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Breaking Cosmic Strings without Monopoles

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Abstract

It is shown that topologically stable cosmic strings can, in fact, appear to end or to break, even in theories without monopoles. This can occur whenever the spatial topology of the universe is nontrivial. For the case of Abelian-Higgs strings, we describe the gauge and scalar field configurations necessary for a string to end on a black hole. We give a lower bound for the rate at which a cosmic string will break via black hole pair production, using an instanton calculation based on the Euclidean C-metric.

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1. Introduction

In the absence of singularities or monopoles, local cosmic strings cannot end, and hence, must either be infinite in extent or form closed loops. It is the purpose of this letter, however, to point out that, if the topology of space is nontrivial, then local cosmic strings may appear to end. In particular, a cosmic string may disappear down the throat of a black hole. Moreover, when topology changing processes are included (as suggested by quantum gravity) a cosmic string can appear to break.

In a functional integral approach to quantum gravity, the leading approximation to such a topology changing process is given by an instanton, or solution to the Euclidean field equations, which interpolates between the initial and final spacetimes. The semiclassical approximation to the rate is then simply related to the Euclidean action for the instanton. One such process, in which a cosmic string splits, with black holes appearing at the two ends, can be described approximately by a gravitational instanton based on the charged C-metric[1].

The Lorentzian charged C-metric describes a pair of charged black holes accelerating away from one another along a symmetry axis [2], say the z-axis. The C-metric then has conical singularities on the z-axis characterized by a deficit angle δ_{in} on the inner part of the axis, between the two black holes, and deficit angle δ_{out} on the outer parts of the axis, extending from each black hole out to $z = \pm\infty$. These conical singularities may be removed by introducing a background magnetic field [3] of the appropriate strength to provide the force necessary to accelerate the black holes. The resulting metric, known as the Ernst metric, has served as the starting point for calculations of the pair creation rate for magnetically charged black holes in a background magnetic field [1],[4],[5].

In this paper, however, we will work with the C-metric directly, interpreting the conical singularities as a model for a thin cosmic string along the z-axis. It has recently been shown that the conical singularity may indeed be filled in with stress energy corresponding to a real cosmic string [6]. For positive black hole mass, one has $\delta_{in} < \delta_{out}$, implying that the mass per unit length of the string is greater on the outer axis than on the inner axis. The corresponding difference in string tension between the inner and outer axis provides the force which accelerates the black holes. The parameters of the C-metric may be chosen so that $\delta_{in} = 0$, corresponding to a string which breaks completely.[2] More generally the string can 'fray'. In a real cosmic string, the magnetic flux is quantized. If the string carries only a single unit of flux, then it must 'break' entirely. If it carries multiple units of flux, than it can fray by discrete amounts, corresponding to a given number of flux quanta.

The Euclidean action for the C-metric is infinite, but the physical quantity determining the rate of pair creation is the difference between this action and that of an appropriate background geometry. As for the Ernst instanton, we find that this difference is given

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[1] This has been previously noted by Gibbons [1], though not the argument which follows below about how the gauge field behaves.

[2] The (nonextreme) black holes which are produced have their horizons identified to form a wormhole in space. If $\mu_{in} = 0$, the cosmic string does not actually break, but simply passes through the wormhole.

by a simple geometrical expression [7], $\Delta I = -\frac{1}{4}(A_{bh} + \Delta A_{acc})$, where A_{bh} is the area of the black hole horizon and ΔA_{acc} is the area of the acceleration horizon relative to the background. For small mass per unit length μ of the string, the relative action determining the rate is given by

$$\Delta I \simeq \frac{\pi m^2}{\mu_{out} - \mu_{in}}. \tag{1.1}$$

The semiclassical approximation to the rate is then $e^{-\Delta I}$.

2. Cosmic Strings and Black Holes

We begin by describing how a cosmic string can appear to end on a black hole. For definiteness, consider the Abelian-Higgs model coupled to gravity. The matter fields are a $U(1)$ gauge field A_μ and a charged scalar field Φ with a Mexican hat potential. The cosmic string is the familiar Nielsen-Olsen vortex. In the simplest case, one unit of magnetic flux runs along the center of the vortex. The scalar field far from the string is $\Phi \approx v \exp(i\phi)$, where v is the vev and $0 \leq \phi \leq 2\pi$ is an angular coordinate around the string. So the phase of Φ has unit winding number going around a large loop linking the string.

