# Symmetry analysis of the hadronic tensor for the semi-inclusive 

 pseudoscalar meson leptoproduction from an unpolarized nucleon targetWei Lu<br>CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China and Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China*

By examining the symmetry constraints on the semi-inclusive pseudoscalar particle production in unpolarized inelastic lepton-hadron scattering, we present a complete, exact Lorentz decomposition for the corresponding hadronic tensor. As a result, we find that it contains five independent terms, instead of the four as have been suggested before. The newly identified one is odd under the naive time reversal transformation, and the corresponding structure function is directly related to the single spin asymmetry in the semi-inclusive pseudoscalar meson production by a polarized lepton beam off an unpolarized target.

PACS Number(s): 11.30.Er, 13.85.Ni, 13.85.+e

Typeset Using REVTEX

[^0]In the particle physics, the symmetry analysis plays a very important role, since it can forbid or allow for the existence of physical quantities before we set about the details of dynamics. Although the principles and methods involved in the symmetry analysis are usually not complicated, in some circumstances it is a highly nontrivial matter to arrive at a complete result. As the time reversal $(\mathcal{T})$ invariance of interactions is involved, this is even the case. In fact, most mistakes associated with the symmetry analysis can be traced to the confusion the so-called naive $\mathcal{T}$ transformation with the full $\mathcal{T}$ transformation. In other words, the constraints due to time reversal invariance are often not properly considered.

In this paper, we clarify an unsatisfying fact [1] [2] existing in the literature, i.e., the early suggested Lorentz decomposition of the hadronic tensor for the spinless particle leptoproduction from unpolarized nucleon target is incomplete. In principle, one may make efforts to attend the more general case in which the target nucleon is polarized and the spin state of the detected particle is specified in the case it has spin. However, such an analysis will be tedious and the phenomenological implications of the corresponding structure functions are not easy to exemplify. Therefore, we choose to study the simplest case in which a pseudoscalar meson is semi-inclusively detected in the deeply inelastic lepton scattering off an unpolarized target. Obviously, the result can also apply to the semi-inclusive baryon production if one does not measure the baryon spin. Nevertheless, we assume the lepton beam is polarized so that the phenomenological consequence of the newly identified structure function can be conveniently demonstrated.

To be specific, we consider

$$
\begin{equation*}
l\left(P_{l}, S_{l}\right)+p\left(P_{p}\right) \rightarrow l\left(P_{l}^{\prime}\right)+\pi\left(P_{\pi}\right)+X, \tag{1}
\end{equation*}
$$

where the particle momenta in the brackets is self-explanatory. We normalize the spin vector in such a way that $S_{l} \cdot S_{l}=-M_{l}^{2}$ and $S_{l} \cdot P_{l}=0$ for a pure lepton state.

In the one-photon exchange approximation, the proton structure is probed by a space-like photon with momentum $q=P_{l}-P_{l}^{\prime}$. The invariant cross section for the process considered can be written as a contraction of the leptonic and hadronic tensors:

$$
\begin{equation*}
E_{l}^{\prime} E_{\pi} \frac{d \sigma\left(S_{l}\right)}{d^{3} P_{l}^{\prime} d^{3} P_{\pi}}=\frac{\alpha^{2}}{16 \pi^{3} M_{p} E_{l} Q^{4}} L_{\mu \nu}\left(P_{l}, S_{l}, q\right) W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right), \tag{2}
\end{equation*}
$$

where $M_{p}$ is the target mass, $E_{l}$ the beam energy in the laboratory frame, and $Q=\sqrt{-q^{2}}$.
Conventionally, the leptonic and hadronic tensors are defined as

$$
\begin{equation*}
L_{\mu \nu}\left(P_{l}, S_{l}, q\right)=\frac{1}{2} \operatorname{Tr}\left[P_{l}^{\prime} \gamma_{\mu} P_{l} \gamma_{\nu} \frac{1+\gamma_{5} \phi_{l}}{2}\right]=\frac{q^{2}}{2}\left(g_{\mu \nu}-\frac{q_{\mu} q_{\nu}}{q^{2}}\right)+2\left(P_{l \mu}-\frac{q_{\mu}}{2}\right)\left(P_{l \nu}-\frac{q_{\nu}}{2}\right)+i \varepsilon_{\mu \nu \rho \sigma} q^{\rho} S_{l}^{\sigma}, \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right)=\frac{1}{4 \pi} \sum_{X} \int d^{4} y e^{i q \cdot y}<P_{p}\left|j_{\mu}(0)\right| \pi\left(P_{\pi}\right), X>_{\text {out out }}<\pi\left(P_{\pi}\right), X\left|j_{\nu}(y)\right| P_{p}>, \tag{4}
\end{equation*}
$$

