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Supersymmetry and Dual String Solitons

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We present new classes of string-like soliton solutions in ($N = 1$; $D = 10$), ($N = 2$; $D = 6$) and ($N = 4$; $D = 4$) heterotic string theory. Connections are made between the solution-generating subgroup of the T-duality group of the compactification and the number of spacetime supersymmetries broken. Analogous solutions are also noted in ($N = 1, 2$; $D = 4$) compactifications, where a different form of supersymmetry breaking arises.

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1. Introduction

The existence of large classes of soliton solutions in string theory is intimately connected with the presence of various dualities in string theory (for recent reviews of string solitons, see [1,2]). Most of the solitonic solutions found so far break half of the spacetime supersymmetries of the theory in which they arise. Examples of string-like solitons in this class are the fundamental string solution of [3] and the dual string solution of [4], which are interchanged once the roles of the strong/weak coupling S-duality and target space T-duality are interchanged. For $N = 4$, $D = 4$ compactifications of heterotic string theory, T-duality corresponds to the discrete group $O(6, 22; Z)$ and is known to be an exact symmetry of the full string theory. For a review, see [5]. There is now a good deal of evidence in favour of the conjecture that the S-duality group $SL(2, Z)$ is also an exact symmetry of the full string theory. For a review, see [6]. The dual string of [4] thus belongs to an $O(6, 22; Z)$ family of dual strings just as there is an $SL(2, Z)$ family of fundamental strings [7]. This accords with the observation that string/string duality interchanges the roles of strong/weak coupling duality and target space duality [8]. For earlier discussions of the string/string duality conjecture see [9,4,10,11,12,1]; for recent ramifications see [13,14,15,16,17].

Interesting examples of solutions which break more than half of the spacetime supersymmetries are the $D = 10$ double-instanton string soliton of [18] (which breaks 3/4), the $D = 10$ octonionic string soliton of [19] (which breaks 15/16) and the $D = 11$ extreme black fourbrane and sixbrane of [20] (which break 3/4 and 7/8 respectively). In this paper we present new classes of string-like solutions which arise in heterotic string theory toroidally compactified to four dimensions. Connections are made between the solution-generating subgroup of the T-duality group and the number of spacetime supersymmetries broken in the $N = 4$ theory. Analogous solutions are also seen to arise in $N = 2$ and $N = 1$ compactifications.

Next, the conjecture [21,22,23,24] that S- and T-duality can be united into $O(8, 24; Z)$ is discussed. In a recent paper by Sen [23], the fundamental string is related to the stringy cosmic string [25] by an $O(8, 24; Z)$ transformation. In this paper, we find an $O(8, 24; Z)$ transformation relating the fundamental string to the dual string of [4], thus supporting the above conjecture.

Finally, we speculate on the significance of these solutions to string/string duality.

2. Generalized T Solutions in $N = 4$

We adopt the following conventions for $N = 1$, $D = 10$ heterotic string theory compactified to $N = 4$, $D = 4$ heterotic string theory: (0123) is the four-dimensional spacetime, $z = x_2 + ix_3 = re^{i\theta}$, (456789) are the compactified directions, $S = e^{-2\Phi} + ia = S_1 + iS_2$, where Φ and a are the four-dimensional dilaton and axion and

$$
\begin{aligned}
T^{(1)} &= T_1^{(1)} + iT_2^{(1)} = \sqrt{\det g_{mn}} - iB_{45}, & m, n &= 4, 5, \\
T^{(2)} &= T_1^{(2)} + iT_2^{(2)} = \sqrt{\det g_{pq}} - iB_{67}, & p, q &= 6, 7, \\
T^{(3)} &= T_1^{(3)} + iT_2^{(3)} = \sqrt{\det g_{rs}} - iB_{89}, & r, s &= 8, 9
\end{aligned}
\tag{2.1}
$$

are the moduli. Throughout this section, and unless specified otherwise in the rest of the paper, we assume dependence only on the coordinates x_2 and x_3 (i.e. x_0 and x_1 are Killing directions), and that no other moduli than the ones above are nontrivial.

