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On Gaugino Condensation with Field-Dependent Gauge Couplings

C.P. Burgess,^a J.-P. Derendinger,^b F. Quevedo^b and M. Quirós^c

^a *Physics Department, McGill University*

3600 University St., Montréal, Québec, Canada, H3A 2T8.

^b *Institut de Physique, Université de Neuchâtel*

CH-2000 Neuchâtel, Switzerland.

^c *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

We study in detail gaugino condensation in globally and locally supersymmetric Yang-Mills theories. We focus on models for which gauge-neutral matter couples to the gauge bosons only through nonminimal gauge kinetic terms, for the cases of one and several condensing gauge groups. Using only symmetry arguments, the low-energy expansion, and general properties of supersymmetry, we compute the low energy Wilson action, as well as the (2PI) effective action for the composite *classical* superfield $U \equiv \langle \text{Tr } W^\alpha W_\alpha \rangle$, with W_α the supersymmetric gauge field strength. The 2PI effective action provides a firmer foundation for the approach of Veneziano and Yankielowicz, who treated the composite superfield, U , as a quantum degree of freedom. We show how to rederive the Wilson action by minimizing the 2PI action with respect to U . We determine, in both formulations and for global and local supersymmetry, the effective superpotential, W , the non-perturbative contributions to the low-energy Kähler potential K , and the leading higher supercovariant derivative terms in an expansion in inverse powers of the condensation scale. As an application of our results we include the string moduli dependence of the super- and Kähler potentials for simple orbifold models.

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1. Introduction

Understanding just how supersymmetry breaks is the biggest present uncertainty for formulating realistic supersymmetric field theories. It is also the principal obstacle to obtaining realistic string models. A tantalizingly attractive possibility is for supersymmetry to be broken dynamically [1], such as by the condensation of an asymptotically-free gauge group [2],[3],[4]. A particularly appealing version of this theme uses the condensation of the gauginos in such a gauge theory, and this option has been explored in many settings, including global supersymmetry [2], supergravity [5] and string theory [6],[7],[8],[9],[10]. Unfortunately, although there is agreement on the behaviour of the simplest systems, the non-perturbative nature of this mechanism has led to a certain amount of disarray in the literature, with several different, not necessarily equivalent, approaches having emerged towards finding the low-energy predictions of these theories, well below the condensation scale [3],[5],[7],[11].

In this article we re-examine the gaugino condensation process yet one more time. One naturally ventures onto such well-trodden territory with trepidation, however we do so here with two goals in mind. Our first goal is to clarify the relationship amongst the results of the original workers, and to do so in a way which makes clear which of their conclusions are robust. In particular, we show how the main two alternative analyses of gaugino condensation can be thought of as equivalent approaches that respectively formulate the problem in terms of the ‘Wilson’, and the ‘effective’ actions (see below for the precise definitions) for the theory. Our second goal is to take advantage of the systematic nature of our approach to see how the standard results can be extended. Besides reproducing the usual expression for the superpotential for the low-energy matter fields, we find also a new result: the contributions to the Kähler potential due to the strongly-interacting gauge physics. Our methods can also be used to find further subleading corrections to these in powers of supercovariant derivatives divided by the condensation scale.

The rest of this article starts with a short preamble, followed by the three sections, which contain our three main results. The preamble, which makes up the contents of the next section, consists of a whirlwind review of the effective and Wilson actions for a generic

field theory. This is introductory material on which the rest of our arguments are based. The guts of our results are presented in section 3, where we derive in detail the effective and Wilson actions for gaugino condensation within global supersymmetry. We confine our analysis to a pure super-Yang-Mills theory coupled to gauge-neutral matter through nonminimal gauge kinetic terms. (We do not pursue here the possibility of additional matter supermultiplets within the strongly-coupled gauge sector.) We keep our treatment completely general, by identifying the most general possible action of each kind that is consistent with the symmetries of the problem, and with the low-energy expansion.