respectively. The electromagnetic current is defined to be $j_{\mu}=\sum_{f} e_{f} \bar{\psi}_{f} \gamma_{\mu} \psi_{f}$, with $f$ being the flavor index and $\epsilon_{f}$ being the quark charge in unit of the electron charge. In defining $W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$, the out-state property of the final states has been labelled explicitly.

The hadronic tensor contains all the information about the nucleon structure and pion production. Because the fundamental vertex of the deeply inelastic scattering in the one-photon approximation is electromagnetic, $W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$ should be subjected to the hermiticity, current conservation, parity conservation and time reversal invariance. To write out the general expression of $W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$ in terms of structure functions, one ought to exhaust all the possible candidate terms by imposing all the symmetry constrains that the electromagnetic interaction satisfies.

Concerning $W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$, its Lorentz decomposition has ever been suggested by Mulders and collaborators [1] [2]. Here we simply recapitulate their result in our language. Choosing $\hat{z}$ axis to be along the direction of the virtual photon momentum, one can define a covariant transverse vector $P_{\pi \perp}$, which is

$$
\begin{equation*}
P_{\pi \perp}^{\mu}=\left(0, P_{\pi x}, P_{\pi y}, 0\right) \tag{5}
\end{equation*}
$$

in the target rest frame. Then, the Mulders decomposition for $W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$ reads

$$
\begin{align*}
W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right) & =\mathcal{W}_{1}\left(-g^{\mu \nu} q^{2}+q^{\mu} q^{\nu}\right)+\mathcal{W}_{2}\left(P_{p}^{\mu}-\frac{P_{p} \cdot q}{q^{2}} q^{\mu}\right)\left(P_{p}^{\nu}-\frac{P_{p} \cdot q}{q^{2}} q^{\nu}\right) \\
& +\mathcal{W}_{3}\left[\left(P_{p}^{\mu}-\frac{P_{p} \cdot q}{q^{2}} q^{\mu}\right) P_{\pi \perp}^{\nu}+P_{\pi \perp}^{\nu}\left(P_{p}^{\nu}-\frac{P_{p} \cdot q}{q^{2}} q^{\nu}\right)\right]+\mathcal{W}_{4} P_{\pi \perp}^{\mu} P_{\pi \perp}^{\nu}, \tag{6}
\end{align*}
$$

where $\mathcal{W}$ 's are called structure functions, dependent on $q^{2}, P_{p} \cdot q, P_{\pi} \cdot q$ and $P_{p} \cdot P_{\pi}$. Our findings to be presented are that this Lorentz expansion is incomplete, and in a more complete decomposition there exists another term, which is imaginary, antisymmetric under $\mu \leftrightarrow \nu$, and odd under the naive $\mathcal{T}$ transformation.

Let us examine in turn the constraints that all the electromagnetic symmetries impose on the hadronic tensor. First, the electromagnetic interaction is gauge invariant, as is reflected by the current conservation condition:

$$
\begin{equation*}
q_{\mu} W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)=q_{\nu} W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)=0 . \tag{7}
\end{equation*}
$$

Second, the hermiticity $j_{\mu}^{\dagger}=j_{\mu}$ of the electromagnetic current leads to

$$
\begin{equation*}
\left[W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)\right]^{*}=W^{\nu \mu}\left(q, P_{p}, P_{\pi}\right) . \tag{8}
\end{equation*}
$$

Thirdly, the electromagnetic interaction is parity conserved. For a generic Lorentz coordinate vector $x^{\mu}$, we follow Itzykson and Zuber [3] and define $\tilde{x}^{\mu}=x_{\mu}$. Then, under a parity ( $\mathcal{P}$ ) transformation, the momentum $P^{\mu}$ and spin vector $S^{\mu}$ behave in the fashion

$$
\begin{equation*}
P^{\mu} \xrightarrow{\mathcal{P}} \tilde{P}^{\mu}, S^{\mu} \xrightarrow{\mathcal{P}}-\tilde{S}^{\mu} \tag{9}
\end{equation*}
$$

