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Spacetime Supersymmetry and Duality in String Theory

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I discuss the role of spacetime supersymmetry in the interplay between strong/weak coupling duality and target space duality in string theory which arises in string/string duality. This can be seen via the construction of string soliton solutions which in $N = 4$ compactifications of heterotic string theory break more than 1/2 of the spacetime supersymmetries but whose analogs in $N = 2$ and $N = 1$ compactifications break precisely 1/2 of the spacetime supersymmetries. As a result, these solutions may be interpreted as stable solitons in the latter two cases, and correspond to Bogomol'nyi-saturated states in their respective spectra.

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1. Introduction

The construction of soliton solutions in string theory is intimately connected with the presence of various dualities in string theory (for recent reviews of string solitons, see [1,2]). Most of the solitonic solutions found so far break half of the spacetime supersymmetries of the theory in which they arise. Examples of string-like solitons (i.e. with one Killing direction) in this class are the fundamental string solution of [3] and the dual string solution of [4], which are interchanged once the roles of the strong/weak coupling S-duality and target space T-duality are interchanged.

In this talk I will first summarize the basic features of S duality, T duality and string/string duality in heterotic string theory. Then I will discuss newly constructed [5] classes of string-like soliton solutions, making connections between the solution-generating subgroup of the T-duality group and the number of spacetime supersymmetries broken in $N = 4$, $D = 4$ compactifications of $N = 1$, $D = 10$ heterotic string theory, as well as the natural realization of these solutions in $N = 1$ and $N = 2$ four-dimensional compactifications. For simplicity, I will restrict myself to solutions in the gravitational sector of the string (i.e. all Yang-Mills fields will be set to zero).

For an interesting discussion of six-dimensional string/string duality see [6]. New and exciting connections between the various dualities in heterotic string theory and type II string theory can be found in [7].

2. S Duality

We adopt the following conventions for $N = 1$, $D = 10$ heterotic string theory compactified to $N = 4$, $D = 4$ heterotic string theory: (0123) is the four-dimensional spacetime, $z = x_2 + ix_3 = re^{i\theta}$, (456789) are the compactified directions, $S = e^{-2\Phi} + ia = S_1 + iS_2$, where Φ and a are the four-dimensional dilaton and axion. S duality generalizes strong-weak coupling duality, since $g = e^{\Phi}$ is the string loop coupling parameter. In $N = 4$, $D = 4$ heterotic string theory S duality corresponds to the group $SL(2, Z)$. In other words, the four-dimensional theory exhibits an invariance under

$$S \to \frac{aS + b}{cS + d}, \tag{2.1}$$

where a, b, c, d are integers and $ad - bc = 1$.

There is now considerable evidence [8–15,4,16] in favor of S duality also being an exact symmetry of the full string theory. One obvious attraction to demonstrating S duality exactly in string theory is that it would allow us to use perturbative string techniques in the strong-coupling regime.

In the absence of nontrivial moduli and Yang-Mills fields, the low-energy four-dimensional bosonic effective action in the gravitational sector of the heterotic string has the form

$$S_4 = \int d^4x\sqrt{-g}\left(R - \frac{g^{\mu\nu}}{2S_1^2}\partial_\mu S\partial_\nu \bar{S}\right). \tag{2.2}$$

A solution of this action is given by [3]

$$\begin{aligned} ds^2 &= -dt^2 + dx_1^2 + ReS(dx_2^2 + dx_3^2) \\ S &= -\frac{1}{2\pi}\sum_{i=1}^{N} n_i \ln\frac{(z-a_i)}{r_{i0}}, \end{aligned} \tag{2.3}$$

where N is an arbitrary number of string-like solitons each with arbitrary location a_i in the complex z-plane and arbitrary winding number n_i respectively. One can replace z by $\bar{z}$ in S, thereby changing the orientation of the windings. There is also an $SL(2,R)$ symmetry manifest in the low-energy action, which is broken down to $SL(2,Z)$ in string theory via axion quantization and from which the above solution can be generalized further. Note that the x_1 Killing direction gives the above solution the structure of a parallel multi-string configuration. Each string is interpreted as a macroscopic fundamental string [3]. For dynamical evidence for this identification see [17].

3. T Duality

T duality in string theory is the target space duality, and generalizes the $R \to \alpha'/R$ duality in compactified string theory. For $N=4$, $D=4$ compactifications of heterotic string theory, T-duality corresponds to the discrete group $O(6,22;Z)$ and is known to be an exact symmetry of the full string theory [18–25].

