{pmatrix} \xi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}. \tag{A.92}$$

Furthermore, in the Weyl representation we have

$$P_L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$
$$P_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

and thus

$$\Psi_L = P_L\Psi = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix},$$
$$\Psi_R = P_R\Psi = \begin{pmatrix} 0 \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}.$$

The Dirac-adjoint spinor of Ψ, is

$$\bar{\Psi} = \Psi^\dagger\gamma_0 = \begin{pmatrix} \eta^\alpha & \bar{\xi}_{\dot{\alpha}} \end{pmatrix}, \tag{A.93}$$

as can be showed by straightforward calculations.

Useful Relations Between Two- and Four-Component Spinors.

Now we shall establish some relations, making the transitions between two- and four-component spinors more explicite and easy later on. Let the Dirac- and Majorana-spinor, $\Psi(x)$ and $\lambda(x)$, be defined as in eqs. $(A.88)$ and $(A.92)$.

[3]For more information on this matrix consider e.g. ref. 56.

Hence we have (in the Weyl representation):

$$\begin{aligned}
\bar{\Psi}_1\Psi_2 &= \eta_1\xi_2 + \bar{\xi}_1\bar{\eta}_2, && \text{(A.94)}\\
\bar{\Psi}_1\gamma^\mu\Psi_2 &= \bar{\xi}_1\bar{\sigma}^\mu\xi_2 - \bar{\eta}_2\bar{\sigma}^\mu\eta_1, && \text{(A.95)}\\
\bar{\Psi}_1\gamma_5\Psi_2 &= -\eta_1\xi_2 + \bar{\eta}_2\bar{\xi}_1, && \text{(A.96)}\\
\bar{\Psi}_1\gamma^\mu\gamma_5\Psi_2 &= -\bar{\xi}_1\bar{\sigma}^\mu\xi_2 - \bar{\eta}_2\bar{\sigma}^\mu\eta_1, && \text{(A.97)}\\
\bar{\Psi}_1\gamma^\mu\partial_\mu\Psi_2 &= \eta_1\sigma^\mu\partial_\mu\bar{\eta}_2 + \bar{\xi}_1\bar{\sigma}^\mu\partial_\mu\xi_2 \\
&= \bar{\eta}_2\bar{\sigma}^\mu\partial_\mu\eta_1 + \bar{\xi}_1\bar{\sigma}^\mu\partial_\mu\xi_2 - \partial_\mu\left(\bar{\eta}_2\bar{\sigma}^\mu\eta_1\right), && \text{(A.98)}\\
\bar{\Psi}_1 P_L\Psi_2 &= \eta_1\xi_2, && \text{(A.99)}\\
\bar{\Psi}_1 P_R\Psi_2 &= \bar{\xi}_1\bar{\eta}_2, && \text{(A.100)}\\
\bar{\Psi}_1\gamma^\mu P_L\Psi_2 &= \bar{\xi}_1\bar{\sigma}^\mu\xi_2, && \text{(A.101)}\\
\bar{\Psi}_1\gamma^\mu P_R\Psi_2 &= -\bar{\eta}_2\bar{\sigma}^\mu\eta_1, && \text{(A.102)}\\
\bar{\Psi}_1\gamma^\mu P_L\partial_\mu\Psi_2 &= \bar{\xi}_1\bar{\sigma}^\mu\partial_\mu\xi_2, && \text{(A.103)}\\
\bar{\Psi}_1\gamma^\mu P_R\partial_\mu\Psi_2 &= \eta_1\sigma^\mu\partial_\mu\bar{\eta}_2, \\
&= \bar{\eta}_2\bar{\sigma}^\mu\partial_\mu\eta_1 - \partial_\mu\left(\bar{\eta}_2\bar{\sigma}^\mu\eta_1\right). && \text{(A.104)}
\end{aligned}$$

A.6 Grassmann Variables.

In this appendix a differentiation and integration calculus for Grassmann variables will be established. The obtained results will be extensively used in the text.

A.6.1 Differentiation with respect to Grassmann Variables.

In supersymmetry the Grassmann variables, which parametrize superspace, are important. Because of their anticommuting properties, they can not be continuos varying variables. However, they have to be discrete objects. Hence, defining differentiation with respect to Grassmann variables in the normal sense, as the ratio of two infinitesimal increments, has no meaning. However, formally we can define differentiation, following common practice, as

$$\begin{aligned}
\frac{\partial\theta_\alpha}{\partial\theta_\beta} &= \delta^\beta{}_\alpha, && \text{(A.105)}\\
\frac{\partial\,\theta^\alpha}{\partial\,\theta^\beta} &= \delta_\beta{}^\alpha, && \text{(A.106)}\\
\frac{\partial\bar{\theta}_{\dot{\alpha}}}{\partial\bar{\theta}_{\dot{\beta}}} &= \delta^{\dot{\beta}}{}_{\dot{\alpha}}, && \text{(A.107)}\\
\frac{\partial\bar{\theta}^{\dot{\alpha}}}{\partial\bar{\theta}^{\dot{\beta}}} &= \delta_{\dot{\beta}}{}^{\dot{\alpha}}. && \text{(A.108)}
\end{aligned}$$

The ε-metric can be used to raise and lower indices of derivatives according to[4]

$$\varepsilon^{\alpha\beta}\frac{\partial}{\partial\,\theta^{\beta}} = -\frac{\partial}{\partial\theta_{\alpha}} \quad (A.109)$$

$$\varepsilon_{\alpha\beta}\frac{\partial}{\partial\theta_{\beta}} = -\frac{\partial}{\partial\,\theta^{\alpha}} \quad (A.110)$$

and

$$\varepsilon^{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}} = -\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} \quad (A.111)$$

$$\varepsilon_{\dot{\alpha}\dot{\beta}}\frac{\partial}{\partial\bar{\theta}_{\dot{\beta}}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \quad (A.112)$$

Proof : Let the eq. $(A.109)$ operates (from the left) on θ^{γ}:

$$\varepsilon^{\alpha\beta}\frac{\partial}{\partial\,\theta^{\beta}}\,\theta^{\gamma} = -\frac{\partial}{\partial\theta_{\alpha}}\,\theta^{\gamma}.$$

Then by comparing each side of this equation we have

$$\begin{aligned}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\,\theta^{\beta}}\,\theta^{\gamma} &= \varepsilon^{\alpha\beta}\,\delta_{\beta}{}^{\gamma}\\ &= \varepsilon^{\alpha\gamma},\end{aligned}$$

$$\begin{aligned}-\frac{\partial}{\partial\theta_{\alpha}}\,\theta^{\gamma} &= -\varepsilon^{\gamma\beta}\,\frac{\partial}{\partial\theta_{\alpha}}\,\theta_{\beta}\\ &= -\varepsilon^{\gamma\beta}\,\delta^{\alpha}{}_{\beta}\\ &= -\varepsilon^{\gamma\alpha}\\ &= \varepsilon^{\alpha\gamma}.\end{aligned}$$

Hence we can conclude eq. $(A.109)$ is fulfilled. The other relations in eqs. $(A.110)$–$(A.112)$ are showed in a similar fashion.

Due to the anticommuting character of θ and $\bar{\theta}$, we shall demand that

$$\left\{\frac{\partial}{\partial\,\theta^{\alpha}},\frac{\partial}{\partial\,\theta^{\beta}}\right\} = \left\{\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}},\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}\right\} = \left\{\frac{\partial}{\partial\,\theta^{\alpha}},\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}}\right\} = 0, \quad (A.113)$$

$$\left\{\frac{\partial}{\partial\theta_{\alpha}},\theta_{\beta}\right\} = \delta^{\alpha}{}_{\beta}, \quad (A.114)$$

$$\left\{\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}},\bar{\theta}^{\dot{\beta}}\right\} = \delta^{\dot{\beta}}{}_{\dot{\alpha}}, \quad (A.115)$$

[4]Take particular notice in the sign on the right-hand side of these equations.

and, since θ and $\bar{\theta}$ are concidered to be independent,

$$\left\{\frac{\partial}{\partial\,\theta^{\alpha}}, \bar{\theta}^{\dot{\beta}}\right\} = \left\{\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}, \theta^{\beta}\right\} = 0. \tag{A.116}$$

These equations yield directly that

$$\frac{\partial\,\theta^{\beta}}{\partial\bar{\theta}^{\dot{\alpha}}} = \frac{\partial\bar{\theta}^{\dot{\beta}}}{\partial\,\theta^{\alpha}} = 0,$$

and also an "unusual" product rule (with a minus sign) like e.g.

$$\frac{\partial}{\partial\,\theta^{\alpha}}\left(\theta_{\beta}\theta_{\gamma}\right) \;=\; \left(\frac{\partial}{\partial\,\theta^{\alpha}}\,\theta_{\beta}\right)\,\theta_{\gamma} - \theta_{\beta}\,\frac{\partial}{\partial\,\theta^{\alpha}}\,\theta_{\gamma} = \delta^{\alpha}{}_{\beta}\theta_{\gamma} - \theta_{\beta}\delta^{\alpha}{}_{\gamma}. \tag{A.117}$$

With the conventions for the differential operators established so far, the following relations are true[5]

$$\frac{\partial}{\partial\,\theta^{\alpha}}\;\theta\theta \;=\; 2\,\theta_{\alpha}, \tag{A.118}$$

$$\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\;\bar{\theta}\bar{\theta} \;=\; -2\,\bar{\theta}_{\dot{\alpha}}, \tag{A.119}$$

$$\frac{\partial}{\partial\theta_{\alpha}}\frac{\partial}{\partial\,\theta^{\alpha}}\;\theta\theta \;=\; 4, \tag{A.120}$$

$$\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}\;\bar{\theta}\bar{\theta} \;=\; 4. \tag{A.121}$$

Proof : We start by proving eqs. $(A.118)$ and $(A.120)$

$$\begin{aligned}
\frac{\partial}{\partial\,\theta^{\alpha}}\;\theta\theta &= \frac{\partial}{\partial\,\theta^{\alpha}}\,\theta^{\beta}\theta_{\beta} \\
&= \delta_{\alpha}{}^{\beta}\theta_{\beta} - \theta^{\beta}\varepsilon_{\beta\gamma}\delta_{\alpha}{}^{\gamma} \\
&= \theta_{\alpha} + \theta_{\gamma}\delta_{\alpha}{}^{\gamma} \\
&= 2\,\theta_{\alpha},
\end{aligned}$$

and

$$\frac{\partial}{\partial\theta_{\alpha}}\frac{\partial}{\partial\,\theta^{\alpha}}\;\theta\theta \;=\; \frac{\partial}{\partial\theta_{\alpha}}\frac{\partial}{\partial\,\theta^{\alpha}}\;\;\theta^{\beta}\theta_{\beta}$$

[5]When spinor indices are suppressed on the differentiation symbols, we will follow the convention

$$\begin{aligned}
\frac{\partial}{\partial\theta}\frac{\partial}{\partial\theta} &= \frac{\partial}{\partial\theta_{\alpha}}\frac{\partial}{\partial\,\theta^{\alpha}}, \\
\frac{\partial}{\partial\bar{\theta}}\frac{\partial}{\partial\bar{\theta}} &= \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}}.
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\partial}{\partial\theta_\alpha}\left(\frac{\partial}{\partial\,\theta^\alpha}\;\;\theta^\beta\theta_\beta\right) \\
&= 2\,\frac{\partial}{\partial\theta_\alpha}\;\;\theta_\alpha \\
&= 2\;\;\delta^\alpha{}_\alpha \\
&= 4.
\end{aligned}
$$

Eqs. $(A.119)$ and $(A.121)$ are proven in a similar way.

A.6.2 The Berezin Integral.

In ordinary field theories, a translation invariant action is constructed (assuming surface terms to vanish), by integrating a Lagrangian density $\mathcal{L}(x)$ over d^4x. In a similar fashion, SUSY invariant actions in superspace can be obtained by an integration over the whole of superspace.

The aim of this section will be to define what we understand by integration with respect to Grassmann variables, i.e. to define the so-called Berezin integral [57].

We will start by considering the simplest situation with only one Grassmann variable ζ. Since $\zeta^n = 0,\; n \geq 2$, due to the anticommuting property of ζ, any function of ζ, $f(\zeta)$, has always the form

$$f(\zeta) \;=\; f(0) + \zeta f^{(1)}. \tag{A.122}$$

Hence it is sufficient to define $\int d\zeta$ and $\int d\zeta\;\zeta$ in order to let $\int d\zeta\;\, f(\zeta)$ be well-defined. Following F.A. Berezin [57] we define

$$\int d\zeta \;=\; 0, \tag{A.123}$$

$$\int d\zeta\;\zeta \;=\; 1. \tag{A.124}$$

Thus

$$\int d\zeta\;\, f(\zeta) \;=\; \int d\zeta\left(f(0) + \zeta f^{(1)}\right) = f^{(1)}, \tag{A.125}$$

and formally differentiation and integration are the same, i.e.

$$\int d\zeta f(\zeta) \;=\; \frac{\partial}{\partial\zeta}\, f(\zeta). \tag{A.126}$$

Two important properties, follow as a consequence of the definitions $(A.123)$, $(A.124)$ and use of eq. $(A.122)$, should be noted

$$\int d\zeta\;\, f(\zeta+\kappa) \;=\; \int d\zeta\;\, f(\zeta), \tag{A.127}$$

$$\int d\zeta\,(a\, f(\zeta) + b\, h(\zeta)) \;=\; a\int d\zeta\;\, f(\zeta) + b\int d\zeta\;\, h(\zeta), \qquad a, b \in C, \tag{A.128}$$

i.e. translation invariance and complex linearity respectively.

Superspace is not parametrized in terms of only one Grassmann variable. However, it "contains" the Grassmann algebras $G_2 = \{\theta^1, \theta^2\}$ and $\bar{G}_2 = \{\bar{\theta}^1, \bar{\theta}^2\}$. To define integration on these algebras, we have to generalize the above results. By demanding

$$\{d\,\theta_\alpha, d\,\theta_\beta\} = \{d\,\theta_\alpha, \theta_\beta\} = 0, \tag{A.129}$$

$$\left\{d\,\bar{\theta}_{\dot{\alpha}}, d\,\bar{\theta}_{\dot{\beta}}\right\} = \left\{d\,\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\right\} = 0, \tag{A.130}$$

and using the definitions $(A.123)$ and $(A.124)$ we have

$$\int d\,\theta_1 d\,\theta_2 \;=\; 0, \tag{A.131}$$

$$\int d\,\theta_1 d\,\theta_2\; \theta_1 \;=\; \int d\,\theta_1 d\,\theta_2\; \theta_2 = 0, \tag{A.132}$$

$$\int d\,\theta_1 d\,\theta_2\; \theta_1\theta_2 \;=\; -1. \tag{A.133}$$

Similar formulae hold for the algebra $\bar{G}_2$. Now the integral of any function on G_2 and/or $\bar{G}_2$ can be obtained by Taylor expansion and linearity.

We now define "volume elements" of the anti-commuting part of superspace

$$d^2\theta \;=\; -\frac{1}{4}\, d\theta^\alpha\, d\theta_\alpha = -\frac{\varepsilon_{\alpha\beta}}{4}\, d\theta^\alpha\, d\theta^\beta, \tag{A.134}$$

$$d^2\bar{\theta} \;=\; -\frac{1}{4}\, d\bar{\theta}_{\dot{\alpha}}\, d\bar{\theta}^{\dot{\alpha}} = -\frac{\varepsilon_{\dot{\alpha}\dot{\beta}}}{4}\, d\bar{\theta}_{\dot{\alpha}}\, d\bar{\theta}_{\dot{\beta}}, \tag{A.135}$$

$$d^4\bar{\theta} \;=\; d^2\theta\, d^2\bar{\theta}. \tag{A.136}$$

With these definitions the following relations are true

$$\int d^2\theta \;=\; \int d^2\bar{\theta} = 0, \tag{A.137}$$

$$\int d^2\theta\;\theta^\alpha \;=\; \int d^2\bar{\theta}\;\bar{\theta}_{\dot{\alpha}} \;=\; 0, \tag{A.138}$$

$$\int d^2\theta\;\theta\theta \;=\; 1, \tag{A.139}$$

$$\int d^2\bar{\theta}\;\bar{\theta}\bar{\theta} \;=\; 1, \tag{A.140}$$

$$\int d^4\theta\;\theta\theta\,\bar{\theta}\bar{\theta} \;=\; 1. \tag{A.141}$$

Proof : Eq. $(A.137)$ and $(A.138)$ follow immediately from eq. $(A.131)$ and $(A.132)$ and the corresponding equations for $\bar{G}_2$.

By using the definition $(A.133)$, $(A.134)$ and the fact that $\theta\theta = \varepsilon_{\alpha\beta}\,\theta^\alpha\theta^\beta = -2\,\theta^1\theta^2$, we have

$$\int d^2\theta\;\theta\theta \;=\; -\frac{\varepsilon_{\alpha\beta}}{4}\int d\,\theta^\alpha\, d\,\theta^\beta\;\left(-2\,\theta^1\,\theta^2\right)$$

$$
\begin{aligned}
&= \frac{\varepsilon_{12}}{2}\int d\,\theta^1\, d\,\theta^2\;\; \theta^1\,\theta^2 + \frac{\varepsilon_{21}}{2}\int d\,\theta^2\, d\,\theta^1\;\; \theta^1\,\theta^2 \\
&= \varepsilon_{12}\int d\,\theta^1\, d\,\theta^2\;\; \theta^1\,\theta^2 \\
&= 1,
\end{aligned}
$$

since $\varepsilon_{\alpha\beta}$ is antisymmetric and $\varepsilon_{12} = -1$. Eq. $(A.140)$ is proved in the same way. With eqs. $(A.139)$ and $(A.140)$ established, it is rather straightforward to prove eq. $(A.141)$

$$
\begin{aligned}
\int d^4\theta\;\theta\theta\,\bar{\theta}\bar{\theta} &= \int d^2\theta\, d^2\bar{\theta}\;\bar{\theta}\bar{\theta}\,\theta\theta \\
&= \int d^2\theta\;\theta\theta \\
&= 1.
\end{aligned}
$$

With the formulae obtained so far, the integral $\int d^4\theta\;\Phi(x,\theta,\bar{\theta})$ of a general superfield can be established. Hence we have

$$
\begin{aligned}
\int d^4\theta\;\Phi(x,\theta,\bar{\theta}) &= \int d^4\theta\;\Big(f(x) + \theta^\alpha\phi_\alpha(x) + \bar{\theta}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}(x) + \theta\theta\, m(x) + \bar{\theta}\bar{\theta}\, n(x) \\
&\qquad + \theta\sigma^\mu\bar{\theta}\, V_\mu(x) + \theta\theta\,\bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}}(x) + \bar{\theta}\bar{\theta}\;\theta^\alpha\psi_\alpha(x) + \theta\theta\,\bar{\theta}\bar{\theta}\, d(x)\Big) \\
&= d(x). \qquad\qquad (A.142)
\end{aligned}
$$

Thus, by integration with respect to Grassmann supercoordinates, the $\theta\theta\;\bar{\theta}\bar{\theta}$-component of any integrand is always outprodjected. This fact, as we will see, is rather useful when supersymmetric Lagrangians are being constructed.

A.6.3 Delta Functions on Grassmann Algebras.

Delta functions on superspace simplify the constructions of SUSY-invariant actions. Let such delta functions on G_2 and $\bar{G}_2$, both two and four dimensional, be defined implicitly by

$$
\int d^2\theta\; f(\theta)\,\delta^2(\theta) = f(0), \qquad f(\theta)\in G_2, \qquad (A.143)
$$

$$
\int d^2\bar{\theta}\; g(\bar{\theta})\,\delta^2(\bar{\theta}) = g(0), \qquad g(\bar{\theta})\in \bar{G}_2, \qquad (A.144)
$$

and

$$
\int d^4\theta\; h(,\theta,\bar{\theta})\,\delta^4(\theta,\bar{\theta}) = h(0,0), \qquad h(\theta,\bar{\theta})\in G_2\times\bar{G}_2. \qquad (A.145)
$$

This implies that

$$
\delta^2(\theta) = \theta\theta, \qquad (A.146)
$$

$$
\delta^2(\bar{\theta}) = \bar{\theta}\bar{\theta}, \qquad (A.147)
$$

and

$$
\delta^4(\theta,\bar{\theta}) = \delta^2(\theta)\,\delta^2(\bar{\theta}) = \theta\theta\,\bar{\theta}\bar{\theta}, \qquad (A.148)
$$

as we now shall show.

Proof : By using the anticommuting properties of the elements of G_2 and eq. $(A.139)$ we have

$$\begin{aligned}\int d^2\theta \; f(\theta)\,\theta\theta &= \int d^2\theta \; \left(f(0) + \theta^\alpha f^{(1)}_\alpha + \theta\theta\, f^{(2)}\right)\theta\theta \\ &= \int d^2\theta \; \theta\theta \; f(0) \\ &= f(0).\end{aligned}$$

Hence, with the identification $\delta^2(\theta) = \theta\theta$, eq. $(A.143)$ is fulfilled, something which shows that our identification is correct.

In a similar way eq. $(A.147)$ is seen to be consistent with eq. $(A.144)$.

For the same reason as above we have

$$\begin{aligned}\int d^4\theta \; h(,\theta,\bar\theta)\delta^4(\theta,\bar\theta) &= \int d^4\theta \; \left(h(0,0) + \theta^\alpha \frac{\partial h}{\partial\,\theta^\alpha} + \bar\theta_{\dot\alpha}\frac{\partial h}{\partial\bar\theta_{\dot\alpha}} + \ldots\right)\theta\theta\;\bar\theta\bar\theta \\ &= h(0,0). \qquad\qquad \text{(A.149)}\end{aligned}$$

Thus, the identification made in eq. $(A.148)$ is correct.

Appendix B

The Two-Component Form of the Off-Shell Lagrangian $\mathcal{L}_{SUSY}$.

In this appendix the expansion of $\mathcal{L}_{SUSY}$, in the two-component formalism, will be performed in detail.

However, before we address this problem, some general calculations will be performed. To be more specific, we will in sect. B.1 calculate the component form of the non-Abelian fieldstrength W_α. In sect. B.2 this expansion will be used in obtaining the component form of the kinetic term of vectorsuperfields. Finally, in sect. B.3, which concludes our general calculations of this appendix, we derive the expansion of the matter Lagrangian of a $\mathcal{G} \times U(1)$- gauge theory, where $\mathcal{G}$ is some non-Abelian gauge group.

B.1 The Non-Abelian Fieldstrength.

In this section we will calculate the component expansion of the non-Abelian fieldstrengths, as defined[1] by

$$W_\alpha = -\frac{1}{8g}\bar{D}\bar{D}e^{-2gV}D_\alpha e^{2gV}, \tag{B.1}$$

$$\bar{W}_{\dot{\alpha}} = -\frac{1}{8g}DDe^{-2gV}\bar{D}_{\dot{\alpha}}e^{2gV}. \tag{B.2}$$

We start by W_α and for simplicity we will work in the WZ-gauge. By hermitian conjugation the corresponding expression for $\bar{W}_{\dot{\alpha}}$ is obtained. It is practical to work in the basis ($y = x + i\theta\sigma\bar{\theta}, \theta, \bar{\theta}$), since then the SUSY covariant derivatives take on a somewhat simpler form

$$D_\alpha(y,\theta,\bar{\theta}) = \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial y^\mu}, \tag{B.3}$$

[1] Here we have made the substitution $g \to 2g$.

$$\bar{D}_{\dot{\alpha}}(y,\theta,\bar{\theta}) \;=\; -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}. \tag{B.4}$$

Hence $(\mathrm{Im}A(x)=0)$[2]

$$\begin{aligned} V^a(x,\theta,\bar{\theta}) &= V^a(y-i\theta\sigma\bar{\theta},\theta,\bar{\theta}) \\ &= -\theta\sigma^\mu\bar{\theta}\,V^a_\mu(y) + i\,\theta\theta\;\bar{\theta}\bar{\lambda}^a(y) - i\,\bar{\theta}\bar{\theta}\;\theta\lambda^a(y) \\ &\quad + \frac{1}{2}\,\theta\theta\;\bar{\theta}\bar{\theta}\;\Big[\,D^a(y) + i\,\partial^\mu V^a_\mu(y)\,\Big], \end{aligned} \tag{B.5}$$

where eq. $(A.76)$ has been used. From now on, the y-dependence of the component fields will be understood and suppressed.