Now suppose the cosmic string enters a black hole. On a constant time slice, the horizon is topologically a 2-sphere. For simplicity, the natural thickness of the string will be taken much smaller than the radius of the black hole. The string pierces the horizon at some point S ("south pole"). Take a loop on the horizon around S much larger than the string thickness but smaller than the black hole. Around this loop, Φ winds once in phase. Deform the loop, and attempt to shrink it to the antipodal point N ("north pole").

It seems as if there will be trouble because of the winding number of Φ in phase. But phase is gauge dependent, and this winding number can be unwound by a suitable gauge transformation

$$\Phi' = U\Phi, \qquad eA'_\mu = eA_\mu + iU^{-1}\partial_\mu U \tag{2.1}$$

which merely implies that we need a nontrivial $U(1)$ bundle.

To be explicit, take a slightly-larger-than-hemispherical gauge patch on the event horizon, about S. Take a similar patch about N. The two patches are to overlap along a closed ("equatorial") strip. To define a bundle we give a gauge transformation U on the overlap, to take us from the S patch to the N patch; a nontrivial bundle is defined by a topologically nontrivial U. To unwind the phase, it suffices to take $U = \exp(-i\phi)$ where $0 \leq \phi \leq 2\pi$ is an angular coordinate ("longitude") on the horizon that runs around the strip. The vector potential can be taken as $A_\mu = 0$ in the N patch, which will gauge-transform in the overlap region into the required vector potential in the S patch. This completes the construction.

We have constructed here a field configuration topologically equivalent to the Wu-Yang monopole [8]. In the Wu-Yang monopole the magnetic flux is spread uniformly over the 2-sphere, whereas here the magnetic flux is all gathered up and concentrated into a narrow flux tube at S. The $U(1)$ bundle we have constructed is precisely the well-known bundle that arises from the Hopf fibration of the 3-sphere.

Now consider possible time dependence. The magnetic flux crossing any closed 2-surface is absolutely conserved, according to a topological conservation law. Thus the flux entering each separate black hole is absolutely conserved, and if a black hole terminates a string at one time, that black hole must always terminate a string. The only way of circumventing this restriction in classical gravity is to allow the black holes themselves to merge, with the total flux remaining conserved. In quantum gravity, black holes themselves can be be created or destroyed in pairs, and the topological conservation law simply constrains the the total magnetic flux of both holes to be zero, while individually the fluxes may be nonzero. Thus, through the creation by quantum tunneling of a black hole pair along a cosmic string, the string can break.

The same process can occur in any gauge theory that admits local cosmic strings, i.e., in which the vacuum manifold has a nontrivial π_1. Some such theories will also admit monopoles on which cosmic strings can end, and in such theories cosmic strings can also break through creation of monopole pairs [9]. However, string breaking by black hole pairs is always possible, even if the theory admits no such monopoles.

3. Splitting Strings

As described in the introduction, the instanton describing the pair creation of black holes along a cosmic string is given by the Euclidean C-metric. This metric and gauge potential V_μ are given by

$$\begin{aligned} ds^2 &= r^2 \left(-G(y) \left(\frac{\beta}{2\pi} \right)^2 d\tau^2 - \frac{dy^2}{G(y)} + \kappa^2 G(x) d\phi^2 + \frac{dx^2}{G(x)} \right) \\ V_\phi &= \kappa q (x - \xi_4) \\ r &= \frac{1}{A(x-y)}, \qquad G(x) = 1 - x^2 - 2mAx^3 - q^2 A^2 x^4 \end{aligned} \tag{3.1}$$

where $0 \leq \phi \leq 2\pi$ and $0 \leq \tau \leq 2\pi$. The function $G(\xi)$ has four roots which we shall label $\xi_1 \leq \xi_2 < \xi_3 < \xi_4$. Hence the coordinate ranges are $\xi_2 \leq y \leq \xi_3$, $\xi_3 \leq x \leq \xi_4$. The black hole horizon is at $y = \xi_2$ and the acceleration horizon is at $y = \xi_3$. The inner strut is at $x = \xi_4$ and the outer strut at $x = \xi_3$. Spatial infinity is at the point where $x = y = \xi_3$, and so $r \to \infty$. The black holes carry magnetic charge q under the unbroken $U(1)$ gauge field V_μ. The reader should note that this gauge field is distinct from the broken gauge field A_μ from which the cosmic string is constructed. The presence of this second gauge field is required below in order to construct a smooth instanton.