Let us consider a simple special compactification, in which the only nontrivial moduli are given by $T = T_1 + iT_2 = \sqrt{\det g_{mn}} - iB_{45}$, where $m,n = 4,5$. For trivial S field, the low-energy four-dimensional bosonic effective action in the gravitational sector has the form

$$S_4 = \int d^4x\sqrt{-g}\left(R - \frac{g^{\mu\nu}}{2T_1^2}\partial_\mu T\partial_\nu \bar{T}\right). \tag{3.1}$$

A solution of this action is given by [4]

$$\begin{aligned} ds^2 &= -dt^2 + dx_1^2 + ReT(dx_2^2 + dx_3^2) \\ T &= -\frac{1}{2\pi}\sum_{j=1}^{M} m_i \ln\frac{(z-b_j)}{r_{j0}}, \end{aligned} \tag{3.2}$$

where M is an arbitrary number of string-like solitons each with arbitrary location b_j in the complex z-plane and arbitrary winding number m_j respectively. Again, one can replace z by $\bar{z}$ in T and reverse the windings, and there is an $SL(2,R)$ symmetry manifest in the low-energy action which is broken down to $SL(2,Z)$, this time due to the presence of instantons, and from which the above solution can be generalized further. Note that the x_1 Killing direction gives the above solution the structure of a parallel multi-string configuration as well, but in this case each string is interpreted as a dual string [4].

4. String/String Duality

Note that in interchanging the S field in the action (2.2) with the T field in the action (3.1), one is interchanging the S (fundamental) string with the T (dual) string and effectively interchanging their respective couplings. In this form, the string/string duality conjecture postulates the existence of a dual string theory, in which the roles of the strong/weak coupling duality and target space duality are interchanged. It follows that the string/string duality conjecture requires the interchange of worldsheet coupling associated with T duality and spacetime coupling associated with S duality.

Of course the full T duality group $O(6,22;Z)$ is much larger than the S duality group $SL(2,Z)$, but from the six-dimensional viewpoint, the strong/weak coupling duality of the fundamental string can be seen to emerge as a subgroup of the target space duality group of the dual string. From the ten-dimensional viewpoint, the dual theory is a theory of fundamental fivebranes ($5+1$-dimensional objects) [26]. However, given the various difficulties in working with fundamental fivebranes (see discussion in [6]) and the fact that the technology of fundamental string theory is reasonably well-developed, it seems natural to prefer string/string duality (in $D=6$ or $D=4$) over string/fivebrane duality in $D=10$.

Both the fundamental (S) and dual (T) string break 1/2 the spacetime supersymmetries, which can be seen either from the $N=1$, $D=10$ uncompactified theory or from the $N=4$, $D=4$ compactified theory. They also both arise in a larger $O(8,24;Z)$ solution generating group (for an explicit $O(8,24;Z)$ transformation that takes one from the fundamental string to the dual string see [5]). As a consequence, they both saturate Bogomol'nyi bounds and correspond to states in the spectrum of the theory [12,16].

5. Generalized Solutions and Supersymmetry Breaking

Now consider the following ansatz, in which the solution-generating subgroup of the

$O(6,22;Z)$ T duality group is contained in $SL(2,Z)^3 = SL(2,Z)\times SL(2,Z)\times SL(2,Z)$:

$$
\begin{aligned}
T^{(1)} &= T^{(1)}_1 + iT^{(1)}_2 = \sqrt{\det g_{mn}} - iB_{45}, \qquad m,n = 4,5,\\
T^{(2)} &= T^{(2)}_1 + iT^{(2)}_2 = \sqrt{\det g_{pq}} - iB_{67}, \qquad p,q = 6,7,\\
T^{(3)} &= T^{(3)}_1 + iT^{(3)}_2 = \sqrt{\det g_{rs}} - iB_{89}, \qquad r,s = 8,9
\end{aligned}
\tag{5.1}
$$

are the moduli. We assume dependence only on the coordinates x_2 and x_3 (i.e. x_1 remains a Killing direction), and that no other moduli than the ones above are nontrivial.