On the other hand, the electromagnetic current satisfies

$$
\begin{equation*}
j^{\mu}(x) \xrightarrow{\mathcal{P}} j_{\mu}(\tilde{x}) . \tag{10}
\end{equation*}
$$

As a result, the parity conservation of the electromagnetic interaction yields

$$
\begin{equation*}
W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)=W_{\mu \nu}\left(\tilde{q}, \tilde{P}_{p}, \tilde{P}_{\pi}\right) \tag{11}
\end{equation*}
$$

Fourthly, the fundamental electromagnetic vertex is $\mathcal{T}$ invariant. Under a $\mathcal{T}$ transformation, one has

$$
\begin{equation*}
P^{\mu} \xrightarrow{\mathcal{T}} \tilde{P}^{\mu}, S^{\mu} \xrightarrow{\mathcal{T}} \tilde{S}^{\mu} . \tag{12}
\end{equation*}
$$

and

$$
\begin{equation*}
j^{\mu}(x) \xrightarrow{\mathcal{T}} j_{\mu}(-\tilde{x}) . \tag{13}
\end{equation*}
$$

In addition, the complex conjugation should be operated and an in state changed into the corresponding out state, or vice versa. In general, an in state is related to its corresponding out state by $S$ matrix (operator) [4]:

$$
\begin{equation*}
\left|>_{\text {in }}=S\right|>_{\text {out }}, \tag{14}
\end{equation*}
$$

where $S$ can be written as

$$
\begin{equation*}
S=1+i T \tag{15}
\end{equation*}
$$

with $T$ being the transition matrix (operator). Unless the state is composed of an individual particle or a set of non-interactive particles, the in state differs from its corresponding out state. Experiences tell us that there exist violent multi-interactions in the hadron system, so we cannot $a$ priori identify the semi-inclusive hadron out state with its corresponding in state. As a consequence, $\mathcal{T}$ invariance can only tell us

$$
\begin{equation*}
\left.W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)=\left[\frac{1}{4 \pi} \sum_{X} \int d^{4} y e^{i q \cdot y}<\tilde{P}_{p}\left|j_{\mu}(0)\right| \pi\left(P_{\pi}\right), X>_{\text {in in }}<\pi\left(P_{\pi}\right), X\left|j_{\nu}(y)\right| \tilde{P}_{p}\right\rangle\right]^{*} . \tag{16}
\end{equation*}
$$

If the final-state interactions are neglected, the in state reduces to its corresponding out state. The resulting simplified time reversal transformation can be called the naive time reversal transformation, under which one has

$$
\begin{equation*}
W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)=\left[W_{\mu \nu}\left(\tilde{q}, \tilde{P}_{p}, \tilde{P}_{\pi}\right)\right]^{*} . \tag{17}
\end{equation*}
$$

Examining Eqs. (11) and (17), one can see that it is more convenient to use the complex $\mathcal{P} T$ transformation instead of the individual discrete transformations $\mathcal{P}$ and $\mathcal{T}$. This is even the case in discussing the single spin asymmetry, which is odd under the naive $\mathcal{P} T$ transformation. In our case, a $\mathcal{P} T$ transformation gives rise to

$$
\begin{equation*}
W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)=\left[\frac{1}{4 \pi} \sum_{X} \int d^{4} y e^{i q \cdot y}<P\left|j^{\mu}(0)\right| \pi\left(P_{\pi}\right), X>_{\text {in in }}\left\langle\pi\left(P_{\pi}\right), X\right| j^{\nu}(y)|P\rangle\right]^{*} . \tag{18}
\end{equation*}
$$

Substituting Eqs. (14) and (15) into (18), one has

$$
\begin{equation*}
W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)=\left[W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)\right]^{*}+T^{\mu \nu}\left(q, P_{p}, P_{\pi}\right) \tag{19}
\end{equation*}
$$

where $T^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$ incorporates the difference between the in and out states. Equation (19) implies that $W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$ can be decomposed into two parts, one even and another odd under the naive $\mathcal{P} T$ transformation.