The canonical four-dimensional bosonic action for the above compactification ansatz in the gravitational sector can be written in terms of $g_{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$, S and $T^{(a)}$, $a = 1, 2, 3$ as

$$
\begin{aligned}
S_4 = \int d^4x \sqrt{-g} \bigg(R &- \frac{g^{\mu\nu}}{2S_1^2} \partial_\mu S \partial_\nu \bar{S} \\
&- \frac{g^{\mu\nu}}{2T_1^{(1)2}} \partial_\mu T^{(1)} \partial_\nu \bar{T}^{(1)} - \frac{g^{\mu\nu}}{2T_1^{(2)2}} \partial_\mu T^{(2)} \partial_\nu \bar{T}^{(2)} - \frac{g^{\mu\nu}}{2T_1^{(3)2}} \partial_\mu T^{(3)} \partial_\nu \bar{T}^{(3)} \bigg).
\end{aligned}
\tag{2.2}
$$

A solution for this action for $S = 1$ $(\Phi = a = 0)$ is given by the metric

$$
ds^2 = -dt^2 + dx_1^2 + T_1^{(1)} T_1^{(2)} T_1^{(3)} (dx_2^2 + dx_3^2),
\tag{2.3}
$$

where three cases with different nontrivial T-duality arise depending on the number n of nontrivial T moduli:

$$
\begin{aligned}
n = 1: && T^{(1)} &= -\frac{1}{2\pi} \ln \frac{z}{r_0}, & T^{(2)} &= T^{(3)} = 1, \\
n = 2: && T^{(1)} &= T^{(2)} = -\frac{1}{2\pi} \ln \frac{z}{r_0}, & T^{(3)} &= 1, \\
n = 3: && T^{(1)} &= T^{(2)} = T^{(3)} = -\frac{1}{2\pi} \ln \frac{z}{r_0}.
\end{aligned}
\tag{2.4}
$$

In each of the expressions for $T^{(a)}$, z may be replaced by $\bar{z}$ independently (i.e. there is a freedom in changing the sign of the axionic part of the modulus). Note that the $n = 1$

case is simply the dual string solution of [4]. Since S_4 has manifest $SL(2,R)$ duality in each of the moduli (broken to $SL(2,Z)$ in string theory), we can generate from the $n = 2$ case an $SL(2,Z)^2$ family of $\infty[5$ solutions and from the $n = 3$ case an $SL(2,Z)^3$ family of solutions. Note that the full T-duality group in all three cases remains $O(6,22;Z)$, but that the subgroup with nontrivial action on the particular solutions (or the solution-generating subgroup referred to above) for $n = 1, 2, 3$ is given by $SL(2,Z)^n$ [26,27].

From the ten-dimensional viewpoint, the $n = 3$ solution, for example, can be rewritten in the string sigma-model metric frame as

$$
e^{2\phi} = (-\frac{1}{2\pi} \ln \frac{r}{r_0})^3,
$$
$$
ds^2 = -dt^2 + dx_1^2 + e^{2\phi}(dx_2^2 + dx_3^2) + e^{2\phi/3}(dx_4^2 + ... + dx_9^2),   \tag{2.5}
$$
$$
B_{45} = \pm B_{67} = \pm B_{89} = \pm\frac{\theta}{2\pi},
$$

where ϕ is the ten-dimensional dilaton.