The canonical four-dimensional bosonic action for the above compactification ansatz in the gravitational sector can be written in terms of $g_{\mu\nu}$ $(\mu,\nu = 0,1,2,3)$, S and $T^{(a)}, a = 1,2,3$ as

$$
\begin{aligned}
S_4 = \int d^4x\sqrt{-g}\Big(R &- \frac{g^{\mu\nu}}{2S_1^2}\partial_\mu S\partial_\nu \bar{S}\\
&- \frac{g^{\mu\nu}}{2{T_1^{(1)}}^2}\partial_\mu T^{(1)}\partial_\nu \bar{T}^{(1)} - \frac{g^{\mu\nu}}{2{T_1^{(2)}}^2}\partial_\mu T^{(2)}\partial_\nu \bar{T}^{(2)} - \frac{g^{\mu\nu}}{2{T_1^{(3)}}^2}\partial_\mu T^{(3)}\partial_\nu \bar{T}^{(3)}\Big).
\end{aligned}
\tag{5.2}
$$

A solution of this action is given by [5]

$$
\begin{aligned}
ds^2 &= -dt^2 + dx_1^2 + ReSReT^{(1)}ReT^{(2)}ReT^{(3)}(dx_2^2 + dx_3^2)\\
S &= -\frac{1}{2\pi}\sum_{i=1}^{N} n_i \ln\frac{(z-a_i)}{r_{i0}},\\
T^{(1)} &= -\frac{1}{2\pi}\sum_{j=1}^{M} m_j \ln\frac{(z-b_j)}{r_{j0}},\\
T^{(2)} &= -\frac{1}{2\pi}\sum_{k=1}^{P} p_k \ln\frac{(z-c_k)}{r_{k0}},\\
T^{(3)} &= -\frac{1}{2\pi}\sum_{l=1}^{Q} q_l \ln\frac{(z-d_l)}{r_{l0}},
\end{aligned}
\tag{5.3}
$$

where N, M, P and Q are arbitrary numbers of string-like solitons in $S, T^{(1)}, T^{(2)}$ and $T^{(3)}$ respectively each with arbitrary location a_i, b_j, c_k and d_l in the complex z-plane and arbitrary winding number n_i, m_j, p_k and q_l respectively. One can replace z by $\bar{z}$ independently in S and in each of the T moduli, and in each of S and the T moduli there is an $SL(2,Z)$ symmetry manifest in the action in each of the moduli, and from which the above solutions can be generalized further. Thus one has an overall effective solution-generating group of $SL(2,Z)^4$.

It can be shown [5] that the solutions with trivial S and $1,2$ and 3 nontrivial T fields preserve $1/2, 1/4$ and $1/8$ of the spacetime supersymmetries respectively, while the solutions with nontrivial S and $0, 1$ and 2 nontrivial T fields preserve $1/2, 1/4$ and $1/8$ spacetime supersymmetries respectively. The solution with nontrivial S and 3 nontrivial T fields preserves $1/8$ of the spacetime supersymmetries for one chirality choice of S, and none of the spacetime supersymmetries for the other, although the ansatz remains a solution to the bosonic action in this latter case. In short, the maximum number of spacetime supersymmetries preserved in the $N = 4$ theory for a solution generating subgroup $SL(2, Z)^n$ of $O(8, 24; Z)$ is given by $(1/2)^n$ [5].

6. Discussion

So what is the interpretation of these new solutions which break more than half the supersymmetries, since they are not expected to arise within the spectrum of Bogomol'nyi-saturated states in the $N = 4$ theory? It turns out that most of the above solutions that break $1/2, 3/4$ or $7/8$ of the spacetime supersymmetries in $N = 4$ have analogs in $N = 1$ or $N = 2$ compactifications of heterotic stirng theory that break only $1/2$ the spacetime supersymmetries*. Of course no solution actually preserves a higher total number of supersymmetries in the lower supersymmetric theory ($N = 1$ or $N = 2$) than in $N = 4$, but the relative number of supersymmetries preserved may be increased in truncating the $N = 4$ theory to $N = 1$ or $N = 2$ by the removal of non-supersymmetric modes. Thus a solution that preserves $1/8$ of the spacetime supersymmetreis in $N = 4$ and $1/2$ of the spacetime supersymmetries in $N = 1$ actually preserves the same total amount of supersymmetry in both theories. The only difference is that in the $N = 4$ case one is starting with four times as many supersymmetries, so that a greater number of those are broken than in the $N = 1$ case.

These solutions are therefore in some sense realized naturally as stable solitons only in the context of either $N = 1$ or $N = 2$ compactifications, and should lead to the construction of the Bogomol'nyi spectrum of these theories. In these two case, however, the situation is complicated by the absence of non-renormalization theorems present in the $N = 4$ case which guarantee the absence of quantum corrections. An exception to this scenario arises for $N = 2$ compactifications with vanishing β-function. The construction of these spectra remains a problem for future research.

* However, when at least one of the fields, either S or one of the T fields, has a different analyticity behaviour from the rest, no supersymmetries are preserved in $N = 1$ [5].

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