Relaxation mechanisms: From Damour-Polyakov to Peccei-Quinn

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Abstract

The relaxation mechanism of Damour-Polyakov for fixing the vacuum expectation value of certain scalar fields (moduli) in string theory could provide a convenient framework for the Peccei-Quinn relaxation mechanism and remove the narrow "axion window".

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I. The Peccei-Quinn mechanism

Relaxation mechanisms, by which some physical parameters can be dynamically relaxed to their (presumably small) values, are not unknown in physics.

In particle physics the most famous example is the Peccei-Quinn mechanism for solving the strong CP problem. Quantum chromodynamics (QCD), the $SU(3)_C$ gauge theory of strong interactions, allows a topological term

$$L_\theta = \frac{\theta}{32\pi^2} G_{\mu\nu}{}^\alpha \tilde{G}^{\prime\alpha\mu\nu} \tag{1}$$

If $\theta \neq 0$, this term induces through non-perturbative QCD-instanton effects violations of P and CP in the strong interactions. However, no such violations have been observed and the upper limit on the electric-dipole moment for the neutron requires $\theta \lesssim 10^{-9}$. The strong CP problem is the question why the parameter θ is so small. The Peccei-Quinn mechanism [1] is based on the idea of making the parameter θ a dynamical field $\theta(x) = \frac{\alpha(x)}{f_\alpha}$, where $\alpha(x)$ is a dynamical pseudo-scalar field called axion and f_α (known as decay constant) is the vacuum expectation value at which the global Peccei-Quinn $U(1)_{PQ}$ symmetry is spontaneously broken. The axion field is taken to reside in the phase of a standard-model (SM) $(SU(3)_C \times SU(2)_L \times U(1)_Y)$-singlet complex scalar field $\varphi = \frac{f_\alpha}{\sqrt{2}} e^{i\alpha/f_\alpha}$ with potential

$$V(\varphi) = \lambda \left(|\varphi|^2 - \frac{f_\alpha^2}{2} \right)^2 \tag{2}$$

The axion α corresponds to the flat $\theta = \frac{\alpha}{f_\alpha}$ degree of freedom and would have been massless (true Nambu-Goldstone boson) if there were not non-perturbative effects that make QCD depend on θ and break explicitly the global $U(1)_{PQ}$ symmetry at the scale Λ_{QCD}. These effects produce an effective potential

$$U(\theta) = U(\frac{\alpha}{f_\alpha}) = \Lambda_{QCD}{}^4 (1 - \cos N_{dw}\theta) = \Lambda_{QCD}{}^4 \left(1 - \cos N_{dw} \frac{\alpha}{f_\alpha} \right) \tag{3}$$

where N_{dw} is an integer depending on the theory and associated with domain walls [2]. One usually takes $N_{dw} = 1$ [3]. The potential (3) allows θ to relax to zero dynamically thus solving the strong CP problem. Moreover, the axion acquires a mass (it becomes a

pseudo-Nambu-Goldstone boson), which scales like $f_\alpha{}^{-1}$: $m_\alpha \sim \frac{\Lambda^2_{QCD}}{f_\alpha} \sim 10^{-5}eV\frac{10^{12}GeV}{f_\alpha}$. Its couplings also scale like $f_\alpha{}^{-1}$. Thus, a very light axion (very large f_α) is also very weakly coupled, hence the term invisible [4].

Various arguments constrain the axion mass m_α and the breaking scale f_α to lie in a very narrow window. In fact, searches for the axion in high-energy and nuclear physics experiments [5] and astrophysical considerations [6] require $m_\alpha \lesssim 10^{-3}eV$ ($f_\alpha \gtrsim 10^{10}GeV$). On the other hand, by asking that axions (through their coherent oscillations around the equilibrium value $\theta = 0$) do not overclose the universe, the famous cosmological constaint $m_\alpha \gtrsim 10^{-5}eV$ ($f_\alpha \lesssim 10^{12}GeV$) is obtained [7]. Moreover, since the Peccei-Quinn symmetry breaking involves the spontaneous breaking of a global U(1) symmetry, strings are produced [8, 3], which decay by radiating (among other things) axions. It was argued [9, 3] that this could strengthen the cosmological constraint $m_\alpha \gtrsim 10^{-4}eV$ ($f_\alpha \lesssim 10^{11}GeV$), although this is a matter of debate [10]. There remains, thus, a narrow "axion window" $10^{-5}eV \lesssim m_\alpha \lesssim 10^{-3}eV$ ($10^{10}GeV \lesssim f_\alpha \lesssim 10^{12}GeV$), to which existing projects of experimental search for axions are oriented [11].