Our program will be to first calculate $e^{-2gV}D_\alpha e^{2gV}$ and then the total (non-Abelian) field strength. In these calculations, expressions for $D_\alpha V^a$, $D_\alpha\left(V^aV^b\right)$, and finally $V^aD_\alpha V^b$ are useful. Hence we start by determine these expressions.

Using the results from appendix A.6, and in particular eqs. $(A.106)$ and $(A.118)$, yields

$$\begin{aligned} D_\alpha V^a &= \left(\frac{\partial}{\partial\,\theta^\alpha} + 2i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial y^\mu}\right) \\ &\quad \times\left(-\theta\sigma^\mu\bar{\theta}\;V^a_\mu + i\,\theta\theta\;\bar{\theta}\bar{\lambda}^a - i\,\bar{\theta}\bar{\theta}\;\theta\lambda^a + \frac{1}{2}\,\theta\theta\;\bar{\theta}\bar{\theta}\;\Big[\,D^a + i\,\partial^\mu V^a_\mu\,\Big]\right) \\ &= -\,\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}V^a_\mu + 2i\;\theta_\alpha\;\bar{\theta}\bar{\lambda}^a - i\;\bar{\theta}\bar{\theta}\;\lambda^a_\alpha + \theta_\alpha\;\bar{\theta}\bar{\theta}\;\Big[\,D^a + i\;\partial^\mu V^a_\mu\,\Big] \\ &\quad -2i\;\sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\;\theta\sigma^\mu\bar{\theta}\;\partial_\nu V^a_\mu - 2\;\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\;\theta\theta\;\bar{\theta}_{\dot{\beta}}\;\partial_\mu\bar{\lambda}^{\dot{\beta}\;a} \\ &= -\,\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}V^a_\mu + 2i\;\theta_\alpha\;\bar{\theta}\bar{\lambda}^a - i\;\bar{\theta}\bar{\theta}\;\lambda^a_\alpha \\ &\quad + \theta_\alpha\;\bar{\theta}\bar{\theta}\;\Big[\,D^a + i\;\partial^\mu V^a_\mu\,\Big] \\ &\quad -2i\;\bar{\theta}\bar{\theta}\;\left[\frac{1}{2}\delta_\alpha{}^\beta g^{\nu\mu} - i\;\left(\sigma^{\nu\mu}\right)_\alpha{}^\beta\right]\;\theta_\beta\;\partial_\nu V^a_\mu \\ &\quad + \theta\theta\;\bar{\theta}\bar{\theta}\;\varepsilon^{\dot{\alpha}\dot{\beta}}\sigma^\mu_{\alpha\dot{\alpha}}\;\partial_\mu\bar{\lambda}^a_{\dot{\beta}}. \end{aligned}$$

Here eq. $(A.65)$ has been used. By utilizing the antisymmetry of $\sigma^{\mu\nu}$ (cf. sect. A.3) and eq. $(A.68)$ (together with a redefinition for the indices μ and ν) one obtains

$$\begin{aligned} D_\alpha V^a &= -\,\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}V^a_\mu + 2i\;\theta_\alpha\;\bar{\theta}\bar{\lambda}^a - i\;\bar{\theta}\bar{\theta}\;\lambda^a_\alpha \\ &\quad + \bar{\theta}\bar{\theta}\;\Big\{\,\theta_\alpha D^a - \left(\sigma^{\mu\nu}\right)_\alpha{}^\beta\,\theta_\beta\;\Big[\,\partial_\mu V^a_\nu - \partial_\nu V^a_\mu\,\Big]\,\Big\} \\ &\quad + \;\theta\theta\;\bar{\theta}\bar{\theta}\;\sigma^\mu_{\alpha\dot{\alpha}}\,\partial_\mu\bar{\lambda}^{\dot{\alpha}\;a}. \end{aligned} \tag{B.6}$$

[2] When we work in the basis $(y,\theta,\bar{\theta})$ the notation ∂_μ will mean $\frac{\partial}{\partial y^\mu}$ if nothing else is said to indicate otherwise.

With eqs. $(A.76)$ and $(A.118)$ we have

$$\begin{aligned} D_\alpha\ \left(V^a V^b\right) &= D_\alpha \left(\frac{1}{2}\,\theta\theta\,\bar{\theta}\bar{\theta}\ V^{a\,\mu} V^b_\mu\right) \\ &= \bar{\theta}\bar{\theta}\ \theta_\alpha\ V^{a\,\mu} V^b_\mu . \end{aligned} \tag{B.7}$$

Note that only the first term of D_α contributes to $D_\alpha\left(V^a V^b\right)$ due to the anticommuting properties of the superspace parameters θ.

Furthermore with eq. $(A.118)$

$$\begin{aligned} V^a D_\alpha V^b &= \left\{-\theta\sigma^\mu\bar{\theta}\, V^a_\mu + i\,\theta\theta\ \bar{\theta}\bar{\lambda}^a - i\,\bar{\theta}\bar{\theta}\ \theta\lambda^a + \frac{1}{2}\ \theta\theta\ \bar{\theta}\bar{\theta}\ D^a\right\} \\ &\quad\times\left\{-\sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} V^b_\nu + 2i\ \theta_\alpha\ \bar{\theta}\bar{\lambda}^b - i\ \bar{\theta}\bar{\theta}\ \lambda^b_\alpha \right. \\ &\qquad \left. +\bar{\theta}\bar{\theta}\left[\theta_\alpha D^b - \left(\sigma^{\sigma\nu}\right)_\alpha^{\ \beta}\theta_\beta\left(\partial_\sigma V^b_\nu - \partial_\nu V^b_\sigma\right)\right] + \theta\theta\ \bar{\theta}\bar{\theta}\ \sigma^\nu_{\alpha\dot{\alpha}}\ \partial_\nu\bar{\lambda}^{\dot{\alpha}\,b}\right\} \\ &= \theta\sigma^\mu\bar{\theta}\ \sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\ V^a_\mu V^b_\nu - 2i\,\theta\sigma^\mu\bar{\theta}\ \theta_\alpha\ \bar{\theta}\bar{\lambda}^b\, V^a_\mu - i\ \theta\theta\ \bar{\theta}\bar{\lambda}^a\ \sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} V^b_\nu . \end{aligned}$$

For later convenience, we rewrite this expression. The first term is rewritten by eq. $(A.65)$, while the two next terms are rewritten as follows:

$$\begin{aligned} -2i\,\theta\sigma^\mu\bar{\theta}\ \theta_\alpha\ \bar{\theta}\bar{\lambda}^b\ V^a_\mu &= 2i\ \bar{\theta}_{\dot{\gamma}}\bar{\sigma}^{\mu\,\dot{\gamma}\gamma}\theta_\gamma\ \theta_\alpha\ \bar{\theta}_{\dot{\alpha}}\bar{\lambda}^{\dot{\alpha}\,b}\ V^a_\mu \\ &= 2i\ \left(-\frac{1}{2}\varepsilon_{\dot{\gamma}\dot{\alpha}}\,\bar{\theta}\bar{\theta}\right)\ \bar{\sigma}^{\mu\,\dot{\gamma}\gamma}\ \left(\frac{1}{2}\varepsilon_{\gamma\alpha}\ \theta\theta\right)\ \bar{\lambda}^{\dot{\alpha}\,b}\ V^a_\mu \\ &= -\ \frac{i}{2}\ \theta\theta\ \bar{\theta}\bar{\theta}\ \sigma^\mu_{\alpha\dot{\alpha}}\ \bar{\lambda}^{\dot{\alpha}\,b}\ V^a_\mu , \end{aligned}$$

and similarly

$$\begin{aligned} -i\ \theta\theta\ \bar{\theta}\bar{\lambda}^a\ \sigma^\nu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} V^b_\nu &= i\ \theta\theta\ \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}}\ \sigma^\nu_{\alpha\dot{\alpha}}\ \bar{\lambda}^a_{\dot{\beta}} V^b_\nu \\ &= \frac{i}{2}\ \theta\theta\ \bar{\theta}\bar{\theta}\ \sigma^\nu_{\alpha\dot{\alpha}}\ \bar{\lambda}^{\dot{\alpha}\,a} V^b_\nu . \end{aligned}$$

By collecting terms, eq. $(B.8)$ reads

$$\begin{aligned} V^a D_\alpha V^b &= i\ \bar{\theta}\bar{\theta}\ \left(\sigma^{\mu\nu}\right)_\alpha^{\ \beta}\theta_\beta V^a_\mu V^b_\nu + \frac{1}{2}\ \bar{\theta}\bar{\theta}\ \theta_\alpha\ V^{a\,\mu} V^b_\mu \\ &\quad + \frac{i}{2}\ \theta\theta\ \bar{\theta}\bar{\theta}\ \sigma^\mu_{\alpha\dot{\alpha}}\ \left[\ \bar{\lambda}^{\dot{\alpha}\,a} V^b_\mu - \bar{\lambda}^{\dot{\alpha}\,b} V^a_\mu\right] . \end{aligned} \tag{B.8}$$

The reader should note that the first term of the above expression is antisymmetric under the combined index transformation $\mu \leftrightarrow \nu$ and $a \leftrightarrow b$, while the second and third terms are symmetric and antisymmetric respectively under $a \leftrightarrow b$.

Hence, by taking advantage of the fact that all powers of three (or higher) of vector superfields in the WZ-gauge always vanish, we have

$$\begin{aligned} e^{-2gV} D_\alpha e^{2gV} &= \left(1 - 2gT^a V^a + 2g^2 T^a T^b V^a V^b\right) D_\alpha \left(1 + 2gT^c V^c + 2g^2 T^c T^d V^c V^d\right) \\ &= 2gT^a D_\alpha V^a + 2g^2\ T^a T^b\ \left[D_\alpha\left(V^a V^b\right) - 2\ V^a D_\alpha V^b\right] . \end{aligned} \tag{B.9}$$

We now rewrite the term in the square brackets, and with eqs. $(B.7)$ and $(B.8)$ we obtain

$$\begin{aligned} T^a T^b & \left[D_\alpha \left(V^a V^b \right) - 2\, V^a D_\alpha V^b \right] \\ &= T^a T^b\, \bar\theta\bar\theta \left[-2i\, (\sigma^{\mu\nu})_\alpha^{\ \beta} \theta_\beta V^a_\mu V^b_\nu - i\, \theta\theta\, \sigma^\mu_{\alpha\dot\alpha} \left(\bar\lambda^{\dot\alpha\, a} V^b_\mu - \bar\lambda^{\dot\alpha\, b} V^a_\mu \right) \right]. \end{aligned}$$

Since the terms in the square brackets are antisymmetric under the index transformation $\mu \leftrightarrow \nu$ and $a \leftrightarrow b$ we have

$$\begin{aligned} T^a T^b & \left[D_\alpha \left(V^a V^b \right) - 2\, V^a D_\alpha V^b \right] \\ &= \frac{1}{2} [T^a, T^b]\, \bar\theta\bar\theta \left[-2i\, (\sigma^{\mu\nu})_\alpha^{\ \beta} \theta_\beta V^a_\mu V^b_\nu - i\, \theta\theta\, \sigma^\mu_{\alpha\dot\alpha} \left(\bar\lambda^{\dot\alpha\, a} V^b_\mu - \bar\lambda^{\dot\alpha\, b} V^a_\mu \right) \right] \\ &= f^{abc} T^c\, \bar\theta\bar\theta \left[(\sigma^{\mu\nu})_\alpha^{\ \beta} \theta_\beta V^a_\mu V^b_\nu + \frac{1}{2}\, \theta\theta\, \sigma^\mu_{\alpha\dot\alpha} \left(\bar\lambda^{\dot\alpha\, a} V^b_\mu - \bar\lambda^{\dot\alpha\, b} V^a_\mu \right) \right] \\ &= T^a\, \bar\theta\bar\theta \left[f^{abc} (\sigma^{\mu\nu})_\alpha^{\ \beta} \theta_\beta V^b_\mu V^c_\nu - f^{abc}\, \theta\theta\, \sigma^\mu_{\alpha\dot\alpha} V^b_\mu \bar\lambda^{\dot\alpha\, c} \right]. \end{aligned} \tag{B.10}$$

Here we have used the antisymmetry of f^{abc}. With this result, eq. $(B.9)$ becomes

$$\begin{aligned} e^{-2gV} D_\alpha e^{2gV} &= 2g\, T^a \Big[-\sigma^\mu_{\alpha\dot\alpha} \bar\theta^{\dot\alpha} V^a_\mu + 2i\, \theta_\alpha\, \bar\theta\bar\lambda^a - i\, \bar\theta\bar\theta\, \lambda^a_\alpha \\ &\quad + \bar\theta\bar\theta \left\{ \theta_\alpha D^a - (\sigma^{\mu\nu})_\alpha^{\ \beta} \theta_\beta\, V^a_{\mu\nu} \right\} \\ &\quad + \theta\theta\, \bar\theta\bar\theta\, \sigma^\mu_{\alpha\dot\alpha} \left\{ \partial_\mu \bar\lambda^{\dot\alpha\, a} - g f^{abc} V^b_\mu \bar\lambda^{\dot\alpha\, c} \right\} \Big], \end{aligned} \tag{B.11}$$

where

$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g\, f^{abc} V^b_\mu V^c_\nu, \tag{B.12}$$

is the non-Abelian, conventional fieldstength[3].

Hence, the total fieldstrength becomes with eq. $(A.121)$

$$\begin{aligned} W_\alpha &= -\frac{1}{8g}\, \bar D \bar D e^{-2gV} D_\alpha e^{2gV} \\ &= -\frac{1}{8g}\, 2g\, T^a \Big[-4i\, \lambda^a_\alpha + 4 \left(\theta_\alpha D^a - (\sigma^{\mu\nu})_\alpha^{\ \beta} \theta_\beta\, V^a_{\mu\nu} \right) \\ &\qquad + 4\, \theta\theta \sigma^\mu_{\alpha\dot\alpha} \left(\partial_\mu \bar\lambda^{\dot\alpha\, a} - g\, f^{abc} V^b_\mu \bar\lambda^{\dot\alpha\, c} \right) \Big] \\ &= T^a \left[i\, \lambda^a_\alpha - \theta_\alpha\, D^a + (\sigma^{\mu\nu})_\alpha^{\ \beta} \theta_\beta\, V^a_{\mu\nu} - \theta\theta\, \sigma^\mu_{\alpha\dot\alpha} \left(\partial_\mu \bar\lambda^{\dot\alpha\, a} - g\, f^{abc} V^b_\mu \bar\lambda^{\dot\alpha\, c} \right) \right]. \end{aligned} \tag{B.13}$$

By hermitian conjugation the component expansion for $\bar W_{\dot\alpha}$ is obtained, and it reads

$$\bar W_{\dot\alpha} = T^a \left[-i\, \bar\lambda^a_{\dot\alpha} - \bar\theta_{\dot\alpha} D^a + \bar\theta_{\dot\beta}\, (\bar\sigma^{\mu\nu})^{\dot\beta}_{\ \dot\alpha}\, V^a_{\mu\nu} - \bar\theta\bar\theta\, \sigma^\mu_{\alpha\dot\alpha} \left(\partial_\mu \lambda^{\alpha\, a} - g\, f^{abc} V^b_\mu \lambda^{\alpha\, c} \right) \right]. \tag{B.14}$$

The fieldstrengths with upper spinor indices are obtained in the usual way by applying $\varepsilon^{\alpha\beta}$ and $\varepsilon^{\dot\alpha\dot\beta}$ to the above expressions

$$W^\alpha = \varepsilon^{\alpha\beta} W_\beta, \tag{B.15}$$

$$\bar W^{\dot\alpha} = \varepsilon^{\dot\alpha\dot\beta} \bar W_{\dot\beta}. \tag{B.16}$$

[3]The factor of two in front of the coupling constant was inserted in order to make this identification possible.

B.2 Calculation $\int d^4\theta \ (1/4k) \ Tr\,(W^\alpha W_\alpha) \ \delta^2(\bar\theta) + h.c.$

Since W_α is Lie-Algebra valued one has

$$\begin{aligned} &\int d^4\theta \, \frac{1}{4k} \, Tr\,(W^\alpha W_\alpha) \ \delta^2(\bar\theta) \\ &\qquad = \int d^4\theta \, \frac{1}{4k} \, Tr\left(T^a T^b\right) W^{\alpha\, a} W^b_\alpha \ \delta^2(\bar\theta) \\ &\qquad = \frac{1}{4} \ W^{a\,\alpha} W^a_\alpha|_{\theta\theta} \, . \end{aligned} \tag{B.17}$$

Here we have used the normalization (in the adjoint representation)

$$Tr\left(T^a T^b\right) \ = \ k\delta^{ab}. \tag{B.18}$$

Furthermore, eqs. $(B.13)$, $(A.69)$, $(A.52)$, $(A.64)$ and $(A.50)$ yield

$$\begin{aligned} &\frac{1}{4} \ W^{a\,\alpha} W^a_\alpha|_{\theta\theta} \\ &\quad = \frac{1}{4} \, \varepsilon^{\alpha\beta} \Big\{ \Big[i\,\lambda^a_\beta - \theta_\beta \, D^a + (\sigma^{\mu\nu})_\beta{}^\delta \, \theta_\delta \, V^a_{\mu\nu} - \theta\theta \, \sigma^\mu_{\beta\dot\beta} \left(\partial_\mu \bar\lambda^{\dot\beta\, a} - g\, f^{abc} V^b_\mu \bar\lambda^{\dot\beta\, c} \right) \Big] \\ &\qquad\qquad \times \Big[i\,\lambda^a_\alpha - \theta_\alpha \, D^a + (\sigma^{\rho\sigma})_\alpha{}^\gamma \, \theta_\gamma \, V^a_{\rho\sigma} - \theta\theta \, \sigma^\mu_{\alpha\dot\alpha} \left(\partial_\rho \bar\lambda^{\dot\alpha\, a} - g\, f^{ab'c'} V^{b'}_\rho \bar\lambda^{\dot\alpha\, c'} \right) \Big] \Big\} \Big|_{\theta\theta} \\ &\quad = -\frac{i}{4} \, \lambda^a \sigma^\mu \left(\partial_\mu \bar\lambda^a - g\, f^{abc} V^b_\mu \bar\lambda^c \right) + \frac{1}{4} \, D^a D^a \\ &\qquad - \frac{1}{4} \, \theta\sigma^{\mu\nu}\theta \ D^a V^a_{\mu\nu} + \frac{1}{8} \, (\sigma^{\mu\nu})_\beta{}^\delta (\sigma^{\rho\sigma})_\alpha{}^\gamma \varepsilon^{\alpha\beta} \varepsilon_{\delta\gamma} V^a_{\mu\nu} V^a_{\rho\sigma} \\ &\qquad - \frac{i}{4} \lambda^a \sigma^\mu \left(\partial_\mu \bar\lambda^a - g\, f^{abc} V^b_\mu \bar\lambda^c \right) \\ &\quad = -\frac{i}{2} \lambda^a \sigma^\mu \left(\partial_\mu \bar\lambda^a - g\, f^{abc} V^b_\mu \bar\lambda^c \right) + \frac{1}{4} \, D^a D^a \\ &\qquad + \frac{1}{8} \, (\sigma^{\mu\nu})_\beta{}^\delta (\sigma^{\rho\sigma})_\alpha{}^\gamma \left(\delta^\alpha{}_\gamma \delta^\beta{}_\delta - \delta^\alpha{}_\delta \delta^\beta{}_\gamma \right) V^a_{\mu\nu} V^a_{\rho\sigma} . \end{aligned} \tag{B.19}$$

By rewriting the last term of eq. $(B.19)$ with use of eqs. $(A.34)$ and $(A.34)$, one obtains

$$
\begin{aligned}
&\frac{1}{8}\,(\sigma^{\mu\nu})_\beta{}^\delta(\sigma^{\rho\sigma})_\alpha{}^\gamma\left(\delta^\alpha{}_\gamma\delta^\beta{}_\delta-\delta^\alpha{}_\delta\delta^\beta{}_\gamma\right)V^a_{\mu\nu}V^a_{\rho\sigma}\\
&\quad=\ \frac{1}{8}\left((\sigma^{\mu\nu})_\beta{}^\beta(\sigma^{\rho\sigma})_\alpha{}^\alpha-(\sigma^{\mu\nu}\sigma^{\rho\sigma})_\beta{}^\beta\right)V^\alpha_{\mu\nu}V^a_{\rho\sigma}\\
&\quad=\ -\frac{1}{16}\,(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho}+i\varepsilon^{\mu\nu\rho\sigma})\,V^a_{\mu\nu}V^a_{\rho\sigma}\\
&\quad=\ -\frac{1}{8}\,V^{a\,\mu\nu}V^a_{\mu\nu}-\frac{i}{16}\,\varepsilon^{\mu\nu\rho\sigma}V^a_{\mu\nu}V^a_{\rho\sigma},
\end{aligned}
\tag{B.20}
$$

where we in the last transition have used the antisymmetry of the conventional fieldstrength. Thus one can conclude

$$
\begin{aligned}
&\int d^4\theta\;\frac{1}{4k}\,Tr\,(W^\alpha W_\alpha)\;\delta^2(\bar\theta)\\
&\quad=\ -\frac{i}{2}\lambda^a\sigma^\mu\left(\partial_\mu\bar\lambda^a-g\,f^{abc}V^b_\mu\bar\lambda^c\right)+\frac{1}{4}\,D^aD^a-\frac{1}{8}\,V^{a\,\mu\nu}V^a_{\mu\nu}\\
&\qquad-\frac{i}{16}\,\varepsilon^{\mu\nu\rho\sigma}V^a_{\mu\nu}V^a_{\rho\sigma}.
\end{aligned}
\tag{B.21}
$$

With eq. $(A.62)$ the first term of the above equation can be rewritten as follows

$$
\begin{aligned}
&-\frac{i}{2}\lambda^a\sigma^\mu\left(\partial_\mu\bar\lambda^a-g\,f^{abc}V^b_\mu\bar\lambda^c\right)\\
&\quad=\ \frac{i}{2}\left(\partial_\mu\bar\lambda^a-gf^{abc}V^b_\mu\bar\lambda^c\right)\bar\sigma^\mu\lambda^a\\
&\quad=\ -\frac{i}{2}\,\bar\lambda^a\bar\sigma^\mu\left(\partial_\mu\lambda^a-gf^{abc}V^b_\mu\lambda^c\right)+\frac{i}{2}\,\partial_\mu\left(\bar\lambda^a\bar\sigma^\mu\lambda^a\right)\\
&\quad=\ -\frac{i}{2}\,\bar\lambda^a\bar\sigma^\mu D_\mu\lambda^a+\frac{i}{2}\,\partial_\mu\left(\bar\lambda^a\bar\sigma^\mu\lambda^a\right).
\end{aligned}
\tag{B.22}
$$

