CERN-TH-95-133
HD-HEP-95-8
IOA-95-323

HEAVY NEUTRINO THRESHOLD EFFECTS IN LOW ENERGY PHENOMENOLOGY

G. K. Leontaris[1,2], **S. Lola**[3], and **G. G. Ross**[4,5⋆]

[1]Centre de Physique Theorique, Ecole Polytechnique, F-91128 Palaiseau, France
[2] *on leave from* Physics Department, Ioannina University, Ioannina, Greece
[3]Institut für Theoretische Physik, Univerisität Heidelberg,
Philosophenweg 16, 69120 Heidelberg, Germany
[4]Department of Physics,Theoretical Physics, University of Oxford,
1 Keble Road, Oxford OX1 3NP
[5]CERN Theory Division, Geneva , Switzerland

ABSTRACT

Right handed neutrinos with mass of $\mathcal{O}$ $(10^{12} - 10^{13})$ GeV are required to implement the see-saw mechanism and generate neutrino masses capable of playing a role in structure formation. Moreover models of fermion masses often relate the Yukawa couplings involving these neutrinos to the up-quark Yukawa couplings. Here we study the effects of such couplings on the radiative corrections to quark masses. We find that $b-\tau$ equality at M_{GUT} may still give the correct $m_b/m_\tau-$ ratio at low energies, but only if there is large $\mu-\tau$ mixing in the charged leptonic sector. We propose specific mass matrix "textures" dictated by a $U(1)$ family symmetry whose structure preserves $m_b = m_\tau$ at M_{GUT}. In these schemes, due to the large $\nu_\mu - \nu_\tau$ mixing, it is possible to give a simultaneous solution to the solar neutrino deficit and the atmospheric neutrino problem.

⋆ SERC Senior Fellow.

1 Introduction

Although the Standard Model is remarkably successful in describing strong and electroweak phenomena it still leaves many questions unanswered. In particular one would like to understand the origin of fermion masses and mixing angles which in the Standard Model appear as arbitrary parameters. An obvious possibility is that some structure additional to that of the Standard Model is responsible for the pattern of masses and mixings that we see at low energies. Support for such a stage of unification has been obtained for, although unification [1, 2] on its own does not agree with experiment, when combined with supersymmetry it leads to very successful predictions [3] for the gauge couplings, the pattern and magnitude of spontaneous symmetry breaking at the elecroweak scale [4] and the bottom − tau ($b-\tau$) unification [3, 5]. A further indication that additional symmetries beyond the Standard Model exist, has been the observation that the fermion mixing angles and masses have values consistent with the appearance of "texture" zeros in the mass matrix [6, 7, 8, 9, 10, 11, 12]. Such zeros may indicate the presence of an additional broken symmetry. When unbroken only the third generation is massive and all mixing angles are zero. However, symmetry breaking terms gradually fill in the mass matrix and generate a hierarchy of mass scales and mixing angles.

In this paper we will consider the implications for the neutrino sector extending the analysis presented in [11]. We shall consider models with right-handed neutrinos in which both Dirac and Majorana masses for the neutrinos are present, and the "see–saw" mechanism automatically explains the lightness of neutrinos relative to the charged fermions.

Neutrino data from various experiments can be explained if there is mixing between the various types of neutrinos. The solar neutrino puzzle can be resolved through matter enhanced oscillations (preferably between ν_e and ν_μ states) with a mixing angle somewhat smaller than the 1/3 of the corresponding Cabbibo angle of the quark sector, V^{12}_{CKM}. For this explanation to work, the squared mass difference of the two types of neutrinos involved in this phenomenon should lie in a very narrow region. The specific ranges for the angle and mass squared given by the latest experimental data are:

For matter enhanced oscillations in the sun:

$$\sin 2\theta_{ex} = (0.39-1.6)\times 10^{-2}, \quad \Delta m^2 = (0.6-1.2)\times 10^{-5} eV^2, \tag{1}$$

and for vacuum oscillations:

$$\sin 2\theta_{ex} \geq 0.75, \quad \Delta m^2 = (0.5-1.1)\times 10^{-5} eV^2. \tag{2}$$

If we wish to avoid fine tuning problems, it seems necessary to assume that such small differences in neutrino masses can be obtained only if the $\nu_{e,\mu}$ neutrino masses themselves are also of the same order. Finally, if neutrinos play a role in structure formation, providing a hot dark matter component, then the heavier neutrino(s) should have mass in the range $\sim (1-6)$ eV, the precise value depending on the number of neutrinos that have masses of this order of magnitude.

What symmetry could explain this pattern of masses? If gauge symmetries have something to do with the hierarchical fermion mass spectrum a similar hierarchy may be expected to hold for the unknown neutrino masses too. In a previous study it was found that the observed hierarchical mass spectrum of the charged fermions (quarks and leptons) follows naturally if we extend the gauge group of the minimal supersymmetric standard model by a $U_{FD}(1)$ family type symmetry. The extension of this model to include the right handed neutrino in the theory resulted in a similar structure of the neutrino sector as well leading to the following general structures [11]:

1.) The solar neutrino puzzle can be explained via $\nu_e \to \nu_\mu$ oscillations. The hierarchical mass spectrum leads to the conclusion that $m_{\nu_\mu} \approx \sqrt{\delta m^2_{\mu,e}}$ while $m_{\nu_e} \ll m_{\nu_\mu}$. This also fixes the right handed neutrino scale M_N through the effective Majorana mass matrix resulting from the usual "see – saw" mechanism

$$m_\nu^{eff} = -\frac{1}{4} m_{\nu_D} M_N^{-1} m_{\nu_D}^T \tag{3}$$

2.) The simultaneous solution of the solar neutrino problem and the interpretation of the ν_τ mass as a hot dark matter component through the effective light Majorana neutrino mass matrix, requires a right handed neutrino scale of the order of $M_{\nu_R} \sim 10^{12} - 10^{13}$ GeV.

3.) There is no natural solution of the atmospheric neutrino problem unless a considerable fine-tuning of the coefficients in the neutrino mass textures occurs. This follows because the U(1) symmetry that was used to derive the above textures together with simple spontaneous breaking gives only a hierarchical mass spectrum and small mixing angles for all fermion mass matrices.

One additional implication of the structure emerging from the $U(1)_{FD}$ symmetry is that the right handed neutrinos have Yukawa couplings of the same order as the up quarks. This in turn affects the radiative corrections in the model and in particular the expectations for gauge unification and for the m_b/m_τ ratio. The implications of such large couplings has already been explored in refs.[13]. Here we develop these analyses in two respects. Firstly we present a semi-analytic analysis of the radiative corrections that allows us to analyse the possibilities for maintaining $b-\tau$ equality at the GUT scale even in the presence of these radiative corrections through large $\mu-\tau$ mixing giving a mechanism to evade conclusion 3.) above[1]. This leads us to consider schemes based on the $U(1)$ family symmetry which naturally generate such mixing. The implications for the neutrino mass spectrum are then explored. In addition the semi-analytic approach is supported by a full numerical calculation. In section 2 we

[1]For another mechanism evading this result through the introduction of spontaneous $U(1)$ breaking via several Higgs scalars see ref.[14].

give a semi–analytical approach to the renormalisation group equations, in the presence of right handed neutrinos. These equations are used in section 3 in order to get some direct intuition on the effects of the heavy neutrinos. The explicit form of the solutions makes it easy to see how $b-\tau$ equality at a GUT scale may be made consistent with the parameter spectrum at low energies, by sufficient $\mu-\tau$ mixing in the charged leptonic sector and for a relatively heavy strange quark. This is discussed in section 4. In section 5 we give the resulting predictions for the heavy and light Majorana neutrino mass matrices and eigenvalues, with a mixing which is of the correct order of magnitude in order to explain the atmospheric neutrino problem. In section 6 we present a numerical approach to the renormalisation group equations, which depicts the observations of section 2. Finally, in section 7 we give a summary of our results.

2 RGE with RH-neutrinos: a semi – analytic approach

From the above it is clear that the interpretation of many important experimental facts is based on the existence of the right – handed partners ν_{R_i} of the three left – handed neutrinos, where the scale of mass of these particles is at least three orders of magnitude smaller than the gauge unification scale, M_U. Thus the running from the Unification scale, $M_U \sim 10^{16}$ GeV, down to the scale of M_{ν_R}, must include radiative corrections from ν_R neutrinos. After that scale, ν_R's decouple from the spectrum, and an effective see – saw mechanism is operative, c.f. eq(3).