In the model of a cosmic string by flat space minus a wedge, the mass per unit length of the string is equal to $\delta/8\pi$, where δ is the deficit angle. In terms of the metric coefficients, the deficit angle on the outer axis is given by

$$\delta_{out} = 2\pi \left(1 - \frac{\kappa}{2} |G'(\xi_3)| \right) \tag{3.2}$$

and the deficit angle on the inner axis is

$$\delta_{in} = 2\pi \left(1 - \frac{\kappa}{2} |G'(\xi_4)| \right). \tag{3.3}$$

Clearly, from the symmetrical form of the metric, there are also nodes in the $\tau - y$ plane for generic choices of parameters. Unlike the conical singularities along the axis, these nodes cannot be interpreted as approximations to a smooth cosmic string. Instead, they represent points where the field equations are no longer satisfied. In the usual instanton approximation, one requires that the equations hold everywhere, and so these singularities must be avoided. There are two ways to achieve this. First, one can set

$$G'(\xi_2) = -G'(\xi_3), \qquad \beta = 4\pi/G'(\xi_3). \tag{3.4}$$

This requires that $q = m$ in the definition of $G(x)$ and implies that $\xi_3 - \xi_1 = \xi_4 - \xi_2$. Geometrically, this corresponds to pair creating nonextreme black holes with their horizons identified to form a wormhole [4]. The surface gravities, or temperatures, of the black hole and acceleration horizons are equal. Alternatively, one can consider extremal black holes where $\xi_1 = \xi_2$ [5]. In this case, the black hole horizon is infinitely far away. The conical singularity on the acceleration horizon will be absent provided we again set $\beta = 4\pi/G'(\xi_3)$.

Consider a cosmic string of a given μ_{out}, or equivalently, a given δ_{out}. We want to compute the rate at which extreme and nonextreme black holes are pair produced with a string of deficit angle $\delta_{in} < \delta_{out}$ between them. So we need to evaluate the Euclidean action for the C-metric with these parameters. The metric (3.1) contains five parameters: m, q, A, β, κ. Two of these are fixed by (3.4) (or the analogous conditions for extreme black holes). Two are fixed by our choice of δ_{out} and δ_{in}. The remaining parameter can be thought of as the charge of the created black holes and remains arbitrary.

The Euclidean action for the Einstein-Maxwell theory is given by

$$I = \frac{1}{16\pi}\int_M [-R + F^2] - \frac{1}{8\pi}\int_{\partial M} K \tag{3.5}$$

This is infinite for (3.1), but the physically meaningful quantity is the difference between the action for the C-metric, and a reference background. The appropriate background here is flat space minus a wedge with deficit angle δ_{out}. As discussed earlier, we are viewing the conical singularity in the C-metric and the background as an approximation to a thin smooth string composed of gauge and scalar fields, which satisfy their field equations everywhere. Thus, in evaluating the action, there is no need to introduce additional boundaries around the conical singularity. As discussed in [10], [11], [12], the action is conveniently evaluated on a solution by rewriting it in Hamiltonian form. The surfaces of constant τ intersect on the horizons, and these points of intersection must be treated separately. Evaluating the action in a neighborhood of the horizon yields a contribution $-A/4$, where A is the horizon area, so one obtains [12]

$$\Delta I = \beta H - \frac{1}{4}\Delta A_{acc} - \frac{1}{4}A_{BH} \tag{3.6}$$

where H is the total energy of the C-metric relative to the background, and ΔA_{acc} is the difference between the area of the acceleration horizons in the C-metric and the background. H is the sum of a term which is pure constraint and, hence, vanishes on a solution, plus an extrinsic curvature boundary term given below. For the extremal black hole of

metric (3.1), the horizon is infinitely far away, and so the surfaces of constant τ do not intersect there. As as result, there is no term $\frac{1}{4}A_{BH}$ in the action.