Here we have introduced the $SU(2)\times U(1)$-covariant derivative

$$
D_\mu\ =\ \partial_\mu+igT^aV^a_\mu+ig'\frac{Y}{2}V'_\mu,\qquad a=1,2,3,
$$

and when it operates on e.g. the gaugino λ^a, which lay in the adjoint representation of the gauge group, i.e.

$$
\begin{aligned}
\left[T^c_{adj}\right]^{ab}\ &=\ -if^{cab},\\
Y_{adj}\ &=\ 0,
\end{aligned}
$$

we have

$$
\begin{aligned}
D_\mu\lambda^a\ &\equiv\ [D_\mu\lambda]^a\\
&=\ [D_\mu]^{ab}\,\lambda^b\\
&=\ \left(\partial_\mu\delta^{ab}+ig\left[T^c_{adj}\right]^{ab}V^c_\mu+ig'\frac{Y_{adj}}{2}V'_\mu\right)\lambda^b\\
&=\ \partial_\mu\lambda^a-gf^{abc}V^b_\mu\lambda^c.
\end{aligned}
\tag{B.23}
$$

This yields for eq. $(B.21)$

$$\int d^4\theta \, \frac{1}{4k} Tr \left(W^\alpha W_\alpha\right) \, \delta^2(\bar\theta) = -\frac{i}{2}\bar\lambda^a \bar\sigma^\mu D_\mu \lambda^a + \frac{1}{4} \, D^a D^a - \frac{1}{8} \, V^{a\,\mu\nu} V^a_{\mu\nu} - \frac{i}{16} \, \varepsilon^{\mu\nu\rho\sigma} V^a_{\mu\nu} V^a_{\rho\sigma} + \frac{i}{2}\partial_\mu \left(\bar\lambda^a \bar\sigma^\mu \lambda^a\right). \quad (B.24)$$

Hence by hermitian conjugation (of eq. $(B.21)$), one has

$$\begin{aligned} \int d^4\theta \, \frac{1}{4k} Tr \left(\bar W_{\dot\alpha} \bar W^{\dot\alpha}\right) \, \delta^2(\theta) &= \frac{i}{2} \, D_\mu \lambda^a \sigma^\mu \bar\lambda^a + \frac{1}{4} \, D^a D^a - \frac{1}{8} \, V^{a\,\mu\nu} V^a_{\mu\nu} \\ &\quad + \frac{i}{16} \, \varepsilon^{\mu\nu\rho\sigma} V^a_{\mu\nu} V^a_{\rho\sigma} \\ &= -\frac{i}{2}\bar\lambda^a \bar\sigma^\mu D_\mu \lambda^a + \frac{1}{4} \, D^a D^a - \frac{1}{8} \, V^{a\,\mu\nu} V^a_{\mu\nu} \\ &\quad + \frac{i}{16} \, \varepsilon^{\mu\nu\rho\sigma} V^a_{\mu\nu} V^a_{\rho\sigma} \end{aligned} \quad (B.25)$$

and by adding eqs. $(B.24)$ and $(B.25)$ we may conclude

$$\begin{aligned} &\frac{1}{4k} \int d^4\theta \, \left\{ Tr \left(W^\alpha W_\alpha\right) \, \delta^2(\bar\theta) + Tr \left(\bar W_{\dot\alpha} \bar W^{\dot\alpha}\right) \, \delta^2(\theta)\right\} \\ &= -i \, \bar\lambda^a \bar\sigma^\mu D_\mu \lambda^a + \frac{1}{2} \, D^a D^a - \frac{1}{4} \, V^{a\,\mu\nu} V^a_{\mu\nu} + \frac{i}{2} \, \partial_\mu \left(\bar\lambda^a \bar\sigma^\mu \lambda^a\right). \end{aligned} \quad (B.26)$$

B.3 Calculating $\int d^4\theta \; \hat\phi^\dagger e^{2g\hat V + g'\hat V'} \hat\phi$.

In this section we will derive the component form of $\int d^4\theta \; \hat\phi^\dagger e^{2g\hat V + g'\hat V'} \hat\phi$. Here $\hat\phi(x,\theta,\bar\theta)$ is a chiral superfield and $\hat V(x,\theta,\bar\theta)$ and $V'(x,\theta,\bar\theta)$ are gauge vector superfields for some non-Abelian group $\mathcal{G}$ and U(1) respectively.

As usual we work in the WZ-gauge with Lie-algebra valued gauge superfields of the form

$$\hat V(x,\theta,\bar\theta) = T^a \hat V^a(x,\theta,\bar\theta), \quad (B.27)$$

$$\hat V'(x,\theta,\bar\theta) = Y \hat v'(x,\theta,\bar\theta), \quad (B.28)$$

and with the following component expansions for the superfields

$$\begin{aligned} \hat\phi(x,\theta,\bar\theta) &= A(x) + i \, \theta\sigma^\mu\bar\theta \, \partial_\mu A(x) - \frac{1}{4} \, \theta\theta \, \bar\theta\bar\theta \, \partial^\mu\partial_\mu A(x) \\ &\quad + \sqrt{2} \, \theta\psi(x) + \frac{i}{\sqrt{2}} \, \theta\theta \, \bar\theta\bar\sigma^\mu \partial_\mu \psi(x) + \theta\theta \, F(x), \end{aligned} \quad (B.29)$$

$$\hat\phi^\dagger(x,\theta,\bar\theta) = A^\dagger(x) - i \, \theta\sigma^\mu\bar\theta \, \partial_\mu A^\dagger(x) - \frac{1}{4} \, \theta\theta \, \bar\theta\bar\theta \, \partial^\mu\partial_\mu A^\dagger(x)$$

$$+\sqrt{2}\,\bar\theta\bar\psi(x)+\frac{i}{\sqrt{2}}\,\bar\theta\bar\theta\;\theta\sigma^\mu\partial_\mu\bar\psi(x)+\bar\theta\bar\theta\;F^\dagger(x), \tag{B.30}$$

$$\hat V^a(x,\theta,\bar\theta) \;=\; -\,\theta\sigma^\mu\bar\theta\;V^a_\mu(x)+i\;\theta\theta\;\bar\theta\bar\lambda^a(x)-i\;\bar\theta\bar\theta\;\theta\lambda^a(x)+\frac{1}{2}\;\theta\theta\,\bar\theta\bar\theta\;D^a(x), \tag{B.31}$$

$$\hat v'(x,\theta,\bar\theta) \;=\; -\,\theta\sigma^\mu\bar\theta\;V'_\mu(x)+i\;\theta\theta\;\bar\theta\bar\lambda'(x)-i\;\bar\theta\bar\theta\;\theta\lambda'(x)+\frac{1}{2}\;\theta\theta\,\bar\theta\bar\theta\;D'(x). \tag{B.32}$$

Furthermore, $\hat\phi(x,\theta,\bar\theta)$ will be taken to lie in a representation of the gauge group $\mathcal{G}\,\times\,U(1)$ described by the matrix representation T^a and the hypercharge quantum number Y. Hence $\hat\phi(x,\theta,\bar\theta)$, and its component fields, are generally matrix-valued. As our notation indicates, we will work in the $(x,\theta,\bar\theta)$-basis, and from now on this dependence will be suppressed.

Since the two gauge super-multiplets are commuting, i.e. $[\hat V,\hat V']=0$, and we are working in the WZ-gauge, we have

$$\begin{aligned} e^{2g\hat V+g'\hat V'} &= \left(1+2gT^a\hat V^a+2g^2T^aT^b\hat V^a\hat V^b\right)\\ &\quad\times\left(1+g'Y\hat v'+\frac{1}{2}g'^2Y^2\hat v'^2\right)\\ &= 1+g'Y\hat v'+2gT^a\hat V^a+\frac{g'^2}{2}Y^2\hat v'^2+2g^2T^aT^b\hat V^a\hat V^b\\ &\quad+2gg'YT^a\hat V^a\hat v.' \end{aligned} \tag{B.33}$$

Here in the last line we have used the fact that third powers of vector superfields in the WZ-gauge always vanish. Furtermore

$$\hat V^a\hat V^b \;=\; \frac{1}{2}\;\theta\theta\;\bar\theta\bar\theta\;V^{a\;\mu}V^b_\mu, \tag{B.34}$$

$$\hat v'^2 \;=\; \frac{1}{2}\;\theta\theta\;\bar\theta\bar\theta\;V'^\mu V'_\mu, \tag{B.35}$$

$$\hat V^a\hat v' \;=\; \frac{1}{2}\;\theta\theta\;\bar\theta\bar\theta\;V^{a\;\mu}V'_\mu, \tag{B.36}$$

and hence with eqs. $(B.31)$, $(B.32)$ and $(B.34)$–$(B.36)$ substituted into eq. $(B.33)$ one obtains

$$\begin{aligned} e^{2g\hat V+g'\hat V'} &= 1-\theta\sigma^\mu\bar\theta\;\left[\,2gT^aV^a_\mu+g'YV'_\mu\,\right]\\ &\quad+i\;\theta\theta\;\bar\theta\;\left[\,2gT^a\bar\lambda^a+g'Y\bar\lambda'\,\right]-i\;\bar\theta\bar\theta\,\theta\;\left[\,2gT^a\lambda^a+g'Y\lambda'\,\right]\\ &\quad+\frac{1}{2}\;\theta\theta\;\bar\theta\bar\theta\;\Big[\,2gT^aD^a+g'YD'+2g^2\;T^aT^bV^{a\;\mu}V^b_\mu\\ &\qquad\qquad+\frac{1}{2}g'^2Y^2V'^\mu V'_\mu+2gg'YT^aV^{a\;\mu}V'_\mu\Big]. \end{aligned} \tag{B.37}$$

Postmultiplying the above expression with $\hat\phi$ yields

$e^{2g\hat V+g'\hat V'}\;\hat\phi$

$$
\begin{aligned}
&= \; 1-\theta\sigma^{\mu}\bar{\theta}\;\Big[\,2gT^aV^a_\mu+g'YV'_\mu\,\Big] \\
&\quad +i\,\theta\theta\,\bar{\theta}\;\Big[\,2gT^a\bar{\lambda}^a+g'Y\bar{\lambda}'\,\Big]-i\,\bar{\theta}\bar{\theta}\,\theta\;[\,2gT^a\lambda^a+g'Y\lambda'\,] \\
&\quad +\frac{1}{2}\,\theta\theta\,\bar{\theta}\bar{\theta}\;\Big[\,2gT^aD^a+g'YD'+2g^2\,T^aT^bV^{a\,\mu}V^b_\mu \\
&\qquad\qquad +\frac{1}{2}g'^2Y^2V'^{\mu}V'_\mu+2gg'YT^aV^{a\,\mu}V'_\mu\Big] \\
&\times\;\Big[A+i\,\theta\sigma^\mu\bar{\theta}\,\partial_\mu A-\frac{1}{4}\,\theta\theta\,\bar{\theta}\bar{\theta}\,\partial^\mu\partial_\mu A \\
&\qquad +\sqrt{2}\,\theta\psi+\frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi+\theta\theta\,F\Big] \\
&= \; A-\theta\sigma^{\mu}\bar{\theta}\;\Big[\,2gT^aV^a_\mu+g'YV'_\mu\,\Big]\;A \\
&\quad +i\,\theta\theta\,\bar{\theta}\Big[\,2gT^a\bar{\lambda}^a+g'Y\bar{\lambda}'\,\Big]\;A-i\,\bar{\theta}\bar{\theta}\,\theta\,[\,2gT^a\lambda^a+g'Y\lambda'\,]\;A \\
&\quad +\frac{1}{2}\,\theta\theta\,\bar{\theta}\bar{\theta}\;\Big[\,2gT^aD^a+g'YD'+2g^2\,T^aT^bV^{a\,\mu}V^b_\mu \\
&\qquad\qquad +\frac{1}{2}g'^2Y^2V'^{\mu}V'_\mu+2gg'YT^aV^{a\,\mu}V'_\mu\Big]\;A \\
&\quad +i\,\theta\sigma^\mu\bar{\theta}\,\partial_\mu A-i\,\theta\sigma^\mu\bar{\theta}\,\theta\sigma^\nu\bar{\theta}\;\Big[\,2gT^aV^a_\mu+g'YV'_\mu\Big]\;\partial_\nu A \\
&\quad -\frac{1}{4}\,\theta\theta\,\bar{\theta}\bar{\theta}\,\partial^\mu\partial_\mu A+\sqrt{2}\,\theta\psi-\sqrt{2}\,\theta\sigma^\mu\bar{\theta}\;\Big[\,2gT^aV^a_\mu+g'YV'_\mu\Big]\;\theta\psi \\
&\quad -\sqrt{2}i\,\bar{\theta}\bar{\theta}\,\theta\,[\,2gT^a\lambda^a+g'Y\lambda'\,]\;\theta\psi+\frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi+\theta\theta\,F.
\end{aligned}
$$

Now we rewrite the fourth and third last term as follows:

$$
\begin{aligned}
-\sqrt{2}\,\theta\sigma^\mu\bar{\theta}\;\Big[\,2gT^aV^a_\mu+g'YV'_\mu\Big]\;\theta\psi &= \sqrt{2}\;\Big[\,2gT^aV^a_\mu+g'YV'_\mu\Big]\;\theta^\alpha\,\theta^\beta\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\psi_\beta \\
&= -\sqrt{2}\;\Big[\,2gT^aV^a_\mu+g'YV'_\mu\Big]\;\frac{1}{2}\,\varepsilon^{\alpha\beta}\theta\theta\,\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\psi_\beta \\
&= -\frac{1}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\sigma}^\mu\psi\;\Big[\,2gT^aV^a_\mu+g'YV'_\mu\Big], \\
-\sqrt{2}i\,\bar{\theta}\bar{\theta}\,\theta\,[\,2gT^a\lambda^a+g'Y\lambda'\,]\;\theta\psi &= -\sqrt{2}i\,\bar{\theta}\bar{\theta}\;\theta^\alpha\,[\,2gT^a\lambda^a_\alpha+g'Y\lambda'_\alpha\,]\;\theta^\beta\psi_\beta \\
&= -\frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\theta}\,[\,2gT^a\lambda^a_\alpha+g'Y\lambda'_\alpha\,]\;\psi^\alpha \\
&= \frac{i}{\sqrt{2}}\,\theta\theta\,\bar{\theta}\bar{\theta}\,[\,2gT^a\lambda^a+g'Y\lambda'\,]\,\psi, \qquad \text{(B.38)}
\end{aligned}
$$

and thus

$$
\begin{aligned}
&e^{2g\hat{V}+g'\hat{V}'}\;\hat{\phi} \\
&\quad= A+\sqrt{2}\,\theta\psi+\theta\theta\,F+\theta\sigma^\mu\bar{\theta}\;\Big\{\,i\partial_\mu A-\Big[\,2gT^aV^a_\mu+g'YV'_\mu\Big]\;A\,\Big\} \\
&\qquad +\theta\theta\,\bar{\theta}\Big\{\,i\,\Big[\,2gT^a\bar{\lambda}^a+g'Y\bar{\lambda}'\,\Big]\;A
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{\sqrt{2}}\ \Big[\,2gT^aV^a_\mu + g'YV'_\mu\,\Big]\,\bar\sigma^\mu\psi + \frac{i}{\sqrt{2}}\ \bar\sigma^\mu\partial_\mu\psi\Big\} \\
& - i\ \bar\theta\bar\theta\ \theta\,[\,2gT^a\lambda^a + g'Y\lambda'\,]\ A \\
& + \frac{1}{2}\ \theta\theta\ \bar\theta\bar\theta\ \Big\{\ \Big[\ 2gT^aD^a + g'YD' + 2g^2\ T^aT^bV^{a\ \mu}V^b_\mu \\
& \qquad + \frac{1}{2}g'^2Y^2V'^\mu V'_\mu + 2gg'YT^aV^{a\ \mu}V'_\mu\Big]\ A \\
& \qquad - i\ \Big[\ 2gT^aV^a_\mu + g'YV'_\mu\Big]\ \partial^\mu A - \frac{1}{2}\ \partial^\mu\partial_\mu A \\
& \qquad + \sqrt{2}i\ \big[\ 2gT^a\lambda^a + g'Y\lambda'\ \big]\ \psi\Big\}\,. \qquad \text{(B.39)}
\end{aligned}
$$

Finally we can address the main purpose of this section. By premultiplying the above result by $\hat\phi^\dagger$ and projecting out the $\theta\theta\ \bar\theta\bar\theta$-component, equivalent to a Grassmann integration, we obtain:

$$
\begin{aligned}
& \int d^4\theta\ \hat\phi^\dagger e^{2g\hat V + g'\hat V'}\hat\phi \\
= & \int d^4\theta\ \Big[A^\dagger - i\ \theta\sigma^\mu\bar\theta\ \partial_\mu A^\dagger - \frac{1}{4}\ \theta\theta\ \bar\theta\bar\theta\ \partial^\mu\partial_\mu A^\dagger \\
& \qquad + \sqrt{2}\ \bar\theta\bar\psi + \frac{i}{\sqrt{2}}\ \bar\theta\bar\theta\ \theta\sigma^\mu\partial_\mu\bar\psi + \bar\theta\bar\theta\ F^\dagger\Big] \\
& \quad \times \Big[A + \sqrt{2}\ \theta\psi + \theta\theta\ F + \theta\sigma^\mu\bar\theta\ \Big\{\ i\partial_\mu A - \Big[\ 2gT^aV^a_\mu + g'YV'_\mu\Big]\ A\ \Big\} \\
& \qquad + \theta\theta\ \bar\theta\Big\{\ i\Big[\ 2gT^a\bar\lambda^a + g'Y\bar\lambda'\ \Big]\ A \\
& \qquad\qquad - \frac{1}{\sqrt{2}}\ \Big[\ 2gT^aV^a_\mu + g'YV'_\mu\Big]\,\bar\sigma^\mu\psi + \frac{i}{\sqrt{2}}\ \bar\sigma^\mu\partial_\mu\psi\Big\} \\
& \qquad - i\ \bar\theta\bar\theta\ \theta\,[\,2gT^a\lambda^a + g'Y\lambda'\,]\ A \\
& \qquad + \frac{1}{2}\ \theta\theta\ \bar\theta\bar\theta\ \Big\{\ \Big[\ 2gT^aD^a + g'YD' + 2g^2\ T^aT^bV^{a\ \mu}V^b_\mu \\
& \qquad\qquad + \frac{1}{2}g'^2Y^2V'^\mu V'_\mu + 2gg'YT^aV^{a\ \mu}V'_\mu\Big]\ A \\
& \qquad\qquad - i\ \Big[\ 2gT^aV^a_\mu + g'YV'_\mu\Big]\ \partial^\mu A - \frac{1}{2}\ \partial^\mu\partial_\mu A \\
& \qquad\qquad + \sqrt{2}i\ \big[\ 2gT^a\lambda^a + g'Y\lambda'\ \big]\ \psi\Big]\Big\}\Big] \\
= & \ A^\dagger\ \Big[\ gT^aD^a + \frac{1}{2}g'YD' + g^2\ T^aT^bV^{a\ \mu}V^b_\mu \\
& \qquad + \frac{1}{4}g'^2Y^2V'^\mu V'_\mu + gg'YT^aV^{a\ \mu}V'_\mu\Big]\ A \\
& - i\ A^\dagger\ \Big[\ gT^aV^a_\mu + \frac{1}{2}g'YV'_\mu\Big]\ \partial^\mu A - \frac{1}{4}\ A^\dagger\partial^\mu\partial_\mu A \\
& + \sqrt{2}i\ A^\dagger\ \Big[\ gT^a\lambda^a + \frac{1}{2}g'Y\lambda'\Big]\ \psi
\end{aligned}
$$

$$
\begin{aligned}
&- i\,\theta\sigma^\nu\bar\theta\;\theta\sigma^\mu\bar\theta\;\partial_\nu A^\dagger\;\left\{ i\partial_\mu A - 2\left[gT^aV^a_\mu + \frac{1}{2}\,g'YV'_\mu \right]\;A\right\}\Big|_{\theta\theta\;\bar\theta\bar\theta} - \frac{1}{4}\;\partial^\mu\partial_\mu A^\dagger A \\
&+ \sqrt{2}\;\bar\theta\bar\psi\;\theta\theta\;\bar\theta\left\{\;2i\left[gT^a\bar\lambda^a + \frac{1}{2}\,g'Y\bar\lambda'\right]\;A\right\}\Big|_{\theta\theta\;\bar\theta\bar\theta} \\
&+ \sqrt{2}\;\bar\theta\bar\psi\;\theta\theta\;\bar\theta\left\{-\sqrt{2}\;\left[gT^aV^a_\mu + \frac{1}{2}\;g'YV'_\mu\right]\bar\sigma^\mu\psi + \frac{i}{\sqrt{2}}\;\bar\sigma^\mu\partial_\mu\psi\right\}\Bigg|_{\theta\theta\;\bar\theta\bar\theta} \\
&+ i\;\bar\theta\bar\theta\;\theta\sigma^\mu\partial_\mu\bar\psi\;\theta\psi\Big|_{\theta\theta\;\bar\theta\bar\theta} + F^\dagger F. \qquad\qquad (\mathrm{B}.40)
\end{aligned}
$$

With eq. $(A.76)$ and the following results

$$
\begin{aligned}
\sqrt{2}\;\bar\theta\bar\psi\;\theta\theta\;\bar\theta_{\dot\alpha} &= \sqrt{2}\;\theta\theta\bar\theta_{\dot\alpha}\bar\theta_{\dot\beta}\bar\psi^{\dot\beta} = -\frac{1}{\sqrt{2}}\,\theta\theta\,\bar\theta\bar\theta\;\bar\psi_{\dot\alpha}, \\
i\;\bar\theta\bar\theta\;\theta\sigma^\mu\partial_\mu\bar\psi\;\theta\psi &= -i\;\bar\theta\bar\theta\;\theta^\alpha\,\theta^\beta\,\sigma^\mu_{\alpha\dot\alpha}\,\partial_\mu\bar\psi^{\dot\alpha}\;\psi_\beta = -\frac{i}{2}\;\theta\theta\,\bar\theta\bar\theta\;\psi\sigma^\mu\partial_\mu\bar\psi,
\end{aligned}
$$

eq. $(B.40)$ becomes

$$
\begin{aligned}
&\int d^4\theta\;\hat\phi^\dagger e^{2g\hat V + g'\hat V'}\,\hat\phi \\
&\quad= A^\dagger\;\left[gT^aD^a + \frac{1}{2}g'YD' + g^2\,T^aT^bV^{a\;\mu}V^b_\mu \right. \\
&\qquad\qquad \left. + \frac{1}{4}g'^2Y^2V'^\mu V'_\mu + gg'YT^aV^{a\;\mu}V'_\mu\right]\;A \\
&\qquad - i\;A^\dagger\;\left[gT^aV^a_\mu + \frac{1}{2}g'YV'_\mu\right]\;\partial^\mu A - \frac{1}{4}\;A^\dagger\partial^\mu\partial_\mu A \\
&\qquad + \sqrt{2}i\;A^\dagger\;\left[gT^a\lambda^a + \frac{1}{2}g'Y\lambda'\right]\;\psi \\
&\qquad + \frac{1}{2}\,\partial^\mu A^\dagger\partial_\mu A + i\,\partial^\mu A^\dagger\left[gT^aV^a_\mu + \frac{1}{2}\,g'YV'_\mu\right]\;A - \frac{1}{4}\;\partial^\mu\partial_\mu A^\dagger A \\
&\qquad - \sqrt{2}i\;\bar\psi\left[gT^a\bar\lambda^a + \frac{1}{2}\,g'Y\bar\lambda'\right]\;A + \bar\psi\left[gT^aV^a_\mu + \frac{1}{2}\;g'YV'_\mu\right]\bar\sigma^\mu\psi \\
&\qquad - \frac{i}{2}\;\bar\psi\bar\sigma^\mu\partial_\mu\psi - \frac{i}{2}\;\psi\sigma^\mu\partial_\mu\bar\psi + F^\dagger F. \qquad\qquad (\mathrm{B}.41)
\end{aligned}
$$

With the following identities

$$
\begin{aligned}
\psi\sigma^\mu\partial_\mu\bar\psi &= -\partial_\mu\psi\sigma^\mu\bar\psi + \partial_\mu\left(\psi\sigma^\mu\bar\psi\right) \\
&= \bar\psi\bar\sigma^\mu\partial_\mu\psi + \partial_\mu\left(\psi\sigma^\mu\bar\psi\right), \\
\partial^\mu\partial_\mu A^\dagger A &= -\partial^\mu A^\dagger\partial_\mu A + \partial_\mu\left(\partial^\mu A^\dagger A\right), \\
A^\dagger\partial^\mu\partial_\mu A &= -\partial^\mu A^\dagger\partial_\mu A + \partial_\mu\left(A^\dagger\partial^\mu A\right),
\end{aligned}
$$

the final expression for the matter Lagrangian reads

$$
\int d^4\theta\;\hat\phi^\dagger e^{2g\hat V + g'\hat V'}\,\hat\phi
$$

$$
\begin{aligned}
= \quad & A^\dagger \left[gT^a D^a + \frac{1}{2} g' Y D' \right] A + \partial^\mu A^\dagger \partial_\mu A \\
& + A^\dagger \left[g^2\, T^a T^b V^{a\,\mu} V^b_\mu + \frac{1}{4} g'^2 Y^2 V'^\mu V'_\mu + g g' Y T^a V^{a\,\mu} V'_\mu \right] A \\
& + i\, \partial^\mu A^\dagger \left[gT^a V^a_\mu + \frac{1}{2} g' Y V'_\mu \right] A - i\, A^\dagger \left[gT^a V^a_\mu + \frac{1}{2} g' Y V'_\mu \right] \partial^\mu A \\
& + \sqrt{2} i\, A^\dagger \left[gT^a \lambda^a + \frac{1}{2} g' Y \lambda' \right] \psi - \sqrt{2} i\, \bar{\psi} \left[gT^a \bar{\lambda}^a + \frac{1}{2} g' Y \bar{\lambda}' \right] A \\
& + \bar{\psi} \bar{\sigma}^\mu \left[gT^a V^a_\mu + \frac{1}{2} g' Y V'_\mu \right] \psi - i\, \bar{\psi} \bar{\sigma}^\mu \partial_\mu \psi + F^\dagger F + t.d. \qquad \text{(B.42)}
\end{aligned}
$$

Here t.d. means a total derivative (t.d.), and it can be neglected if we like. This is so due to the four dimensional Gauss-theorem[4] which implies that the total derivative does not contribute to the action.