II.   The Damour-Polyakov mechanism

In superstring theory the Damour-Polyakov mechanism [12] offers another example of a relaxation mechanism by which various moduli fields Φ are attracted towards their present vacuum expectation values due to string-loop effects. The idea is that non-perturbative effects, associated with higher genus corrections, may naturally generate different non-monotonic coupling functions $B_i(\Phi)$ of Φ to the other fields, labelled i, of the form

$$B_i(\Phi) = e^{-2\Phi} + c_0{}^{(i)} + c_1{}^{(i)}e^{2\Phi} + ... \tag{4}$$

Note that in the case of the dilaton such a coupling function already starts at the tree level (the first term in equation (4)), whereas for the other moduli fields it will arise at the one loop level and beyond. Under the assumption that the different coupling functions $B_i(\Phi)$ have extrema at some common point $\Phi = \Phi_m$ (which is guaranteed if they coincide $B(\Phi) \equiv B_i(\Phi)$), the expanding universe drives the vacuum expectation value of Φ towards the value Φ_m at which its interactions with matter become very weak [12].

In fact, under an appropriate rescaling $\Phi \rightarrow \phi = \phi(\Phi)$, all relevant couplings are $\propto \delta\phi$, where $\delta\phi = \phi - \phi_m$ is the relaxation shift of the moduli field towards ϕ_m. Deviations from general relativity are proportional to $(\delta\phi)^2$ and the present high-precision tests of the equivalence principle require $\delta\phi \lesssim 10^{-6}$. All these deviations have been actually estimated in this scheme to be sufficiently small at the present cosmological epoch [12]. Additional astrophysical and cosmological considerations may require a further strong suppression [13].

III.   Implications of an inflationary era

Inflation [14] has been extensively discussed in the past in relation with the Peccei-Quinn mechanism. One possibility is to have (either no inflation at all or) the Peccei-Quinn symmetry breaking down after inflation. The narrow "axion window" mentioned above is now relevant and it remains to be seen if it is realized in nature. The most serious problem in this case is the axionic domain wall problem [2]. For $N_{dw} = 1$ the problem does not exist, since then the domain walls are bounded by axionic strings and this can lead to their decay before they dominate the universe causing, thus, no considerable cosmological effects [8].

On the other hand, if the Peccei-Quinn symmetry breaks down before the end of inflation and the reheating temperature after inflation is lower than the Peccei-Quinn symmetry breaking scale, then the domain wall problem disappears, since the domain walls problem are inflated away.(It has been argued [15] that quantum fluctuations of the axion field during inflation may still lead to a domain wall problem even for $N_{dw} = 1$, but again domain walls are inflated away except if they are produced late enough.)[2]

If the Peccei-Quinn symmetry breaking occurs during inflation, anthropic principle arguments have been invoked to relax the cosmological constraint on the axion mass and open the "axion window" [18]. However, it was subsequently argued [19] that, even with inflation, it is rather difficult to avoid the constraint $m_\alpha \gtrsim 10^{-5} eV$ ($f_\alpha \lesssim 10^{12} GeV$), mainly due to

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[2]Although there could exist solutions of the axionic domain wall problem relying purely on particle physics, see e.g. ref. [16], or cases with the cosmological constraints on the axion mass relaxed simply due to possible physical processes of a large entropy increase at late stages in the evolution of the universe, see e.g. ref. [17], inflation being an influential idea per se remains the most appealing solution in many particle physics problems related in one way or another to cosmology.

considerations of isocurvature density perturbations produced during inflation by quantum fluctuations of the axion field [20]. (Nevertheless, it was pointed out [21] that inflationary models exist where the constraint $m_\alpha \gtrsim 10^{-5} eV$ ($f_\alpha \lesssim 10^{12} GeV$) may still be avoided.)

Coming now to the Damour-Polyakov mechanism, inflation may be a necessity [13]. A detailed analysis of a primordial inflationary era within this mechanism has been done in ref. [22]. It was there shown that such an era could easily solve the Polonyi-moduli problem [23] and, moreover, the produced quantum fluctuations $\delta\phi$ of the relevant moduli fields during this era are naturally compatible with the observational requirements from general relativity.

IV. A possible scenario

We will now present a possible scenario in which the Peccei-Quinn mechanism is realized in a superstring-theory context with the Damour-Polyakov ansätz and examine the consequences.

First, we notice that in superstring theory with N = 1 supergravity the potential axions are massless scalars closely connected with the anomaly cancellation mechanism [24]. They originate from the two form B residing in the supergravity multiplet. We encounter a model-independent scalar zero-mode (it arises in a way that does not depend on the details of compactification), as well as a model-dependent one. They exhibit couplings to $\text{tr}G\tilde{G}$ and give, thus, a four-dimensional scalar behaving as an axion α. The $\text{tr}G\tilde{G}$ coupling is the dominant term violating the axionic Peccei-Quinn symmetry non-linearly realized : $\alpha \rightarrow \alpha$ + c, c = constant. (Cosmological implications of domain walls in superstring theory have been discussed in ref. [25].)