To evaluate the first two terms in (3.6), we need to match the C-metric and the background metric on a large sphere near infinity. The sphere is defined by $x-y=\epsilon$, and we are interested in the limit $\epsilon \to 0$. As in [7], we change to new coordinates χ, ϵ with $x = \xi_3 + \epsilon\chi,\ y = \xi_3 + \epsilon(\chi - 1)$, where $0 \le \chi \le 1$. Then the induced metric on the two surface $d\tau = d\epsilon = 0$ is

$$ {}^2ds^2 = \frac{1}{\epsilon A^2 G'(\xi_3)} \left[4(1-\frac{1}{2\pi}\delta_{out})^2 \chi \left(1 + \frac{\epsilon}{2}\frac{G''(\xi_3)}{G'(\xi_3)}\chi \right) d\phi^2 - \frac{d\chi^2}{\chi(\chi-1)} \right] \tag{3.7} $$

The background metric can be described by (3.1) with $m = q = 0$. We now require that the metric (3.7) agree with the metric induced on the surface $x - y = \bar{\epsilon}$ in the background where $\bar{G}(x) = 1 - x^2$ and $\bar{\xi_3} = -1$. This will be the case provided

$$ G'(\xi_3)A^2\epsilon = 2\bar{A}^2\bar{\epsilon}, \quad \text{and} \quad -\epsilon\frac{G''(\xi_3)}{G'(\xi_3)} = \bar{\epsilon} \tag{3.8} $$

where $\bar{A}$ is the parameter appearing in the background metric.

On a solution, the Hamiltonian in (3.6) is given by $H = \int N({}^{(2)}K - {}^{(2)}\bar{K})$, where ${}^{(2)}K$ is the extrinsic curvature of the boundary in the $\tau =$ constant surface. The components of the normal to this surface are given by $n^x = -\frac{G(x)}{r\sqrt{G(x)-G(y)}}$, $n^y = -\frac{G(y)}{r\sqrt{G(x)-G(y)}}$ and one finds

$$ {}^{(2)}K = D_i n^i = A\sqrt{\epsilon G'(\xi_3)} \left(1 + \epsilon\frac{G''(\xi_3)}{G'(\xi_3)}(\chi - \frac{3}{4}) \right). \tag{3.9} $$

Subtracting the analogous expression for the extrinsic curvature in the background, and using the matching conditions (3.8), one finds that ${}^{(2)}K - {}^{(2)}\bar{K} = 0(\epsilon^2)$. From (3.7), we see that $\sqrt{{}^{(2)}g}$ goes like ϵ^{-1}. The lapse behaves like $N = O(\epsilon^{-1/2})$. Therefore the energy term in the action vanishes as $\epsilon \to 0$.

We now compute ΔA_{acc}. Since the area of each acceleration horizon is infinite, we integrate out to the surface $x = \xi_3 + \epsilon$, subtract, and then take ϵ to zero:

$$ A_{acc} = \int_0^{2\pi} \kappa d\phi \int_{\xi_3+\epsilon}^{\xi_4} \frac{dx}{A^2(x-\xi_3)^2} = \frac{2(2\pi - \delta_{out})}{\epsilon A^2 G'(\xi_3)} \left(1 - \frac{\epsilon}{\xi_4 - \xi_3} \right) \tag{3.10} $$

Subtracting the similar expression for $\bar{A}_{acc}$ and using (3.8) gives

$$ \Delta A_{acc} = A_{acc} - \bar{A}_{acc} = -\frac{2(2\pi - \delta_{out})}{A^2 G'(\xi_3)} \left(\frac{1}{\xi_3 - \xi_1} + \frac{1}{\xi_3 - \xi_2} \right) \tag{3.11} $$

The area of the black hole horizon is

$$ A_{BH} = \frac{2(2\pi - \delta_{out})}{A^2 G'(\xi_3)} \left(\frac{1}{\xi_3 - \xi_2} - \frac{1}{\xi_4 - \xi_2} \right) \tag{3.12} $$

Combining these and using (3.4), gives the total physical action,

$$\Delta I = \frac{2\pi - \delta_{out}}{A^2 G'(\xi_3)(\xi_3 - \xi_1)} \tag{3.13}$$

This formula is also valid for the extremal instanton since in this case $\Delta I = -\frac{1}{4}\Delta A_{acc}$ and $\xi_2 = \xi_1$.