B.3.1 Introducing the Covariant Derivative.

Eq. $(B.42)$ can be simplified even further if we introduce the $\mathcal{G} \times U(1)$-covariant derivative, defined by

$$
D_\mu \quad = \quad \partial_\mu + igT^a V^a_\mu + ig' \frac{Y}{2} V'_\mu. \qquad \text{(B.43)}
$$

Here T^a and Y have the same meaning as in the previous section.

With this definition one has

$$
\begin{aligned}
& (D^\mu A)^\dagger (D_\mu A) \\
= \quad & \left(\partial^\mu A + igT^a V^{a\,\mu} A + ig' \frac{Y}{2} V'^\mu A \right)^\dagger \left(\partial_\mu A + igT^b V^b_\mu A + ig' \frac{Y}{2} V'_\mu A \right) \\
= \quad & \partial^\mu A^\dagger \partial_\mu A \\
& + i\, \partial^\mu A^\dagger \left(gT^a V^a_\mu + g' \frac{Y}{2} V'_\mu \right) A - i\, A^\dagger \left(gT^a V^a_\mu + g' \frac{Y}{2} V'_\mu \right) \partial^\mu A \\
& + A^\dagger \left(g^2 T^a T^b V^{a\,\mu} V^b_\mu + g g'\, T^a Y V^{a\,\mu} V'_\mu + \frac{g'^2}{4} Y^2 V'^\mu V'_\mu \right) A, \qquad \text{(B.44)}
\end{aligned}
$$

and substituting eqs. $(B.43)$ and $(B.44)$ into eq. $(B.42)$ yields

$$
\int d^4\theta\ \hat{\phi}^\dagger e^{2g\hat{V} + g'\hat{V}'} \hat{\phi}
$$

[4] The four dimensional Gauss theorem states that

$$
\int_V d^4x\, F(x) \quad = \quad \int_S d^3 S^\mu\, \partial_\mu F(x),
$$

where S is a 3-dimmentional surface enclosing the 4-dimensional volume V. In our case, V denotes the total 4-space, and hence S is an surface at infinity. Since the fields are assumed to vanish at infinity, the right hand side vanish because F(x) is some function of quantum fields.

$$
\begin{aligned}
&= (D^\mu A)^\dagger (D_\mu A) - i\,\bar\psi\bar\sigma^\mu D_\mu\psi \\
&\quad + A^\dagger \left(gT^a D^a + g'\frac{Y}{2}D' \right) A \\
&\quad + \sqrt{2}i\; A^\dagger \left[\, gT^a\lambda^a + g'\frac{Y}{2}\lambda' \,\right]\psi - \sqrt{2}i\;\bar\psi\left[\, gT^a\bar\lambda^a + g'\frac{Y}{2}\bar\lambda' \,\right] A \\
&\quad + F^\dagger F + t.d. \qquad\qquad \text{(B.45)}
\end{aligned}
$$

This concludes this section.

B.4 Component Expansion of $\mathcal{L}_{SUSY}$.

Now we will leave the general situation, and instead consider, what is the purpose of this thesis, the electroweak $SU(2)\times U(1)$-theory. From chapter 1 we recall that the unbroken theory is described by the Lagrangian

$$
\mathcal{L}_{SUSY} = \mathcal{L}_{Lepton} + \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs}, \tag{B.46}
$$

where

$$
\mathcal{L}_{Lepton} = \int d^4\theta\;\left[\hat{L}^\dagger e^{2g\hat V + g'\hat V'}\hat L + \hat R^\dagger e^{2g\hat V + g'\hat V'}\hat R\right], \tag{B.47}
$$

$$
\mathcal{L}_{Gauge} = \frac{1}{4}\int d^4\theta\;\left[\, W^{a\,\alpha}W^a_\alpha + W'^{\,\alpha}W'_\alpha \right]\delta^2(\bar\theta) + h.c., \tag{B.48}
$$

$$
\mathcal{L}_{Higgs} = \int d^4\theta\;\left[\hat H_1^\dagger e^{2g\hat V + g'\hat V'}\hat H_1 + \hat H_2^\dagger e^{2g\hat V + g'\hat V'}\hat H_2 + W\,\delta^2(\bar\theta) + W^\dagger\,\delta^2(\theta)\right]. \tag{B.49}
$$

Here the superpotential W is given by

$$
\begin{aligned}
W &= W_H + W_Y \\
&= \mu\,\varepsilon^{ij}\hat H_1^i\hat H_2^j + f\,\varepsilon^{ij}\hat H_1^i\hat L^j\,\hat R. \qquad \text{(B.50)}
\end{aligned}
$$

From chapter 2 we recall the component expansions of the various superfields of $\mathcal{L}_{SUSY}$. They are

$$
\begin{aligned}
\hat L(x,\theta,\bar\theta) &= \tilde L(x) + i\,\theta\sigma^\mu\bar\theta\;\partial_\mu\tilde L(x) - \frac{1}{4}\,\theta\theta\,\bar\theta\bar\theta\;\partial^\mu\partial_\mu\tilde L(x) \\
&\quad + \sqrt{2}\;\theta L^{(2)}(x) + \frac{i}{\sqrt{2}}\,\theta\theta\;\bar\theta\bar\sigma^\mu\partial_\mu L^{(2)}(x) + \theta\theta\;F_L(x), \qquad \text{(B.51)}
\end{aligned}
$$

$$
\begin{aligned}
\hat R(x,\theta,\bar\theta) &= \tilde R(x) + i\,\theta\sigma^\mu\bar\theta\;\partial_\mu\tilde R(x) - \frac{1}{4}\,\theta\theta\,\bar\theta\bar\theta\;\partial^\mu\partial_\mu\tilde R(x) \\
&\quad + \sqrt{2}\;\theta R^{(2)}(x) + \frac{i}{\sqrt{2}}\,\theta\theta\;\bar\theta\bar\sigma^\mu\partial_\mu R^{(2)}(x) + \theta\theta\;F_R(x), \qquad \text{(B.52)}
\end{aligned}
$$

$$
\begin{aligned}
\hat H_1(x,\theta,\bar\theta) &= H_1(x) + i\,\theta\sigma^\mu\bar\theta\;\partial_\mu H_1(x) - \frac{1}{4}\,\theta\theta\,\bar\theta\bar\theta\;\partial^\mu\partial_\mu H_1(x) \\
&\quad + \sqrt{2}\;\theta\tilde H_1^{(2)}(x) + \frac{i}{\sqrt{2}}\,\theta\theta\;\bar\theta\bar\sigma^\mu\partial_\mu\tilde H_1^{(2)}(x) + \theta\theta\;F_1(x), \qquad \text{(B.53)}
\end{aligned}
$$

and finally

$$\hat{H}_2(x,\theta,\bar{\theta}) = H_2(x) + i\ \theta\sigma^\mu\bar{\theta}\ \partial_\mu H_2(x) - \frac{1}{4}\ \theta\theta\ \bar{\theta}\bar{\theta}\ \partial^\mu\partial_\mu H_2(x) + \sqrt{2}\ \theta\tilde{H}_2^{(2)}(x) + \frac{i}{\sqrt{2}}\ \theta\theta\ \bar{\theta}\bar{\sigma}^\mu\partial_\mu\tilde{H}_2^{(2)}(x) + \theta\theta\ F_2(x). \tag{B.54}$$

The various component fields are fully defined in the chapter mentioned above and the quantum numbers are listed in table 1.1.

With the results from the previous sections of this appendix together with

$$\Phi_i(y,\theta)\Phi_j(y,\theta) = A_i(y)A_j(y) + \sqrt{2}\theta\left[A_i(y)\psi_j(y) + \psi_i(y)A_j(y)\right] + \theta\theta\ \left[A_i(y)F_j(y) + F_i(y)A_j(y) - \psi_i(y)\psi_j(y)\right], \tag{B.55}$$

and

$$\begin{aligned}\Phi_i(y,\theta)\Phi_j(y,\theta)\Phi_k(y,\theta)\\ = A_i(y)A_j(y)A_k(y)\\ + \sqrt{2}\theta\left[\psi_i(y)A_j(y)A_k(y) + \psi_j(y)A_k(y)A_i(y) + \psi_k(y)A_i(y)A_j(y)\right]\\ + \theta\theta\left[F_i(y)A_j(y)A_k(y) + F_j(y)A_k(y)A_i(y) + F_k(y)A_i(y)A_j(y)\right.\\ \left. -\psi_i(y)\psi_j(y)A_k(y) - \psi_j(y)\psi_k(y)A_i(y) - \psi_k(y)\psi_i(y)A_j(y)\right],\end{aligned} \tag{B.56}$$

it is easy to calculate the expansion of $\mathcal{L}_{SUSY}$. Note that the $\theta\theta$-component of eqs. $(B.55)$ and $(B.56)$ is independent of basis. We will now give the component expansions of the different terms of eq. $(B.46)$.

B.4.1 The Component Form of $\mathcal{L}_{Lepton}$.

With eqs. $(B.45)$, $(B.51)$, $(B.52)$ and table 1.1 one has

$$\begin{aligned}\mathcal{L}_{Lepton} &= \int d^4\theta\ \left[\hat{L}^\dagger e^{2g\hat{V}+g'\hat{V}'}\hat{L} + \hat{R}^\dagger e^{2g\hat{V}+g'\hat{V}'}\hat{R}\right]\\ &= \left(D^\mu\tilde{L}\right)^\dagger\left(D_\mu\tilde{L}\right) + \left(D^\mu\tilde{R}\right)^\dagger\left(D_\mu\tilde{R}\right) - i\ \bar{L}^{(2)}\bar{\sigma}^\mu D_\mu L^{(2)} - i\ \bar{R}^{(2)}\bar{\sigma}^\mu D_\mu R^{(2)}\\ &+ \tilde{L}^\dagger\left(gT^aD^a - \frac{1}{2}g'D'\right)\tilde{L} + \tilde{R}^\dagger g'D'\tilde{R}\\ &+ \sqrt{2}i\ \tilde{L}^\dagger\left(gT^a\lambda^a - \frac{1}{2}g'\lambda'\right)L^{(2)} - \sqrt{2}i\ \bar{L}^{(2)}\left(gT^a\bar{\lambda}^a - \frac{1}{2}g'\bar{\lambda}'\right)\tilde{L}\\ &+ \sqrt{2}i\ \tilde{R}^\dagger g'\lambda' R^{(2)} - \sqrt{2}i\ \bar{R}^{(2)}g'\bar{\lambda}'\tilde{R}\\ &+ F_L^\dagger F_L + F_R^\dagger F_R + t.d.\end{aligned} \tag{B.57}$$

Here D_μ is the $SU(2)\times U(1)$-covariant derivative given in complete agreement with eq. $(B.43)$. Furthermore $T^a = \sigma^a/2$ $(a = 1,\ldots,3)$ and this will be understood from now on.

B.4.2 The Component Form of $\mathcal{L}_{Gauge}$.

$\mathcal{L}_{Gauge}$ contains both an SU(2)- and an U(1)-piece. The SU(2)-piece can be taken directly from eq. $(B.26)$, while the U(1)-piece is obtained by taking the non-Abelian limit of the same equation. Hence we may conclude

$$
\begin{aligned}
\mathcal{L}_{Gauge} &= \frac{1}{4}\int d^4\theta \; \left[W^{a\,\alpha} W^a_\alpha + W'^{\,\alpha} W'_\alpha \right] \delta^2(\bar\theta) + h.c. \\
&= -i\; \bar\lambda^a \bar\sigma^\mu \left(\partial_\mu \lambda^a - g f^{abc} V^b_\mu \lambda^c\right) - i\; \bar\lambda' \bar\sigma^\mu \partial_\mu \lambda' \\
&\quad - \frac{1}{4}\; \left(V^{a\;\mu\nu} V^a_{\mu\nu} + V'^{\mu\nu} V'_{\mu\nu} \right) + \frac{1}{2}\; (\; D^a D^a + D' D' \;) + t.d. \qquad \text{(B.58)}
\end{aligned}
$$

Here $V^a_{\mu\nu}$ and $V'_{\mu\nu}$ are the (non-SUSY) fieldstrengths for the SU(2)- and U(1)-gauge group respectively.

B.4.3 The Component Form of $\mathcal{L}_{Higgs}$.

The expansion of the kinetic terms of $\hat H_1$ and $\hat H_2$ are obtained in a complete analogous way to what we did in the subsect B.4.1.

However, in order to give the full expression for $\mathcal{L}_{Higgs}$, the component form of the super-potential piece has to be obtained. This is done with eqs. $(B.55)$ and $(B.56)$ and reads

$$
\begin{aligned}
\int d^4\theta\; W\; \delta^2(\bar\theta) &= \int d^4\theta\; \left\{ \mu\; \varepsilon^{ij} \hat H^i_1 \hat H^j_2 + f\; \varepsilon^{ij} \hat H^i_1 \hat L^j \hat R \right\} \delta^2(\bar\theta) \\
&= \mu \varepsilon^{ij}\; \left[H^i_1 F^j_2 + F^i_1 H^j_2 - \tilde H^{(2)\,i}_1 \tilde H^{(2)\,j}_2 \right] \\
&\quad + f \varepsilon^{ij}\; \Big[F^i_1 \tilde L^j \tilde R + H^i_1 F^j_L \tilde R + H^i_1 \tilde L^j F_R \\
&\qquad - \tilde H^{(2)\,i}_1 L^{(2)\,j} \tilde R - H^i_1 L^{(2)\,j} R^{(2)} - R^{(2)} \tilde H^{(2)\,i}_1 \tilde L^j \Big] . \qquad \text{(B.59)}
\end{aligned}
$$

The corresponding expression for $W^\dagger$ is, of course, obtained by hermitian conjugation.

Thus the expression for $\mathcal{L}_{Higgs}$ becomes

$$
\begin{aligned}
\mathcal{L}_{Higgs} &= \int d^4\theta\; \left[\hat H^\dagger_1 e^{2g\hat V + g'\hat V'} \hat H_1 + \hat H^\dagger_2 e^{2g\hat V + g'\hat V'} \hat H_2 + W\; \delta^2(\bar\theta) + W^\dagger\; \delta^2(\theta) \right] \\
&= (D^\mu H_1)^\dagger (D_\mu H_1) + (D^\mu H_2)^\dagger (D_\mu H_2) \\
&\quad - i\; \bar{\tilde H}^{(2)}_1 \bar\sigma^\mu D_\mu \tilde H^{(2)}_1 - i\; \bar{\tilde H}^{(2)}_2 \bar\sigma^\mu D_\mu \tilde H^{(2)}_2 \\
&\quad + H^\dagger_1 \left(g T^a D^a - \frac{1}{2} g' D' \right) H_1 + H^\dagger_2 \left(g T^a D^a + \frac{1}{2} g' D' \right) H_2 \\
&\quad + \sqrt{2} i\; H^\dagger_1 \left(g T^a \lambda^a - \frac{1}{2} g' \lambda' \right) \tilde H^{(2)}_1 - \sqrt{2} i\; \bar{\tilde H}^{(2)}_1 \left(g T^a \bar\lambda^a - \frac{1}{2} g' \bar\lambda' \right) H_1
\end{aligned}
$$

$$
\begin{aligned}
&+ \sqrt{2}i\ H_2^\dagger \left(gT^a\lambda^a + \frac{1}{2}g'\lambda' \right) \tilde{H}_2^{(2)} - \sqrt{2}i\ \bar{\tilde{H}}_2^{(2)} \left(gT^a\bar{\lambda}^a + \frac{1}{2}g'\bar{\lambda}' \right) H_2 \\
&+ F_1^\dagger F_1 + F_2^\dagger F_2 \\
&+ \mu\varepsilon^{ij}\ \Big[\ H_1^i F_2^j + H_1^{i\,\dagger} F_2^{j\,\dagger} + F_1^i H_2^j + F_1^{i\,\dagger} H_2^{j\,\dagger} - \tilde{H}_1^{(2)\,i}\tilde{H}_2^{(2)\,j} - \bar{\tilde{H}}_1^{(2)\,i}\bar{\tilde{H}}_2^{(2)\,j}\ \Big] \\
&+ f\varepsilon^{ij}\ \Big[\ F_1^i\tilde{L}^j\tilde{R} + F_1^{i\,\dagger}\tilde{L}^{j\,\dagger}\tilde{R}^\dagger + H_1^i F_L^j\tilde{R} + H_1^{i\,\dagger}F_L^{j\,\dagger}\tilde{R}^\dagger \\
&\qquad + H_1^i\tilde{L}^jF_R + H_1^{i\,\dagger}\tilde{L}^{j\,\dagger}F_R^\dagger - \tilde{H}_1^{(2)\,i}L^{(2)\,j}\tilde{R} - \bar{\tilde{H}}_1^{(2)\,i}\bar{L}^{(2)\,j}\tilde{R}^\dagger \\
&\qquad - H_1^iL^{(2)\,j}R^{(2)} - H_1^{i\,\dagger}\bar{L}^{(2)\,j}\bar{R}^{(2)} - R^{(2)}\tilde{H}_1^{(2)\,i}\tilde{L}^j - \bar{R}^{(2)}\bar{\tilde{H}}_1^{(2)\,i}\tilde{L}^{j\,\dagger}\ \Big] \\
&+ t.d. \qquad \text{(B.60)}
\end{aligned}
$$

B.4.4 Conclusion — The Two-Component Form of $\mathcal{L}_{SUSY}$.

By adding the results from the three previous subsections, the expansion of $\mathcal{L}_{SUSY}$ is obtained.

Hence

$$
\begin{aligned}
\mathcal{L}_{SUSY} \ = \ & \left(D^\mu\tilde{L}\right)^\dagger\left(D_\mu\tilde{L}\right) + \left(D^\mu\tilde{R}\right)^\dagger\left(D_\mu\tilde{R}\right) - i\ \bar{L}^{(2)}\bar{\sigma}^\mu D_\mu L^{(2)} - i\ \bar{R}^{(2)}\bar{\sigma}^\mu D_\mu R^{(2)} \\
&+ \tilde{L}^\dagger\left(gT^aD^a - \frac{1}{2}g'D'\right)\tilde{L} + \tilde{R}^\dagger g'D'\tilde{R} \\
&+ \sqrt{2}i\ \tilde{L}^\dagger\left(gT^a\lambda^a - \frac{1}{2}g'\lambda'\right)L^{(2)} - \sqrt{2}i\ \bar{L}^{(2)}\left(gT^a\bar{\lambda}^a - \frac{1}{2}g'\bar{\lambda}'\right)\tilde{L} \\
&+ \sqrt{2}i\ \tilde{R}^\dagger g'\lambda' R^{(2)} - \sqrt{2}i\ \bar{R}^{(2)}g'\bar{\lambda}'\tilde{R} \\
&+ F_L^\dagger F_L + F_R^\dagger F_R \\
&- i\ \bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - i\ \bar{\lambda}'\bar{\sigma}^\mu D_\mu\lambda' \\
&- \frac{1}{4}\ \left(\ V^{a\ \mu\nu}V^a_{\mu\nu} + V'^{\mu\nu}V'_{\mu\nu}\ \right) + \frac{1}{2}\ (\ D^aD^a + D'D'\ ) \\
&+ (D^\mu H_1)^\dagger(D_\mu H_1) + (D^\mu H_2)^\dagger(D_\mu H_2) \\
&- i\ \bar{\tilde{H}}_1^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_1^{(2)} - i\ \bar{\tilde{H}}_2^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_2^{(2)} \\
&+ H_1^\dagger\left(gT^aD^a - \frac{1}{2}g'D'\right)H_1 + H_2^\dagger\left(gT^aD^a + \frac{1}{2}g'D'\right)H_2 \\
&+ \sqrt{2}i\ H_1^\dagger\left(gT^a\lambda^a - \frac{1}{2}g'\lambda'\right)\tilde{H}_1^{(2)} - \sqrt{2}i\ \bar{\tilde{H}}_1^{(2)}\left(gT^a\bar{\lambda}^a - \frac{1}{2}g'\bar{\lambda}'\right)H_1 \\
&+ \sqrt{2}i\ H_2^\dagger\left(gT^a\lambda^a + \frac{1}{2}g'\lambda'\right)\tilde{H}_2^{(2)} - \sqrt{2}i\ \bar{\tilde{H}}_2^{(2)}\left(gT^a\bar{\lambda}^a + \frac{1}{2}g'\bar{\lambda}'\right)H_2 \\
&+ F_1^\dagger F_1 + F_2^\dagger F_2 \\
&+ \mu\varepsilon^{ij}\ \Big[\ H_1^iF_2^j + H_1^{i\,\dagger}F_2^{j\,\dagger} + F_1^iH_2^j + F_1^{i\,\dagger}H_2^{j\,\dagger} - \tilde{H}_1^{(2)\,i}\tilde{H}_2^{(2)\,j} - \bar{\tilde{H}}_1^{(2)\,i}\bar{\tilde{H}}_2^{(2)\,j}\ \Big] \\
&+ f\varepsilon^{ij}\ \Big[\ F_1^i\tilde{L}^j\tilde{R} + F_1^{i\,\dagger}\tilde{L}^{j\,\dagger}\tilde{R}^\dagger + H_1^iF_L^j\tilde{R} + H_1^{i\,\dagger}F_L^{j\,\dagger}\tilde{R}^\dagger
\end{aligned}
$$

$$
\begin{aligned}
& + H_1^i \tilde{L}^j F_R + H_1^{i\,\dagger} \tilde{L}^{j\,\dagger} F_R^\dagger - \tilde{H}_1^{(2)\,i} L^{(2)\,j} \tilde{R} - \bar{\tilde{H}}_1{}^{(2)\,i} \bar{L}^{(2)\,j} \tilde{R}^\dagger \\
& -H_1^i L^{(2)\,j} R^{(2)} - H_1^{i\,\dagger} \bar{L}^{(2)\,j} \bar{R}^{(2)} - R^{(2)} \tilde{H}_1^{(2)\,i} \tilde{L}^j - \bar{R}^{(2)} \bar{\tilde{H}}_1{}^{(2)\,i} \tilde{L}^{j\,\dagger} \Big] \\
& + t.d. \qquad \text{(B.61)}
\end{aligned}
$$

and this appendix is concluded.