The main purpose of section 3's analysis is twofold. The first purpose is to make contact with the earlier literature. Our treatment of the superpotential in the Wilson action duplicates that of Dine *et al.* [7], deriving the standard form using the anomalous R invariance of the underlying theory. The same analysis, when applied to the 2PI effective action for the gaugino bilinear field, reproduces the results of Veneziano and Yankielowicz (VY) [3], although with a few important differences. We therefore provide a more solid basis for their approach, as well as show how it can be generalized to include other effects, such as subleading corrections in Λ_c/M , the influence of more complicated field-dependent couplings, or more complicated gauge groups. We also show how to re-derive the Wilson action from the 2PI effective action, and thereby clarify the relation between these two approaches.

Even though the 2PI effective action reproduces the VY results, there are a number of important differences between their approach and ours which are instructive to identify. First, at a technical level, the treatment using the 2PI formalism permits us to show quite generally that the effective action must be *linear* in the gauge-coupling field, S , something that is an *ad hoc* assumption in earlier discussions [3],[12]. Second, although we appeal to the same symmetries as in Ref. [3], our approaches differ fundamentally in how these symmetries are implemented. In the spirit of anomaly matching, VY write an effective action for which the anomalies in the underlying theory are expressed by the non-invariance of the effective action. In particular, based on a physical motivation, they choose one particular noninvariant form for this purpose, but this choice is not unique. The

uncertainty in the argument therefore enters in quantifying the validity of the assumed form for the low-energy non-invariance. By contrast, our own approach follows the old ‘spurion’ ideas by identifying an exact symmetry of the underlying theory by cancelling the anomaly against the classical transformation of some of the low-energy ‘external’ fields (a similar method was also used in Ref. [12]). The requirement that the low-energy theory be *invariant* with respect to this symmetry then dictates the form for the 2PI superpotential, as well as the low-energy limit of the Kähler potential. We therefore avoid the ambiguity of the earlier approach.

A more conceptual difference between our analysis and that of VY comes from the role that is played by the composite gaugino condensate field, U , in the two approaches. In the VY approach, this field is a *bona-fide* quantum field — much like the η particle in QCD — which must be integrated out to obtain the Wilson action for energies well below the condensation scale (see [12], [13] for a recent discussion). In our 2PI approach, however, U arises purely as a classical field which represents the expectation value of a particular composite field on which we have chosen to focus (see also [14] for a similar treatment). As a result, the Wilson action for the low-energy fields is obtained simply by evaluating the effective action at its stationary point, with no further path integration over strongly-coupled degrees of freedom required at all.

The second, and final, purpose of this section (still section 3) is to proceed beyond the standard results. To do so we use the constraints which follow from the theory’s anomalous R symmetry and scale invariance to determine the first subleading terms in the low-energy action for the light fields. We are led in this way to the nonperturbative contributions to the Kähler potential of the low-energy theory. We identify additional corrections, which arise as an expansion in powers of supercovariant derivatives of the light fields, divided by the condensation scale.

Moving on to section 4, we generalize the previous arguments to supergravity, using the formalism of superconformal fields. The principal new complication is that the scale and R symmetries of the global case are now part of the full, local superconformal symmetry, whose anomalies *must* be cancelled by a Green-Schwarz counterterm. Even though the

formalism is completely different from the global case since, in particular, we have to work with a compensating field (as usual in conformal supergravity), we arrive at similar results, both for the Wilson and the 2PI actions, as in the global case. We verify that the results of global supersymmetry are reobtained from those of supergravity in an inverse Planck mass expansion. Within this framework we repeat the derivation of the Wilson action from the effective action, and so find the Planck-mass-suppressed corrections to the low-energy action.

In section 5, we specialize the supergravity results to derive explicit formulae for the super- and Kähler-potentials of the low-energy theory, as functions of the moduli (T) for (2,2) string vacua. We find in this way explicit results, for single- and multiple-gauge-group condensations, for which all of the low-energy symmetries are manifest, even after including threshold corrections.

Our conclusions are briefly summarized in section 6. We present in an appendix, useful formulae concerning the component expression of the superconformal action, details about fixing the compensating field to obtain standard Poincaré supergravity as well as the derivation of its correspondence with the globally supersymmetric limit.

2. The Effective vs Wilson Actions

The two main tools for our analysis of gaugino condensation are the theory's Wilson and effective actions. Each of these quantities contains a specific kind of information about the nature of the theory. Since much of our discussion hinges on the properties of, and the relationship between, these two kinds of effective actions, we pause here to outline their definitions and differences.