We note that the metric tensor $g_{\mu \nu}$ gets into work in our decomposition. As for the full antisymmetric rank-four tensor $\varepsilon_{\mu \nu \rho \sigma}$, it is not involved because we are dealing with the unpolarized target. Keep in mind that $W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$ satisfies the current conservation condition, which greatly constrains the forms in which momenta and spin vectors contribute. Regarding the metric tensor, its only independent, gauge invariant combination is $g_{\mu \nu} q^{2}-q_{\mu} q_{\nu}$. In our case, we can construct two independent momentum combinations $P_{p}^{\mu}-q^{\mu}\left(P_{p} \cdot q\right) / q^{2}$ and $P_{\pi}^{\mu}-q^{\mu}\left(P_{\pi} \cdot q\right) / q^{2}$, whose contractions with the photon momentum $q^{\mu}$ vanish. Another combination $P_{p}^{\mu}-P_{\pi}^{\mu}\left(P_{p} \cdot q\right) /\left(P_{\pi} \cdot q\right)$ can be expressed in terms of $P_{p}^{\mu}-q^{\mu}\left(P_{p} \cdot q\right) / q^{2}$ and $P_{\pi}^{\mu}-q^{\mu}\left(P_{\pi} \cdot q\right) / q^{2}$. Exhausting all the possible candidate terms, we attain

$$
\begin{align*}
W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right) & =F_{1}\left(g^{\mu \nu} q^{2}-q^{\mu} q^{\nu}\right)+F_{2}\left(P_{p}^{\mu}-\frac{P_{p} \cdot q}{q^{2}} q^{\mu}\right)\left(P_{p}^{\nu}-\frac{P_{p} \cdot q}{q^{2}} q^{\nu}\right) \\
& +F_{3}\left(P_{\pi}^{\mu}-\frac{P_{\pi} \cdot q}{q^{2}} q^{\mu}\right)\left(P_{\pi}^{\nu}-\frac{P_{\pi} \cdot q}{q^{2}} q^{\nu}\right) \\
& +F_{4}\left[\left(P_{p}^{\mu}-\frac{P_{p} \cdot q}{q^{2}} q^{\mu}\right)\left(P_{\pi}^{\nu}-\frac{P_{\pi} \cdot q}{q^{2}} q^{\nu}\right)+\left(P_{\pi}^{\mu}-\frac{P_{\pi} \cdot q}{q^{2}} q^{\mu}\right)\left(P_{p}^{\nu}-\frac{P_{p} \cdot q}{q^{2}} q^{\nu}\right)\right] \\
& +i \hat{F}\left[\left(P_{p}^{\mu}-\frac{P_{p} \cdot q}{q^{2}} q^{\mu}\right)\left(P_{\pi}^{\nu}-\frac{P_{\pi} \cdot q}{q^{2}} q^{\nu}\right)-\left(P_{\pi}^{\mu}-\frac{P_{\pi} \cdot q}{q^{2}} q^{\mu}\right)\left(P_{p}^{\nu}-\frac{P_{p} \cdot q}{q^{2}} q^{\nu}\right)\right] \tag{20}
\end{align*}
$$

If the Mulders parameterization is used, one has equivalently

$$
\begin{align*}
W^{\mu \nu}\left(q, P_{p}, P_{\pi}\right) & =\mathcal{W}_{1}\left(-g^{\mu \nu} q^{2}+q^{\mu} q^{\nu}\right)+\mathcal{W}_{2}\left(P_{p}^{\mu}-\frac{P_{p} \cdot q}{q^{2}} q^{\mu}\right)\left(P_{p}^{\nu}-\frac{P_{p} \cdot q}{q^{2}} q^{\nu}\right) \\
& +\mathcal{W}_{3}\left[\left(P_{p}^{\mu}-\frac{P_{p} \cdot q}{q^{2}} q^{\mu}\right) P_{\pi \perp}^{\nu}+P_{\pi \perp}^{\nu}\left(P_{p}^{\nu}-\frac{P_{p} \cdot q}{q^{2}} q^{\nu}\right)\right]+\mathcal{W}_{4} P_{\pi \perp}^{\mu} P_{\pi \perp}^{\nu} \\
& +\mathcal{W}\left[\left(P_{p}^{\mu}-\frac{P_{p} \cdot q}{q^{2}} q^{\mu}\right) P_{\pi \perp}^{\nu}-P_{\pi \perp}^{\nu}\left(P_{p}^{\nu}-\frac{P_{p} \cdot q}{q^{2}} q^{\nu}\right)\right] \tag{21}
\end{align*}
$$