Appendix C

The Four Component-Form of the On-Shell Lagrangian $\mathcal{L}_{SUSY}$.

In this appendix the two component Lagrangian (2.26), i.e.

$$\begin{aligned}
\mathcal{L}_{SUSY} \;=\; & \left(D^\mu \tilde{L}\right)^\dagger \left(D_\mu \tilde{L}\right) + \left(D^\mu \tilde{R}\right)^\dagger \left(D_\mu \tilde{R}\right) - i\, \bar{L}^{(2)} \bar{\sigma}^\mu D_\mu L^{(2)} - i\, \bar{R}^{(2)} \bar{\sigma}^\mu D_\mu R^{(2)} \\
& + \sqrt{2} i\, \tilde{L}^\dagger \left(g T^a \lambda^a - \frac{1}{2} g' \lambda'\right) L^{(2)} - \sqrt{2} i\, \bar{L}^{(2)} \left(g T^a \bar{\lambda}^a - \frac{1}{2} g' \bar{\lambda}'\right) \tilde{L} \\
& + \sqrt{2} i\, \tilde{R}^\dagger g' \lambda' R^{(2)} - \sqrt{2} i\, \bar{R}^{(2)} g' \bar{\lambda}' \tilde{R} \\
& - i\, \bar{\lambda}^a \bar{\sigma}^\mu \left(\partial_\mu \lambda^a - g f^{abc} V^b_\mu \lambda^c\right) - i\, \bar{\lambda}' \bar{\sigma}^\mu \partial_\mu \lambda' - \frac{1}{4} \left(V^{a\,\mu\nu} V^a_{\mu\nu} + V'^{\mu\nu} V'_{\mu\nu} \right) \\
& + \left(D^\mu H_1\right)^\dagger \left(D_\mu H_1\right) + \left(D^\mu H_2\right)^\dagger \left(D_\mu H_2\right) \\
& - i\, \bar{\tilde{H}}_1^{(2)} \bar{\sigma}^\mu D_\mu \tilde{H}_1^{(2)} - i\, \bar{\tilde{H}}_2^{(2)} \bar{\sigma}^\mu D_\mu \tilde{H}_2^{(2)} \\
& + \sqrt{2} i\, H_1^\dagger \left(g T^a \lambda^a - \frac{1}{2} g' \lambda'\right) \tilde{H}_1^{(2)} - \sqrt{2} i\, \bar{\tilde{H}}_1^{(2)} \left(g T^a \bar{\lambda}^a - \frac{1}{2} g' \bar{\lambda}'\right) H_1 \\
& + \sqrt{2} i\, H_2^\dagger \left(g T^a \lambda^a + \frac{1}{2} g' \lambda'\right) \tilde{H}_2^{(2)} - \sqrt{2} i\, \bar{\tilde{H}}_2^{(2)} \left(g T^a \bar{\lambda}^a + \frac{1}{2} g' \bar{\lambda}'\right) H_2 \\
& - \varepsilon^{ij} \left[\mu \left(\tilde{H}_1^{(2)\,i} \tilde{H}_2^{(2)\,j} + \bar{\tilde{H}}_1^{(2)\,i} \bar{\tilde{H}}_2^{(2)\,j} \right) + f \left(\tilde{H}_1^{(2)\,i} L^{(2)\,j} \tilde{R} + \bar{\tilde{H}}_1^{(2)\,i} \bar{L}^{(2)\,j} \tilde{R}^\dagger \right) \right. \\
& \qquad \left. + f \left(H_1^i L^{(2)\,j} R^{(2)} + H_1^{i\,\dagger} \bar{L}^{(2)\,j} \bar{R}^{(2)} + R^{(2)} \tilde{H}_1^{(2)\,i} \tilde{L}^j + \bar{R}^{(2)} \bar{\tilde{H}}_1^{(2)\,i} \tilde{L}^{j\,\dagger} \right) \right] \\
& - \mu^2\, H_1^\dagger H_1 - \mu^2\, H_2^\dagger H_2 - \mu f \left[H_2^\dagger \tilde{L}\, \tilde{R} + \tilde{L}^\dagger H_2\, \tilde{R}^\dagger \right] \\
& - f^2 \left[\tilde{L}^\dagger \tilde{L}\, \tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 \left(\tilde{L}^\dagger \tilde{L} + \tilde{R}^\dagger \tilde{R}\right) - H_1^\dagger \tilde{L} \left(H_1^\dagger \tilde{L}\right)^\dagger \right] \\
& - \frac{g^2}{2} \left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2 \right) \left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2 \right) \\
& - \frac{g'^2}{8} \left(\tilde{L}^\dagger \tilde{L} - 2 \tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 - H_2^\dagger H_2 \right)^2 + t.d., \qquad \text{(C.1)}
\end{aligned}$$

will be transformed into four-component notation (i.e. to introduce four-component spinors).

Our strategy will be as follows. First the following well known gauge boson combinations

$$A_\mu(x) = \cos\theta_{\rm W}\, V'_\mu(x) + \sin\theta_{\rm W}\, V^3_\mu(x), \quad (C.2)$$

$$Z_\mu(x) = -\sin\theta_{\rm W}\, V'_\mu(x) + \cos\theta_{\rm W}\, V^3_\mu(x), \quad (C.3)$$

$$W^\pm_\mu(x) = \frac{V^1_\mu(x) \mp i V^2_\mu(x)}{\sqrt{2}}, \quad (C.4)$$

and corresponding relations for the spin-1/2 gauginos

$$\lambda_A(x) = \cos\theta_{\rm W}\, \lambda'(x) + \sin\theta_{\rm W}\, \lambda^3(x), \quad (C.5)$$

$$\lambda_Z(x) = -\sin\theta_{\rm W}\, \lambda'(x) + \cos\theta_{\rm W}\, \lambda^3(x), \quad (C.6)$$

$$\lambda^\pm(x) = \frac{\lambda^1(x) \mp i\lambda^2(x)}{\sqrt{2}}, \quad (C.7)$$

will be introduced. Next, the two component spinors will be arranged in various (four-component) Majorana- and Dirac-spinors. As we will see, the S-QFD theory contains Photino- ($\tilde{A}$), Zino- ($\tilde{Z}$) and two neutral Higgsino-states ($\tilde{H}_1$, $\tilde{H}_2$) defined in terms of two-component spinors as follows

$$\tilde{A}(x) = \begin{pmatrix} -i\lambda_A(x) \\ i\bar{\lambda}_A(x) \end{pmatrix}, \quad (C.8)$$

$$\tilde{Z}(x) = \begin{pmatrix} -i\lambda_Z(x) \\ i\bar{\lambda}_Z(x) \end{pmatrix}, \quad (C.9)$$

$$\tilde{H}_1 = \begin{pmatrix} \psi^1_{H_1} \\ \bar{\psi}^1_{H1} \end{pmatrix}, \quad (C.10)$$

$$\tilde{H}_2 = \begin{pmatrix} \psi^2_{H_2} \\ \bar{\psi}^2_{H2} \end{pmatrix}. \quad (C.11)$$

These spinors are all of the Majorana type.

For the Dirac-spinors, we have the Winos ($\tilde{W}$) and the charged Higgsinos ($\tilde{H}$) given by

$$\tilde{W}(x) = \begin{pmatrix} -i\lambda^+(x) \\ i\bar{\lambda}^-(x) \end{pmatrix}, \qquad \tilde{W}^c(x) = \begin{pmatrix} -i\lambda^-(x) \\ i\bar{\lambda}^+(x) \end{pmatrix}, \quad (C.12)$$

$$\tilde{H}(x) = \begin{pmatrix} \psi^1_{H_2} \\ \bar{\psi}^2_{H_1} \end{pmatrix}, \qquad \tilde{H}^c(x) = \begin{pmatrix} \psi^2_{H_1} \\ \bar{\psi}^1_{H_2} \end{pmatrix}. \quad (C.13)$$

Here the upper "c" on $\tilde{W}^c$ and $\tilde{H}^c$ means charge conjugation (cf. eq.(A.91)).

Finally we have the leptons which as usual are arranged in four-component Dirac-spinors defined by

$$l = \begin{pmatrix} l^{(2)}_L \\ \bar{l}^{(2)}_R \end{pmatrix}. \quad (C.14)$$

After introducing all necessary notation, one is in position to show attention to the main purpose of this appendix — the four-component formulation of the Lagrangian $\mathcal{L}_{SUSY}$.

The coming calculations rely heavily on the results of subsect. A.5.2, and in order to avoid clutter in our description, these results will be used without any further reference. Those readers not familiar with the connection between two- and four-component spinors are guided to study this subsection most carefully.

C.1 Rewriting Kinetic Terms.

C.1.1 Slepton and Higgs Kinetic Terms.

From eq. $(C.1)$ we see that the transcription of the kinetic terms of sleptons and Higgses is completed once the $SU(2) \times U(1)$-covariant derivative is written in terms of the new field-combinations eqs. $(C.2)$–$(C.4)$.

Hence

$$
\begin{aligned}
D_\mu &= \partial_\mu + igT^a V^a_\mu + ig'\frac{Y}{2}V'_\mu \\
&= \partial_\mu + \frac{ig}{\sqrt{2}}\left(T^1 + iT^2\right)\left(\frac{V^1_\mu(x) - iV^2_\mu(x)}{\sqrt{2}}\right) + \frac{ig}{\sqrt{2}}\left(T^1 - iT^2\right)\left(\frac{V^1_\mu(x) + iV^2_\mu(x)}{\sqrt{2}}\right) \\
&\quad + igT^3\left(\lambda_A \sin\theta_{\mathrm{W}} + \lambda_Z \cos\theta_{\mathrm{W}}\right) + ig'\frac{Y}{2}\left(\lambda_A \cos\theta_{\mathrm{W}} - \lambda_Z \sin\theta_{\mathrm{W}}\right) \\
&= \partial_\mu + \frac{ig}{\sqrt{2}}T^+W^+_\mu + \frac{ig}{\sqrt{2}}T^-W^-_\mu \\
&\quad + i\left(g\sin\theta_{\mathrm{W}}T^3 + g'\cos\theta_{\mathrm{W}}\frac{Y}{2}\right)A_\mu + i\left(g\cos\theta_{\mathrm{W}}T^3 - g'\sin\theta_{\mathrm{W}}\frac{Y}{2}\right)Z_\mu \\
&= \partial_\mu + \frac{ig}{\sqrt{2}}T^+W^+_\mu + \frac{ig}{\sqrt{2}}T^-W^-_\mu \\
&\quad + ie\left(T^3 + \frac{Y}{2}\right)A_\mu + \frac{ig}{\cos\theta_{\mathrm{W}}}\left[T^3 - \left(T^3 + \frac{Y}{2}\right)\sin^2\theta_{\mathrm{W}}\right]Z_\mu \\
&= \partial_\mu + \frac{ig}{\sqrt{2}}T^+W^+_\mu + \frac{ig}{\sqrt{2}}T^-W^-_\mu + ieQA_\mu + \frac{ig}{\cos\theta_{\mathrm{W}}}\left[T^3 - Q\sin^2\theta_{\mathrm{W}}\right]Z_\mu, \qquad \text{(C.15)}
\end{aligned}
$$

where we have used the SM-relations

$$e \;=\; g\sin\theta_{\mathrm{W}} \;=\; g'\cos\theta_{\mathrm{W}}, \qquad \text{(C.16)}$$

and introduced the operators

$$T^\pm \;=\; T^1 \pm iT^2, \qquad \text{(C.17)}$$

$$Q \quad = \quad T^3 + \frac{Q}{2}. \tag{C.18}$$

Here the operator Q is the charge operator, with eigenvalues in units of the elementary charge e.

C.1.2 Lepton and Higgsinos Kinetic Terms.

After completing the rewriting of the covariant derivative in the previous subsection, we have for the kinetic term of left-handed leptons[1]

$$\begin{aligned}
-i\,\bar{L}^{(2)}\bar{\sigma}^\mu D_\mu L^{(2)} &= -i\,\bar{L}^{(2)\,i}\bar{\sigma}^\mu D_\mu^{ij} L^{(2)\,j}, \qquad\qquad i,j=1,2, \\
&= -i\,\bar{\nu}_l^{(2)}\bar{\sigma}^\mu D_\mu^{11}\nu_l^{(2)} - i\,\bar{\nu}_l^{(2)}\bar{\sigma}^\mu D_\mu^{12} l_L^{(2)} \\
&\quad - i\,\bar{l}_L^{(2)}\bar{\sigma}^\mu D_\mu^{21}\nu_l^{(2)} - i\,\bar{l}_L^{(2)}\bar{\sigma}^\mu D_\mu^{22} l_L^{(2)} \\
&= -i\,\bar{\nu}_l\gamma^\mu D_\mu^{11}\nu_l - i\,\bar{\nu}_l\gamma^\mu D_\mu^{12} l_L \\
&\quad - i\,\bar{l}_L\gamma^\mu D_\mu^{21}\nu_l - i\,\bar{l}_L\gamma^\mu D_\mu^{22} l_L \\
&= -i \begin{pmatrix} \bar{\nu}_l & \bar{l}_L \end{pmatrix} \gamma^\mu \begin{pmatrix} D_\mu^{11} & D_\mu^{12} \\ D_\mu^{21} & D_\mu^{22} \end{pmatrix} \begin{pmatrix} \nu_l \\ l_L \end{pmatrix} \\
&= -i\,\bar{L}\gamma^\mu D_\mu L.
\end{aligned} \tag{C.19}$$

Here $L = \begin{pmatrix} \nu_l & l \end{pmatrix}_L^T$ is the SU(2)-doublet of four-component Dirac-spinors, well known from the SM.

In a similar way, we can show that $(R = l_R)$

$$-i\,\bar{R}^{(2)}\bar{\sigma}^\mu D_\mu R^{(2)} \quad = \quad -i\,\bar{R}\bar{\sigma}^\mu D_\mu R, \tag{C.20}$$

for the right-handed leptons.

Furthermore, one has for the kinetic term of the two-component Higgsino $\tilde{H}_1^{(2)}$

$$\begin{aligned}
-i\,\bar{\tilde{H}}_1^{(2)}\bar{\sigma}^\mu D_\mu \tilde{H}_1^{(2)} &= -i\,\bar{\psi}_{H_1}^1\bar{\sigma}^\mu D_\mu^{11}\psi_{H_1}^1 - i\,\bar{\psi}_{H_1}^1\bar{\sigma}^\mu D_\mu^{12}\psi_{H_1}^2 \\
&\quad - i\,\bar{\psi}_{H_1}^2\bar{\sigma}^\mu D_\mu^{21}\psi_{H_1}^1 - i\,\bar{\psi}_{H_1}^2\bar{\sigma}^\mu D_\mu^{22}\psi_{H_1}^2 \\
&= -i\,\bar{\psi}_{H_1}^1\bar{\sigma}^\mu\partial_\mu\psi_{H_1}^1 + \frac{g}{2\cos\theta_{\mathrm{W}}}\,\bar{\psi}_{H_1}^1\bar{\sigma}^\mu\psi_{H_1}^1\,Z_\mu \\
&\quad + \frac{g}{\sqrt{2}}\,\bar{\psi}_{H_1}^1\bar{\sigma}^\mu\psi_{H_1}^2\,W_\mu^+ + \frac{g}{\sqrt{2}}\,\bar{\psi}_{H_1}^2\bar{\sigma}^\mu\psi_{H_1}^1\,W_\mu^- \\
&\quad - i\,\bar{\psi}_{H_1}^2\bar{\sigma}^\mu\partial_\mu\psi_{H_1}^2 - e\,\bar{\psi}_{H_1}^2\bar{\sigma}^\mu\psi_{H_1}^2\,A_\mu \\
&\quad - \frac{g}{2\cos\theta_{\mathrm{W}}}\left(1 - 2\sin^2\theta_{\mathrm{W}}\right)\bar{\psi}_{H_1}^2\bar{\sigma}^\mu\psi_{H_1}^2\,Z_\mu
\end{aligned}$$

[1] Keep in mind that the covariant derivative has $SU(2)\times U(1)$-indices, and that the neutrinos are assumed to be completely left-handed.

$$
\begin{aligned}
= \; & -\frac{i}{2}\,\bar{\tilde{H}}_1\gamma^\mu\partial_\mu\tilde{H}_1 - \frac{g}{4\cos\theta_\mathrm{W}}\,\bar{\tilde{H}}_1\gamma^\mu\gamma_5\tilde{H}_1\,Z_\mu \\
& - \frac{g}{\sqrt{2}}\,\bar{\tilde{H}}\gamma^\mu P_R\tilde{H}_1\,W^+_\mu - \frac{g}{\sqrt{2}}\,\bar{\tilde{H}}_1\gamma^\mu P_R\tilde{H}\,W^-_\mu \\
& - i\,\bar{\tilde{H}}\gamma^\mu P_R\partial_\mu\tilde{H} + e\,\bar{\tilde{H}}\gamma^\mu P_R\tilde{H}\,A_\mu \\
& + \frac{g}{2\cos\theta_\mathrm{W}}\left(1 - 2\sin^2\theta_\mathrm{W}\right)\bar{\tilde{H}}\gamma^\mu P_R\tilde{H}\,Z_\mu + t.d. \qquad (C.21)
\end{aligned}
$$

Here the charge of the various (two-component) fields, recapitulated in table 2.1, has been taken advantage of.

In a complete analogous way, we obtain for the kinetic term of $\tilde{H}_2^{(2)}$

$$
\begin{aligned}
-i\,\bar{\tilde{H}}_2^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_2^{(2)} = \; & -\frac{i}{2}\,\bar{\tilde{H}}_2\gamma^\mu\partial_\mu\tilde{H}_2 + \frac{g}{4\cos\theta_\mathrm{W}}\,\bar{\tilde{H}}_2\gamma^\mu\gamma_5\tilde{H}_2\,Z_\mu \\
& + \frac{g}{\sqrt{2}}\,\bar{\tilde{H}}\gamma^\mu P_L\tilde{H}_2\,W^+_\mu + \frac{g}{\sqrt{2}}\,\bar{\tilde{H}}_2\gamma^\mu P_L\tilde{H}\,W^-_\mu \\
& - i\,\bar{\tilde{H}}\gamma^\mu P_L\partial_\mu\tilde{H} + e\,\bar{\tilde{H}}\gamma^\mu P_L\tilde{H}\,A_\mu \\
& + \frac{g}{2\cos\theta_\mathrm{W}}\left(1 - 2\sin^2\theta_\mathrm{W}\right)\bar{\tilde{H}}\gamma^\mu P_L\tilde{H}\,Z_\mu + t.d. \qquad (C.22)
\end{aligned}
$$

By adding eqs. $(C.21)$ and $(C.22)$, and using eq. $(A.82)$, one may conclude

$$
\begin{aligned}
& -i\,\bar{\tilde{H}}_1^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_1^{(2)} - i\,\bar{\tilde{H}}_2^{(2)}\bar{\sigma}^\mu D_\mu\tilde{H}_2^{(2)} \\
& \quad = -i\,\bar{\tilde{H}}\gamma^\mu\partial_\mu\tilde{H} - \frac{i}{2}\,\bar{\tilde{H}}_1\gamma^\mu\partial_\mu\tilde{H}_1 - \frac{i}{2}\,\bar{\tilde{H}}_2\gamma^\mu\partial_\mu\tilde{H}_2 \\
& \quad - \frac{g}{\sqrt{2}}\left[\left(\bar{\tilde{H}}\gamma^\mu P_R\tilde{H}_1 - \bar{\tilde{H}}\gamma^\mu P_L\tilde{H}_2\right)W^+_\mu + h.c.\right] + e\,\bar{\tilde{H}}\gamma^\mu\tilde{H}\,A_\mu \\
& \quad + \frac{g}{2\cos\theta_\mathrm{W}}\left[\left(1 - 2\sin^2\theta_\mathrm{W}\right)\bar{\tilde{H}}\gamma^\mu\tilde{H} - \frac{1}{2}\left(\bar{\tilde{H}}_1\gamma^\mu\gamma_5\tilde{H}_1 - \bar{\tilde{H}}_2\gamma^\mu\gamma_5\tilde{H}_2\right)\right]Z_\mu \\
& \quad + t.d. \qquad (C.23)
\end{aligned}
$$

C.1.3 Gaugino Kinetic Terms.