In the presence of the right handed neutrino, the renormalization group equations for the Yukawa couplings at the one–loop level are

$$16\pi^2\frac{d}{dt}h_U = \left(3h_Uh_U^\dagger + h_Dh_D^\dagger + I\cdot Tr[h_Nh_N^\dagger] + I\cdot Tr[3h_Uh_U^\dagger] - I\cdot G_U\right)h_U, \quad (4)$$

$$16\pi^2\frac{d}{dt}h_D = \left(3h_Dh_D^\dagger + h_Uh_U^\dagger + I\cdot Tr[3h_Dh_D^\dagger] + I\cdot Tr[h_Eh_E^\dagger] - I\cdot G_D\right)h_D, \quad (5)$$

$$16\pi^2\frac{d}{dt}h_E = \left(3h_Eh_E^\dagger + h_Nh_N^\dagger + I\cdot Tr[h_Eh_E^\dagger] + I\cdot Tr[3h_Dh_D^\dagger] - I\cdot G_E\right)h_E, \quad (6)$$

$$16\pi^2\frac{d}{dt}h_N = \left(h_Eh_E^\dagger + 3h_Nh_N^\dagger + I\cdot Tr[3h_Uh_U^\dagger] + I\cdot Tr[h_Nh_N^\dagger] - I\cdot G_N\right)h_N. \quad (7)$$

where h_α, $\alpha = U, D, E, N$, represent the $3\otimes 3$ Yukawa matrices for the up and down quarks, charged lepton and Dirac neutrinos, while I is the $3\otimes 3$ identity matrix. Finally, $G_\alpha = \sum_{i=1}^{3} c_\alpha^i g_i(t)^2$ are functions which depend on the gauge couplings with the coefficients c_α^i's given by [15, 13].

$$\{c_U^i\}_{i=1,2,3} = \left\{\frac{13}{15}, 3, \frac{16}{3}\right\}, \qquad \{c_D^i\}_{i=1,2,3} = \left\{\frac{7}{15}, 3, \frac{16}{3}\right\}, \quad (8)$$

$$\{c_E^i\}_{i=1,2,3} = \left\{\frac{9}{5}, 3, 0\right\}, \qquad \{c_N^i\}_{i=1,2,3} = \left\{\frac{3}{5}, 3, 0\right\}. \quad (9)$$

Consider initially the simple case where only the top and Dirac – type neutrino Yukawa couplings are large at the GUT scale (i.e. the case of small $tan\beta$ scenario). Let us start assuming that the top and neutrino Yukawa couplings are equal at the Unification scale, $h_t(M_U) = h_N(M_U)$, a relation which arises naturally not only in our case but in most of the Grand Unified Models which predict the existence of the right handed neutrino. As in the case of the charged fermions, we will consider only hierarchical textures [11] for the right handed neutrino Majorana mass matrices, i.e. $M_{\nu_1} \ll M_{\nu_2} \ll M_{\nu_3}$. If we work in a diagonal basis we can considerably simplify the above equations. Then, for the range M_U to M_N, the renormalization group evolution of the Yukawa couplings of third generation, can be written as follows

$$\begin{aligned} 16\pi^2 \frac{d}{dt} h_t &= \left(6h_t^2 + h_N^2 - G_U\right) h_t, & (10) \\ 16\pi^2 \frac{d}{dt} h_N &= \left(4h_N^2 + 3h_t^2 - G_N\right) h_N, & (11) \\ 16\pi^2 \frac{d}{dt} h_b &= \left(h_t^2 - G_D\right) h_b, & (12) \\ 16\pi^2 \frac{d}{dt} h_\tau &= \left(h_N^2 - G_E\right) h_\tau, & (13) \end{aligned}$$

Below M_N, the right handed neutrino decouples from the massless spectrum and we are left with the standard spectrum of the MSSM. For scales $Q \leq M_N$ the gauge and Yukawa couplings evolve according to the standard renormalisation group equations. In addition, the effective Yukawa coupling of the light neutrino mass matrix (3) evolves according to [16]

$$16\pi^2 \frac{d}{dt} h_\nu = h_\nu (I \cdot Tr[6 h_U h_U^\dagger] - G_\nu) + h_\nu h_E h_E^\dagger + h_E^\dagger h_E h_\nu \qquad (14)$$

with $G_\nu = 2g_1^2 + 6g_2^2$. In order to gain an insight into the effects of new couplings associated with the ν_R in the renormalisation group running we integrate the above equations in the region $M_N \leq Q \leq M_U$. We denote the top and ν_R Yukawas by h_G at the unification scale, while bottom and τ are denoted with h_{b_0}, h_{τ_0} respectively. We get

$$\begin{aligned} h_t(t) &= \gamma_U(t) h_G \xi_t^6 \xi_N & (15) \\ h_N(t) &= \gamma_N(t) h_G \xi_t^3 \xi_N^4 & (16) \\ h_b(t) &= \gamma_D(t) h_{b_0} \xi_t & (17) \\ h_\tau(t) &= \gamma_E(t) h_{\tau_0} \xi_N & (18) \end{aligned}$$

where the functions $\gamma_\alpha(t)$ depend purely on gauge coupling constants and are given by

$$\begin{aligned} \gamma_\alpha(t) &= \exp\left(\frac{1}{16\pi^2} \int_{t_0}^{t} G_\alpha(t)\, dt\right) & (19) \\ &= \prod_{j=1}^{3} \left(\frac{\alpha_{j,0}}{\alpha_j}\right)^{c_\alpha^j / 2b_j} & (20) \end{aligned}$$

$$= \prod_{j=1}^{3}\left(1-\frac{b_{j,0}\alpha_{j,0}(t-t_0)}{2\pi}\right)^{c_\alpha^j/2b_j} , \tag{21}$$

The ξ_i's $(i = t, N, b, \tau)$ are given by the integrals

$$\xi_i = \exp(\frac{1}{16\pi^2}\int_{t_0}^{t}\lambda_i^2 dt) \tag{22}$$

The values of the parameters ξ_i can be determined at any scale by numerically solving the renormalization group equations. As a general remark, we note that the initial condition for ξ_i is $\xi_i(t_U) = 1$, while at any lower scale $Q < M_U$, $\xi_i(Q) < 1$.

3 Heavy neutrino effects : an insight

We start by investigating the $b-\tau$ Yukawa coupling solutions. Thus, in the case of small $tan\beta$, we can relate their solutions at the scale M_N in terms of the initial values, by the following equation

$$h_b(t_N) = \rho\xi_t\frac{\gamma_D}{\gamma_E}h_\tau(t_N) \tag{23}$$

with $\rho = \frac{h_{b_0}}{h_{\tau_0}\xi_N}$. In the case of $b-\tau$ unification at M_U, we have $h_{\tau_0} = h_{b_0}$, while in the absence of the right − handed neutrino $\xi_N \equiv 1$, thus $\rho = 1$ and the m_b mass has the phenomenologically reasonable prediction at low energies given by the approximate one-loop formula

$$m_b = \eta_b\xi_t\frac{\gamma_D}{\gamma_E}m_\tau \tag{24}$$

where η_b is the renormalization group coefficient in the m_t–m_b range. In the presence of ν_R however, if $h_{\tau_0} = h_{b_0}$ at the GUT scale, the parameter ρ is no longer equal to unity since $\xi_N < 1$. In fact the parameter ξ_N becomes smaller for lower M_N scales. Therefore in order to restore the correct m_b/m_τ prediction at low energies we need $\rho = 1$ corresponding to

$$h_{b_0} = h_{\tau_0}\xi_N \tag{25}$$

Hence it is obvious that we need a $\tau-$Yukawa coupling h_{τ_0}, larger than h_{b_0} at M_U to compensate for the factor ξ_N arising from the presence of ν_R.

It is interesting that ξ_N can be given in this case in terms of the values of gauge and Yukawa ratios at M_N only, irrespective of the initial conditions

$$\begin{aligned}\xi_N &= \frac{h_{b_0}}{h_{\tau_0}}\\ &= \frac{h_b\gamma_E}{h_\tau\gamma_D}(\frac{h_t\gamma_N}{h_N\gamma_U})^{-1/3}\end{aligned} \tag{26}$$

On the other hand, the top mass at the scale $t_N = ln(M_N)$ can also be expressed formally as follows

$$m_t(t_N) = h_G\gamma_U(t_N)\xi_t^6(t_N)\xi_N(t_N)\frac{\upsilon}{\sqrt{2}}sin\beta \tag{27}$$

where $\upsilon = 246GeV$ and $tan\beta$ is the ratio of the two Higgs vev's. Again, in the absence of ν_R this reduces to the well known one loop approximate formula which coincides with the above for $\xi_N = 1$. In the present case however, this prediction corresponds effectively to a smaller initial value of the top Yukawa coupling of the order

$$h'_G = h_G \xi_N(t_N) \tag{28}$$

For $h_G > 1$, however, due to the infrared fixed point property of the top – Yukawa solution [17], the m_t – prediction is not going to alter significantly. For the same $tan\beta$, one will get almost the previous top mass prediction, reduced at most by 2%. In contrast, in the small $tan\beta$ scenario where $h_b \ll 1$, one naturally expects that bottom mass will be affected by the presence of ν_R. For $M_N \approx 10^{13}GeV$ for example and $h_G \geq 1$, we can estimate that $\xi(t_N) \approx 0.89$ thus, there is a corresponding $\sim 10\%$ deviation of the $\tau - b$ equality at the GUT scale.