The solution (2.4) can in fact be generalized to include an arbitrary number of string-like sources in each $T^{(i)}$

$$
ds^2 = -dt^2 + dx_1^2 + T_1^{(1)}T_1^{(2)}T_1^{(3)}(dx_2^2 + dx_3^2)
$$
$$
T^{(1)} = -\frac{1}{2\pi} \sum_{j=1}^{M} m_j \ln \frac{(z-b_j)}{r_{j0}},
$$
$$
T^{(2)} = -\frac{1}{2\pi} \sum_{k=1}^{P} p_k \ln \frac{(z-c_k)}{r_{k0}},   \tag{2.6}
$$
$$
T^{(3)} = -\frac{1}{2\pi} \sum_{l=1}^{Q} q_l \ln \frac{(z-d_l)}{r_{l0}},
$$

where M, P and Q are arbitrary numbers of string-like solitons in $T^{(1)}, T^{(2)}$ and $T^{(3)}$ respectively each with arbitrary location b_j, c_k and d_l locations in the complex z-plane and arbitrary winding number m_j, p_k and q_l respectively. The solutions with $1, 2$ and 3 nontrivial T fields break $1/2, 3/4$ and $7/8$ of the spacetime supersymmetries respectively. Again, one can make the replacement $z \to \bar{z}$ independently in each of the moduli, so that each $T^{(i)}$ is either analytic or anti-analytic in z.

3. Supersymmetry Breaking

We claim that the above solutions for the massless fields in the gravitational sector when combined with a Yang-Mills field given by $A_M^{PQ} = \Omega_M^{PQ} = \omega_M^{PQ} \pm 1/2 H_M{}^{PQ}$ (the usual expedient of equating the gauge to the generalized connection) solve the tree-level supersymmetry equations of the heterotic string for zero fermi fields and can be argued to be exact solutions of heterotic string theory [28,4]. The supersymmetry equations in $D = 10$ are given by

$$\delta\psi_M = \left(\partial_M + \tfrac{1}{4}\Omega_{MAB}\Gamma^{AB}\right)\epsilon = 0,$$
$$\delta\lambda = \left(\Gamma^A\partial_A\phi - \tfrac{1}{12}H_{ABC}\Gamma^{ABC}\right)\epsilon = 0, \qquad (3.1)$$
$$\delta\chi = F_{AB}\Gamma^{AB}\epsilon = 0,$$

where $A, B, C, M = 0, 1, 2, ..., 9$ and where ψ_M, λ and χ are the gravitino, dilatino and gaugino fields. The Bianchi identity is given by

$$dH = \frac{\alpha'}{4}\left(\mathrm{tr}R \wedge R - \frac{1}{30}\mathrm{Tr}F \wedge F\right), \qquad (3.2)$$

and is satisfied automatically for this ansatz. We further claim that the $n = 1, 2, 3$ solutions break $1/2, 3/4$ and $7/8$ of the spacetime supersymmetries respectively. We will show this to be true for the most general case of $n = 3$.

$\delta\lambda = 0$ follows from scaling, since the dilaton can be written as the sum of three parts (the moduli) each of which produces a contribution which cancels against the contribution of the H term coming from the appropriate four-dimensional subspace. In other words, each of the subspaces (2345), (2367) and (2389) effectively contains a four-dimensional axionic (anti) instanton [29,30,31] with the appropriate (anti) self-duality in the generalized connection in the respective subspace, depending on whether the corresponding modulus is analytic or anti-analytic in z. Another way of saying this is that there are three independent parts of $\delta\lambda$, each of which vanishes as in the simple $n = 1$ case, for the appropriate chirality choice of ϵ in the respective four-dimensional subspace.

$\delta\psi_M = 0$ is a little more subtle. For the $n = 1$ case, the generalized connection is an instanton [30,31], and for constant chiral spinor ϵ with chirality in the four-space of the instanton opposite to that of the instanton (e.g. negative for instanton and positive for anti-instanton), it is easy to show that $\Omega_M^{AB}\Gamma_{AB}\epsilon = 0$. In the more general $n = 3$ case, we proceed as follows. It is sufficient to show that $\delta\psi_M = 0$ for $M = 2$ and $M = 4$ (i.e. for a spacetime and for a compactified index), as for $M = 0, 1$ the supersymmetry variation

