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SOME THOUGHTS ON THE FOCUSING REGIME FOR AN ALVAREZ LINAC

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Abstract

When designing an Alvarez linac the perennial problem arises as to what is the optimum phase advance and focusing structure. This report discusses this question and makes recommendations based on simple criteria. The general conclusions are valid over a wide range of parameters, but the detailed simulations given in the report can be used to calculate the optima more carefully for a particular beam current and energy range. This study is complementary to the AUSTRON Feasibility Study for a fast-cycling, synchrotron-driven neutron spallation source.

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1. Introduction

This note discusses the transverse focusing in an Alvarez drift-tube linac and considers:

- the choice between the FODO and FOFODODO focusing structures[*] and
- the optimisation of the transverse phase advance per cell.

The underlying aims are to minimise the drift-tube aperture in order to have a good transit-time factor and to minimise the power consumption of the quadrupoles, which are assumed to be pulsed in this study. It is tacitly assumed that the frequency is 200 Mhz, which is a *de facto* standard in high-energy, particle-physics accelerators. However, the conclusions presented are largely independent of the longitudinal parameters, except for technical limitations that would become apparent at very high frequencies due to the small dimensions of the structure. The effects of space charge have been included by the use of the program TRACE 3-D [1].

2. Considerations

The transverse focusing of the beam in a linac is maintained by quadrupoles housed inside the drift tubes. The focusing structure and strength of these quadrupoles should be optimised for ***minimum beam size*** and ***minimum power consumption*** while taking into account the RF defocusing and space charge effects. Added to these general points there is a technical limitation on the quadrupole gradient that turns out to be about 80 Tm^{-1} for the dimensions that are typical in this type of linac.

2.1 Beam size

The beam radius, r, is related to the transverse emittance, ε_t, and the betatron amplitude, β_t, by

$$r = \sqrt{\beta_t \frac{\varepsilon_t}{\pi}} \,. \tag{1}$$

β_t depends on:

- the transverse focusing structure, which can either be a FD or a FFDD,
- the transverse phase advance in a single cell of the focusing structure, μ_0 (i.e. two RF cells of length L for the FD and four RF cells for the FFDD lattices),
- the space charge conditions.

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[*] The quadrupoles are housed in consecutive drift tubes of the linac. For brevity, the FODO and FOFODODO structures will be referred to as FD and FFDD.

An indication of what this implies is given by the well-known thin-lens analysis of a FD cell for the non-space-charge case [2]. This analysis expresses the maximum of the betatron amplitude function as

$$\hat{\beta}_t = 2L\frac{1+\sin(\mu_0/2)}{\sin(\mu_0)} . \tag{2}$$

This function has a minimum at $\mu_0 \approx 76.4^0$, at which point $\beta_{t,min} \approx 3.23L$. The form of the minimum is rather broad. The transverse phase advance per focusing period for zero space charge is adjusted by the focal strength of the quadrupole, g, according to

$$\sin\left(\frac{\mu_0}{2}\right) = \frac{1}{2}gL . \tag{3}$$

The inclusion of space charge results in a tune depression, which increases the beam radius. This can be compensated by increasing the quadrupole strengths, but the general behaviour of the betatron amplitude function remains very similar.

The inclusion of the RF defocusing also results in a tune depression that requires higher quadrupole gradients for its compensation. The influence of RF defocusing, however, is strongest at the low-energy end of the linac where the drift tubes are shortest. This may mean that the optimum phase advance cannot be achieved at the low-energy end owing to the technical limitation of 80 Tm^{-1} mentioned above.

The FD and FFDD lattices have similar behaviours, but the FFDD lattice leads to larger beam sizes in general, but requires less focusing power.

2.2 Quadrupole power

The dissipation in pulsed quadrupoles is due mainly to the cyclic creation of the field. Thus, the power dissipation, P, is proportional to the square of the field, G^2r^2, and to the volume, which is proportional to the beam radius squared, r^2 (assuming a constant magnet length). With use of (1),

$$P \propto G^2\beta_t^{\,2} . \tag{4}$$

3. Simulations

Simulations were carried out with the program TRACE 3-D. These calculations include space charge and RF defocusing effects. Four beam energies (0.75, 2, 50 and 130 MeV), five beam current settings (0, 50, 100, 150, and 200 mA) and the FD and FFDD structures were investigated. As an approximation the F and D quadrupole settings per focusing period were kept constant. This causes a slight asymmetry due to the increasing beam energy. Quadrupoles were assumed to be mounted in every drift tube. Therefore, an FD focusing period includes two RF cells and an FFDD focusing period includes 4 RF cells.

