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GLUON STRUCTURE FUNCTION
FOR DEEPLY INELASTIC SCATTERING
WITH NUCLEUS IN QCD

M. B. G A Y D U C A T I and A L V A R O L. A Y A L A Fº

Instituto de Fisica, Univ. Federal do Rio Grande do Sul
Caixa Postal 15051, 91500 Porto Alegre, RS, BRASIL

and

E U G E N E L E V I N [†]

LAFEX, Centro Brasileiro de Pesquisas Físicas (CNPq)
Rua Dr. Xavier Sigaud 150, 22290 - 180 Rio de Janeiro, RJ, BRASIL
and
Theory Department, Petersburg Nuclear Physics Institute
188350, Gatchina, St. Petersburg, RUSSIA

Abstract: In this talk we present the first calculation of the gluon structure function for nucleus in QCD. We discuss the Glauber formula for the gluon structure function and the violation of this simple approach that we anticipate in QCD [*].

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[†]) Email: levin@lafex.cbpf.br; levin@fnalv.fnal.gov; levin@ccsg.tau.ac.il
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1 Introduction.

The subject of the talk is the gluon structure function for DIS with nucleus. The gluon structure function is the most important physical observable that governs the physics at high energy (low Bjorken x) in the DIS. Dealing with nucleus we have to take into account the shadowing correction, which is the main point of interest in this talk. We show that the shadowing correction in the region of small x can be treated theoretically in QCD and can be reliably calculated using the information on the behaviour of the gluon structure function for the nucleon. We organize the presentation in the following way: first, we discuss the Glauber approach to the nucleus gluon structure function and answer the question what information on nucleon structure function we need to provide a reliable calculation using the Glauber formula; second, we briefly consider the corrections to Glauber approach that have been anticipated in QCD. It should be stressed that this is the first presentation of our results and the lack of space does not allow us to discuss the issue in details. This is why we are going to outline our strategy and to present the first estimates rather than to give the complete study of the problem which will be published elsewhere.

2 Glauber approach in QCD .

The idea how to write the Glauber formula in QCD has been first formulated in ref. [1] and was carefully developed by Mueller in ref. [2]. It is easier to explain the idea considering the penetration of quark-antiquark pair through the target. Indeed, during the time of passage through the target the transverse distance r_t between quark and antiquark can vary by the amount $\Delta r_t \propto R \frac{k_t}{E}$, where E is the energy of the pair and R is the size of the target (see Fig.1). The quark transverse momentum (k_t) is $k_t \propto \frac{1}{r_t}$ due to uncertainty principle. Therefore

$$\Delta r_t R \frac{k_t}{E} \ll r_t \tag{1}$$

holds if

$$r_t^2 \cdot s \gg 2 m R \tag{2}$$

In terms of Bjorken x the above condition looks as follows:

$$x \ll \frac{1}{2 m R} \tag{3}$$

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It means that the transverse distance between quark and antiquark is a good degree of freedom [1][2][3]. As has been shown by Mueller not only quark - antiquark pair can be considered in such way. The propagation of a gluon through the target can be treated in a similar way as the interaction of gluon - gluon pair with definite transverse separation r_t with the target. The total cross section of the absorbtion of gluon(G^*) with virtuality Q^2 and Bjorken x can be written in the form:

$$\sigma_{G^*} \quad = \tag{4}$$

$$\int_0^1 dz \ \int \frac{d^2 r_t}{2\pi} \ \int \frac{d^2 b_t}{2\pi} \Psi_\perp^{G^*}(Q^2, r_t, x, z) \ 2 \cdot \{1 \ - \ exp[- \ \sigma(r_t^2) \ S(b_t^2)]\} \cdot \Psi_\perp^{G^**}(Q^2, r_t, x, z)$$