With eqs. (2.13) and (2.14) we have

$$
\begin{aligned}
& -i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - i\bar{\lambda}'\bar{\sigma}^\mu D_\mu\lambda' \\
& \qquad = -i\bar{\lambda}^a\bar{\sigma}^\mu\partial_\mu\lambda^a - i\bar{\lambda}'\bar{\sigma}^\mu\partial_\mu\lambda' + ig\,f^{abc}\bar{\lambda}^a\bar{\sigma}V^b_\mu\lambda^c. \qquad (C.24)
\end{aligned}
$$

Using the inverse of the transformations $(C.5)$–$(C.7)$ yields for the two first terms of eq. $(C.24)$

$$
\begin{aligned}
& -i\bar{\lambda}^a\bar{\sigma}^\mu\partial_\mu\lambda^a - i\bar{\lambda}'\bar{\sigma}^\mu\partial_\mu\lambda' \\
& \qquad = -i\left(\frac{\bar{\lambda}^- + \bar{\lambda}^+}{\sqrt{2}}\right)\bar{\sigma}^\mu\partial_\mu\left(\frac{\lambda^- + \lambda^+}{\sqrt{2}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& - i \left(\frac{\bar\lambda^- - \bar\lambda^+}{-i\sqrt{2}} \right) \bar\sigma^\mu \partial_\mu \left(\frac{\lambda^- - \lambda^+}{i\sqrt{2}} \right) \\
& - i \left(\bar\lambda_A \sin\theta_{\rm W} + \bar\lambda_Z \cos\theta_{\rm W} \right) \bar\sigma^\mu \partial_\mu \left(\lambda_A \sin\theta_{\rm W} + \lambda_Z \cos\theta_{\rm W} \right) \\
& - i \left(\bar\lambda_A \cos\theta_{\rm W} - \bar\lambda_Z \sin\theta_{\rm W} \right) \bar\sigma^\mu \partial_\mu \left(\lambda_A \cos\theta_{\rm W} - \lambda_Z \sin\theta_{\rm W} \right) \\
= \; & -i\,\bar\lambda^+ \bar\sigma^\mu \partial_\mu \lambda^+ - i\,\bar\lambda^- \bar\sigma^\mu \partial_\mu \lambda^- - i\,\bar\lambda_A \bar\sigma^\mu \partial_\mu \lambda_A - i\,\bar\lambda_Z \bar\sigma^\mu \partial_\mu \lambda_Z . \qquad \text{(C.25)}
\end{aligned}
$$

For the last term of eq. $(C.24)$ we have

$$
\begin{aligned}
& ig\, f^{abc} \bar\lambda^a \bar\sigma^\mu V^b_\mu \lambda^c \\
= \; & ig\, f^{3ij} \bar\lambda^3 \bar\sigma^\mu V^i_\mu \lambda^j + ig\, f^{i3j} \bar\lambda^i \bar\sigma^\mu V^3_\mu \lambda^j + ig\, f^{ij3} \bar\lambda^i \bar\sigma^\mu V^j_\mu \lambda^3 \\
= \; & ig\, \varepsilon^{ij} \left[\bar\lambda^3 \bar\sigma^\mu V^i_\mu \lambda^j - \bar\lambda^i \bar\sigma^\mu V^3_\mu \lambda^j + \bar\lambda^i \bar\sigma^\mu V^j_\mu \lambda^3 \right], \quad i,j = 1,2. \qquad \text{(C.26)}
\end{aligned}
$$

Here we have used that

$$
f^{ij3} = \varepsilon^{ij},
$$

where ε^{ij} is the usual antisymmetric tensor defined by $\varepsilon^{12} = 1$.

Now each term in square brackets of eq. $(C.26)$ will be rewritten separately. The results are:

$$
\begin{aligned}
\varepsilon^{ij}\, \bar\lambda^3 \bar\sigma^\mu V^i_\mu \lambda^j &= \bar\lambda^3 \bar\sigma^\mu \left(\lambda^2 V^1_\mu - \lambda^1 V^2_\mu \right) \\
&= i \left(\bar\lambda_A \sin\theta_{\rm W} + \bar\lambda_Z \cos\theta_{\rm W} \right) \bar\sigma^\mu \left(\lambda^+ W^-_\mu - \lambda^- W^+_\mu \right), \\
-\varepsilon^{ij}\, \bar\lambda^i \bar\sigma^\mu V^3_\mu \lambda^j &= \left(\bar\lambda^2 \bar\sigma^\mu \lambda^1 - \bar\lambda^1 \bar\sigma^\mu \lambda^2 \right) V^3_\mu \\
&= i \left(\bar\lambda^- \bar\sigma^\mu \lambda^- - \bar\lambda^+ \bar\sigma^\mu \lambda^+ \right) \left(A_\mu \sin\theta_{\rm W} + Z_\mu \cos\theta_{\rm W} \right), \\
\varepsilon^{ij}\, \bar\lambda^i \bar\sigma^\mu V^j_\mu \lambda^3 &= \left(\bar\lambda^1 \bar\sigma^\mu V^2_\mu - \bar\lambda^2 \bar\sigma^\mu V^1_\mu \right) \lambda^3 \\
&= i \left(\bar\lambda^+ \bar\sigma^\mu W^+_\mu - \bar\lambda^- \bar\sigma^\mu W^-_\mu \right) \left(\lambda_A \sin\theta_{\rm W} + \lambda_Z \cos\theta_{\rm W} \right).
\end{aligned}
$$

Hence, collecting terms yields

$$
\begin{aligned}
& ig\, f^{abc} \bar\lambda^a \bar\sigma^\mu V^b_\mu \lambda^c \\
= \; & -g \left(\bar\lambda_A \sin\theta_{\rm W} + \bar\lambda_Z \cos\theta_{\rm W} \right) \bar\sigma^\mu \left(\lambda^+ W^-_\mu - \lambda^- W^+_\mu \right) \\
& - g \left(\bar\lambda^- \bar\sigma^\mu \lambda^- - \bar\lambda^+ \bar\sigma^\mu \lambda^+ \right) \left(A_\mu \sin\theta_{\rm W} + Z_\mu \cos\theta_{\rm W} \right) \\
& - g \left(\bar\lambda^+ \bar\sigma^\mu W^+_\mu - \bar\lambda^- \bar\sigma^\mu W^-_\mu \right) \left(\lambda_A \sin\theta_{\rm W} + \lambda_Z \cos\theta_{\rm W} \right) \\
= \; & g \cos\theta_{\rm W} \Big[\left(\bar\lambda_Z \bar\sigma^\mu \lambda^- - \bar\lambda^+ \bar\sigma^\mu \lambda_Z \right) W^+_\mu - \left(\bar\lambda_Z \bar\sigma^\mu \lambda^+ - \bar\lambda^- \bar\sigma^\mu \lambda_Z \right) W^-_\mu \\
& \qquad + \left(\bar\lambda^+ \bar\sigma^\mu \lambda^+ - \bar\lambda^- \bar\sigma^\mu \lambda^- \right) Z_\mu \Big] \\
& + e \Big[\left(\bar\lambda_A \bar\sigma^\mu \lambda^- - \bar\lambda^+ \bar\sigma^\mu \lambda_A \right) W^+_\mu - \left(\bar\lambda_A \bar\sigma^\mu \lambda^+ - \bar\lambda^- \bar\sigma^\mu \lambda_A \right) W^-_\mu \\
& \quad + \left(\bar\lambda^+ \bar\sigma^\mu \lambda^+ - \bar\lambda^- \bar\sigma^\mu \lambda^- \right) A_\mu \Big], \qquad \text{(C.27)}
\end{aligned}
$$

and thus

$$
\begin{aligned}
&-i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - i\bar{\lambda}'\bar{\sigma}^\mu D_\mu\lambda' \\
&\quad = \; -i\,\bar{\lambda}^+\bar{\sigma}^\mu\partial_\mu\lambda^+ - i\,\bar{\lambda}^-\bar{\sigma}^\mu\partial_\mu\lambda^- - i\,\bar{\lambda}_A\bar{\sigma}^\mu\partial_\mu\lambda_A - i\,\bar{\lambda}_Z\bar{\sigma}^\mu\partial_\mu\lambda_Z \\
&\qquad + g\cos\theta_{\rm W}\left[\left(\bar{\lambda}_Z\bar{\sigma}^\mu\lambda^- - \bar{\lambda}^+\bar{\sigma}^\mu\lambda_Z\right)W^+_\mu - \left(\bar{\lambda}_Z\bar{\sigma}^\mu\lambda^+ - \bar{\lambda}^-\bar{\sigma}^\mu\lambda_Z\right)W^-_\mu\right. \\
&\qquad\qquad \left. + \left(\bar{\lambda}^+\bar{\sigma}^\mu\lambda^+ - \bar{\lambda}^-\bar{\sigma}^\mu\lambda^-\right)Z_\mu\right] \\
&\qquad + e\left[\left(\bar{\lambda}_A\bar{\sigma}^\mu\lambda^- - \bar{\lambda}^+\bar{\sigma}^\mu\lambda_A\right)W^+_\mu - \left(\bar{\lambda}_A\bar{\sigma}^\mu\lambda^+ - \bar{\lambda}^-\bar{\sigma}^\mu\lambda_A\right)W^-_\mu\right. \\
&\qquad\quad \left. + \left(\bar{\lambda}^+\bar{\sigma}^\mu\lambda^+ - \bar{\lambda}^-\bar{\sigma}^\mu\lambda^-\right)A_\mu\right]. \qquad (C.28)
\end{aligned}
$$

With eqs. (C.8), (C.9), (C.12) and (A.64), the four-component form of (C.28) is easily obtained, and it reads

$$
\begin{aligned}
&-i\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - i\bar{\lambda}'\bar{\sigma}^\mu D_\mu\lambda' \\
&\quad = \; -i\,\bar{\tilde{W}}\gamma^\mu\partial_\mu\tilde{W} - \frac{i}{2}\,\bar{\tilde{A}}\gamma^\mu\partial_\mu\tilde{A} - \frac{i}{2}\,\bar{\tilde{Z}}\gamma^\mu\partial_\mu\tilde{Z} \\
&\qquad - g\cos\theta_{\rm W}\left[\,\bar{\tilde{Z}}\gamma^\mu\tilde{W}\;W^-_\mu + \bar{\tilde{W}}\gamma^\mu\tilde{Z}\;W^+_\mu - \bar{\tilde{W}}\gamma^\mu\tilde{W}\;Z_\mu\right] \\
&\qquad - e\left[\,\bar{\tilde{A}}\gamma^\mu\tilde{W}\;W^-_\mu + \bar{\tilde{W}}\gamma^\mu\tilde{A}\;W^+_\mu - \bar{\tilde{W}}\gamma^\mu\tilde{W}\;A_\mu\right] + t.d. \qquad (C.29)
\end{aligned}
$$

C.1.4 Gauge-Boson Kinetic Terms.

By introducing the practical "scripted" quantities

$$
\begin{aligned}
\mathcal{A}_{\mu\nu} &= \cos\theta_{\rm W}V'_{\mu\nu} + \sin\theta_{\rm W}V^3_{\mu\nu}, \qquad (C.30)\\
\mathcal{Z}_{\mu\nu} &= -\sin\theta_{\rm W}V'_{\mu\nu} + \cos\theta_{\rm W}V^3_{\mu\nu}, \qquad (C.31)\\
\mathcal{W}^\pm_{\mu\nu} &= \frac{V^1_{\mu\nu} \mp iV^2_{\mu\nu}}{\sqrt{2}}, \qquad (C.32)
\end{aligned}
$$

defined in complete analogy with the eqs. (C.2)–(C.4), the kinetic terms of the gauge-bosons can be rewritten in a compact form as we will see in a moment. However, first the explicit form of these "scripted" fieldstrengths will be derived. Hence

$$
\begin{aligned}
\mathcal{A}_{\mu\nu} &= V'_{\mu\nu}\cos\theta_{\rm W} + V^3_{\mu\nu}\sin\theta_{\rm W} \\
&= \partial_\mu A_\nu - \partial_\nu A_\mu - g\sin\theta_{\rm W}\,f^{312}\left(V^1_\mu V^2_\nu - V^2_\mu V^1_\nu\right) \\
&= A_{\mu\nu} + ie\left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right), \qquad (C.33)\\
\mathcal{Z}_{\mu\nu} &= -V'_{\mu\nu}\sin\theta_{\rm W} + V^3_{\mu\nu}\cos\theta_{\rm W} \\
&= \partial_\mu Z_\nu - \partial_\nu Z_\mu - g\cos\theta_{\rm W}\,f^{312}\left(V^1_\mu V^2_\nu - V^2_\mu V^1_\nu\right) \\
&= Z_{\mu\nu} + ig\cos\theta_{\rm W}\left(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu\right), \qquad (C.34)
\end{aligned}
$$

with

$$\begin{aligned} A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ Z_{\mu\nu} &= \partial_\mu Z_\nu - \partial_\nu Z_\mu. \end{aligned}$$

Furthermore

$$\begin{aligned} \mathcal{W}^{\pm}_{\mu\nu} &= \frac{V^1_{\mu\nu} \mp i V^2_{\mu\nu}}{\sqrt{2}} \\ &= \partial_\mu \left(\frac{V^1_\nu \mp i V^2_\nu}{\sqrt{2}} \right) - \partial_\nu \left(\frac{V^1_\mu \mp i V^2_\mu}{\sqrt{2}} \right) \\ &\quad - \frac{g}{\sqrt{2}} \left\{ f^{123} \left(V^2_\mu V^3_\nu - V^3_\mu V^2_\nu \right) \mp i f^{231} \left(V^3_\mu V^1_\nu - V^1_\mu V^3_\nu \right) \right\} \\ &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \\ &\quad - \frac{ig}{2} \Big[\left(W^+_\mu - W^-_\mu \right) \left(A_\nu \sin\theta_{\rm W} + Z_\nu \cos\theta_{\rm W} \right) \\ &\qquad - \left(A_\mu \sin\theta_{\rm W} + Z_\mu \cos\theta_{\rm W} \right) \left(W^+_\nu - W^-_\nu \right) \\ &\qquad \mp \Big\{ \left(A_\mu \sin\theta_{\rm W} + Z_\mu \cos\theta_{\rm W} \right) \left(W^+_\nu + W^-_\nu \right) \\ &\qquad - \left(W^+_\mu + W^-_\mu \right) \left(A_\nu \sin\theta_{\rm W} + Z_\nu \cos\theta_{\rm W} \right) \Big\} \Big] \\ &= \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \\ &\quad - \frac{ig}{2} \Big[W^+_\mu \left\{ (A_\nu \pm A_\nu) \sin\theta_{\rm W} + (Z_\nu \pm Z_\nu) \cos\theta_{\rm W} \right\} \\ &\qquad - W^-_\mu \left\{ (A_\nu \mp A_\nu) \sin\theta_{\rm W} + (Z_\nu \mp Z_\nu) \cos\theta_{\rm W} \right\} \\ &\qquad - \left\{ (A_\mu \pm A_\mu) \sin\theta_{\rm W} + (Z_\mu \pm Z_\mu) \cos\theta_{\rm W} \right\} W^+_\nu \\ &\qquad + \left\{ (A_\mu \mp A_\mu) \sin\theta_{\rm W} + (Z_\mu \mp Z_\mu) \cos\theta_{\rm W} \right\} W^-_\nu \Big], \end{aligned} \tag{C.35}$$

where

$$W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu, \tag{C.36}$$

is the “normal” fieldstrength of the W-bosons.

Writing eq. $(C.35)$ out in full yields ($e = g \sin\theta_{\rm W}$)

$$\begin{aligned} \mathcal{W}^+_{\mu\nu} &= W^+_{\mu\nu} + i\, e \left(A_\mu W^+_\nu - W^+_\mu A_\nu \right) \\ &\quad + i\, g \cos\theta_{\rm W} \left(Z_\mu W^+_\nu - W^+_\mu Z_\nu \right), \end{aligned} \tag{C.37}$$

and

$$\begin{aligned} \mathcal{W}^-_{\mu\nu} &= W^-_{\mu\nu} - i\, e \left(A_\mu W^-_\nu - W^-_\mu A_\nu \right) \\ &\quad - i\, g \cos\theta_{\rm W} \left(Z_\mu W^-_\nu - W^-_\mu Z_\nu \right). \end{aligned} \tag{C.38}$$

Note that $\mathcal{W}^{\pm}_{\mu\nu}$ contains neither $W^{\mp}_{\mu}$ nor $W^{\mp}_{\nu}$ (reversed signs) as we may have guessed in advance.

With the above relations established, we have for the kinetic terms of gauge-bosons

$$\begin{aligned} -\frac{1}{4}\left(V^{a\,\mu\nu}V^{a}_{\mu\nu} + V'^{\mu\nu}V'_{\mu\nu}\right) \\ = \; -\frac{1}{4}\,\mathcal{W}^{+\,\mu\nu}\mathcal{W}^{-}_{\mu\nu} - \frac{1}{4}\,\mathcal{W}^{-\,\mu\nu}\mathcal{W}^{+}_{\mu\nu} - \frac{1}{4}\,\mathcal{Z}^{\mu\nu}\mathcal{Z}_{\mu\nu} - \frac{1}{4}\,\mathcal{A}^{\mu\nu}\mathcal{A}_{\mu\nu}. \end{aligned} \tag{C.39}$$

This concludes this subsection.

C.2 Rewriting Interaction terms.

In this section the various interaction terms of eq. (C.1) will be rewritten.

C.2.1 Rewriting Interaction Terms Containing Gauginos.

Before proceeding, a useful general calculation will be performed. From the no-shall Lagrangian (2.26), or equivalently from eq. (B.45), we see that the transcription of the matter field Lagrangian is completed once the expression (adopting the general notation of sect. B.3)

$$\sqrt{2}i\; A^{\dagger}\left[\, gT^{a}\lambda^{a} + \frac{1}{2}g'Y\lambda' \,\right]\psi - \sqrt{2}i\;\bar{\psi}\left[\, gT^{a}\bar{\lambda}^{a} + \frac{1}{2}g'Y\bar{\lambda}' \,\right]A, \tag{C.40}$$

is rewritten. The first term of eq. (C.40), in square brackets, can in analogy with the covariant derivative, be written as

$$\begin{aligned} gT^{a}\lambda^{a} + \frac{1}{2}g'Y\lambda' \\ = \; \frac{g}{\sqrt{2}}\left(T^{+}\lambda^{+} + T^{-}\lambda^{-}\right) + eQ\,\lambda_{A} + \frac{g}{\cos\theta_{\mathrm{W}}}\left[T^{3} - Q\sin^{2}\theta_{\mathrm{W}}\right]\lambda_{Z}. \end{aligned} \tag{C.41}$$

Here T^3 and Q are the representations of T^3 and Q respectively.

By hermitian conjugation, one obtains for eq. (C.40)

$$\begin{aligned} \sqrt{2}i\; A^{\dagger}\left[\, gT^{a}\lambda^{a} + \frac{1}{2}g'Y\lambda' \,\right]\psi - \sqrt{2}i\;\bar{\psi}\left[\, gT^{a}\bar{\lambda}^{a} + \frac{1}{2}g'Y\bar{\lambda}' \,\right]A \\ = \; ig\left(A^{\dagger}T^{+}\psi\,\lambda^{+} - \bar{\lambda}^{+}\,\bar{\psi}T^{-}A\right) + ig\left(A^{\dagger}T^{-}\psi\,\lambda^{-} - \bar{\lambda}^{-}\,\bar{\psi}T^{+}A\right) \\ + \sqrt{2}ie\left(A^{\dagger}Q\psi\,\lambda_{A} - \bar{\lambda}_{A}\,\bar{\psi}QA\right) \\ + \frac{\sqrt{2}ig}{\cos\theta_{\mathrm{W}}}\left(A^{\dagger}\left[T^{3} - Q\sin^{2}\theta_{\mathrm{W}}\right]\psi\,\lambda_{Z} - \bar{\lambda}_{Z}\,\bar{\psi}\left[T^{3} - Q\sin^{2}\theta_{\mathrm{W}}\right]A\right) \end{aligned}$$

$$
\begin{aligned}
&= ig\left(A^{\dagger}T^{+}\psi\;\lambda^{+} - \bar{\lambda}^{+}\;\bar{\psi}T^{-}A\right) + ig\left(A^{\dagger}T^{-}\psi\;\lambda^{-} - \bar{\lambda}^{-}\;\bar{\psi}T^{+}A\right) \\
&\quad + \sqrt{2}ieQ_i\left(A^{\dagger i}\psi^i\;\lambda_A - \bar{\lambda}_A\;\bar{\psi}^i A^i\right) \\
&\quad + \frac{\sqrt{2}ig}{\cos\theta_{\mathrm{W}}}\left[\mathcal{T}_i^3 - Q_i\sin^2\theta_{\mathrm{W}}\right]\left(A^{\dagger i}\psi^i\;\lambda_Z - \bar{\lambda}_Z\;\bar{\psi}^i A^i\right), \qquad i = 1, 2. \qquad \text{(C.42)}
\end{aligned}
$$

Here $\mathcal{T}_i^3$ and Q_i are the eigenvalues of T^3 and Q respectively.

To introducing the new two-component spinors $(\lambda^{\pm}, \lambda_{\mathrm{A}}, \lambda_{\mathrm{Z}})$ in the various interaction terms is thus straightforward in view of the general expression $(C.42)$. Hence we have

$$
\begin{aligned}
&\sqrt{2}i\;\tilde{L}^{\dagger}\left(gT^a\lambda^a - \frac{1}{2}g'\lambda'\right)L^{(2)} - \sqrt{2}i\;\bar{L}^{(2)}\left(gT^a\bar{\lambda}^a - \frac{1}{2}g'\bar{\lambda}'\right)\tilde{L} \\
&= ig\left(\tilde{L}^{\dagger\,1}\;L^{(2)\,2}\lambda^{+} - \bar{\lambda}^{+}\bar{L}^{(2)\,2}\;\tilde{L}^1\right) + ig\left(\tilde{L}^{\dagger\,2}\;L^{(2)\,1}\lambda^{-} - \bar{\lambda}^{-}\bar{L}^{(2)\,1}\;\tilde{L}^2\right) \\
&\quad - \sqrt{2}ie\left(\tilde{L}^{\dagger\,2}\;L^{(2)\,2}\lambda_A - \bar{\lambda}_A\bar{L}^{(2)\,2}\;\tilde{L}^2\right) \\
&\quad + \frac{\sqrt{2}ig}{\cos\theta_{\mathrm{W}}}\left(\mathcal{T}_i^3 - Q_i\sin^2\theta_{\mathrm{W}}\right)\left(\tilde{L}^{\dagger\,i}\;L^{(2)\,i}\lambda_Z - \bar{\lambda}_Z\bar{L}^{(2)\,i}\;\tilde{L}^i\right) \\
&= -g\left(\tilde{L}^{\dagger\,1}\;\bar{\tilde{W}}^c P_L L^2 + \bar{L}^2 P_R\tilde{W}^c\;\tilde{L}^1\right) - g\left(\tilde{L}^{\dagger\,2}\;\bar{\tilde{W}}P_L L^1 + \bar{L}^1 P_R\tilde{W}\;\tilde{L}^2\right) \\
&\quad + \sqrt{2}e\left(\tilde{L}^{\dagger\,2}\;\bar{\tilde{A}}P_L L^2 + \bar{L}^2 P_R\tilde{A}\;\tilde{L}^2\right) \\
&\quad - \frac{\sqrt{2}g}{\cos\theta_{\mathrm{W}}}\left(\mathcal{T}_i^3 - Q_i\sin^2\theta_{\mathrm{W}}\right)\left(\tilde{L}^{\dagger\,i}\;\bar{\tilde{Z}}P_L L^i + \bar{L}^i P_R\tilde{Z}\;\tilde{L}^i\right) \\
&= -g\left(\bar{L}^2\tilde{W}^c\;\tilde{L}^1 + \bar{L}^1\tilde{W}\;\tilde{L}^2\right) + \sqrt{2}e\;\bar{L}^2\tilde{A}\;\tilde{L}^2 \\
&\quad - \frac{\sqrt{2}g}{\cos\theta_{\mathrm{W}}}\left(\mathcal{T}_i^3 - Q_i\sin^2\theta_{\mathrm{W}}\right)\bar{L}^i\tilde{Z}\;\tilde{L}^i + h.c. \qquad \text{(C.43)}
\end{aligned}
$$

Here in the last line we have utilized that $P_L L = L$.