3.1 FD focusing structure

Figure 1 shows how the normalised maximum betatron function (and therefore corresponding to (1) the beam size) varies with phase advance in the FD focusing structure. In each figure there are four graphs for the four energy levels and each graph has five lines corresponding to increasing beam current.

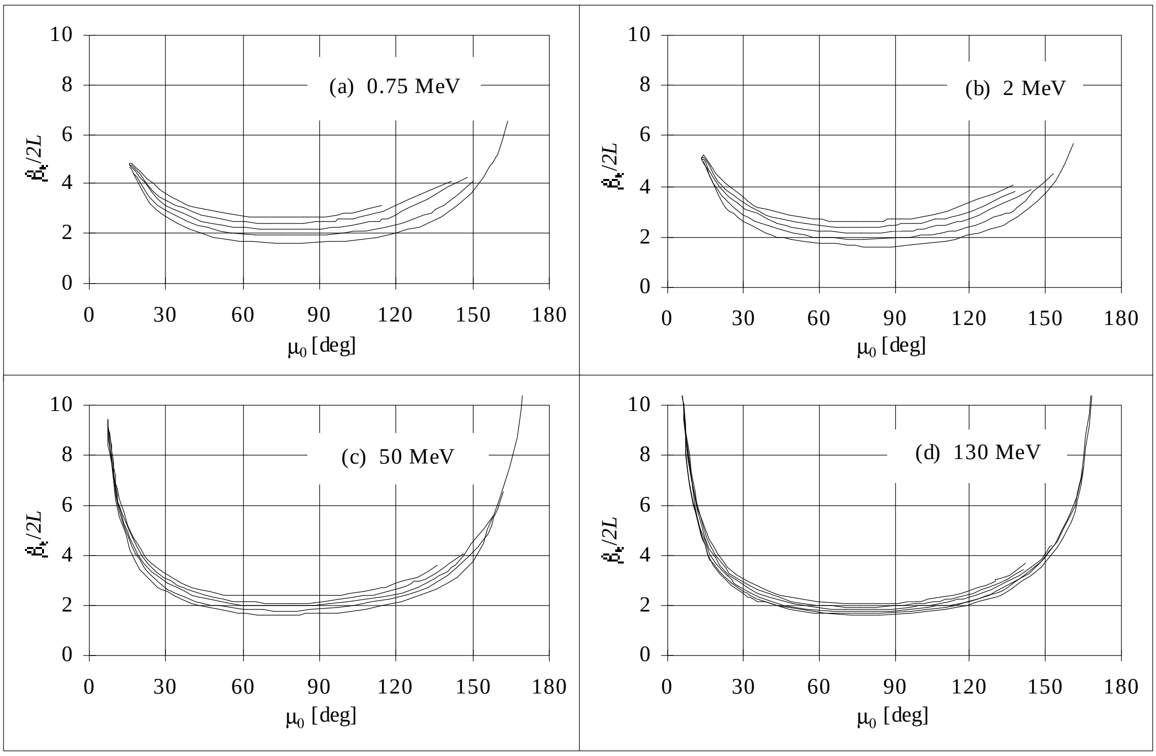

Figure 1 The normalised maximum betatron function vs. the transverse phase advance for four beam energies (a)..(d) for FD focusing; each graph contains five lines corresponding to increasing beam current, starting from 0 mA (bottom line) to 200 mA (top line) in 50 mA steps

Figure 2 shows the quadrupole power dissipation in arbitrary units for the same cases as in Figure 1.