In quantum field theory the generating functional of one-particle-irreducible (1PI) correlation functions — the ‘effective action’ — is the relevant quantity with which to ask questions related to the expectation values of the field operators in the vacuum state (vev 's). This is because the vev of these fields must minimize the effective action. It is therefore the most useful tool for analysing issues of symmetry breaking. A similar, two-particle-irreducible (2PI) generalization is of interest to determine the vev of composite

field bilinears [15], [16].

On the other hand, ‘the’ Wilson action is obtained by integrating out all of those modes whose masses lie above some reference energy scale.¹ This provides a useful way of organizing the relative effects for low-energy observables of physics associated with much higher scales. In particular, for gaugino condensation it encapsulates the effective description of physics well below the condensation scale, after all composites of the strongly interacting fields are integrated out. The Wilson action is a local functional of the ‘light’ quantum fields. In supersymmetric theories it is the Wilson action which has many useful properties, such as the requirement that some interactions must depend only holomorphically on chiral superfields.²

2.1) The Effective Action

We start with the definition of the effective action. Consider a field theory, whose fields we generically call ϕ . In order to determine the vacuum expectation value of an operator $\mathcal{O}[\phi]$, we couple an external current to it and compute the generating functional for its connected Green functions as follows³

$$\exp\{i\mathcal{W}[J]\} = \int \mathcal{D}\phi \exp\left\{i \int d^4x \left[\mathcal{L}[\phi] + J\mathcal{O}[\phi]\right]\right\}. \quad (1)$$

We define the effective action for the operator \mathcal{O} by performing a Legendre transformation on the functional $\mathcal{W}[J]$. That is, we define the average field, u , by:

$$u(J) \equiv \frac{\delta\mathcal{W}}{\delta J} = \langle\mathcal{O}[\phi]\rangle_J, \quad (2)$$

¹ We put the word ‘the’ here in quotes, since there are as many kinds of Wilson actions as there are ways of distinguishing the low from the high energies within a theory.

² For a discussion, see for instance ref. [17].

³ In ref. [15] the composite operators $\mathcal{O}(\phi)$ are taken in such a way that each of the fields ϕ were evaluated at different spacetime points and coupled to a non-local current $J(x_1, x_2)$. This generalization is not relevant for our purposes of finding the vacuum state and so we consider the full composite operator at a single point x .

where the average in this last equation denotes the quantity

$$\langle \mathcal{O}[\phi] \rangle_J \equiv e^{-i\mathcal{W}[J]} \int \mathcal{D}\phi \mathcal{O}[\phi] \exp \left\{ i \int d^4x \left[\mathcal{L}[\phi] + J\mathcal{O}[\phi] \right] \right\}. \quad (3)$$

The Legendre transform of $\mathcal{W}[J]$ is then the functional $\Gamma[u]$, defined by

$$\Gamma[u] \equiv \mathcal{W}[J(u)] - \int d^4x u J, \quad (4)$$

where we imagine $J(u)$ to be the external current that is required to obtain the expectation value $\langle \mathcal{O} \rangle_J = u$, and which may be found, in principle, by inverting eq. (2) for $u(J)$. If $\Gamma[u]$ is known, $J(u)$ can be found by directly differentiating the defining equation for $\Gamma[u]$:

$$\frac{\delta\Gamma[u]}{\delta u} + J = 0. \quad (5)$$

A path-integral expression for $\Gamma[u]$ may be obtained by combining the definitions of eqs. (1) and (4):

$$\exp\{i\Gamma[u]\} = \int \mathcal{D}\phi \exp \left\{ i \int d^4x \left[\mathcal{L}[\phi] + J(\mathcal{O}[\phi] - u) \right] \right\}. \quad (6)$$

It is important to keep in mind that this equation is somewhat self-referential since the current J that appears on the right-hand side is meant to be that function of u given by $J = -(\delta\Gamma/\delta u)$. This does not make the above equation useless for computing $\Gamma[u]$. On the contrary, this choice for the current merely ensures that, in perturbation theory, the appropriate reducible graphs get cancelled, making $\Gamma[u]$ the sum of a set of irreducible graphs. For example, if $\mathcal{O}(\phi) = \phi$, then Γ is simply the sum of all one-particle irreducible (1PI) graphs. If $\mathcal{O}(\phi) = \phi^2$, then it is the sum of two-particle irreducible (2PI) graphs [15], and so on.