The newly identified term, associated with $\hat{F}$ in Eq. (20) or with $\hat{\mathcal{W}}$ in (21), is imaginary, antisymmetric under $\mu \leftrightarrow \nu$, and odd under the naive $\mathcal{T}$ transformation. Physically, it incorporates
the distinction between the inclusively detected hadron state and its corresponding in state. Since such a difference is caused by the nontrivial final-state interaction described by the transition operator $T$ [see Eqs. (14)-(16), (18) and (19)], it can be said that the new term reflects the effects of final-state multi-interactions.

To exemplify the phenomenological consequences, we call for the polarized incident lepton beam, as has been specified in our kinematics. Obviously, when the hadronic and leptonic tensors contact, their imaginary parts co-work to make a contribution to the cross section. Using our parameterization for $W_{\mu \nu}\left(q, P_{p}, P_{\pi}\right)$, we have

$$
\begin{align*}
E_{l}^{\prime} E_{\pi} \frac{d \sigma\left(S_{l}\right)}{d^{3} P_{l}^{\prime} d^{3} P_{\pi}} & =\frac{\alpha^{2}}{16 \pi^{3} M_{p} E_{l} Q^{4}}\left\{Q^{4} F_{1}+M_{p}^{2}\left(2 E_{l} E_{l}^{\prime}-\frac{Q^{2}}{2}\right) F_{2}+\left(2\left(P_{l} \cdot P_{\pi}\right)\left(P_{l}^{\prime} \cdot P_{\pi}\right)-\frac{1}{2} M_{\pi}^{2} Q^{2}\right) F_{3}\right. \\
& \left.+M_{p}\left(2 E_{l}^{\prime} P_{l} \cdot P_{\pi}+2 E_{l} P_{l}^{\prime} \cdot P_{\pi}-E_{\pi}^{\prime} Q^{2}\right) F_{4}+2 M_{p} S_{l} \cdot q \times P_{\pi} \hat{F}\right\} . \tag{22}
\end{align*}
$$

Consequently, structure function $\hat{F}$ can be related to the correlation among the lepton polarization vector, the momentum of the virtual photon and that of the semi-inclusively detected pion. Experimentally, of more interest are the single spin asymmetries. Considering that large spin asymmetries of the order of ten or more percent have been observed in inclusive pion production [5] [6] from hadron-nucleus fixed target experiments, we discuss briefly in the following the feasibilities to measure the longitudinal and transverse spin asymmetries in the process we are considering.

We first discuss the case in which the lepton beam is longitudinally polarized. Neglecting the lepton mass effects, we may replace $S_{l}$ with $\mathcal{H}_{l} P_{l}$, where $\mathcal{H}_{l}$ is the lepton helicity. The lepton scattering plane is symmetric under the rotation about the beam axis, so there are only five independent degrees of freedom. We work in the target rest frame, with the $\hat{z}$ axis along the beam direction and the $\hat{x}$ axis in the lepton scattering plane. Call the outgoing angle of the scattered lepton $\theta_{l}^{\prime}$. When the detected pion falls into the solid angle $d \Omega_{\pi}=d \cos \theta_{\pi} d \phi_{\pi}$, the differential cross section reads