Furthermore, we have seen that in order to recover the correct m_b/m_τ relation at low energies, it is necessary that $h_{b,0}/h_{\tau,0} < 1$ as long as $M_N < M_U$. This can be done in two ways: Either we can keep the same value of the b – Yukawa and increase the τ-Yukawa by a factor ξ_N^{-1}, or decrease the bottom coupling by a factor ξ_N. In the first case, the angle β remains the same and the top mass unaffected. In the second case, in order to retain the same absolute value of the initial parameters for the b, τ masses we need to increase $cos\beta$. This results to a corresponding decrease of the top mass prediction.

We will present a detailed numerical analysis of the above in section 5, where two loop effects from the gauge couplings are taken into account. In the next section we first propose a scheme in which the bottom-tau unification is restored. We will show that this may result in a solution of the solar neutrino deficit and the atmospheric neutrino problem.

4 Restoration of bottom – tau unification

Given the results of section 3, it is natural to ask if Grand Unified models which predict the $b - \tau$ equality at the Unification scale, exclude the experimentally required and cosmologically interesting region for the neutrino masses. To answer this question, we should first recall that the $b - \tau$ – equality at the GUT scale refers to the (33) entries of the corresponding charged lepton and down quark mass matrices. The detailed structure of the mass matrices is not predicted, at least by the Grand Unified Group itself, unless additional structure is imposed. It is possible then to assume $(m_E^0)_{33} = (m_D^0)_{33}$ and a specific structure of the corresponding mass matrices such that after the diagonalisation at the GUT scale, the $(m_E^\delta)_{33}$ and $(m_D^\delta)_{33}$ entries are no-longer equal.

To illustrate this point, let us present here a simple 2×2 example. Assume a diagonal form of m_D^0 at the GUT scale , $m_D^0 = diagonal(cm_0, m_0)$, while the corresponding entries of charged lepton mass matrix have the form

$$m_E^0 = \begin{pmatrix} d & \epsilon \\ \epsilon & 1 \end{pmatrix} m_0 \tag{29}$$

These forms of m_D^0, m_E^0 ensure that at the GUT scale $(m_D^0)_{33} = (m_E^0)_{33}$. However, at the low energies one should diagonalize the renormalised Yukawa matrices to obtain the correct eigenmasses. Equivalently, one can diagonalise the quark and charged lepton Yukawa matrices at the GUT scale and evolve separately the eigenstates and the mixing angles. Since m_D^0 has been chosen diagonal there is no need of diagonalization and the mass eigenstates which are to be identified with the s, b – quark masses at low energies are given by

$$m_s = c\gamma_D m_0, \quad m_b = \gamma_D m_0 \xi_t \tag{30}$$

with $m_0 = h_{b_0} \frac{v}{\sqrt{2}} cos\beta$. To find the charged lepton mass eigenstates we need first to diagonalise m_E^0 at M_{GUT}. We can obtain the following relations between the entries ϵ, d of m_E^0 and the mass eigenstates m_μ^0, m_τ^0 at the GUT scale.

$$d = (\frac{m_\tau^0 - m_\mu^0}{m_0} - 1) \tag{31}$$

$$\epsilon^2 = (\frac{m_\mu^0}{m_0} + 1)(\frac{m_\tau^0}{m_0} - 1) \tag{32}$$

In the presence of right handed neutrino, the evolution of the above $\tau-$ eigenstate down to low energies is that described by (13) with $m_{\tau_0} = h_{\tau_0} \frac{v}{\sqrt{2}} cos\beta$. By simple comparison of the obtained formulae, we conclude that, to obtain the correct m_τ/m_b ratio at m_W while preserving the $b - \tau$ unification at m_{GUT}, the m_E^0 entries should satisfy the following relations

$$\epsilon = \sqrt{\frac{1}{\xi_N} - 1}, \quad d \approx (\tfrac{1}{\xi_N} - 1) = \epsilon^2 \tag{33}$$

The above result deserves some discussion. Firstly we see that it is possible to preserve $b - \tau$ unification by assuming $2 - 3$ generation mixing in the lepton sector, even if the effects of the ν_R states are included. Secondly, this mixing is related to a very simple parameter which depends only on the scale M_N and the initial h_N condition. The range of the coefficient c in the diagonal form of the m_D^0 – matrix, can also be estimated using the experimental values of the quark masses m_s, m_b. An interesting observation is that the usual GUT – relation for the (22) – matrix elements of the charged lepton and down quark mass matrices, i.e., $(m_E)_{22} = -3(m_D)_{22}$, which in our case is satisfied for $c = -d/3$, implies here a relatively heavy strange quark mass $m_s \sim 200$MeV. Smaller m_s values are obtained if $-3c/d < 1$. [2]

[2] An alternative mechanism which restores the correct m_b/m_τ ratio in the presence of ν_R was proposed in [18].

We turn now to a consideration whether the hierarchical structure of the lepton mass matrix corresponding to eq(33) can be obtained by a simple $U(1)$ symmetry [10].

In [10] a viable fermion mass matrix structure was constructed following from a spontaneously broken $U(1)$ gauge symmetry. In this the form of the down mass matrix is

$$\frac{M_d}{m_b} \approx \begin{pmatrix} \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon} & 1 \end{pmatrix} \tag{34}$$

(We have concentrated for simplicity in the case of the two heavier generations which are relevant here.) Note that we have suppressed all Yukawa couplings in writing this mass matrix – only the order of the matrix elements allowed by the broken symmetry are given. These Yukawa couplings are assumed to be of order 1 and the object of the exercise is to demonstrate that the hierarchical structure of the fermion masses may come only from symmetry considerations. Eq(34) is diagonalised by the orthogonal matrix

$$V \approx \begin{pmatrix} 1 & \bar{\epsilon} \\ -\bar{\epsilon} & 1 \end{pmatrix} \tag{35}$$

where $\bar{\epsilon} = 0.23$, in order to fit the down quark masses and mixing angles.

The structure of the lepton mass matrix following from the $U(1)$ symmetry (again for the heavier generations) is

$$\frac{M_l}{m_b} \approx \begin{pmatrix} \bar{\epsilon}^{2|\beta|} & \bar{\epsilon}^{|\beta|} \\ \bar{\epsilon}^{|\beta|} & 1 \end{pmatrix} \tag{36}$$

where $\beta \equiv 1 - b = \frac{a_2 - \alpha_1}{\alpha_2 - \alpha_1}$. and in [10], [11] the cases $\beta = 1/2$ and $\beta = 1$ had been considered. Both possibilities are in very good agreement with the measured masses and mixing angles. The important fact here is that β can be determined by the requirement that $b - \tau$ mass ratio be correctly given when heavy neutrinos, which become massive at an intermediate scale $M_N < M_U$, are present. Allowing for the unknown coefficients of $O(1)$ we see (cf. eqs(36) and (33)) that both $\beta = 1/2$ and $\beta = 1$ are in reasonable agreement with the above expectation[3]. Now the diagonalising matrix is given by

$$V \approx \begin{pmatrix} \sqrt{1-\bar{\epsilon}^2} & \bar{\epsilon} \\ -\bar{\epsilon} & \sqrt{1-\bar{\epsilon}^2} \end{pmatrix} \tag{37}$$

Obviously, there is a large mixing in the $2-3$ lepton sector of the obtained solution which may lead to interesting effects in the rare processes like $\tau \to \mu\gamma$ and neutrino oscillations. In the simplest realisation of this scheme we expect $h_b \approx h_t$ because in the limit ϵ, $\bar{\epsilon} \to 0$ the $U(1)$ quantum numbers of the light Higgs $H_{1,2}$ allow them to couple to the third generation and a $SU(2)_l \otimes SU(2)_R$ symmetry of the couplings ensures equal Yukawa coupling. Thus this model applies only to the large $\tan\beta$ regime. However if

[3] Here we assume the field spontaneously breaking $U(1)$ carries half – integral $U(1)$ charge so we do not have the Z_2 symmetry of [10].

there is an additional heavy state, $H_i,\ \bar{H}_i,\ i=1$ or 2, with the same $U(1)$ quantum number then mixing effects can generate different h_b and h_t couplings allowing for any value of $\tan\beta$.