For our purposes, the most important property of the functional $\Gamma[u]$, is that its stationary point specifies the *vev* of the original operator, $\mathcal{O}(\phi)$. This can be seen from eqs. (2) and (5) above. Eq. (2) shows that for an arbitrary current, J , u gives the value of

$\langle \mathcal{O}(\phi) \rangle$ in the presence of this current. But we are really interested in this expectation in the absence of all external currents, and by eq. (5) this corresponds to choosing u to satisfy $\delta\Gamma/\delta u = 0$. For time-independent field configurations this argument can be sharpened to show that $\Gamma[u]$ is the minimum expectation value of the system's Hamiltonian given that the expectation of the field $\mathcal{O}(\phi)$ is constrained to equal u [18].

2.2) The Wilson Action

It is often the case that we are only concerned with the properties of very low-energy phenomena in the field theory of interest. For example, we might imagine only being interested in the effective action, $\Gamma[u]$, for fields u which vary only over distances that are much longer than $\ell = 1/\mu$. (For example, we might demand, in a Euclidean-space formulation, that only Fourier components of u for which the four-momentum, p_α , satisfies $p^2 \leq \mu^2$ are of interest.) We can ensure that such a condition is true for $u(x)$ by demanding the same of the external current, $J(x)$, with which we probe the system. In this case, we can partition the integration over ϕ into an integration over modes $\phi_\ell(\mu)$ and $\phi_h(\mu)$, where the modes $\phi_\ell(\mu)$ satisfy $p^2 \leq \mu^2$, and the modes $\phi_h(\mu)$ do not. Typically there are many ways to define this split between high and low energies, and there is in principle a different Wilson action for each. This ‘scheme dependence’ does not concern us in detail here, although we assume in later sections that it is possible to define this split in a manifestly supersymmetric way.

The above construction is particularly simple when $\mathcal{O}(\phi) = \phi$ (for which we use the notation φ in place of u), because in this case the integration over ϕ_h completely decouples from the external current, allowing us to write eq. (6) in the following, suggestive, form:

$$\exp\{i\Gamma[\varphi]\} = \int \mathcal{D}\phi_\ell \exp\left\{i \int d^4x \left[\mathcal{L}_w[\phi_\ell, \mu] + J(\phi_\ell - \varphi)\right]\right\}. \quad (7)$$

Here \mathcal{L}_w is the Lagrangian density for the Wilson action, $I_w[\phi_\ell, \mu] \equiv \int d^4x \mathcal{L}_w(\phi_\ell, \mu)$, which is defined by

$$\exp\{iI_w[\phi_\ell, \mu]\} \equiv \int \mathcal{D}\phi_h \exp\left\{i \int d^4x \mathcal{L}[\phi_\ell, \phi_h]\right\}. \quad (8)$$

By writing the Wilson action in terms of a Lagrangian density, we anticipate one of its most important properties: that it is a local functional of the fields once it is written as a power series expansion in $1/\mu$. The same need not be true, in general, of the effective action, $\Gamma[u]$, since this is defined to include the integration over *all* modes of the underlying fields, including any massless modes from which potentially nonlocal contributions can come.

There is an important situation for which the effective and Wilson actions are very simply related to one another. If we evaluate the effective action at its stationary point: $\Gamma[\bar{u}]$, with $(\delta\Gamma/\delta u)_{u=\bar{u}} = 0$, then the corresponding current vanishes, $J(\bar{u}) = 0$. In this case we have

$$\exp\{i\Gamma[\bar{u}]\} = \int \mathcal{D}\phi_\ell \exp\left\{i \int d^4x \mathcal{L}_w[\phi_\ell, \mu]\right\}. \quad (9)$$

Integrating over the light modes ϕ_ℓ corresponds formally to the $\mu \rightarrow 0$ limit. This expression states that the result obtained by completely integrating out *all* of the modes of the field gives the same answer as would be obtained by evaluating the effective action at its minimum. This equality should be thought of as an equivalence in the dependence of both sides on whatever other parameters (background fields, *etc.*) may characterize the system.