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is trivial, while for the rest of the indices the arguments are identical to one of the above two representative cases. For $M = 2$ this can be written out explicitly as

$$4\delta\psi_2 = \left(\frac{1}{3}\omega_2^{23}\Gamma_{23} + \omega_2^{24}\Gamma_{24} + \omega_2^{25}\Gamma_{25} - \frac{1}{2}H_2{}^{45}\Gamma_{45}\right)\epsilon$$
$$+ \left(\frac{1}{3}\omega_2^{23}\Gamma_{23} + \omega_2^{26}\Gamma_{26} + \omega_2^{27}\Gamma_{27} - \frac{1}{2}H_2{}^{67}\Gamma_{67}\right)\epsilon \qquad (3.3)$$
$$+ \left(\frac{1}{3}\omega_2^{23}\Gamma_{23} + \omega_2^{28}\Gamma_{28} + \omega_2^{29}\Gamma_{29} - \frac{1}{2}H_2{}^{89}\Gamma_{89}\right)\epsilon.$$

Each line in (3.3) acts on only a four-dimensional component of ϵ and can be shown to exactly correspond to the contribution of the supersymmetry equation of a single $n = 1$ axionic instanton. So in effect the configuration carries three such instantons in the generalized curvature in the spaces (2345), (2367) and (2389). Therefore for the appropriate chirality of the four-dimensional components of ϵ (depending on the choices of analyticity of the T fields), $\delta\psi_2 = 0$. Since we are making three such choices, 1/8 of the spacetime supersymmetries are preserved and 7/8 are broken. Another, perhaps simpler, way to understand this is to write $\epsilon = \epsilon_{(01)} \otimes \epsilon_{(23)} \otimes \epsilon_{(45)} \otimes \epsilon_{(67)} \otimes \epsilon_{(89)}$. Then the chiralities of $\epsilon_{(45)}, \epsilon_{(67)}$ and $\epsilon_{(89)}$ are all correlated with that of $\epsilon_{(23)}$, so it follows that 7/8 of the supersymmetries are broken.

We also need to check $\delta\psi_4 = 0$. In this case, it is easy to show that the whole term reduces exactly to the contribution of a single $n = 1$ axionic instanton:

$$4\delta\psi_4 = \left(\omega_4^{42}\Gamma_{42} + \omega_4^{43}\Gamma_{43} - \frac{1}{2}H_4{}^{25}\Gamma_{25} - \frac{1}{2}H_4{}^{35}\Gamma_{35}\right)\epsilon = 0, \qquad (3.4)$$

as in this case there is only the contribution of the instanton in the (2345) subspace. $\delta\psi_4$ then vanishes for the same chirality choice of ϵ as in the paragraph above.

There remains to show that $\delta\chi = 0$. This can be easily seen by noting that, as in the $\delta\psi_M$ case, the term $F_{23}\Gamma^{23}$ splits into three equal pieces, each of which combines with the rest of a $D = 4$ instanton (since the Yang-Mills connection is equated to the generalized connection and is also effectively an instanton in each of the three four-dimensional subspaces) to give a zero contribution for the same chirality choices in the four-dimensional subspaces as above.

For the $n = 2$ case, it is even easier to show that 3/4 of the supersymmetries are broken. Tree-level neutral versions ($A_M = 0$) of these solutions also follow immediately and reduce to (2.3) and (2.4) on compactification to $D = 4$, where, of course, the same degree of supersymmetry breaking for each class of solutions may be verified directly. Henceforth we will consider only neutral solutions.