5 The Effective Light Majorana Mass Matrix

We have seen in section 3, that we can obtain with a $U(1)$ family symmetry a charged lepton mass matrix with the required large mixing in the two heavier generations by choosing the one free parameter, b. The choice $b=1/2$ gives a very good agreement with the charged lepton masses and the bottom-tau relation in the presence of ν_R with mass $M_N \approx 10^{13} GeV$. Our next step is to determine the Dirac and heavy Majorana mass matrices. The general form of the Dirac neutrino mass matrix for arbitrary $\alpha, \beta \equiv 1-b$ is given by [11]

$$M_\nu^D = \left(\begin{array}{ccc} \epsilon^{2|3\alpha+\beta|} & \epsilon^{3|\alpha|} & \epsilon^{|3\alpha+\beta|} \\ \epsilon^{3|\alpha|} & \epsilon^{2|\beta|} & \epsilon^{|\beta|} \\ \epsilon^{|3\alpha+\beta|} & \epsilon^{|\beta|} & 1 \end{array} \right) \tag{38}$$

The Majorana masses for the right – handed neutrinos are generated by terms of the form $\nu_R \nu_R \Sigma$, where Σ is a singlet scalar field –invariant under the $SU(3)\otimes SU(2)_L \otimes U(1)_Y$ gauge group– with charge Q_Σ under the $U(1)_{FD}$ family symmetry. For the various choices of Q_Σ we may then form the possible "textures" for the heavy Majorana mass matrix. For example, when $Q_\Sigma = -2a_1$, that is when the field Σ has the same charge with the Higgs fields, only the $(3,3)$ entry of the mass matrix appears for an exact $U(1)$ symmetry [4] and is of order unity.

However, at a later stage the Abelian symmetry is broken by standard model singlet fields θ and $\bar\theta$ with $U(1)$ charge $\pm 1/2$, which acquire vacuum expectation values along a D – flat direction. At this stage, invariant combinations involving the θ, $\bar\theta$ fields are generated, filling the rest of the entries in the mass matrices in terms of an expansion parameter [10]. Depending on which elements of the Majorana mass matrix we require to appear before the $U(1)$ symmetry breaking, we can make six different choices of the charge Q_Σ which result to the $M_{\nu_R}^{maj.}$ – "textures" shown in Table 1.

For $\alpha = 1$, $\beta = 1/2$, we can obtain the specific forms for Dirac and Majorana textures compatible with the correct fermion mass predictions in the presence of the intermediate neutrino scale. In Table 2 we present the eigenvalues of the heavy Majorana mass matrix for this choice of α and β.

The analysis of the resulting m_ν^{eff} follows the same steps as in ref.[11]. In the matrices

[4] The full anomaly free Abelian group $U(1)$ involves an additional family independent component $U(1)_{FD}$.

$\begin{pmatrix} \bar{\epsilon}^{2\lvert 3\alpha+\beta\rvert} & \bar{\epsilon}^{3\lvert\alpha\rvert} & \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} \\ \bar{\epsilon}^{\lvert 3\alpha\rvert} & \bar{\epsilon}^{2\lvert\beta\rvert} & \bar{\epsilon}^{\lvert\beta\rvert} \\ \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} & \bar{\epsilon}^{\lvert\beta\rvert} & 1 \end{pmatrix}$	$\begin{pmatrix} \bar{\epsilon}^{3\lvert 2\alpha+\beta\rvert} & \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} & \bar{\epsilon}^{\lvert 3\alpha+2\beta\rvert} \\ \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} & \bar{\epsilon}^{\lvert\beta\rvert} & 1 \\ \bar{\epsilon}^{\lvert 3\alpha+2\beta\rvert} & 1 & \bar{\epsilon}^{\lvert\beta\rvert} \end{pmatrix}$
$\begin{pmatrix} \bar{\epsilon}^{2\lvert 3\alpha+2\beta\rvert} & \bar{\epsilon}^{\lvert 3\alpha+2\beta\rvert} & \bar{\epsilon}^{3\lvert\alpha+\beta\rvert} \\ \bar{\epsilon}^{\lvert 3\alpha+2\beta\rvert} & 1 & \bar{\epsilon}^{\lvert\beta\rvert} \\ \bar{\epsilon}^{3\lvert\alpha+\beta\rvert} & \bar{\epsilon}^{\lvert\beta\rvert} & \bar{\epsilon}^{2\lvert\beta\rvert} \end{pmatrix}$	$\begin{pmatrix} \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} & \bar{\epsilon}^{\lvert\beta\rvert} & 1 \\ \bar{\epsilon}^{\lvert\beta\rvert} & \bar{\epsilon}^{3\lvert\alpha+\beta\rvert} & \bar{\epsilon}^{3\lvert\alpha+2\beta\rvert} \\ 1 & \bar{\epsilon}^{3\lvert\alpha+2\beta\rvert} & \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} \end{pmatrix}$
$\begin{pmatrix} 1 & \bar{\epsilon}^{\lvert 3\alpha+2\beta\rvert} & \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} \\ \bar{\epsilon}^{\lvert 3\alpha+2\beta\rvert} & \bar{\epsilon}^{2\lvert 3\alpha+2\beta\rvert} & \bar{\epsilon}^{3\lvert 2\alpha+\beta\rvert} \\ \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} & \bar{\epsilon}^{3\lvert 2\alpha+\beta\rvert} & \bar{\epsilon}^{2\lvert 3\alpha+\beta\rvert} \end{pmatrix}$	$\begin{pmatrix} \bar{\epsilon}^{\lvert 3\alpha+2\beta\rvert} & 1 & \bar{\epsilon}^{\lvert\beta\rvert} \\ 1 & \bar{\epsilon}^{\lvert 3\alpha+2\beta\rvert} & \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} \\ \bar{\epsilon}^{\lvert\beta\rvert} & \bar{\epsilon}^{\lvert 3\alpha+\beta\rvert} & \bar{\epsilon}^{\lvert 3\alpha\rvert} \end{pmatrix}$

Table 1: General forms of heavy Majorana mass matrix textures. The specific textures of the text arise for $\alpha = 1, \beta = 1/2$.

$\begin{pmatrix} e^{10} & & \\ & e^{2} & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} e^{15} & & \\ & -1+e & \\ & & 1+e \end{pmatrix}$
$\begin{pmatrix} e^{16} & & \\ & e^{2} & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} e^{9} & & \\ & -1-e^{2} & \\ & & 1+e^{2} \end{pmatrix}$
$\begin{pmatrix} e^{16} & & \\ & e^{14} & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} e^{6} & & \\ & -1-e^{2} & \\ & & 1+e^{2} \end{pmatrix}$

Table 2: Eigenvalues of Heavy Majorana mass matrix textures, for $\alpha = 1$ and $\beta = 1/2$

$\begin{pmatrix} e^{26} & & \\ & e^{10} & \\ & & 1 \end{pmatrix}$	$\begin{pmatrix} e^{25} & & \\ & e^{9} & \\ & & 1/e \end{pmatrix}$
$\begin{pmatrix} e^{24} & & \\ & e^{8} & \\ & & 1/e^{2} \end{pmatrix}$	$\begin{pmatrix} e^{33} & & \\ & e^{13} & \\ & & 1/e^{7} \end{pmatrix}$
$\begin{pmatrix} e^{40} & & \\ & 1/e^{8} & \\ & & 1/e^{14} \end{pmatrix}$	$\begin{pmatrix} e^{32} & & \\ & e^{6} & \\ & & 1/e^{6} \end{pmatrix}$

Table 3: Eigenvalues of light Majorana mass matrix textures, for $\alpha = 1$ and $\beta = 1/2$

we use

$$e = \bar{\epsilon}^{1/2}, \ \bar{\epsilon} = 0.23 \tag{39}$$

The eigenvalues of m_{eff} are given in Table 3. The order of the matrices in Tables 2 and 3 corresponds to the one of Table 1.