where $\Psi_\perp^{G^*}$ is the wave function of the virtual gluon with transverse polarization. As was
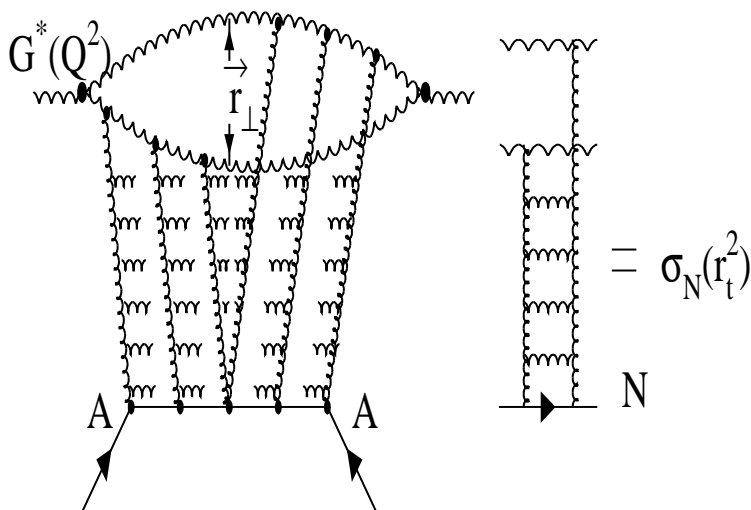

Figure 1: *The structure of the parton cascade in the Glauber formula.*

shown in ref. [2] within leading log approximation of perturbative QCD (pQCD) we can

safely replace this function by $\frac{1}{r_t^2}$ after integration over z in eq.(3). Finally the Glauber formula for the gluon structure function reads (for $N_c = N_f = 3$):

$$xG(Q^2, x) \; = \; \frac{4}{\pi^2} \int_x^1 \frac{dx'}{x'} \int \frac{d^2 b_t}{\pi} \int_{\frac{4}{Q^2}}^{\infty} \frac{d^2 r_t}{\pi} \frac{1}{r_t^4} \, 2 \left\{ 1 \; - \; e^{-\frac{1}{2} \sigma^{GG}(r_t^2, x') \, S(b_t^2)} \right\} \qquad (5)$$

where

$$\sigma^{GG} \; = \; \frac{3\alpha_s}{4} \, \pi^2 \, r_t^2 \, xG(\frac{4}{r_t^2}, x) \qquad (6)$$

and $S(b_t^2)$ is the profile function in impact parameter space for the interaction of the gluon-gluon pair with the target. We use for the calculation the Gaussian parameterization for S, namely:

$$S(b_t^2) \; = \; \frac{A}{\pi R_A^2} \, e^{-\frac{b_t^2}{R_A^2}} \qquad (7)$$

where A is the number of the nucleons in a nucleus and R_A^2 is the mean radius of a nucleus, which is equal to

$$R_A^2 \; = \; \frac{2}{5} \, R_{WS}^2$$

R_{WS} is the size of the nucleus in the Wood - Saxon parameterization, which we chose $R_{WS} \; = \; r_0 A^{\frac{1}{3}}$ with $r_0 = 1.3 fm$. Using the Gaussian parameterization for S we can take the integral over b_t and get the answer:

$$xG(Q^2, x) \; = \; \frac{2}{\pi^2} \int_x^1 \frac{dx'}{x'} \int_{\frac{1}{Q^2}}^{\infty} \frac{d^2 r_t'}{\pi} \frac{R_A^2}{r_t'^4} \left\{ C \; + \; \ln \kappa_G(x', r_t'^2) \; + \; E_1(\kappa_G(x', r_t'^2) \, \right\} \quad (8)$$

where C is the Euler constant and E_1 is the exponential integral (see ref.[4] **5.1.11**) and

$$\kappa_G(x', r_t'^2) \; = \; \frac{3\alpha_s A\pi \, r_t'^2}{2R_A^2} \, x' G_N(x', \frac{1}{r_t'^2}) \qquad (9)$$

3 Theory status of the Glauber formula.

In this section we would like to recall the main assumptions that have been made to get the Glauber formula:

1.Energy (x) should be so high (small) to satisfy eqs.(2) and (3) and $\alpha_s \ln(1/x) \approx 1$. The last condition means that we are doing the calculation in leading $\log(1/x)$ approximation of perturbative QCD (pQCD).

2.The GLAP [5] evolution equation holds in the region of small x. It means that $\alpha_s \ln(1/r_t^2) \approx 1$. One of the lessons that we have learned at this workshop is the fact that the GLAP equation is able to describe the HERA data quite well.