$$
\begin{align*}
E_{\pi} \frac{d \sigma\left(\mathcal{H}_{l}\right)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} & =\frac{\alpha^{2}}{8 \pi^{2} M_{p} Q^{4}} \frac{E_{l}^{\prime}}{E_{l}}\left\{Q^{4} F_{1}+M_{p}^{2}\left(2 E_{l} E_{l}^{\prime}-\frac{Q^{2}}{2}\right) F_{2}+\left(2\left(P_{l} \cdot P_{\pi}\right)\left(P_{l}^{\prime} \cdot P_{\pi}\right)-\frac{1}{2} M_{\pi}^{2} Q^{2}\right) F_{3}\right. \\
& \left.+M_{p}\left(2 E_{l}^{\prime} P_{l} \cdot P_{\pi}+2 E_{l} P_{l}^{\prime} \cdot P_{\pi}-E_{\pi}^{\prime} Q^{2}\right) F_{4}-2 \mathcal{H}_{l} M_{p} E_{l} E_{l}^{\prime}\left|P_{\pi}\right| \sin \theta_{l}^{\prime} \sin \theta_{\pi} \cos \phi_{\pi} \hat{F}\right\} . \tag{23}
\end{align*}
$$

The longitudinal spin asymmetry for the inclusive pion production is defined as

$$
\begin{equation*}
A_{L}=\frac{\int E_{\pi} \frac{d \sigma(+)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}-\int E_{\pi} \frac{d \sigma(-)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}}{\int E_{\pi} \frac{d \sigma(+)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}+\int E_{\pi} \frac{d \sigma(-)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}}, \tag{24}
\end{equation*}
$$

which can be cast into

$$
\begin{align*}
A_{L}= & -2 E_{l} M_{p}\left|P_{\pi}\right| \sin \theta_{\pi} \cos \phi_{\pi} \int d E^{\prime} d \cos \theta_{l}^{\prime} \hat{F} \frac{E_{l}^{\prime 2}}{Q^{4}} \sin \theta_{l}^{\prime} \times\left\{\int d E _ { l } ^ { \prime } d \operatorname { c o s } \theta _ { l } ^ { \prime } \frac { E _ { l } ^ { \prime } } { Q ^ { 4 } } \left[Q^{4} F_{1}+M_{p}^{2}\left(2 E_{l} E_{l}^{\prime}-\frac{Q^{2}}{2}\right) F_{2}\right.\right. \\
& \left.\left.+\left(2\left(P_{l} \cdot P_{\pi}\right)\left(P_{l}^{\prime} \cdot P_{\pi}\right)-\frac{1}{2} M_{\pi}^{2} Q^{2}\right) F_{3}+M_{p}\left(2 E_{l}^{\prime} P_{l} \cdot P_{\pi}+2 E_{l} P_{l}^{\prime} \cdot P_{\pi}-E_{\pi}^{\prime} Q^{2}\right) F_{4}\right]\right\}^{-1} . \tag{25}
\end{align*}
$$

Notice that the integrations over $E_{l}^{\prime}$ and $\sin \theta_{l}^{\prime}$ cannot be completed at present unless we have known how structure functions depend on them.

Now we attend the transverse polarization case of the incident lepton beam, in which there are six independent degrees of freedom. Again, we work in the target rest frame, letting the $\hat{z}$ axis be along the beam direction but specifying the $\hat{x}$ axis as the lepton polarization direction. Then, the spin vector is $S_{l \uparrow, \downarrow}^{\mu}= \pm M_{l}(0,1,0,0)$ for the lepton polarized parallel and antiparallel to the $\hat{x}$ axis, respectively. Provided the scattered lepton and the detected pion fly into the solid angles $d \Omega_{l}^{\prime}=d \cos \theta_{l}^{\prime} d \phi_{l}^{\prime}$ and $d \Omega_{\pi}=d \cos \theta_{\pi} d \phi_{\pi}$ respectively, the cross section can be written as

$$
\begin{align*}
E_{\pi} \frac{d \sigma\left(S_{l \uparrow, l}\right)}{d^{3} P_{\pi} d E_{l}^{\prime} d \Omega_{l}^{\prime}}= & \frac{\alpha^{2}}{16 \pi^{3} M_{p} Q^{4}} \frac{E_{l}^{\prime}}{E_{l}}\left\{Q^{4} F_{1}+M_{p}^{2}\left(2 E_{l} E_{l}^{\prime}-\frac{Q^{2}}{2}\right) F_{2}\right. \\
& +\left(2\left(P_{l} \cdot P_{\pi}\right)\left(P_{l}^{\prime} \cdot P_{\pi}\right)-\frac{1}{2} M_{\pi}^{2} Q^{2}\right) F_{3}+M_{p}\left(2 E_{l}^{\prime} P_{l} \cdot P_{\pi}+2 E_{l} P_{l}^{\prime} \cdot P_{\pi}-E_{\pi}^{\prime} Q^{2}\right) F_{4} \\
& \left.\mp 2 M_{l} M_{p}\left|P_{\pi}\right|\left[E_{l} \sin \theta_{\pi} \cos \phi_{\pi}+E_{l}^{\prime}\left(\sin \theta_{l}^{\prime} \sin \phi_{l}^{\prime} \cos \theta_{\pi}-\cos \theta_{l}^{\prime} \sin \theta_{\pi} \cos \phi_{\pi}\right)\right] \hat{F}\right\} \tag{26}
\end{align*}
$$