Note the interesting feature that the large mixing in the (2,3) entries of the charged leptons which we introduced in order to restore the $b - \tau$ unification leads to a similar large mixing in the neutrino sector[5]. This mixing is of the correct order of magnitude for a possible solution to the atmospheric neutrino problem. Indeed, the atmospheric neutrino problem may be explained in the case that large mixing and small mass splitting involving the muon neutrino exists [19]. Taking into account the bounds from accelerator and reactor disappearance experiments one finds that for $\nu_e - \nu_\mu$ or $\nu_\tau - \nu_\mu$ oscillations

$$\delta m^2_{\nu_\alpha \nu_\mu} \geq 10^{-2} \text{ eV}^2 \tag{40}$$

$$sin^2 2\theta_{\mu\alpha} \geq 0.51 - 0.6 \tag{41}$$

In [11], such a large mixing was not present due to a residual discrete symmetry assumed for the $b = 1/2$ case. In that case, from the resulting m_ν^{eff} mass matrix we had been able to fit the COBE results and solve the solar neutrino problem (solution A).

In the case of the large mixing discussed here we may also have a similtaneous solution to the solar and the atmospheric neutrino problems (solution B). However, the small mass splitting which is required between the neutrinos that mix in both the solar and atmospheric neutrino oscillations, make it impossible to account for the COBE measurements at the same time. This is a result of working in the minimal scheme with only one Σ field present, which naturally leads to a large splitting between the neutrino masses. However, in the case that we add more singlets Σ in the theory, it is possible to obtain a heavy Majorana mass matrix that leads to a solution of all three problems similtaneously [14].

Going back to the case of a single Σ field, whether we obtain the solution (A) or (B) depends on the predicted mass splitting between the two heavy neutrinos in each of the six choices of the heavy Majorana mass matrix. For a $\nu_\tau \approx 5$ eV and $x_i \equiv e^6$, e^8 and e^{10}, for $i = 1, 2, 3$, we obtain a muon neutrino mass $m_{\nu_\mu} = m_{\nu_\tau} x_i = 0.06, 0.014$ and 0.003 eV respectively. This indicates that our solutions with a total splitting e^{10} naturally lead to a solution of the COBE measurements and the solar neutrino problem. On the other hand, for $m_{\nu_\tau} \approx 0.1$ eV and $x_1 = e^6$, $m_{\nu_\mu} = m_{\nu_\tau} x_1 = 0.0012$ eV, which may be marginally consistent with a solution to the atmospheric and solar neutrino problems (remember that coefficients of order unity have not yet been defined in the solutions). Since there are alternative schemes which lead to an explanation of

[5]The mixing in the (1,2) is of the $\mathcal{O}((\frac{m_e}{m_\mu})^{1/2})$ and negligible in (1,3).

the COBE measurements, other than hot and cold dark matter [6] we believe that the scheme (B) should be considered on equivalent grounds with the scheme (A).

6 Numerical analysis

In this section, we present the results of a numerical analysis of the effects of the heavy neutrinos in the renormalisation group analysis of masses, concentrating on its implications for lepton mass matrices with a large $\mu - \tau$ mixing. We start by giving a brief description of the procedure we are following. We compute numerically the low energy values of gauge and Yukawa couplings, taking into account two – loop effects of the gauge couplings, one loop RGEs for the Yukawas assuming an effective SUSY scale to account for low energy threshold effects.

First, we check the procedure by reproducing the standard results when no right handed neutrino is present in the theory. We start at the unification scale M_U using as inputs M_U itself, the values of the common gauge coupling α_U, the top coupling h_{t_G} and h_{b_0}, h_{τ_0}. In obtaining the low energy values of α_{em}, a_3, and $sin^2\theta_W$, we use the following ranges

$$\alpha_{em}{}^{-1} = 127.9 \pm .1,\ a_3 = .12 \pm .01,\ sin^2\theta_W = .2319 \pm 0.0004 \tag{42}$$

The top mass m_t is obtained in consistency with the correlation[21]

$$sin^2\theta_W(m_Z) = 0.2324 - 10^{-7} \times \left\{(m_t/GeV)^2 - 143^2\right\} \pm 0.0003 \tag{43}$$

We have converted this correlation into a relation between $sin^2\theta_W$ and $tan\beta$, using the relation of m_t^{fxd} and m_t , i.e.

$$m_t = m_t^{fxd} sin\beta \tag{44}$$

We first search for the $tan\beta$'s satisfying the above correlation. Then, this range is further constrained by the requirement $h_{b_0} = h_{\tau_0}$ at M_U. In the present work, we have concentrated in the small $tan\beta$ scenario, i.e. when $h_t \gg h_{b,\tau}$ and we comment for the large $tan\beta$ case later.

At the next stage, we introduce the Dirac neutrino RGE and run all of them together from M_U down to the right handed neutrino scale M_N. We compare the predictions with those of the previous running (i.e. when there is no right–handed neutrino in the theory) and calculate the deviation from the equality of the $\tau - b$ unification for the same inputs at M_U. Below m_N we add the RGE for the effective light neutrino

[6] For example, we have found that domain walls may give structure at medium and large scales if, either they are unstable, or the minima of the potentials of the relevant scalar field appear with different probabilities [20].

Majorana mass matrix. We assume that we are in a diagonal basis, so we can run the three eigenvalues of M_N independently.

Let us start with the low $tan\beta$ regime, assuming an effective SUSY scale $M_S^{eff} \leq 1TeV$. We vary a_U in a range close to a central value $\frac{1}{25}$ and M_U close to 10^{16} GeV. Our first observation is that the introduction of an intermediate scale where the right handed neutrino gets a mass, shifts slightly the range of the parameter space for which unification is possible. For example, assuming $h_{t_G} \approx 3$, i.e., close to its infrared fixed point, and assuming a unification point ranging in $M_G \sim (1.2-2.2)\times 10^{16}GeV$, with $\frac{1}{\alpha_U} \approx (23.81-25.64)$, the effective scale ranges from $M_S^{eff} = (100)GeV$ to $1TeV$. Introducing the right–handed neutrino, we find $M_S^{eff} \geq 110GeV$. Some particular cases with the corresponding low energy predictions are shown in Table 1.

$\frac{M_N}{GeV}$	$\frac{1}{\alpha_U}$	$\frac{M_U}{10^{16}GeV}$	M_S^{eff}	$\frac{1}{\alpha_{em}}$	$sin^2_{\theta_W}$	α_3
10^{16}	23.81	2.18	100	127.9	0.2325	0.121
	23.81	2.41	110	128.96	0.2320	0.123
	24.39	1.97	221	128.09	0.2320	0.120
	25.00	1.46	493	127.98	0.2321	0.118
	25.64	1.08	1212	127.82	0.2319	0.116
10^{11}	23.81	2.18	110	127.83	0.2323	0.122
	23.81	2.41	122	127.90	0.2318	0.124
	24.39	1.97	270	127.89	0.2315	0.122
	25.00	1.46	493	128.05	0.2321	0.118
	25.64	1.08	1212	127.89	0.2320	0.115

We should point out that, the presence of ν_R in the spectrum has little effect in the low energy values of a_{em}, $sin^2\theta_W$, α_3 parameters. Moreover, for the above initial conditions the $sin^2\theta_W - m_t$ correlation, restricts $tan\beta$ very close to unity $tan\beta \leq 2$. Of course, the biggest effects from the ν_R threshold are found in the $b-\tau$ unification.

For values in the perturbative regime of the top Yukawa coupling, h_{t_G}, at M_{GUT} we find it impossible to obtain the correct m_b, m_τ masses starting with $h_b = h_\tau$ at M_{GUT}, even if the neutrino threshold is as high as $M_N = \mathcal{O}(10^{15}GeV)$. For example, using $h_{t_G} \approx 3.2$, (i.e. very close to its non-perturbative regime) and $h_{b_0} \approx h_{\tau_0} \approx .0125$, one can hardly obtain a running mass for the bottom $m_b(m_b) \approx 4.5GeV$ while the upper experimental bound is $m_b(m_b) \leq 4.4GeV$. However, the solution of the solar puzzle needs $M_N \leq 10^{13}GeV$.

Therefore, in the following we do a complete two loop analysis and calculate the exact deviation from the $b-\tau-$ universality for a reasonable range of the scale M_N. In our approach we first require the τ – mass to be 1.749 ± 0.001 at m_Z. Then we search for the correct bottom mass and top mass as well as the required $tan\beta$. We choose the biggest possible coefficient for which we have a solution, which corresponds to a bottom mass $4.4GeV$. The variation of this coefficient as a function of M_N is plotted in Fig *1* ($h_{\tau_0} = 0.012$), for $h_{t_G} = 3.2$ and 2.0 denoted in the plot by stars and crosses respectively. For the rest of the input parameters we take:

$$M_S^{eff} = 544GeV, \quad M_U = 1.46 \times 10^{16} GeV, \quad a_U = \frac{1}{25} \tag{45}$$

We see, in agreement with the qualitative analysis, that for this parameter range and small h_{t_G} it is not possible to obtain solutions for the $b-\tau$ ratio at unification being unity. The larger the Yukawa coupling for the top, the lower the neutrino scale for which we find solutions.