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4. Generalized Solutions with Nontrivial S

It turns out that these solutions generalize even further to solutions which include a nontrivial S field. The net result of adding a nontrivial S (with $SL(2, Z)$ symmetry) is to break half again of the remaining spacetime supersymmetries preserved by the corresponding T configuration with trivial S, except for the case of $n = 3$ nontrivial moduli, which is a bit more subtle and will be discussed below. In particular, the simplest solution of the action (2.2) with one nontrivial S and three nontrivial T moduli has the form

$$
ds^2 = -dt^2 + dx_1^2 + S_1 T_1^{(1)} T_1^{(2)} T_1^{(3)} (dx_2^2 + dx_3^2),
$$
$$
S = T^{(1)} = T^{(2)} = T^{(3)} = -\frac{1}{2\pi} \ln \frac{z}{r_0},
$$

(4.1)

where again we have an $SL(2, Z)$ symmetry in S and in each of the T fields.

It is interesting to note that the real parts of the S and T fields can be arbitrary as long as they satisfy the box equation in the two-dimensional subspace (23). In particular, each can be generalized to multi-string configurations independently, with arbitrary number of strings each with arbitrary winding number. The corresponding imaginary part can most easily be found by going to ten dimensions, where the corresponding B-field follows from the modulus. So there is nothing special about the choice $\ln z$. It is merely the simplest case.

The ten-dimensional form of the most general solution can be written in the string sigma-model metric frame as

$$
ds^2 = e^{2\Phi}(-dt^2 + dx_1^2) + e^{2(\sigma_1 + \sigma_2 + \sigma_3)}(dx_2^2 + dx_3^2)
$$
$$
+ e^{2\sigma_1}(dx_4^2 + dx_5^2) + e^{2\sigma_2}(dx_6^2 + dx_7^2) + e^{2\sigma_3}(dx_8^2 + dx_9^2),
$$
$$
S = e^{-2\Phi} + ia = -\frac{1}{2\pi} \sum_{i=1}^{N} n_i \ln \frac{(z - a_i)}{r_{i0}},
$$
$$
T^{(1)} = e^{2\sigma_1} - iB_{45} = -\frac{1}{2\pi} \sum_{j=1}^{M} m_j \ln \frac{(z - b_j)}{r_{j0}},
$$
$$
T^{(2)} = e^{2\sigma_2} - iB_{67} = -\frac{1}{2\pi} \sum_{k=1}^{P} p_k \ln \frac{(z - c_k)}{r_{k0}},
$$
$$
T^{(3)} = e^{2\sigma_3} - iB_{89} = -\frac{1}{2\pi} \sum_{l=1}^{Q} q_l \ln \frac{(z - d_l)}{r_{l0}},
$$
$$
\phi = \Phi + \sigma_1 + \sigma_2 + \sigma_3,
$$

(4.2)

where ϕ is the ten-dimensional dilaton, Φ is the four-dimensional dilaton, σ_i are the metric moduli, a is the axion in the four-dimensional subspace (0123) and N, M, P and Q are arbitrary numbers of string-like solitons in $S, T^{(1)}, T^{(2)}$ and $T^{(3)}$ respectively each with arbitrary location a_i, b_j, c_k and d_l in the complex z-plane and arbitrary winding number n_i, m_j, p_k and q_l respectively. Again one can replace z by $\bar{z}$ independently in S and in each of the moduli.

The solutions with nontrivial S and $0, 1$ and 2 nontrivial T fields preserve $1/2, 1/4$ and $1/8$ spacetime supersymmetries respectively. This follows from the fact that the nontrivial S field breaks half of the remaining supersymmetries by imposing a chirality choice on the spinor ϵ in the (01) subspace of the ten-dimensional space. The solution with nontrivial S and 3 nontrivial T fields breaks $7/8$ of the spacetime supersymmetries for one chirality choice of S, and all the spacetime supersymmetries for the other. This can be seen as follows: the three nontrivial T fields, when combined with an overall chirality choice of the Majorana-Weyl spinor in ten dimensions, impose a chirality choice on ϵ_{01}. If this choice agrees with the chirality choice imposed by S, then no more supersymmetries are broken, and so $1/8$ are preserved (or $7/8$ are broken). When these two choices are not identical, all the supersymmetries are broken, although the ansatz remains a solution to the bosonic action.