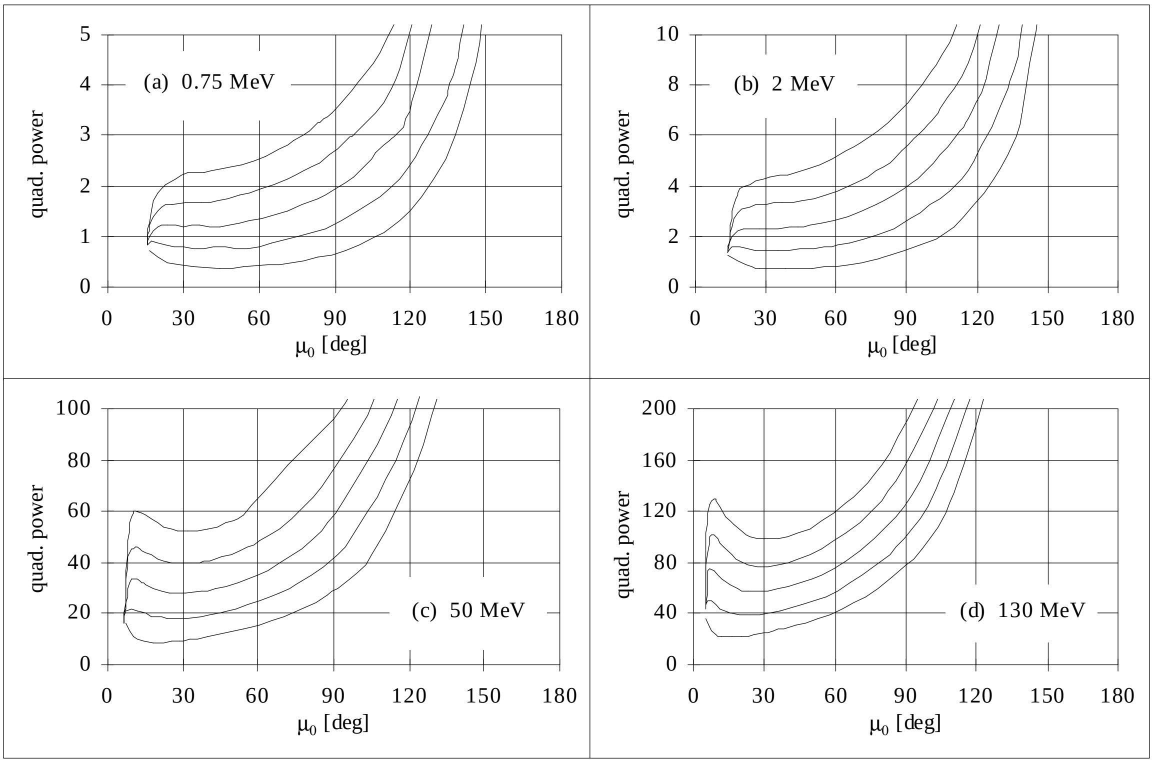

Figure 2 Quadrupole power dissipation (in arbitrary units) vs. the transverse phase advance for FD focusing for four beam energies (a)..(d) and five current settings starting from 0 mA (bottom line) to 200 mA (top line) in 50 mA steps

3.2 Comparison between FD and FFDD focusing structures

Figure 3 shows how the maximum betatron function varies with transverse phase advance for the FD and FFDD focusing structures for 130 MeV and 0 mA. The minima are rather flat and appear at $\mu_0 \approx 80^0$ in both cases. The ratio of the betatron functions at the minima is about 1.65. Therefore the minimum beam size for the FD structure is about 78% of the FFDD structure.

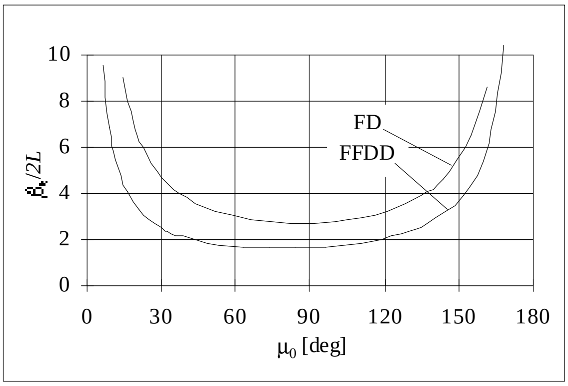

Figure 3 The normalised betatron amplitude vs. the transverse phase advance for FD and FFDD focusing structures

A comparison of the required quadrupole gradients, *G*, is shown in Figure 4 for FD and FFDD focusing for 130 MeV and 0 mA. The FD focusing requires quadrupole gradients which are approximately three times higher compared to the FFDD focusing for equivalent transverse phase advance.