The transverse spin asymmetry is defined as

$$
\begin{equation*}
A_{T}=\frac{\int E_{\pi} \frac{d \sigma\left(S_{l \uparrow}\right)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} d E_{l}^{\prime} d \Omega_{l}^{\prime}-\int E_{\pi} \frac{d \sigma\left(S_{l \downarrow}\right)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} d E_{l}^{\prime} d \Omega_{l}^{\prime}}{\int E_{\pi} \frac{d \sigma\left(S_{l \uparrow}\right)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} d E_{l}^{\prime} d \cos \Omega_{l}^{\prime}+\int E_{\pi} \frac{d \sigma\left(S_{l \downarrow}\right)}{d^{3} P_{\pi} d E_{l}^{\prime} d \cos \theta_{l}^{\prime}} d E_{l}^{\prime} d \cos \Omega_{l}^{\prime}} \tag{27}
\end{equation*}
$$

From Eq. (26), one has

$$
\begin{align*}
A_{T}= & -2 M_{l} M_{p}\left|P_{\pi}\right| \int d E_{l}^{\prime} d \cos \Omega_{l}^{\prime} \frac{E_{l}^{\prime}}{Q^{4}}\left[E_{l} \sin \theta_{\pi} \cos \phi_{\pi}+E_{l}^{\prime}\left(\sin \theta_{l}^{\prime} \sin \phi_{l}^{\prime} \cos \theta_{\pi}-\cos \theta_{l}^{\prime} \sin \theta_{\pi} \cos \phi_{\pi}\right)\right] \hat{F} \\
& \times\left\{\int d E _ { l } ^ { \prime } d \operatorname { c o s } \Omega _ { l } ^ { \prime } \frac { E _ { l } ^ { \prime } } { Q ^ { 4 } } \left[Q^{4} F_{1}+M_{p}^{2}\left(2 E_{l} E_{l}^{\prime}-\frac{Q^{2}}{2}\right) F_{2}+\left(2\left(P_{l} \cdot P_{\pi}\right)\left(P_{l}^{\prime} \cdot P_{\pi}\right)-\frac{1}{2} M_{\pi}^{2} Q^{2}\right) F_{3}\right.\right. \\
& \left.\left.+M_{p}\left(2 E_{l}^{\prime} P_{l} \cdot P_{\pi}+2 E_{l} P_{l}^{\prime} \cdot P_{\pi}-E_{\pi}^{\prime} Q^{2}\right) F_{4}\right]\right\}^{-1} \tag{28}
\end{align*}
$$

Comparing Eqs. (25) and (28), we observe that the single transverse spin asymmetry is $O\left(M_{l} / E_{l}\right)$ suppressed relative to the longitudinal spin asymmetry. So measuring the transverse spin asymmetry is infeasible in practice. At moderate energies, however, we can anticipate that the longitudinal spin asymmetry is be measured.

In summary, we examined the hadronic tensor for the semi-inclusive pion leptoproduction in the case of unpolarized target. It is found that a term has been missed in the early suggested Lorentz decomposition of the hadronic tensor. Such an incompleteness is simply due to the fact the constraint of time reversal invariance on the inclusive one-particle production is not properly considered. By carefully examining the contents of time reversal transformation, we suggest a complete Lorentz decomposition for the hadronic tensor considered, which contain five independent terms instead of the four in the literature. The newly identified term is imaginary, antisymmetric under $\mu \leftrightarrow \nu$, and odd under the naive time reversal transformation. Moreover, the possible phenomenology pertinent to the new structure function is discussed.

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