5. Dyonic Strings in $D = 6$ and $D = 10$

A special case of the above generalized S and T solutions is the one with nontrivial S and only one nontrivial T. This is in fact a "dyonic" solution which interpolates between the fundamental S string of [3] and the dual T string of [4]. It turns out that in going to higher dimensions, one still has a solution even if the box equation covers the whole transverse four-space (2345) (the remaining four directions are flat even in $D = 10$, as $\sigma_2 = \sigma_3 = 0$). The $D = 10$ form in fact reduces to a $D = 6$ dyonic solution ($i = 2, 3, 4, 5$)

$$\phi = \Phi_E + \Phi_M,$$
$$ds^2 = e^{2\Phi_E}(-dt^2 + dx_1^2) + e^{2\Phi_M} dx_i dx^i,$$
$$e^{-2\Phi_E} = 1 + \frac{Q_E}{y^2}, \qquad e^{2\Phi_M} = 1 + \frac{Q_M}{y^2}, \qquad (5.1)$$
$$H_3 = 2Q_M \epsilon_3, \qquad e^{-2\phi} {}^* H_3 = 2Q_E \epsilon_3$$

for the special case of a single electric and single magnetic charge at $y = 0$. Again this solution generalizes to one with an arbitrary number of arbitrary (up to dyonic quantization

conditions) charges at arbitrary locations in the transverse four-space. For $Q_M = 0$ (5.1) reduces to the solution of [3] in $D = 6$, while for $Q_E = 0$ (5.1) reduces to the $D = 6$ dual string of [10] (which can be obtained from the fivebrane soliton [32] simply by compactifying four flat directions). This solution breaks 3/4 of the spacetime supersymmetries. The self-dual limit $Q_E = Q_M$ of this solution has already been found in [10] in the context of $N = 2$, $D = 6$ self-dual supergravity, where the solution was shown to break 1/2 the spacetime supersymmetries. This corresponds precisely to breaking 3/4 of the spacetime supersymmetries in the non self-dual theory in this paper [1].

Finally, one can generalize the dyonic solution to the following solution in $D = 10$:

$$ds^2 = e^{2\Phi_E}\left(-dt^2 + dx_1^2\right) + e^{2\Phi_{M1}}\delta_{ij}dx^i dx^j + e^{2\Phi_{M2}}\delta_{ab}dx^a dx^b,$$

$$\phi = \Phi_E + \Phi_{M1} + \Phi_{M2}, \qquad \Phi_E = \Phi_{E1} + \Phi_{E2},$$

$$e^{2\Phi_{E1}}\Box_1\, e^{-2\Phi_{E1}} = e^{2\Phi_{E2}}\Box_2\, e^{-2\Phi_{E2}} = e^{-2\Phi_{M1}}\Box_1\, e^{2\Phi_{M1}} = e^{-2\Phi_{M2}}\Box_2\, e^{2\Phi_{M2}} = 0,$$

$$B_{01} = e^{2\Phi_E}, \qquad H_{ijk} = 2\epsilon_{ijkm}\partial^m\Phi_{M1}, \qquad H_{abc} = 2\epsilon_{abcd}\partial^d\Phi_{M1},$$

(5.2)

where $i, j, k, l = 2, 3, 4, 5$, $a, b, c, d = 6, 7, 8, 9$, $\Box_1$ and $\Box_2$ represent the Laplacians in the subspaces (2345) and (6789) respectively and ϕ is the ten-dimensional dilaton. This solution with all fields nontrivial breaks 7/8 of the spacetime supersymmetries. For $\Phi_{E2} = \Phi_{M2} = 0$ we recover the dyonic solution (5.1) which breaks 3/4 of the super-symmetries, for $\Phi_{E1} = \Phi_{E2} = 0$ we recover the double-instanton solution of [18] which also breaks 3/4 of the supersymmetries, while for $\Phi_{M1} = \Phi_{M2} = 0$ we obtain the dual of the double-instanton solution, and which, however, breaks only 1/2 of the supersymmetries.