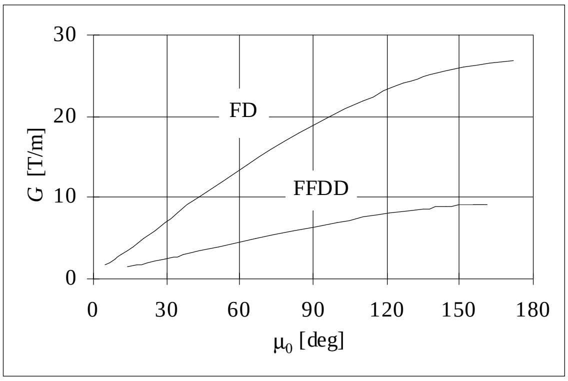

Figure 4 The quadrupole gradient vs. the transverse phase advance for FD and FFDD focusing structures

Figure 5 compares the quadrupole power for 130 MeV and 0 mA for the FD and FFDD focusing structures. There are minima at $\mu_0 \approx 30^0$ in the FD and $\mu_0 \approx 40^0$ in the FFDD focusing case. The corresponding minimum power requirements in the FFDD case are about 50% of the FD case.

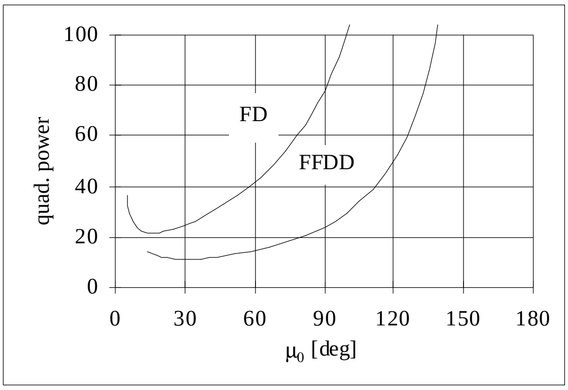

Figure 5 Quadrupole power (in arbitrary units) vs. the transverse phase advance for FD and FFDD focusing structures

4. Conclusions

- The general features of the simulations show that minimum beam size is obtained with a phase advance in a focusing cell of around 80^0. In general, the FD curves yield minimum beam sizes of about 78% of the FFDD beam size.

- The quadrupole power consumption shows a minimum in all cases around 30^0 phase advance in a FD focusing structure and 40^0 in a FFDD structure. The minimum power requirements in the FFDD case are about 50% of the FD case.

The optimum phase advances for beam size and power consumption appear very different, but since the minima are flat ***a phase advance of about*** 40^0 ***in a FD structure puts the beam size and power consumption within 10% of their minimum values.*** Figure 6 illustrates this for the FD cell at 130 MeV with a beam current of 50 mA. ***Similary, a good compromise occurs at*** 50^0 ***for the FFDD structure.***

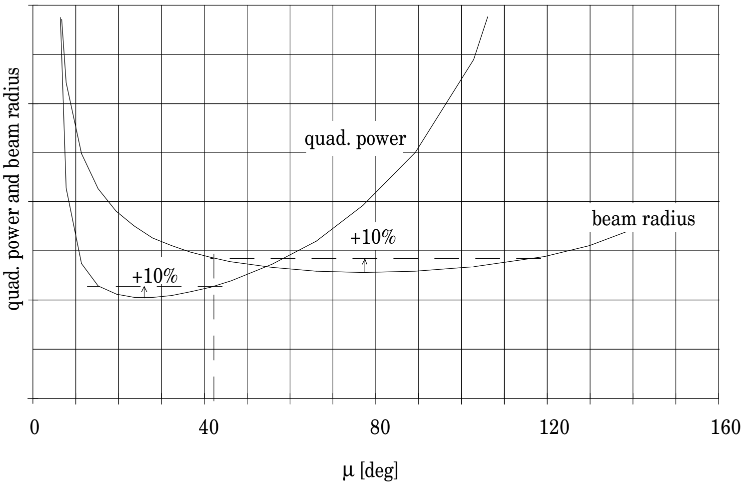

Figure 6 Compromise for the phase advance in a FD cell (130 MeV and 50 mA)

This work was undertaken for the AUSTRON feasibility study [3]. The geometrical parameters and the beam parameters that were used in the numerical simulations were taken from this study.

5. Acknowledgements

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References

[1] K.R. Crandall, *TRACE 3-D Documentation*, LA-UR-90-4146, LANL, 1987.

[2] E. Keil, *Single-particle dynamics - linear machine lattices*, Proc. Int. School of Part. Accel. Ettore Majorana, Centre for Scientific Culture, Erice, 1976, CERN 77-13 (July 1977), 22-36.

[3] P.J. Bryant, M. Regler, M. Schuster eds., *'AUSTRON Feasibility Study'*, (AUSTRON Planning Office, c/o Atominstitut der Österreichischen Universitäten, Vienna), Nov. 1994, also to be published as a CERN PS Divisional Report.

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