It is useful to compare the mass and other parameter predictions with respect to those obtained without the inclusion of ν_R. As has been pointed out in the previous section, in the presence of ν_R correct predictions for $b-\tau$ masses can be restored either by increasing h_{τ_0} or by shifting h_{b_0} to smaller values at M_U with a simultaneous change in the β – angle. In this latter case, we show in Fig *2* the curves corresponding to the values of $tan\beta$ as a function of M_N, needed to compensate for the decrease of the bottom mass. We find that as M_N decreases there is a large effect on $tan\beta$, which drops for the two different choices for the top Yukawa coupling, from a common value of 1.35 at $M_N \sim 10^{16}$GeV to 1.02 and 1.13 for $h_{t_G} = 3.2$ and 2.0 respectively, at $M_N \sim 10^{11}$GeV. This, combined with the running of the top Yukawa coupling to the fixed point (Fig 3), implies that we expect in this case a decrease in the top mass, as the qualitative description of the previous section has indicated (Fig *4*). The larger the initial value of the top Yukawa coupling and the smaller the initial value of h_τ, the biggest the effect on $tan\beta$ through the running, and the larger the effect on the top mass.

In Fig. *5* we see the effect of M_N on $1/a_{em}$, which increases slightly as M_N drops. At the same time, $sin^2\theta_w$ also increases slowly (Fig *6*), while the strong coupling decreases (Fig *7*). In all cases the effect is much smaller than the errors on these quantities, however it is enough to eliminate some of the solutions that were in the border of the allowed region.

We would like to stress that, in the case where the h_{b_0} is the same while h_{τ_0} is slightly increased to compensate for the reduction caused by ξ_N, there is no need to change the angle β. For this reason there is no significant effect on the top mass, which preserves its value as in the standard case.

Finally, in Fig *8* we plot the light τ–Majorana neutrino mass versus h_{t_G} coupling, for three different values of the heavy Majorana scale $M_N = 10^{12}$, 10^{13} and 10^{14} GeV.

This analysis can be applied also in the case of the large $tan\beta$ regime. However in this case there are important corrections to the bottom mass from one-loop graphs involving susy scalar masses and the μ parameter. These graphs might induce corrections to m_b even of the order of $(30-50)\%$.

In view of the considerable uncertainties involved we have not extended the numerical analysis to cover this case.

7 Conclusions

In this paper we have discussed the implications for low energy physics of right-handed neutrinos with masses at an intermediate scale M_N. For $M_N \approx 10^{12-13} GeV$ (required to give a τ neutrino mass $\mathcal{O}(1eV)$ to serve as a hot dark matter component) a 10% deviation from $b-\tau$ mass equality at the GUT scale is needed to give an acceptable value for the ratio m_b/m_τ as measured in the laboratory. We showed that it is possible to retain the $m_b^0 = m_\tau^0$ GUT prediction of the $(3,3)$ – elements of the corresponding mass matrices provided there is sufficient mixing in the charged lepton mass matrix between the two heavier generations. The scenario we propose can be realised in a simple extension of the standard symmetry of electroweak interactions to include a $U(1)$ family symmetry. Consideration of the implications of this symmetry for neutrino masses shows that the large mixing implied allows for a simultaneous explanation of the atmospheric neutrino problem and the solar neutrino problem. This complements our previous discussion of solutions to the solar neutrino deficit while having a neutrino mass in the range needed to fit the COBE measurements in a hot plus cold dark matter universe. We have also presented detailed numerical solutions of the renormalisation group equations for the case of a heavy right-handed neutrino to support the analytical analysis.

References

[1] G. G. Ross, *Grand Unified Theories*, Benjamin Cummings (1985); S. Ferrara, ed. in 'Supersymmetry', (North Holland, Amsterdam, 1987); G. Costa, J. Ellis, G.L. Fogli, D.V. Nanopoulos and F. Zwirner, Nucl.Phys. **B297** (1988) 244; J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. **B249** (1990)441; Phys. Lett. **B260** (1991) 131; U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. **B260** (1991) 447;P. Langacker and M. Luo, Phys.Rev. **D44** (1991) 817.

[2] for recent works see e.g. G. L. Kane, C. Kolda, L. Roszkowski and J. D. Wells, Phys. Rev. **D49** (1994) 49; M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. **B426** (1994) 269; P. Langacker and N. Polonsky, Phys. Rev. **D49** (1994) 1454 and UPR-0594T (1994) preprint and references therein.

[3] H. Arason, D.J. Castaño, B. Keszthelyi, S. Mikaelian, E.J. Piard, P. Ramond and B.D. Wright, Phys. Rev. Lett. **67** (1991) 2933; B. Anathanathan, G. Lazarides and Q. Shafi, Phys.Rev. **D44** (1991)1613; A. Anderson et al, Phys. Rev. **D 49** (1994)3660.

[4] K. Inoue et al., Prog. Theor. Phys. **68** (1982) 927; L.E. Ibáñez, Nucl.Phys. **B218** (1983) 514; L.E. Ibáñez and C. López, Phys. Lett. **B126** (1983) 54; Nucl.Phys. **B233** (1984) 511; L. Alvarez-Gaume, J. Polchinsky and M. Wise, Nucl.Phys. **B221** (1983) 495; L.E Ibáñez, C. López and C. Muñoz, Nucl. Phys. **B256** (1985) 218.

[5] M.S. Chanowitz, J. Ellis and M.K. Gaillard, Nucl. Phys. B **128** (1977) 506. A. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B **135** (1978) 66.

[6] N. Cabibbo, Phys. Rev. Lett. **10** (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652; S. Weinberg, in " A Festschrift for I.I. Rabi" [Trans. N.Y. Acad. Sci., Ser. II (1977), v. 38], p. 185; F. Wilczek and A. Zee, Phys. Lett. B **70** (1977) 418. T. Maehara and T. Yanagida, Prog. Theor. Phys. **60** (1978) 822 J. Chakrabarti, Phys. Rev. **D20** (1979) 2411 F. Wilczek and A. Zee, Phys. Rev. Lett. **42** (1979) 421; H. Fritsch,Phys. Lett. B **70** (1977) 436; Phys. Lett. B **73** (1978) 317.

[7] J. Harvey, P. Ramond and D. Reiss, Phys. Lett. B **92** (1980) 309; M.E. Machacek and M.T. Vaughn, Phys. Lett. **B103** (1981) 427; C. Wetterich, Nucl. Phys. **B261** (1985) 461; Nucl. Phys. **B279** (1987) 711; J. Bijnens and C. Wetterich, Phys. Lett. **B176** (1986) 431; Nucl. Phys. **B283** (1987) 237; Phys. Lett. **B199** (1987) 525; P. Kaus and S. Meshkov, Mod. Phys. Lett. **A3** (1988) 1251; F.J. Gilman and Y. Nir, Ann. Rev. Nucl. Part. Sci. **40** (1990) 213; C.D.Froggatt and H.B. Nielsen, Origin of symmetries, World Scientific (1991); S. Dimopoulos, L. J. Hall and S. Raby, Phys. Rev. Lett. **68** (1992) 1984; Phys. Rev. D **45** (1992) 4195; H. Arason, D. J. Castaño, P. Ramond and E. J. Piard, Phys. Rev. D **47** (1993) 232; G. F. Giudice, Mod. Phys. Lett. **A7** (1992)2429; G. K. Leontaris and N. D.

Tracas, Phys. Lett.**B 303** (1993)50; K. Babu and Q. Shafi, hep-ph/9503313; C. D. Froggatt, hep-ph/9504323.

[8] P.Ramond, R.G. Roberts and G.G. Ross, Nucl. Phys. **B406** (1993) 19.

[9] S. Dimopoulos *et al*, Phys. Rev. **D47** (1993)R3697; K. Babu and Q. Shafi, Phys.Lett. **B311** (1993) 172; E. Papageorgiu, Orsay preprint, LPTHE-ORSAY, hep-ph/9504208.

[10] L. E. Ibáñez, Phys. Lett. **B 303** (1993) 55; L.E. Ibáñez and G.G. Ross, Phys. Lett. **B 332** (1994)100.

[11] H. Dreiner, G. K. Leontaris, S. Lola, G. G. Ross and C. Scheich, Nucl. Phys.**B346** (1995)461.

[12] P. Binetruy and P. Ramond, Phys. Lett. B 350(1995)49; hep-ph/9412385; Y. Grossman and Y. Nir, hep-ph/9502418.