3.Only the fastest partons (GG pair) interacts with the target and there are no correlations between partons from different parton cascades (see Fig.1).

4.There are no correlations between different nucleons in a nucleus.

5.The average b_t for GG pair - nucleon interaction is much smaller than R_A.

We are going to discuss how well all the above assumptions work in the last section of the talk.

4 Results.

In our calculations we use the GRV parameterization [6] for the nucleon gluon structure function. This parameterization describes the data quite well and it is suited for our purpose because (i) the initial virtuality for the GLAP equation is small and we can discuss the contribution of the large distances having some support in the experimental data; (ii) the parameterization uses the GLAP equation and the most essential contribution comes from the region where $\alpha_s \ln Q^2 \approx 1$ and $\alpha_s \ln(1/x) \approx 1$.

4.1 Where the shadowing corrections are big.

Fig.2 shows the kinematic region of the deeply inelastic scattering. The curves are the solution of the equation $\kappa_G = 1$ for N (nucleon), Ca and Au. Above each of these curves the value of $\kappa_G > 1$ and the shadowing correction (SC) are big, below $\kappa_G < 1$ and the SC are rather small.

4.2 What we are able to calculate in QCD.

From the master equation (5) one can see that the large distances contribute to the value of the gluon structure function. Such contributions we are not able to calculate in pQCD and the value of the gluon structure function crucially depends on the hypothesis about nonperturbative behaviour of the gluon structure function that we have to assume to treat the large distances contribution. In pQCD we can safely calculate only the difference
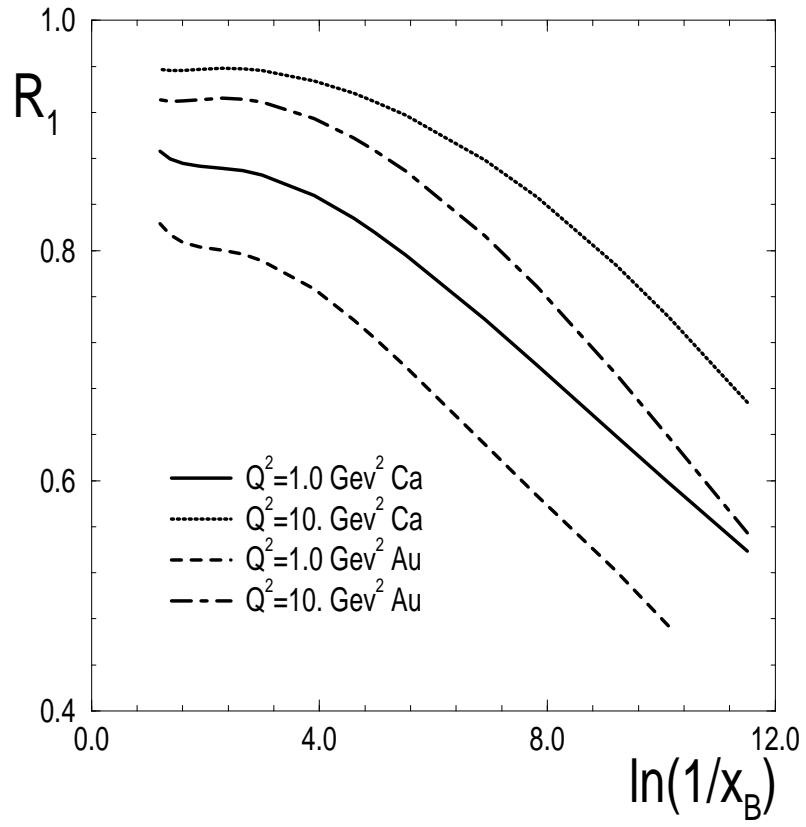
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Figure 2: *Solution for $\kappa = 1$.*

$xG_A(x, Q^2) - xG_A(x, Q^2 = Q_0^2)$ where Q_0^2 is the initial virtuality. In Fig. 3 one can find the calculation for the ratio:

$$R_1 = \frac{xG_A(x, Q^2) - xG_A(x, Q^2 = Q_0^2)}{A\left(xG_N(x, Q^2) - xG_N(x, Q^2 = Q_0^2)\right)} \tag{10}$$