The corresponding term for the right-handed leptons is rewritten as follows

$$
\begin{aligned}
&\sqrt{2}i\;\tilde{R}^{\dagger}g'\lambda' R^{(2)} - \sqrt{2}i\;\bar{R}^{(2)}g'\bar{\lambda}'\tilde{R} \\
&\quad = \sqrt{2}ig'\;\tilde{R}^{\dagger}\left(\lambda_A\cos\theta_{\mathrm{W}} - \lambda_Z\sin\theta_{\mathrm{W}}\right)R^{(2)} \\
&\qquad - \sqrt{2}ig'\;\bar{R}^{(2)}\left(\bar{\lambda}_A\cos\theta_{\mathrm{W}} - \bar{\lambda}_Z\sin\theta_{\mathrm{W}}\right)\tilde{R} \\
&\quad = \sqrt{2}ie\left(\tilde{R}^{\dagger}\;R^{(2)}\lambda_A - \bar{\lambda}_A\bar{R}^{(2)}\;\tilde{R}\right) \\
&\qquad - \sqrt{2}ig\frac{\sin^2\theta_{\mathrm{W}}}{\cos\theta_{\mathrm{W}}}\left(\tilde{R}^{\dagger}\;R^{(2)}\lambda_Z - \bar{\lambda}_Z\bar{R}^{(2)}\;\tilde{R}\right) \\
&\quad = -\sqrt{2}e\left(\tilde{R}^{\dagger}\;\bar{R}P_L\tilde{A} + \bar{\tilde{A}}P_R R\;\tilde{R}\right)
\end{aligned}
$$

$$+\sqrt{2}g\frac{\sin^2\theta_W}{\cos\theta_W}\left(\tilde{R}^\dagger\ \bar{R}P_L\tilde{Z}+\bar{\tilde{Z}}P_R R\ \tilde{R}\right)$$

$$= -\sqrt{2}e\ \bar{\tilde{A}}R\ \tilde{R}+\sqrt{2}g\frac{\sin^2\theta_W}{\cos\theta_W}\ \bar{\tilde{Z}}R\ \tilde{R}+h.c. \tag{C.44}$$

Here we have used that $\hat{R}$ is a right-handed gauge-singlet (and thus also the component fields), and that $g'=g\tan\theta_W$.

Hence, adding eqs. (C.43) and (C.44) yields

$$\begin{aligned}
&\sqrt{2}i\ \tilde{L}^\dagger\left(gT^a\lambda^a-\frac{1}{2}g'\lambda'\right)L^{(2)}-\sqrt{2}i\ \bar{L}^{(2)}\left(gT^a\bar{\lambda}^a-\frac{1}{2}g'\bar{\lambda}'\right)\tilde{L}\\
&+\sqrt{2}i\ \tilde{R}^\dagger g'\lambda' R^{(2)}-\sqrt{2}i\ \bar{R}^{(2)}g'\bar{\lambda}'\tilde{R}\\
&\quad= -g\left[\left\{\bar{L}^1\tilde{W}\ \tilde{L}^2+\bar{L}^2\tilde{W}^c\ \tilde{L}^1\right\}+h.c.\right]+\sqrt{2}e\left[\left\{\bar{L}^2\tilde{A}\ \tilde{L}^2-\bar{\tilde{A}}R\ \tilde{R}\right\}+h.c.\right]\\
&\qquad-\frac{\sqrt{2}g}{\cos\theta_W}\left[\left\{\left(T_i^3-Q_i\sin^2\theta_W\right)\bar{L}^i\tilde{Z}\ \tilde{L}^i-\sin^2\theta_W\ \bar{\tilde{Z}}R\ \tilde{R}\right\}+h.c.\right].
\end{aligned} \tag{C.45}$$

With eq. (C.42) and the fact that $QH_1=\begin{pmatrix}0 & -H_1^2\end{pmatrix}^T$ we have

$$\begin{aligned}
&\sqrt{2}i\ H_1^\dagger\left(gT^a\lambda^a-\frac{1}{2}g'\lambda'\right)\tilde{H}_1^{(2)}-\sqrt{2}i\ \bar{\tilde{H}}_1^{(2)}\left(gT^a\bar{\lambda}^a-\frac{1}{2}g'\bar{\lambda}'\right)H_1\\
&= ig\left(H_1^{1\,\dagger}\ \psi_{H_1}^2\lambda^+-\bar{\lambda}^+\bar{\psi}_{H_1}^2\ H_1^1\right)+ig\left(H_1^{2\,\dagger}\ \psi_{H_1}^1\lambda^--\bar{\lambda}^-\bar{\psi}_{H_1}^1\ H_1^2\right)\\
&\quad-\sqrt{2}ie\left(H_1^{2\,\dagger}\ \psi_{H_1}^2\lambda_A-\bar{\lambda}_A\bar{\psi}_{H_1}^2\ H_1^2\right)\\
&\quad+\frac{ig}{\sqrt{2}\cos\theta_W}\left(H_1^{1\,\dagger}\ \psi_{H_1}^1\lambda_Z-\bar{\lambda}_Z\bar{\psi}_{H_1}^1\ H_1^1\right)\\
&\quad-\frac{ig}{\sqrt{2}\cos\theta_W}\left(1-2\sin^2\theta_W\right)\left(H_1^{2\,\dagger}\ \psi_{H_1}^2\lambda_Z-\bar{\lambda}_Z\bar{\psi}_{H_1}^2\ H_1^2\right)\\
&= -g\left(H_1^{1\,\dagger}\ \bar{\tilde{H}}P_L\tilde{W}+\bar{\tilde{W}}P_R\tilde{H}\ H_1^1\right)-g\left(H_1^{2\,\dagger}\ \bar{\tilde{W}}P_L\tilde{H}_1+\bar{\tilde{H}}_1P_R\tilde{W}\ H_1^2\right)\\
&\quad+\sqrt{2}e\left(H_1^{2\,\dagger}\ \bar{\tilde{H}}P_L\tilde{A}+\bar{\tilde{A}}P_R\tilde{H}\ H_1^2\right)\\
&\quad-\frac{g}{\sqrt{2}\cos\theta_W}\left(H_1^{1\,\dagger}\ \bar{\tilde{H}}_1P_L\tilde{Z}+\bar{\tilde{Z}}P_R\tilde{H}_1\ H_1^1\right)\\
&\quad+\frac{g}{\sqrt{2}\cos\theta_W}\left(1-2\sin^2\theta_W\right)\left(H_1^{2\,\dagger}\ \bar{\tilde{H}}P_L\tilde{Z}+\bar{\tilde{Z}}P_R\tilde{H}\ H_1^2\right)\\
&= -g\left(\bar{\tilde{W}}P_R\tilde{H}\ H_1^1+\bar{\tilde{H}}_1P_R\tilde{W}\ H_1^2\right)+\sqrt{2}e\ \bar{\tilde{A}}P_R\tilde{H}\ H_1^2\\
&\quad-\frac{g}{\sqrt{2}\cos\theta_W}\ \bar{\tilde{Z}}P_R\tilde{H}_1\ H_1^1+\frac{g}{\sqrt{2}\cos\theta_W}\left(1-2\sin^2\theta_W\right)\bar{\tilde{Z}}P_R\tilde{H}\ H_1^2\\
&\quad+h.c.
\end{aligned} \tag{C.46}$$

A corresponding calculation for the H_2-term yields

$$\sqrt{2}i\ H_2^\dagger \left(gT^a\lambda^a + \frac{1}{2}g'\lambda'\right)\tilde{H}_2^{(2)} - \sqrt{2}i\ \bar{\tilde{H}}_2^{(2)}\left(gT^a\bar{\lambda}^a + \frac{1}{2}g'\bar{\lambda}'\right)H_2$$

$$\begin{aligned} = & -g\left(\bar{\tilde{W}}P_R\tilde{H}_2\ H_2^1 + \bar{\tilde{H}}P_R\tilde{W}\ H_2^2\right) - \sqrt{2}e\ \bar{\tilde{H}}P_R\tilde{A}\ H_2^1 \\ & - \frac{g}{\sqrt{2}\cos\theta_{\mathrm{W}}}\left(1 - 2\sin^2\theta_{\mathrm{W}}\right)\bar{\tilde{H}}P_R\tilde{Z}\ H_2^1 + \frac{g}{\sqrt{2}\cos\theta_{\mathrm{W}}}\bar{\tilde{H}}_2P_R\tilde{Z}\ H_2^2 \\ & + h.c. \end{aligned} \tag{C.47}$$

Here we have used that for Majorana spinors $\bar{\Psi}_1\Psi_2 = \bar{\Psi}_2\Psi_1$ and $\bar{\Psi}_1\gamma_5\Psi_2 = \bar{\Psi}_2\gamma_5\Psi_1$ (cf. eqs. (A.94) and (A.96))

Adding eqs. (C.46) and (C.47) yields

$$\begin{aligned} & \sqrt{2}i\ H_1^\dagger \left(gT^a\lambda^a - \frac{1}{2}g'\lambda'\right)\tilde{H}_1^{(2)} - \sqrt{2}i\ \bar{\tilde{H}}_1^{(2)}\left(gT^a\bar{\lambda}^a - \frac{1}{2}g'\bar{\lambda}'\right)H_1 \\ & +\sqrt{2}i\ H_2^\dagger \left(gT^a\lambda^a + \frac{1}{2}g'\lambda'\right)\tilde{H}_2^{(2)} - \sqrt{2}i\ \bar{\tilde{H}}_2^{(2)}\left(gT^a\bar{\lambda}^a + \frac{1}{2}g'\bar{\lambda}'\right)H_2 \end{aligned}$$

$$\begin{aligned} = & -g\left[\left(\bar{\tilde{W}}P_R\tilde{H}\ H_1^1 + \bar{\tilde{H}}P_R\tilde{W}\ H_2^2 + \bar{\tilde{H}}_1P_R\tilde{W}\ H_1^2 + \bar{\tilde{W}}P_R\tilde{H}_2\ H_2^1\right) + h.c.\right] \\ & + \sqrt{2}e\left[\left(\bar{\tilde{A}}P_R\tilde{H}\ H_1^2 - \bar{\tilde{H}}P_R\tilde{A}\ H_2^1\right) + h.c.\right] \\ & - \frac{g}{\sqrt{2}\cos\theta_{\mathrm{W}}}\left[\left\{\bar{\tilde{Z}}P_R\tilde{H}_1\ H_1^1 - \bar{\tilde{H}}_2P_R\tilde{Z}\ H_2^2\right.\right. \\ & \left.\left. - \left(1 - 2\sin^2\theta_{\mathrm{W}}\right)\left(\bar{\tilde{Z}}P_R\tilde{H}\ H_1^2 - \bar{\tilde{H}}P_R\tilde{Z}\ H_2^1\right)\right\} + h.c.\right] \end{aligned} \tag{C.48}$$

This completes this subsection.

C.2.2 Rewriting the Cubic Interaction Terms.

In the previous subsection, cubic interaction terms containing gauginos were transcripted. The aim of the present subsection is to perform a paraphrase of the remaining cubic interaction terms of the Lagrangian (C.1). The calculations go like this

$$\begin{aligned} & -f\varepsilon^{ij}\left(\tilde{H}_1^{(2)\,i}L^{(2)\,j}\tilde{R} + \bar{\tilde{H}}_1^{(2)\,i}\bar{L}^{(2)\,j}\tilde{R}^\dagger + H_1^iL^{(2)\,j}R^{(2)}\right. \\ & \left. +H_1^{i\,\dagger}\bar{L}^{(2)\,j}\bar{R}^{(2)} + R^{(2)}\tilde{H}_1^{(2)\,i}\tilde{L}^j + \bar{R}^{(2)}\bar{\tilde{H}}_1^{(2)\,i}\tilde{L}^{j\,\dagger}\right) \end{aligned}$$

$$\begin{aligned} = & -f\left[\ \tilde{H}_1^{(2)\,1}L^{(2)\,2}\,\tilde{R} - \tilde{H}_1^{(2)\,2}L^{(2)\,1}\,\tilde{R} + \bar{\tilde{H}}_1^{(2)\,1}\bar{L}^{(2)\,2}\,\tilde{R}^\dagger - \bar{\tilde{H}}_1^{(2)\,2}\bar{L}^{(2)\,1}\,\tilde{R}^\dagger\right. \\ & + L^{(2)\,2}R^{(2)}\,H_1^1 - L^{(2)\,1}R^{(2)}\,H_1^2 + \bar{L}^{(2)\,2}\bar{R}^{(2)}\,H_1^{1\,\dagger} - \bar{L}^{(2)\,1}\bar{R}^{(2)}\,H_1^{2\,\dagger} \\ & \left. +R^{(2)}\tilde{H}_1^{(2)\,1}\,\tilde{L}^2 - R^{(2)}\tilde{H}_1^{(2)\,2}\,\tilde{L}^1 + \bar{R}^{(2)}\bar{\tilde{H}}_1^{(2)\,1}\,\tilde{L}^{2\,\dagger} - \bar{R}^{(2)}\bar{\tilde{H}}_1^{(2)\,2}\,\tilde{L}^{1\,\dagger}\ \right] \end{aligned}$$

$$
\begin{aligned}
= \; & f \Big[\; \psi^2_{H_1} \nu^{(2)}_l \, \tilde{R} - \psi^1_{H_1} l^{(2)}_L \, \tilde{R} + \bar{\psi}^2_{H_1} \bar{\nu}^{(2)}_l \, \tilde{R}^\dagger - \bar{\psi}^1_{H_1} \bar{l}^{(2)}_L \, \tilde{R}^\dagger \\
& \quad + \nu^{(2)}_l l^{(2)}_R \, H^2_1 - l^{(2)}_L l^{(2)}_R \, H^1_1 + \bar{\nu}^{(2)}_l \bar{l}^{(2)}_R \, H^{2\,\dagger}_1 - \bar{l}^{(2)}_L \bar{l}^{(2)}_R \, H^{1\,\dagger}_1 \\
& \quad + l^{(2)}_R \psi^2_{H_1} \, \tilde{L}^1 - l^{(2)}_R \psi^1_{H_1} \, \tilde{L}^2 + \bar{l}^{(2)}_R \bar{\psi}^2_{H_1} \, \tilde{L}^{1\,\dagger} - \bar{l}^{(2)}_R \bar{\psi}^1_{H_1} \, \tilde{L}^{2\,\dagger} \; \Big] \\
= \; & f \Big[\; \bar{\tilde{H}} P_L \nu_l \, \tilde{R} + \bar{\nu}_l P_R \tilde{H} \, \tilde{R}^\dagger - \bar{\tilde{H}}_1 P_L l_L \, \tilde{R} - \bar{l}_L P_R \tilde{H}_1 \, \tilde{R}^\dagger \\
& \quad + \bar{l}_R P_L \nu_l \, H^2_1 + \bar{\nu}_l P_R l_R \, H^{2\,\dagger}_1 - \bar{l}_R P_L l_L \, H^1_1 - \bar{l}_L P_R l_R \, H^{1\,\dagger}_1 \\
& \quad + \bar{l}_R P_L \tilde{H}^c \, \tilde{L}^1 + \bar{\tilde{H}}^c P_R l_R \, \tilde{L}^{1\,\dagger} - \bar{l}_R P_L \tilde{H}_1 \, \tilde{L}^2 - \bar{\tilde{H}}_1 P_R l_R \, \tilde{L}^{2\,\dagger} \; \Big] \\
= \; & f \Big[\; \Big\{ \bar{\tilde{H}} L^1 \, \tilde{R} - \bar{\tilde{H}}_1 L^2 \, \tilde{R} + \bar{R} L^1 \, H^2_1 - \bar{R} L^2 \, H^1_1 \\
& \qquad + \bar{R} \tilde{H}^c \, \tilde{L}^1 - \bar{R} \tilde{H}_1 \, \tilde{L}^2 \Big\} + h.c. \Big] \, . \qquad \text{(C.49)}
\end{aligned}
$$

C.2.3 Rewriting the Higgsino Mass Terms.

In order to complete the rewriting of the Lagrangian $(C.1)$, one has to transform the terms $\mu\varepsilon^{ij} \; \tilde{H}^{(2)\,i}_1 \tilde{H}^{(2)\,j}_2$, and their hermitian conjugated, into four-component notation. This is done like this

$$
\begin{aligned}
-\mu\varepsilon^{ij} & \Big[\; \tilde{H}^{(2)\,i}_1 \tilde{H}^{(2)\,j}_2 + \bar{\tilde{H}}^{(2)\,i}_1 \bar{\tilde{H}}^{(2)\,j}_2 \; \Big] \\
& = \mu \left[\psi^2_{H_1} \psi^1_{H_2} + \bar{\psi}^2_{H_1} \bar{\psi}^1_{H_2} - \psi^1_{H_1} \psi^2_{H_2} - \bar{\psi}^1_{H_1} \bar{\psi}^2_{H_2} \right] \\
& = \mu \, \bar{\tilde{H}} \tilde{H} - \frac{\mu}{2} \, \bar{\tilde{H}}_1 \tilde{H}_2 - \frac{\mu}{2} \, \bar{\tilde{H}}_2 \tilde{H}_1 , \qquad \text{(C.50)}
\end{aligned}
$$

and finally the rewriting procedure is completed.

C.3 Summation — The On-Shell Lagrangian.

In the two previous sections the transcription from two- to four-component notation of the various terms of the Lagrangian $(C.1)$ was completed. In this section we will collect the results, and with eqs. $(C.15)$, $(C.19)$, $(C.20)$, $(C.23)$, $(C.29)$, $(C.39)$, $(C.45)$, $(C.48)$, $(C.49)$ and finally eq. $(C.50)$ we obtain

$$
\begin{aligned}
\mathcal{L}_{SUSY} = \; & \left(D^\mu \tilde{L} \right)^\dagger \left(D_\mu \tilde{L} \right) + \left(D^\mu \tilde{R} \right)^\dagger \left(D_\mu \tilde{R} \right) - i \, \bar{L} \gamma^\mu D_\mu L - i \, \bar{R} \gamma^\mu D_\mu R \\
& - g \left[\left\{ \bar{L}^1 \tilde{W} \, \tilde{L}^2 + \bar{L}^2 \tilde{W}^c \, \tilde{L}^1 \right\} + h.c. \right] + \sqrt{2} e \left[\left\{ \bar{L}^2 \tilde{A} \, \tilde{L}^2 - \bar{\tilde{A}} R \, \tilde{R} \right\} + h.c. \right] \\
& - \frac{\sqrt{2} g}{\cos\theta_{\mathrm{W}}} \left[\left\{ \left(\mathcal{T}^3_i - Q_i \sin^2\theta_{\mathrm{W}} \right) \bar{L}^i \tilde{Z} \, \tilde{L}^i - \sin^2\theta_{\mathrm{W}} \, \bar{\tilde{Z}} R \, \tilde{R} \right\} + h.c. \right] \\
& - i \, \bar{\tilde{W}} \gamma^\mu \partial_\mu \tilde{W} - \frac{i}{2} \, \bar{\tilde{A}} \gamma^\mu \partial_\mu \tilde{A} - \frac{i}{2} \, \bar{\tilde{Z}} \gamma^\mu \partial_\mu \tilde{Z}
\end{aligned}
$$

$$
\begin{aligned}
& - g\cos\theta_{\rm W}\left[\ \bar{\tilde{Z}}\gamma^\mu\tilde{W}\ W^-_\mu + \bar{\tilde{W}}\gamma^\mu\tilde{Z}\ W^+_\mu - \bar{\tilde{W}}\gamma^\mu\tilde{W}\ Z_\mu\right] \\
& - e\left[\ \bar{\tilde{A}}\gamma^\mu\tilde{W}\ W^-_\mu + \bar{\tilde{W}}\gamma^\mu\tilde{A}\ W^+_\mu - \bar{\tilde{W}}\gamma^\mu\tilde{W}\ A_\mu\right] \\
& - \frac{1}{4}\mathcal{W}^{+\,\mu\nu}\mathcal{W}^-_{\mu\nu} - \frac{1}{4}\mathcal{W}^{-\,\mu\nu}\mathcal{W}^+_{\mu\nu} - \frac{1}{4}\mathcal{Z}^{\mu\nu}\mathcal{Z}_{\mu\nu} - \frac{1}{4}\mathcal{A}^{\mu\nu}\mathcal{A}_{\mu\nu} \\
& + \left(D^\mu H_1\right)^\dagger\left(D_\mu H_1\right) - \mu^2 H_1^\dagger H_1 + \left(D^\mu H_2\right)^\dagger\left(D_\mu H_2\right) - \mu^2 H_2^\dagger H_2 \\
& - \bar{\tilde{H}}\left(i\gamma^\mu\partial_\mu - \mu\right)\tilde{H} - \frac{i}{2}\ \bar{\tilde{H}}_1\gamma^\mu\partial_\mu\tilde{H}_1 - \frac{i}{2}\ \bar{\tilde{H}}_2\gamma^\mu\partial_\mu\tilde{H}_2 - \frac{\mu}{2}\bar{\tilde{H}}_1\tilde{H}_2 - \frac{\mu}{2}\bar{\tilde{H}}_2\tilde{H}_1 \\
& - \frac{g}{\sqrt{2}}\left[\ \left(\bar{\tilde{H}}\gamma^\mu P_R\tilde{H}_1 - \bar{\tilde{H}}\gamma^\mu P_L\tilde{H}_2\right)W^+_\mu + h.c.\right] + e\ \bar{\tilde{H}}\gamma^\mu\tilde{H}\ A_\mu \\
& + \frac{g}{2\cos\theta_{\rm W}}\left[\ \left(1-2\sin^2\theta_{\rm W}\right)\bar{\tilde{H}}\gamma^\mu\tilde{H} - \frac{1}{2}\left(\bar{\tilde{H}}_1\gamma^\mu\gamma_5\tilde{H}_1 - \bar{\tilde{H}}_2\gamma^\mu\gamma_5\tilde{H}_2\right)\ \right]Z_\mu \\
& - g\left[\left(\bar{\tilde{W}}P_R\tilde{H}\ H^1_1 + \bar{\tilde{H}}P_R\tilde{W}\ H^2_2 + \bar{\tilde{H}}_1P_R\tilde{W}\ H^2_1 + \bar{\tilde{W}}P_R\tilde{H}_2\ H^1_2\right) + h.c.\right] \\
& + \sqrt{2}e\left[\left(\bar{\tilde{A}}P_R\tilde{H}\ H^2_1 - \bar{\tilde{H}}P_R\tilde{A}\ H^1_2\right) + h.c.\right] \\
& - \frac{g}{\sqrt{2}\cos\theta_{\rm W}}\left[\left\{\bar{\tilde{Z}}P_R\tilde{H}_1\ H^1_1 - \bar{\tilde{H}}_2P_R\tilde{Z}\ H^2_2\right.\right. \\
& \qquad\qquad \left.\left. - \left(1-2\sin^2\theta_{\rm W}\right)\left(\bar{\tilde{Z}}P_R\tilde{H}\ H^2_1 - \bar{\tilde{H}}P_R\tilde{Z}\ H^1_2\right)\right\} + h.c.\right] \\
& + f\left[\left\{\bar{\tilde{H}}L^1\,\tilde{R} - \bar{\tilde{H}}_1L^2\,\tilde{R} + \bar{R}L^1\,H^2_1 - \bar{R}L^2\,H^1_1 + \bar{R}\tilde{H}^c\,\tilde{L}^1 - \bar{R}\tilde{H}_1\,\tilde{L}^2\right\} + h.c.\right] \\
& - \mu f\left[H_2^\dagger\tilde{L}\,\tilde{R} + h.c.\right] - f^2\left[\tilde{L}^\dagger\tilde{L}\,\tilde{R}^\dagger\tilde{R} + H_1^\dagger H_1\left(\tilde{L}^\dagger\tilde{L} + \tilde{R}^\dagger\tilde{R}\right) - H_1^\dagger\tilde{L}\left(H_1^\dagger\tilde{L}\right)^\dagger\right] \\
& - \frac{g^2}{2}\left(\tilde{L}^\dagger T^a\tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right)\left(\tilde{L}^\dagger T^a\tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right) \\
& - \frac{g^2\tan^2\theta_{\rm W}}{8}\left(\tilde{L}^\dagger\tilde{L} - 2\tilde{R}^\dagger\tilde{R} + H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + t.d. \qquad\qquad \text{(C.51)}
\end{aligned}
$$

Here $\tilde{W}^c$ is the charge conjugated (defined in eq. $(A.91)$) of the spinor (2.46) and P_L and P_R are the left- and right-handed projection operators given by eqs. $(A.80)$ and $(A.81)$, i.e.

$$
P_L = \frac{1}{2}\left(1-\gamma_5\right), \qquad \text{(C.52)}
$$

$$
P_R = \frac{1}{2}\left(1+\gamma_5\right). \qquad \text{(C.53)}
$$

Appendix D

The Two-Component Form of the On-Shell Lagrangian $\mathcal{L}_{SUSY}$.