[13] F. Vissani and A. Yu. Smirnov, Phys. Lett. **B341** (1994) 173; A. Brignole, H. Murayama and R. Rattazzi, Phys. Lett. **B335** (1994) 345.

[14] E. Papageorgiu, Zeit. Phys. **C**64(1995)509; *ibid* **C65**(1995)135.

[15] H. Dreiner, G. K. Leontaris and N. D. Tracas, Mod. Phys. Lett. **A9** (1993) 2099.

[16] P. H. Chankowski and Z. Pluciennik, Phys. Lett. **B316** (1993) 312; K. Babu , C. N. Leung and J. Pantaleone, Phys. Lett. **B319** (1993) 191;

[17] B. Pendleton and G. G. Ross, Phys. Lett. **B98** (1981)291; C. T. Hill, Phys. Rev. **D24** (1981)691.

[18] S. Dimopoulos and A. Pomarol, CERN preprint, hep-ph/9502397.

[19] K. S. Hirata et al., Phys. Lett. **B280** (1992) 164; R. Becker-Szendy et al., Phys. Rev. **D46** (1992) 3720; Y. Fukuda et al., Phys. Lett. **B335** (1994) 237.

[20] S. Lola and G. G. Ross, Nucl. Phys. B 406 (1993) 452; Z. Lalak, S. Lola, B. Ovrut and G. G. Ross, Nucl. Phys. B 434 (1995) 675.

[21] P. Langacker and N. Polonsky, Phys.Rev. **D47** (1993) 4028; M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys.**B426** (1994) 269.

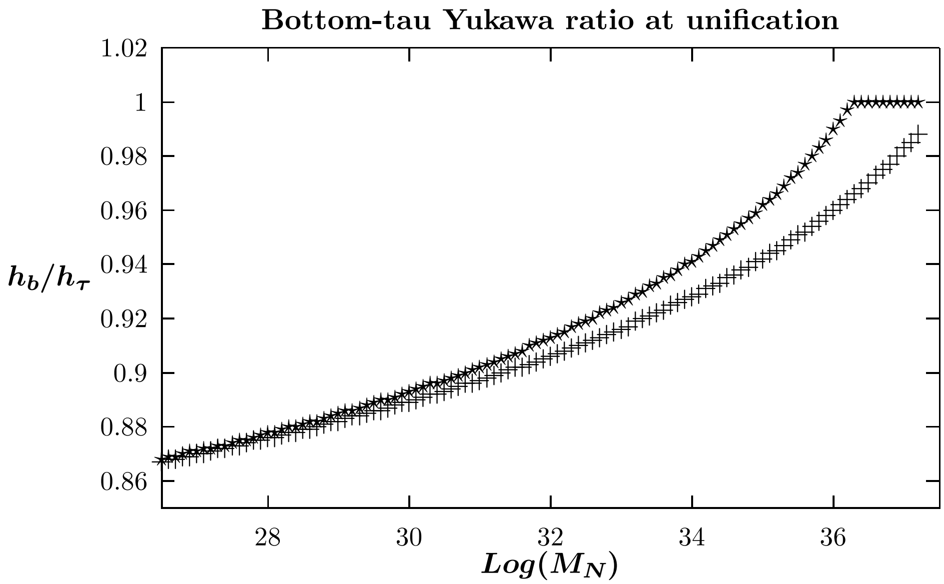

Bottom-tau Yukawa ratio at unification
1.02
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
h_b/h_τ
28
30
32
34
36
$Log(M_N)$

Fig. 1

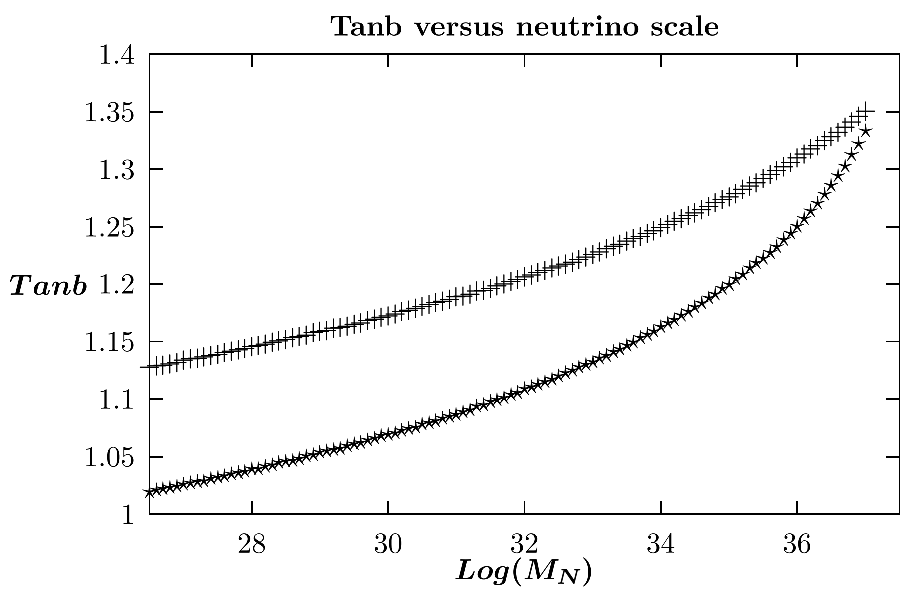
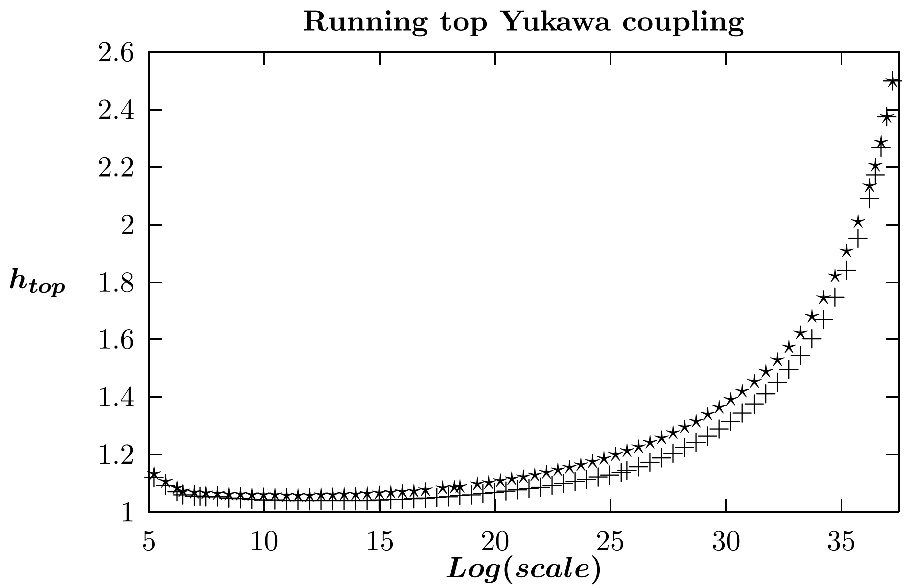
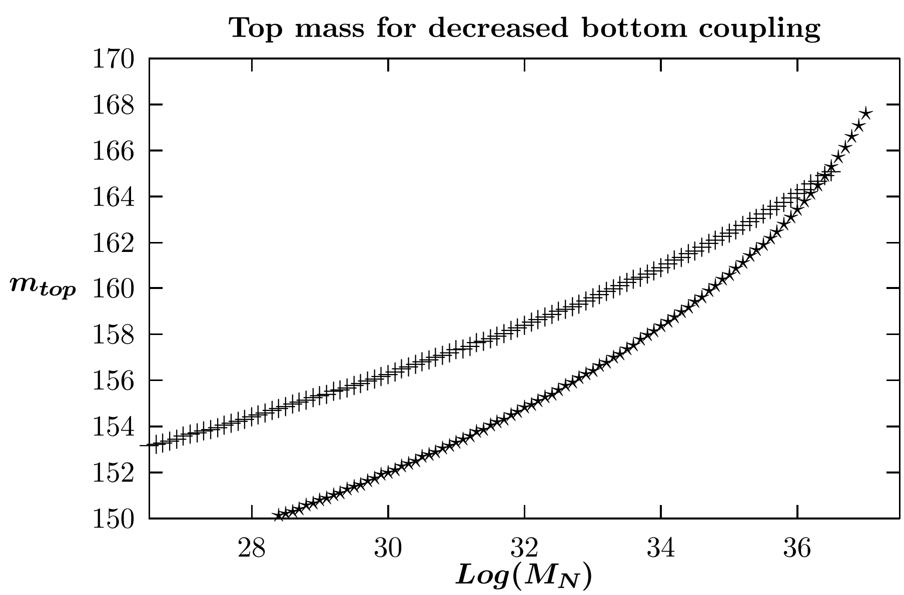