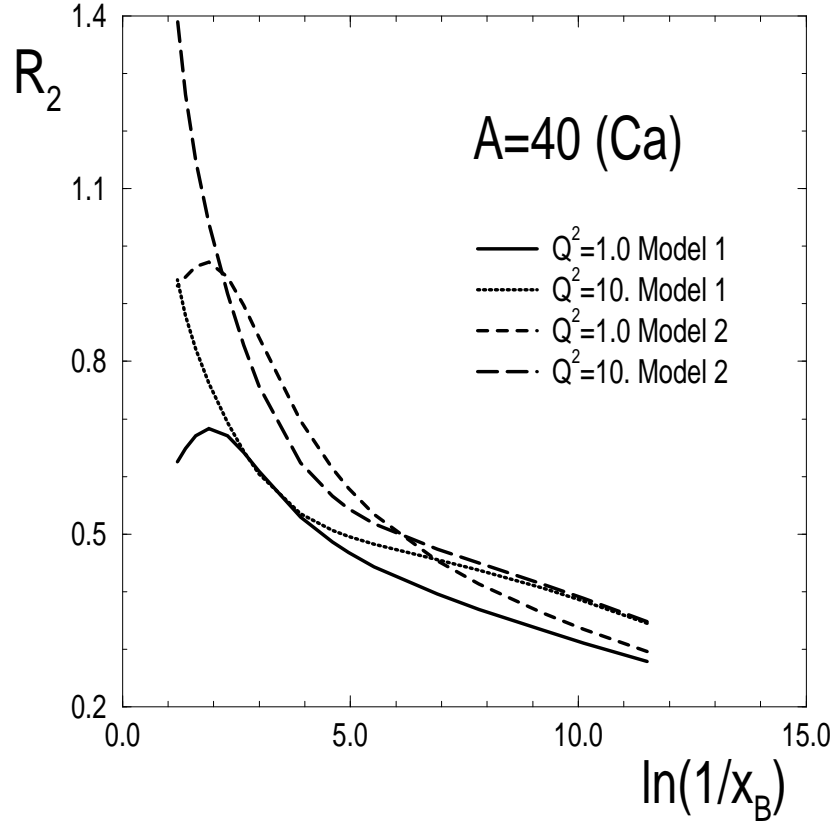as function of x for Ca and Au ($Q_0^2 = 0.25 \ GeV^2$).

Figure 3: R_1 as a function of $ln(1/x_B)$ for Ca and Au.

4.3 Contribution of the large distances.

As has been mentioned we are able to treat this problem only using some model for large distance behaviour of xG_N. Fig.4 shows the ratio:

$$R_2 = \frac{xG_A(x, Q^2)}{A\left(xG_N(x, Q^2)\right)} \tag{11}$$

for two models:

1. $xG_N(x, Q^2 < Q_0^2) = \frac{Q^2}{Q_0^2} xG_N(x, Q^2 = Q_0^2)$. This model takes into account the correct limit of the gluon structure function at small value of Q^2, which follows from the gauge invariance of QCD.

2. $xG_N(x, Q^2 < Q_0^2) = xG_N(x, Q^2 = Q_0^2)$. In this model we assume that the scale for the behaviour $xG_N \propto Q^2$ is much smaller than Q_0^2. We look at the difference in the value of R_2 as the estimates of possible errors that originated from our poor knowledge of long distance behaviour of the gluon structure function. The conclusion is that we cannot calculate the value of R_2 even at $x = 10^{-3}$ at $Q^2 = 1 GeV^2$ with better accuracy that 20%, while at larger value of Q^2 ($Q^2 \sim 10 GeV^2$) the accuracy is better (about 5%).

5 Correction to the Glauber formula.

To abandon the main assumptions which have been made in the Glauber formula we have to develop a technique to include (i) the interactions of all partons (not only the fastest one) with a nucleus; (ii) the parton interaction inside a nucleon and (iii) the nucleon correlation inside a nucleus. Such a technique has been suggested in ref.[8] and it is based on new evolution equation that takes into account the parton interaction inside the parton cascade as well as the parton interaction with the different nucleons. The lack of space does not allow us to discuss this problem in details but we want to point out that the Glauber formula shall be used as initial condition to the new evolution equation of ref.[8].