In this appendix, starting with the off-shell Lagrangian (2.11), we will construct the corresponding (two-component) on-shell Lagrangian, i.e. we have to eliminate the auxiliary fields.

D.1 The Auxiliary Fields.

In sect. 2.2 we obtained, by using the Euler-Lagrange equations, the following relations for the auxiliary fields

$$F_L^{j\,\dagger} = -f\,\varepsilon^{ij} H_1^i \tilde{R}, \tag{D.1}$$

$$F_R^{\dagger} = -f\,\varepsilon^{ij} H_1^i \tilde{L}^j, \tag{D.2}$$

$$F_1^{i\,\dagger} = -\mu\,\varepsilon^{ij} H_2^j - f\,\varepsilon^{ij} \tilde{L}^j \tilde{R}, \tag{D.3}$$

$$F_2^{j\,\dagger} = -\mu\,\varepsilon^{ij} H_1^i, \tag{D.4}$$

and

$$D^a = -g\left[\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right], \tag{D.5}$$

$$D' = \frac{g'}{2}\,\tilde{L}^\dagger \tilde{L} - g'\,\tilde{R}^\dagger \tilde{R} + \frac{g'}{2}\,H_1^\dagger H_1 - \frac{g'}{2}\,H_2^\dagger H_2. \tag{D.6}$$

In this appendix, the detailed calculations for the back-substitution of these relations into $\mathcal{L}_{Aux}$, given by eq. (2.16), will be performed, and we start by eliminating the auxiliary F-fields.

D.1.1 Auxiliary F-fields.

With eqs. $(D.1)$–$(D.4)$ we have

$$
\begin{aligned}
\mathcal{L}_{Aux-F} &= F_L^\dagger F_L + F_R^\dagger F_R + F_1^\dagger F_1 + F_2^\dagger F_2 \\
&\quad + \mu\, \varepsilon^{ij} \left[H_1^i F_2^j + H_1^{i\,\dagger} F_2^{j\,\dagger} + F_1^i H_2^j + F_1^{i\,\dagger} H_2^{j\,\dagger} \right] \\
&\quad + f\, \varepsilon^{ij} \left[F_1^i \tilde{L}^j \tilde{R} + F_1^{i\,\dagger} \tilde{L}^{j\,\dagger} \tilde{R}^\dagger + H_1^i F_L^j \tilde{R} + H_1^{i\,\dagger} F_L^{j\,\dagger} \tilde{R}^\dagger \right. \\
&\qquad\qquad \left. + H_1^i \tilde{L}^j F_R + H_1^{i\,\dagger} \tilde{L}^{j\,\dagger} F_R^\dagger \right] \\
&= \left(-f\varepsilon^{ij}\, H_1^i \tilde{R}\right) \left(-f\varepsilon^{kj}\, H_1^{k\,\dagger} \tilde{R}^\dagger\right) + \left(-f\varepsilon^{ij}\, H_1^i \tilde{L}^j\right) \left(-f\varepsilon^{kl}\, H_1^{k\,\dagger} \tilde{L}^{l\,\dagger}\right) \\
&\quad + \left(-\mu\varepsilon^{ij}\, H_2^j - f\varepsilon^{ij} \tilde{L}^j \tilde{R}\right) \left(-\mu\varepsilon^{ik}\, H_2^{k\,\dagger} - f\varepsilon^{ik} \tilde{L}^{k\,\dagger} \tilde{R}^\dagger\right) \\
&\quad + \left(-\mu\varepsilon^{ij}\, H_1^i\right) \left(-\mu\varepsilon^{kj}\, H_1^{k\,\dagger}\right) \\
&\quad + \mu\, \varepsilon^{ij}\, H_1^i \left(-\mu\varepsilon^{kj}\, H_1^{k\,\dagger}\right) + \mu\, \varepsilon^{ij}\, H_1^{i\,\dagger} \left(-\mu\varepsilon^{kj}\, H_1^k\right) \\
&\quad + \mu\, \varepsilon^{ij} \left(-\mu\varepsilon^{ik}\, H_2^{k\,\dagger} - f\varepsilon^{ik} \tilde{L}^{k\,\dagger} \tilde{R}^\dagger\right) H_2^j \\
&\quad + \mu\, \varepsilon^{ij} \left(-\mu\varepsilon^{ik}\, H_2^k - f\varepsilon^{ik} \tilde{L}^k \tilde{R}\right) H_2^{j\,\dagger} \\
&\quad + f\, \varepsilon^{ij} \left(-\mu\varepsilon^{ik}\, H_2^{k\,\dagger} - f\varepsilon^{ik} \tilde{L}^{k\,\dagger} \tilde{R}^\dagger\right) \tilde{L}^j \tilde{R} \\
&\quad + f\, \varepsilon^{ij} \left(-\mu\varepsilon^{ik}\, H_2^k - f\varepsilon^{ik} \tilde{L}^k \tilde{R}\right) \tilde{L}^{j\,\dagger} \tilde{R}^\dagger \\
&\quad + f\, \varepsilon^{ij}\, H_1^i \left(-f\varepsilon^{kj}\, H_1^{k\,\dagger} \tilde{R}^\dagger\right) \tilde{R} + f\, \varepsilon^{ij}\, H_1^{i\,\dagger} \left(-f\varepsilon^{kj}\, H_1^k \tilde{R}\right) \tilde{R}^\dagger \\
&\quad + f\, \varepsilon^{ij}\, H_1^i \tilde{L}^j \left(-f\varepsilon^{kl}\, H_1^{k\,\dagger} \tilde{L}^{l\,\dagger}\right) + f\, \varepsilon^{ij}\, H_1^{i\,\dagger} \tilde{L}^{j\,\dagger} \left(-f\varepsilon^{kl}\, H_1^k \tilde{L}^l\right) \\
&= -\mu^2\, H_1^\dagger H_1 - \mu^2\, H_2^\dagger H_2 - \mu f \left[H_2^\dagger \tilde{L}\, \tilde{R} + \tilde{L}^\dagger H_2\, \tilde{R}^\dagger \right] \\
&\quad - f^2 \left[\tilde{L}^\dagger \tilde{L}\, \tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 \left(\tilde{L}^\dagger \tilde{L} + \tilde{R}^\dagger \tilde{R}\right) - H_1^\dagger \tilde{L} \left(H_1^\dagger \tilde{L}\right)^\dagger \right]. \qquad \text{(D.7)}
\end{aligned}
$$

Here in the last transition the following relations have been used:

$$
\begin{aligned}
\varepsilon^{ij}\varepsilon^{kj} &= \delta^{ik}, \\
\varepsilon^{ij}\varepsilon^{kl} &= \delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}.
\end{aligned}
$$

D.1.2 Auxiliary D-fields.

When one is going to rewrite $\mathcal{L}_{Aux-D}$, given by

$$
\begin{aligned}
\mathcal{L}_{Aux-D} &= \frac{1}{2}\, (\, D^a D^a + D' D'\,) \\
&\quad + \tilde{L}^\dagger \left(g T^a D^a - \frac{1}{2} g' D' \right) \tilde{L} + \tilde{R}^\dagger g' D' \tilde{R}
\end{aligned}
$$

$$+ H_1^\dagger \left(gT^a D^a - \frac{1}{2} g' D' \right) H_1 + H_2^\dagger \left(gT^a D^a + \frac{1}{2} g' D' \right) H_2, \qquad \text{(D.8)}$$

it is practical to introduce the following temporary abbreviations

$$\begin{aligned} A &= \tilde{L}^\dagger T^a \tilde{L}, \\ B &= H_1^\dagger T^a H_1, \\ C &= H_2^\dagger T^a H_2, \\ D &= \tilde{L}^\dagger \tilde{L}, \\ E &= \tilde{R}^\dagger \tilde{R}, \\ F &= H_1^\dagger H_1, \\ G &= H_2^\dagger H_2. \end{aligned}$$

Here the SU(2)-index "a" has been suppressed for convenience.

With these abbreviations eqs. $(D.5)$ and $(D.6)$ take on the form

$$\begin{aligned} D^a &= -g\left[A + B + C\right], \\ D' &= g' \left[\frac{D}{2} - E + \frac{F}{2} - \frac{G}{2} \right]. \end{aligned}$$

We will now rewrite each term of eq. $(D.8)$. Hence

$$\begin{aligned} \frac{1}{2} D^a D^a &= \frac{g^2}{2} (A + B + C)(A + B + C), \\ \frac{1}{2} D' D' &= \frac{g'^2}{2} \left(\frac{D}{2} - E + \frac{F}{2} - \frac{G}{2} \right) \left(\frac{D}{2} - E + \frac{F}{2} - \frac{G}{2} \right), \\ \tilde{L}^\dagger & \left(gT^a D^a - \frac{1}{2} g' D' \right) \tilde{L} \\ &= -g^2 A \left[A + B + C\right] - \frac{1}{2} g'^2 D \left[\frac{D}{2} - E + \frac{F}{2} - \frac{G}{2} \right], \\ \tilde{R}^\dagger g' D' \tilde{R} &= g'^2 \, E \left(\frac{D}{2} - E + \frac{F}{2} - \frac{G}{2} \right), \\ H_1^\dagger & \left(gT^a D^a - \frac{1}{2} g' D' \right) H_1 \\ &= -g^2 B \left[A + B + C\right] - \frac{1}{2} g'^2 F \left[\frac{D}{2} - E + \frac{F}{2} - \frac{G}{2} \right], \\ H_2^\dagger & \left(gT^a D^a + \frac{1}{2} g' D' \right) H_2 \\ &= -g^2 C \left[A + B + C\right] + \frac{1}{2} g'^2 G \left[\frac{D}{2} - E + \frac{F}{2} - \frac{G}{2} \right]. \end{aligned}$$

For $\mathcal{L}_{Aux-D}$ this implies

$$
\begin{aligned}
\mathcal{L}_{Aux-D} &= -\frac{g^2}{2}(A+B+C)(A+B+C) \\
&\quad -\frac{g'^2}{2}\left(\frac{D}{2}-E+\frac{F}{2}-\frac{G}{2}\right)\left(\frac{D}{2}-E+\frac{F}{2}-\frac{G}{2}\right),
\end{aligned}
$$

or in terms of the S-QFD fields

$$
\begin{aligned}
\mathcal{L}_{Aux-D} &= -\frac{g^2}{2}\left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right)\left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right) \\
&\quad -\frac{g'^2}{8}\left(\tilde{L}^\dagger \tilde{L} - 2\tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 - H_2^\dagger H_2\right)^2. \qquad \text{(D.9)}
\end{aligned}
$$

D.1.3 Conclusion.

From the two previous subsections, we can conclude that the expression for the “auxiliary” Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{Aux} &= \mathcal{L}_{Aux-F} + \mathcal{L}_{Aux-D} \\
&= -\mu^2\, H_1^\dagger H_1 - \mu^2\, H_2^\dagger H_2 - \mu f\left[H_2^\dagger \tilde{L}\,\tilde{R} + \tilde{L}^\dagger H_2\,\tilde{R}^\dagger\right] \\
&\quad - f^2\left[\tilde{L}^\dagger \tilde{L}\,\tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1\left(\tilde{L}^\dagger \tilde{L} + \tilde{R}^\dagger \tilde{R}\right) - H_1^\dagger \tilde{L}\left(H_1^\dagger \tilde{L}\right)^\dagger\right] \\
&\quad -\frac{g^2}{2}\left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right)\left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right) \\
&\quad -\frac{g'^2}{8}\left(\tilde{L}^\dagger \tilde{L} - 2\tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 - H_2^\dagger H_2\right)^2. \qquad \text{(D.10)}
\end{aligned}
$$

This concludes this section.

D.2 The On-Shell Lagrangian.

The on-shell Lagrangian $\mathcal{L}_{SUSY}$ is with the results of the previous section, easily obtained from the corresponding off-shell Lagrangian (2.11) by substituting for eq. $(D.10)$.

The result is:

$$
\begin{aligned}
\mathcal{L}_{SUSY} &= \left(D^\mu \tilde{L}\right)^\dagger\left(D_\mu \tilde{L}\right) + \left(D^\mu \tilde{R}\right)^\dagger\left(D_\mu \tilde{R}\right) - i\,\bar{L}^{(2)}\bar{\sigma}^\mu D_\mu L^{(2)} - i\,\bar{R}^{(2)}\bar{\sigma}^\mu D_\mu R^{(2)} \\
&\quad + \sqrt{2}i\,\tilde{L}^\dagger\left(gT^a\lambda^a - \frac{1}{2}g'\lambda'\right)L^{(2)} - \sqrt{2}i\,\bar{L}^{(2)}\left(gT^a\bar{\lambda}^a - \frac{1}{2}g'\bar{\lambda}'\right)\tilde{L} \\
&\quad + \sqrt{2}i\,\tilde{R}^\dagger g'\lambda' R^{(2)} - \sqrt{2}i\,\bar{R}^{(2)} g'\bar{\lambda}'\tilde{R} \\
&\quad - i\,\bar{\lambda}^a\bar{\sigma}^\mu D_\mu\lambda^a - i\,\bar{\lambda}'\bar{\sigma}^\mu D_\mu\lambda' - \frac{1}{4}\left(V^{a\,\mu\nu}V^a_{\mu\nu} + V'^{\mu\nu}V'_{\mu\nu}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+ \left(D^\mu H_1\right)^\dagger \left(D_\mu H_1\right) + \left(D^\mu H_2\right)^\dagger \left(D_\mu H_2\right) \\
&- i\; \bar{\tilde{H}}_1^{(2)} \bar{\sigma}^\mu D_\mu \tilde{H}_1^{(2)} - i\; \bar{\tilde{H}}_2^{(2)} \bar{\sigma}^\mu D_\mu \tilde{H}_2^{(2)} \\
&+ \sqrt{2} i\; H_1^\dagger \left(g T^a \lambda^a - \frac{1}{2} g' \lambda' \right) \tilde{H}_1^{(2)} - \sqrt{2} i\; \bar{\tilde{H}}_1^{(2)} \left(g T^a \bar{\lambda}^a - \frac{1}{2} g' \bar{\lambda}' \right) H_1 \\
&+ \sqrt{2} i\; H_2^\dagger \left(g T^a \lambda^a + \frac{1}{2} g' \lambda' \right) \tilde{H}_2^{(2)} - \sqrt{2} i\; \bar{\tilde{H}}_2^{(2)} \left(g T^a \bar{\lambda}^a + \frac{1}{2} g' \bar{\lambda}' \right) H_2 \\
&- \varepsilon^{ij} \;\Big[\mu \left(\tilde{H}_1^{(2)\,i} \tilde{H}_2^{(2)\,j} + \bar{\tilde{H}}_1^{(2)\,i} \bar{\tilde{H}}_2^{(2)\,j} \right) + f \left(\tilde{H}_1^{(2)\,i} L^{(2)\,j} \tilde{R} + \bar{\tilde{H}}_1^{(2)\,i} \bar{L}^{(2)\,j} \tilde{R}^\dagger \right) \\
&\qquad + f \left(H_1^i L^{(2)\,j} R^{(2)} + H_1^{i\,\dagger} \bar{L}^{(2)\,j} \bar{R}^{(2)} + R^{(2)} \tilde{H}_1^{(2)\,i} \tilde{L}^j + \bar{R}^{(2)} \bar{\tilde{H}}_1^{(2)\,i} \tilde{L}^{j\,\dagger} \right) \Big] \\
&- \mu^2\, H_1^\dagger H_1 - \mu^2\, H_2^\dagger H_2 - \mu f \left[H_2^\dagger \tilde{L}\, \tilde{R} + \tilde{L}^\dagger H_2\, \tilde{R}^\dagger \right] \\
&- f^2 \left[\tilde{L}^\dagger \tilde{L}\, \tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 \left(\tilde{L}^\dagger \tilde{L} + \tilde{R}^\dagger \tilde{R} \right) - H_1^\dagger \tilde{L} \left(H_1^\dagger \tilde{L} \right)^\dagger \right] \\
&- \frac{g^2}{2} \left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2 \right) \left(\tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2 \right) \\
&- \frac{g'^2}{8} \left(\tilde{L}^\dagger \tilde{L} - 2 \tilde{R}^\dagger \tilde{R} + H_1^\dagger H_1 - H_2^\dagger H_2 \right)^2 + t.d. \qquad \text{(D.11)}
\end{aligned}
$$

Hence this appendix is concluded.

Appendix E

Transcription of the Scalar Higgs Potential.

The aim of this appendix is to eliminate the SU(2) representation matrices T^a appearing in the scalar Higgs potential given by eq. (3.5), i.e.

$$\begin{aligned} V_{Higgs} &= \left(m_1^2+\mu^2\right) H_1^\dagger H_1 + \left(m_2^2+\mu^2\right) H_2^\dagger H_2 - m_3^2\,\varepsilon^{ij}\left(H_1^i H_2^j + h.c.\right) \\ &+ \frac{g^2}{2}\left(H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right)\left(H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2\right) \\ &+ \frac{g'^2}{8}\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 . \end{aligned} \tag{E.1}$$

Our starting point is the following general calculation

$$\begin{aligned} H^\dagger{}_m T^a H_m\, H^\dagger{}_n T^a H_n &= \frac{1}{4}\, H^\dagger{}_m \sigma^a H_m\, H^\dagger{}_n \sigma^a H_n \qquad\qquad m,n=1,2 \quad \text{(no sum)} \\ &= \frac{1}{4}\Bigg[H^\dagger{}_m \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} H_m\, H^\dagger{}_n \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} H_n \\ &\qquad + H^\dagger{}_m \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} H_m\, H^\dagger{}_n \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} H_n \\ &\qquad + H^\dagger{}_m \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} H_m\, H^\dagger{}_n \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} H_n\Bigg] \\ &= \frac{1}{4}\Big[\left(H_m^1{}^\dagger H_m^2 + H_m^2{}^\dagger H_m^1\right)\left(H_n^1{}^\dagger H_n^2 + H_n^2{}^\dagger H_n^1\right) \\ &\qquad - \left(H_m^2{}^\dagger H_m^1 - H_m^1{}^\dagger H_m^2\right)\left(H_n^2{}^\dagger H_n^1 - H_n^1{}^\dagger H_n^2\right) \\ &\qquad + \left(H_m^1{}^\dagger H_m^1 - H_m^2{}^\dagger H_m^2\right)\left(H_n^1{}^\dagger H_n^1 - H_n^2{}^\dagger H_n^2\right)\Big] \end{aligned}$$

$$= \frac{1}{4} \Big[2 \; H_m^{1\,\dagger} H_m^2 \;\; H_n^{2\,\dagger} H_n^1 + 2 \; H_m^{2\,\dagger} H_m^1 \;\; H_n^{1\,\dagger} H_n^2$$
$$+ H_m^{1\,\dagger} H_m^1 \;\; H_n^{1\,\dagger} H_n^1 + H_m^{2\,\dagger} H_m^2 \;\; H_n^{2\,\dagger} H_n^2$$
$$- H_m^{1\,\dagger} H_m^1 \;\; H_n^{2\,\dagger} H_n^2 - H_m^{2\,\dagger} H_m^2 \;\; H_n^{1\,\dagger} H_n^1 \Big] .$$

With this result we have

$$\frac{g^2}{2} \left(H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2 \right) \left(H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2 \right)$$
$$= \frac{g^2}{8} \Big[\left(H_1^{1\,\dagger} H_1^1 \right)^2 + \left(H_1^{2\,\dagger} H_1^2 \right)^2 + \left(H_2^{1\,\dagger} H_2^1 \right)^2 + \left(H_2^{2\,\dagger} H_2^2 \right)^2$$
$$+ 2 \; H_1^{2\,\dagger} H_1^1 \;\; H_1^{1\,\dagger} H_1^2 + 2 \; H_2^{2\,\dagger} H_2^1 \;\; H_2^{1\,\dagger} H_2^2$$
$$+ 2 \; H_1^{1\,\dagger} H_1^1 \;\; H_2^{1\,\dagger} H_2^1 + 2 \; H_1^{2\,\dagger} H_1^2 \;\; H_2^{2\,\dagger} H_2^2$$
$$- 2 \; H_1^{1\,\dagger} H_1^1 \;\; H_2^{2\,\dagger} H_2^2 - 2 \; H_1^{2\,\dagger} H_1^2 \;\; H_2^{1\,\dagger} H_2^1$$
$$+ 4 \; H_1^{1\,\dagger} H_1^2 \;\; H_2^{2\,\dagger} H_2^1 + 4 \; H_1^{2\,\dagger} H_1^1 \;\; H_2^{1\,\dagger} H_2^2 \; \Big]$$

$$= \frac{g^2}{8} \Big[\, 2 \left| H_1^{i\,\dagger} H_2^i \right|^2 - 2 \left| \varepsilon^{ij} \, H_1^i H_2^j \right|^2$$
$$+ \left(H_1^{i\,\dagger} H_1^i - \; H_2^{j\,\dagger} H_2^j \right)^2 + 2 \left(H_1^{i\,\dagger} H_1^i \right) \left(H_2^{j\,\dagger} H_2^j \right) \Big]$$

$$= \frac{g^2}{8} \left[\left(H_1^{i\,\dagger} H_1^i - \; H_2^{j\,\dagger} H_2^j \right)^2 + 4 \left| H_1^{i\,\dagger} H_2^i \right|^2 \right] .$$

Here in the last transition we have used the identity

$$\left| H_1^{i\,\dagger} H_2^j \right|^2 + \left| e^{ij} \; H_1^i H_2^j \right|^2 \;\; = \;\; \left(H_1^{i\,\dagger} H_1^i \right) \left(H_2^{j\,\dagger} H_2^j \right) ,$$

which can be derived by straightforward calculations.

Hence the scalar Higgs potential $(E.1)$ reads

$$V_{Higgs} \;\; = \;\; \left(m_1^2 + \mu^2 \right) H_1^\dagger H_1 + \left(m_2^2 + \mu^2 \right) H_2^\dagger H_2 - m_3^2 \, \varepsilon^{ij} \left(H_1^i H_2^j + h.c. \right)$$
$$+ \frac{1}{8} \left(g^2 + g'^2 \right) \left(H_1^\dagger H_1 - H_2^\dagger H_2 \right)^2 + \frac{g^2}{2} \left| H_1{}^\dagger H_2 \right|^2 , \qquad \text{(E.2)}$$

and this concludes this appendix.