The important thing for us is that a potential axion field resides among the moduli fields of a superstring theory. So, for that the Damour-Polyakov ansätz is applicable. Then, we can imagine the following picture.

The Peccei-Quinn symmetry is broken when a SM-singlet complex scalar field acquires a vacuum expectation value $\sim f_\alpha$ minimizing a potential as in (2). In a superstring theory f_α is naturally of the order of the Planck mass $\sim M_P$. The $\theta = \frac{\alpha}{f_\alpha}$ degree of freedom is a flat direction (flat directions naturally arise in the effective supergravity theories anyway)

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and corresponds to the axion field α. Being a moduli field, this degree of freedom develops a coupling function $B(\theta)$ à la Damour-Polyakov as in (4) (we consider a common coupling function). The Peccei-Quinn symmetry is broken before the end of inflation, which is driven by some scalar field σ interacting with the Peccei-Quinn field (it could be that the Peccei-Quinn field itself is the inflaton, as in " modular cosmology" [26]; we assume here that the inflaton is some other field). Note that, although the non-vanishing vacuum energy present during inflation can lift the flat directions of the effective supergravity theory, this is not necessarily the case [27]. Then, as explained in ref. [22, 13], at the end of inflation the dynamical variable θ, irrespective of its initial value, is quicly relaxed extremely close to its equilibrium point θ_m :

$$\delta\theta = (\theta - \theta_m) \sim e^{-cH\tau} \lesssim 10^{-30} \tag{5}$$

for $c \sim O(1)$ and $H\tau \gtrsim 70$, where H is the approximately constant Hubble parameter during the slow-roll period τ of inflation. The equilibrium value θ_m is naturally guaranteed to be $\theta_m = 0$ if there exist a discrete duality symmetry. Discrete duality symmetries are known to hold for moduli fields and, in fact, motivate the Damour-Polyakov mechanism.

The result is that in this case the axion angle θ in the early universe - the so-called "misalignment" angle - is quickly settled down to $\theta = 0$ at the end of an inflationary era within a causal region from which our entire presently observable universe has originated.

However, in addition there are quantum fluctuations arisen at the late stages of inflation. They set an absolute minimum to the effective misalignment angle and give rise to isocurvature axion fluctuations (fluctuations in the local axion-to-photon ratio) [20], which later evolve into density perturbations of the same maginitude leading to fluctuations in the temperature of the cosmic microwave background radiation (CMBR). The relevant quantum fluctuations in the axion field in the scheme under discussion can be extracted from ref. [22]. The largest possible ones have a size

$$\delta\theta \sim 10^{-7} 10^{-13\kappa} \left(\frac{10^5 H_*}{M_P} \right)^{1-\kappa/4} \tag{6}$$

where H_* is the expansion rate at the end of inflation t_* and $\kappa \equiv -B''(\theta_m)/B(\theta_m)$ is a parameter of the model expected to be $\lesssim 1$. For $H_* \lesssim 10^{-5} M_P$ (larger values of H_* lead to

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excessive amount of relic gravitational waves), the fluctuations (6) induce anisotropies of the CMBR temperatures (order of magnitude estimates) $\delta\theta/\theta \sim \delta\rho/\rho \sim \delta T/T$ safely smaller than the experimental constraint $\delta T/T \lesssim 10^{-5}$.

After inflation, the universe is left with a misalignment angle very close to zero. The thermalization temperature is estimated [28] to be $T_* \lesssim N_*^{-1/2} (H_* M_P)^{1/2}$, where N_* is the number of the effective relativistic degrees of freedom. A representative value is $H_* \sim 10^{-7} M_P$, for which $T_* \lesssim 10^{15} GeV$. The QCD-instanton effects are not operative until sufficiently small temperatures $T \sim \Lambda_{QCD}$, at which the field θ starts its coherent oscillations around the equilibrium value $\theta = 0$ of a potential as in (3). However, because of the very small value (6) of the effective axion angle left after inflation, the contribution of axions produced by the misalignment mechanism [7] to the present mass density of the universe is suppressed, due to the fact that it is proportional to the square of the effective misalignment angle. As a result, it is no longer necessary to lead to the constraint $m_\alpha \gtrsim 10^{-5} eV$, ($f_\alpha \lesssim 10^{12} GeV$).

For the present scenario, the value $f_\alpha \sim M_P$ is both possible and consistent: the satisfied condition $H_* < f_\alpha$ can prevent potential restoration of the Peccei-Quinn symmetry (by the Hawking temperature) before the end of inflation [29] and the also satisfied condition $T_* < f_\alpha$ is necessary for not restoring the symmetry after inflation. With $f_\alpha \sim M_P$ the axions develop a very small mass $m_\alpha \sim 10^{-12} eV$ and are precluded from being the dark matter. Their couplings are also very small. So, in this case the axions are invisible indeed.

Acknowledgements

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