6. Solutions in $N = 1$ and $N = 2$

It turns out that most of the above solutions that break $1/2, 3/4, 7/8$ or all of the spacetime supersymmetries in $N = 4$ have analogs in $N = 1$ or $N = 2$ that break only 1/2 the spacetime supersymmetries.

For simplicity, let us consider the case of $N = 1$, as the $N = 2$ case is similar. It turns out that the number of nontrivial T fields with the same analyticity and the inclusion of a nontrivial S field with the same analyticity does not affect the number of supersymmetries broken, as in the supersymmetry equations the contribution of each field is independent. In particular, the presence of an additional field produces no new condition on the chiralities, so that the number of supersymmetries broken is the same for

any number of fields, provided the fields have the same analyticity or anti-analyticity in z, corresponding to different chirality choices on the four-dimensional spinor. This can be seen as follows below.

The supersymmetry transformations in $N = 1$ for nonzero metric, S and moduli fields are given by [33,34]

$$\delta \psi_{\mu L} = \left(\partial_\mu + \frac{1}{2} \omega_{\mu mn} \sigma^{mn} \right) \epsilon_L - \frac{\epsilon_L}{4} \left(\frac{\partial G}{\partial z_i} \partial_\mu z_i - \frac{\partial G}{\partial \bar{z}_i} \partial_\mu \bar{z}_i \right) = 0,$$

$$\delta \chi_{iL} = \frac{1}{2} \hat{D} z_i \epsilon_R = 0,$$

(6.1)

where ω is the spin connection, $\sigma^{mn} = (1/4)[\gamma^m, \gamma^n]$, $\epsilon_{L,R} = (1/2)(1 \pm \gamma^5)\epsilon$, $\hat{D} = \gamma^\mu D_\mu$, $z_i = S, T^{(1)}, T^{(2)}, T^{(3)}$, and where

$$G = -\ln(S + \bar{S}) - \sum_{j=1}^{3} \ln(T^{(j)} + \bar{T}^{(j)}).$$

(6.2)

Consider the case of a single nontrivial $T = T^{(1)}$ field (i.e. the dual string) in $N = 1, D = 4$

$$ds^2 = -dt^2 + dx_1^2 + T_1(dx_2^2 + dx_3^2),$$

$$T = T_1 + iT_2 = -\frac{1}{2\pi} \ln z.$$

(6.3)

Then it is easy to show that this configuration breaks precisely half the spacetime supersymmetries of (6.1) by imposing two conditions on the components of ϵ. A quick check shows that the presence of additional nontrivial S and T fields with the same analyticity behaviour also lead to solutions of (6.1), and this scenario generalizes to multi-string solutions. The number, location and winding numbers of the multi-string solitons is not relevant, but the fact the fields have $\ln z$ or $\ln \bar{z}$ behaviour is. Provided the S and various T fields all have the same analyticity (i.e. either all analytic or all anti-analytic) in z, then no new chirality choice is imposed by the addition of more fields. This can be seen simply from the fact that the spin connection and potential G both scale logarithmically with the fields, while $\delta \chi_{iL}$ is satisfied in the identical manner for each i. Unlike the $N = 4$ case, the presence of these additional fields produces no new conditions on ϵ, as the supersymmetry variations act on ϵ in precisely the same manner for all the fields. It follows then that the $N = 1$ analogs of these particular $N = 4$ solutions discussed above break only half the spacetime supersymmetries in $N = 1$, and in some sense are realized naturally as stable solitons only in this context. When at least one of the fields, either S or one of the T fields, has a different analyticity behaviour from the rest, opposite chirality conditions are imposed on ϵ, and no supersymmetries are preserved.

A similar analysis can be done in the $N = 2$ case, at least for those solutions which can arise in $N = 2$.