The numerical estimates [8] shows that the most essential contribution at least in HERA kinematic region is generated by the interaction of all partons with the target which corresponds to so called "fan" diagrams (see ref.[7]) while the dynamic correlation (see refs.[9] [10]) due to the interaction inside the parton cascade remains to be rather small.

However, reliable estimates can be done only after extracting from HERA data the value of the scale for the SC for nucleon structure function. In our numerical estimates
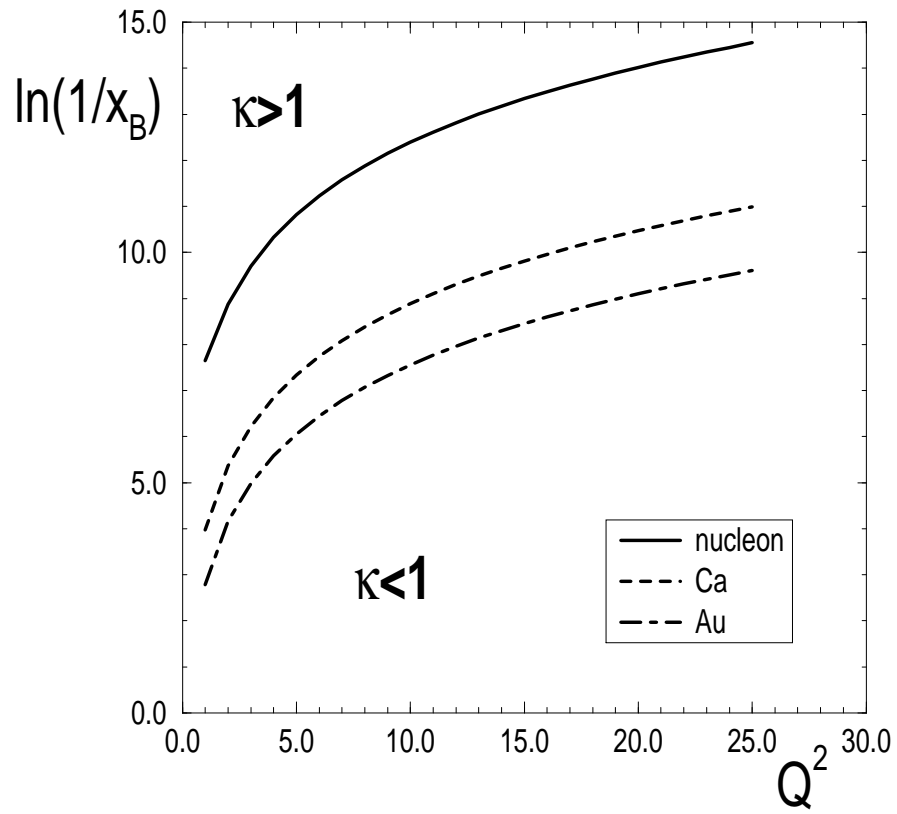
Figure 4: R_2 ratio for two models.

which will be published elsewhere we follow the following strategy: we neglect the dynamic gluon correlations and iterate the master equation (5) several times. Since there is strong ordering in rapidity of parton in each parton cascade the "$i - th$" iteration means that we take into account the interaction with the target of all partons with the value of rapidity (y) larger than y_i. It turns out that we have to make only two iterations for $x > 10^{-3}$ to get convergent result.

6   Conclusions.

We know the Glauber formula and the technique how to find the corrections to the Glauber formula in QCD. However we cannot provide reliable predictions for the gluon structure function for nucleus until we will get more data on low Q^2 and low x behaviour of the nucleon structure function. Unfortunately, we have to know not only the behaviour of the nucleon structure function but also extract from the experimental data the scale for the shadowing corrections to the value of the gluon structure function in the nucleon. We are going to check how the information from DIS with nucleus can reduce these uncertainties. At the moment we suggest to measure the ratio R_1 which can be calculated with much better accuracy than the value of the gluon structure function.

Acknowledgements

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