Tanb versus neutrino scale
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
$Tanb$
28
30
32
34
36
$Log(M_N)$

Fig. 2



Running top Yukawa coupling
h_{top}
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
5
10
15
20
25
30
35
$Log(scale)$

Fig. 3



Top mass for decreased bottom coupling
m_{top}
170
168
166
164
162
160
158
156
154
152
150
28
30
32
34
36
$Log(M_N)$

Fig. 4

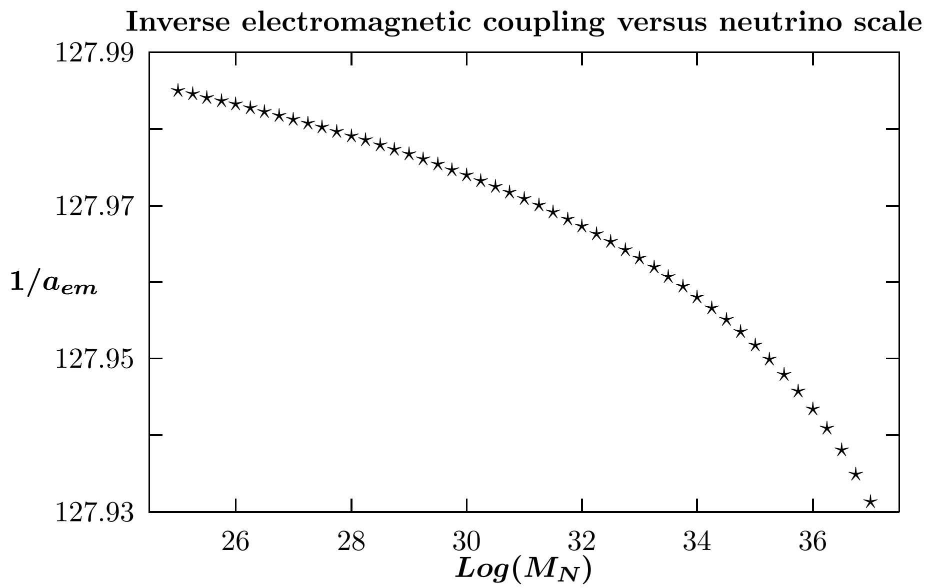

Inverse electromagnetic coupling versus neutrino scale
127.99
127.97
$1/a_{em}$
127.95
127.93
26
28
30
32
34
36
$Log(M_N)$

Fig. 5

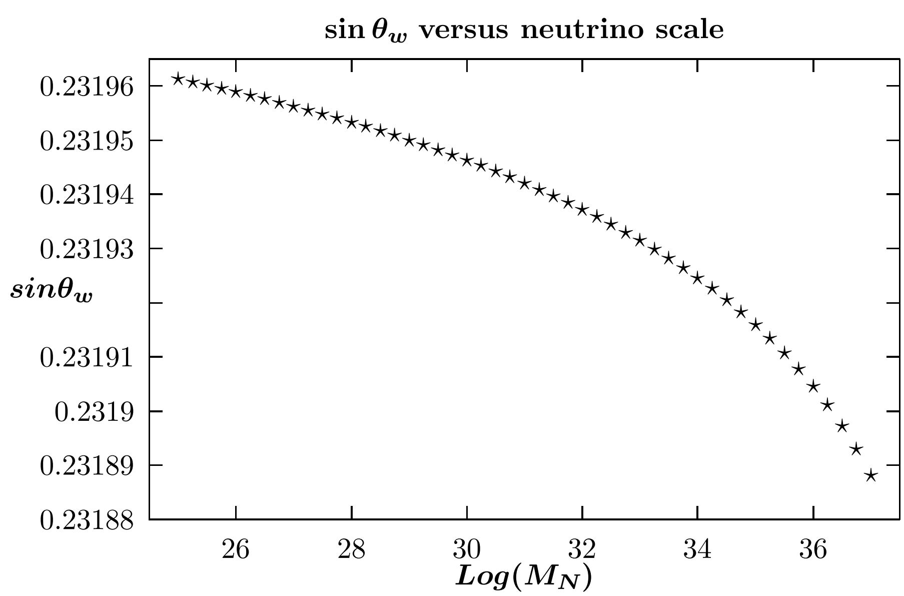

$\sin\theta_w$ versus neutrino scale
0.23196
0.23195
0.23194
0.23193
$sin\theta_w$
0.23191
0.2319
0.23189
0.23188
26
28
30
32
34
36
$Log(M_N)$

Fig. 6

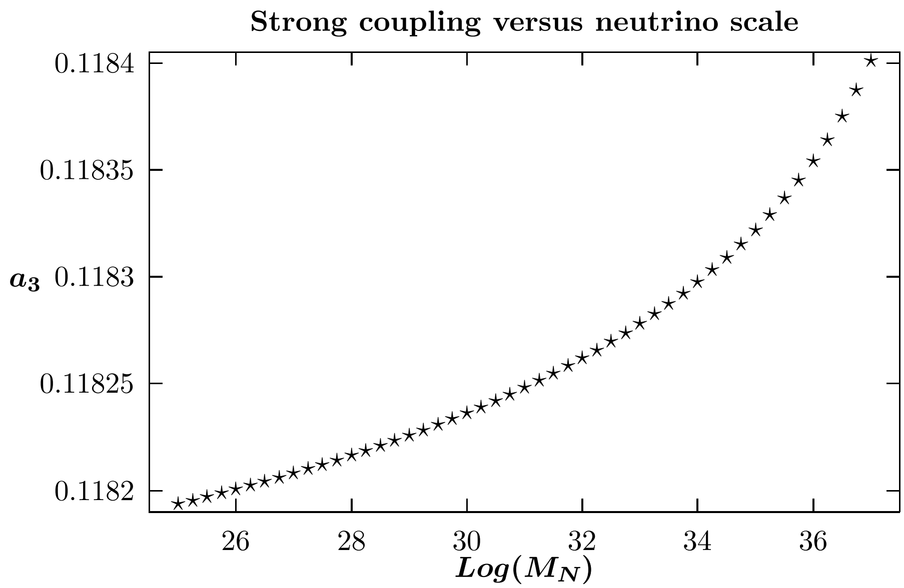

Strong coupling versus neutrino scale
0.1184
0.11835
a_3 0.1183
0.11825
0.1182
26
28
30
32
34
36
$Log(M_N)$

Fig. 7

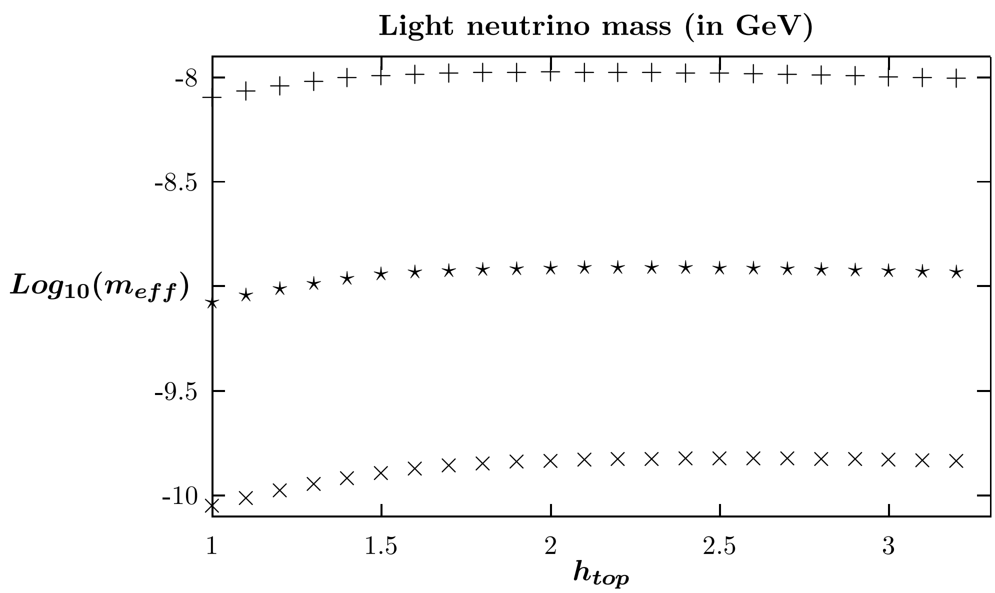

Light neutrino mass (in GeV)
-8
-8.5
$Log_{10}(m_{eff})$
-9.5
-10
1
1.5
2
2.5
3
h_{top}

Fig. 8