For small mA we can find a simple expression for this action. If we fix δ_{out}, the deficit angle of the string at infinity, and δ_{in}, the deficit angle of the string connecting the black holes, then we can expand $G'(\xi_3)$ and $G'(\xi_4)$ to first order in mA and use (3.2) and (3.3) to solve for mA. The result is

$$mA = \frac{1}{8\pi}(\delta_{out} - \delta_{in}) = \mu_{out} - \mu_{in} \tag{3.14}$$

This says that the black holes satisfy Newton's law. The acceleration is determined by the net tension in the strings connecting the black holes. Expanding the terms in the action (3.13) in powers of mA and using this result we obtain

$$\Delta I \simeq \frac{\pi m^2}{\mu_{out} - \mu_{in}} \tag{3.15}$$

The rate, $e^{-\Delta I}$, is largest for the string breaking $\mu_{in} = 0$. This makes sense because roughly the mass of the black holes must come from the missing mass of the string, so $\mu_{in} = 0$ corresponds to the black holes tunneling out at the smallest separation, which one expects for a quantum event. The rate increases for a more massive external string, and the rate vanishes when $\mu_{out} = \mu_{in}$, which says that one cannot pair create black holes without taking some energy away from the cosmic string.

4. Real Strings

The process we have discussed could have cosmological significance. It is well known that any process that turns cosmic strings into black holes (or other massive remnants) might seriously disrupt cosmic string cosmology. Note that black holes are always left behind; in a closed loop of strings, a nucleated black hole pair will race around the string, consume it entirely, and collide to leave behind one (or perhaps more) black holes. If multiple nucleations happen, multiple collisions will occur.

One can, however, substitute numbers corresponding to grand unified strings into (3.15) and find that the rate for breaking cosmic strings by this mechanism is far too small to be of cosmological significance. For a Higgs vacuum expectation value $v \sim 10^{16}$GeV and self coupling $\lambda \sim 1$, we must take the black hole to have mass $m >> 10^3\ m_{pl}$ in order for the thin string limit implicit in the use of the C-metric to be valid. This implies $\mu \sim v^2 \sim 10^{-6} m_{pl}^2$. We then have $\Delta I >> 10^{12}$, yielding an infinitesimally small rate.

However, this estimate of the rate is only a lower limit. The most likely tunneling event actually falls outside the class described by the C-metric. This would be to pair

create the smallest possible black holes which can swallow the flux from the string. One can estimate the size of such a black hole as having mass equal to a single quanta of magnetic charge, making it extremal. For the parameters assumed above, such a black hole would be small on the scale of a flux tube, so we would need another method for estimating the rate of production.

It is interesting to speculate about the production rate for black holes with mass not equal to charge. For a general choice of q and m in (3.1), there is a nodal singularity at the Euclidean black hole horizon. However, this singularity is integrable–it is only a two dimensional delta-function in the curvature. Evaluating the action (3.5) in the neighborhood of a horizon, one still finds that the contribution is $\frac{1}{4}A_H$, using the Gauss-Bonnet theorem [11]. Therefore, the action evaluated on any of the C-metrics is given by the basic formulae (3.6). Further, combining (3.11) and (3.12), one finds that for any of the C-metrics *except* the extremal black hole case, the action is given by

$$\Delta I = \frac{(2\pi - \delta_{out})}{2A^2 G'(\xi_3)} \left(\frac{1}{\xi_3 - \xi_1} + \frac{1}{\xi_4 - \xi_2} \right) \tag{4.1}$$

Finally, one finds that the value of ΔI for small mA given in (3.15) is the same for all values of q, m.

These nonsmooth C-metrics are not solutions everywhere, and so they do not have the usual instanton interpretation. However, since they fail to be a solution in a very mild way, and the "answer" they give for the rates is of exactly the same form as the smooth case, it is tempting to speculate that they do give the leading contribution to the pair production rate for general q, m. This is an issue for further consideration.

NOTE ADDED: After this work was completed, two papers appeared which discuss black hole pair creation and cosmic strings. The first [13] considers the case $\mu_{in} = 0$, and asserts that the calculation does not apply to topologically stable strings. We clearly disagree with this statement. The second [14] adds a background magnetic field and sets $\mu_{out} = 0$ (leaving $\mu_{in} \neq 0$), but does not discuss the applicability to real cosmic strings.

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