7. S, T and $O(8,24;Z)$ Duality

We now turn to the issue of whether S-duality and T-duality can be combined in a larger duality group $O(8,24;Z)$ [21,22,23,24]. Their combination into $O(2,2;Z)$ was discussed in [35]. In particular, in heterotic string theory compactified on a seven-torus the $SL(2,Z)$ S-duality should be combined with the $O(7,23;Z)$ T-duality into the group $O(8,24;Z)$. The existence of a Killing direction in all of the solitons discussed in this paper means that they may all be viewed as point-like solutions of three-dimensional heterotic string theory. Thus from the viewpoint of three-dimensional heterotic string theory $O(8,24;Z)$ appears as a duality group, whereas from the point of view of four-dimensional heterotic string theory it appears as a solution-generating group.

Central to the issue of combining S and T-duality is whether there exists an $O(8,24;Z)$ transformation that maps the fundamental S string of [3] to the dual T string of [4]. In [23], Sen finds a transformation that takes the fundamental string to a limit of the stringy "cosmic" string solution of [25] (with the slight subtlety that the two strings point in different directions from the ten-dimensional viewpoint). We call this solution the U string, since its nontrivial field is (with a change of coordinates) $U = g_{44}^{-1}(\sqrt{\det g_{mn}} - ig_{45})$ with $m,n = 4,5$. This modulus field transforms under $SL(2,Z)$ U-duality, just like S and T do under S- and T-duality. It is related to the T field by an $O(2,2;Z)$ transformation. Therefore, it is no surprise that we can map the S string to the T string by an $O(8,24;Z)$ transformation. The explicit transformation doing this is given as follows. Following Sen's notation [23], $\mathcal{M}_T$, the 32×32 matrix that corresponds to the T string, is obtained from $\mathcal{M}_S$, the 32×32 matrix that corresponds to the S string, simply by exchanging (rows and columns) 1 with 10, 2 with 31, 3 with 8 and 9 with 32. The transformation matrix is therefore, effectively, an $O(4,4;Z)$ matrix. It follows that all three strings, fundamental, dual and cosmic are related by $O(8,24;Z)$. As will be discussed in a subsequent publication [36], the S, T and U strings are related by a four-dimensional string/string/string triality.

Repeating Sen's and other arguments on three-dimensional reduction for $N = 2$ superstring theory, we can infer that the larger duality group (or solution-generating group) for $N = 2$ containing S and T-duality is connected to a dual quaternionic manifold. In the case of n moduli, this group is $SO(n + 2, 4; Z)$ [37,38].

Of course, all the $N = 4$ soliton discussed in this paper (and many other solutions) are related by generalized Geroch group transformations, because the $D = 4$ space-time admits two Killing directions. Therefore, the theory is effectively two-dimensional and the equations of motion have an affine $o(8,24)$ symmetry [39,22].

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8. Conclusions

The previously-known string soliton solutions, which break half of the supersymmetries, have played a crucial part in understanding heterotic/Type IIA string/string duality. It is not yet clear what part will be played by the new string soliton solutions presented here which break more than half the supersymmetries. These solutions are in some sense realized naturally as stable solitons only in the context of either $N = 1$ or $N = 2$ compactifications, and should lead to a better understanding of duality in $N = 1$ and $N = 2$ compactifications and to the construction of the Bogomol'nyi spectra of these theories. In these two cases, however, the situation is complicated by the absence of non-renormalization theorems present in the $N = 4$ case which guarantee the absence of quantum corrections. An exception to this scenario arises for $N = 2$ compactifications with vanishing β-function. The construction of these spectra remains a problem for future research.

In heterotic/Type IIA duality, we have $dH \neq 0$, but $d\tilde{H} = 0$. It is tempting to speculate that the dyonic solution, for which both dH and $d\tilde{H}$ are non-zero, will be important for the conjectured [11], but as yet little explored, six-dimensional heterotic/heterotic duality.

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