The above connection between the Wilson and effective actions has concrete applications to systems for which an entire sector of the theory is much more massive than the scale μ which defines the Wilson action. In this case it is useful to define the Wilson action in such a way that all fields from this sector are completely integrated out. If A collectively denotes these massive fields, and ϕ denotes the light fields whose masses are lighter than μ , then the Wilson action for ϕ can be written as:

$$\begin{aligned} \exp\{iI_w[\phi_\ell]\} &= \int \mathcal{D}\phi_h \mathcal{D}A \exp\left\{i \int d^4x \mathcal{L}[\phi_\ell, \phi_h, A]\right\} \\ &= \int \mathcal{D}\phi_h \exp\{i\Gamma_A[\phi_\ell, \phi_h]\}, \end{aligned} \quad (10)$$

$$\text{where} \quad \exp\{i\Gamma_A[\phi_\ell, \phi_h]\} = \int \mathcal{D}A \exp\left\{i \int d^4x \mathcal{L}[\phi_\ell, \phi_h, A]\right\}.$$

$\Gamma_A[\phi]$ is the result obtained after the complete integration only over the fields A . The other fields, ϕ_ℓ and ϕ_h , can be considered to be simply background fields for this part of the

integration. Keeping in mind eq. (9), it can therefore equally well be regarded as the result obtained by evaluating the effective action for some operator involving only the fields A , evaluated at its minimum — again with ϕ_ℓ and ϕ_h regarded as fixed background fields. The full Wilson action at scale μ is then simply found by integrating this result over ϕ_h . It is in this vein that we use the effective action in this paper, where the heavy sector is made up of the strongly-coupled gauge theory.

3. Gaugino Condensation in Global Supersymmetry

This section is devoted to applying the above definitions to determine the low-energy Wilson action for a collection of gauge-singlet chiral matter superfields — in components: $\Sigma_I = \{z_I, \psi_I, f_I\}$ with $I = 1, \dots, N$ — well below the condensation scale, Λ_c , for some non-abelian gauge theory, having gauge group \mathcal{G} (*e.g.* the hidden E_8 sector of $(2, 2)$ compactifications). Although we work with chiral superfields, we certainly do not exclude the possibility of there also being gauge interactions [including the standard $SU_c(3) \times SU_L(2) \times U_Y(1)$] in the low-energy theory below Λ_c , so long as these are weakly coupled at scales Λ_c and higher. In general, the Σ_I can carry gauge quantum numbers for these other gauge interactions, whose effects at Λ_c can be computed perturbatively. Since this low-energy gauge dynamics plays no role in what follows, for simplicity we ignore it from here on.

We next write down our starting action, \mathcal{I} . We do not restrict ourselves to renormalizable couplings above the condensation scale, Λ_c , since we regard our initial classical action to be itself a Wilson action obtained by integrating out more physics — perhaps string physics — at still higher scales, $M \gg \Lambda_c$. We suppose that this action is supersymmetric, and in the present section we also assume that M is sufficiently small in comparison to the Planck scale, M_P , to justify a globally supersymmetric treatment. (Relaxing this last assumption is the topic of section 4, below). To leading order in $1/M$ we must neglect all terms which are suppressed by any inverse powers of M . As is typical for supersymmetric theories, since we do *not* know *a priori* whether the scalar fields, z_I , have *vev*'s which are small compared to M , we do *not* assume these to be small and so do *not* expand \mathcal{I} in powers of Σ_I . It is therefore convenient to scale the appropriate power of M out of the

z_I to ensure that they, and so also the Σ_I , are dimensionless. We *do*, however, assume all supersymmetry-breaking *vev*'s to be much smaller than M , and so we *do* neglect all higher supercovariant derivatives of the various fields.

We first consider the case of a strongly-interacting simple gauge sector. The leading terms in $1/M$ then are⁴ :

$$\mathcal{I} = \int d^4x \left[(K_p(\Sigma_I, \Sigma_J^*))_D + \left[\left(W_p(\Sigma_I) + \frac{1}{4} f_p(\Sigma_I) \text{Tr} W^\alpha W_\alpha \right)_F + \text{c.c.} \right] \right], \quad (11)$$

where K_p , W_p and f_p are arbitrary functions of their arguments, and where W_p and f_p both must be holomorphic functions. The subscript ' p ' on these functions could stand for 'prior' (or, for string theory, 'perturbative', since in the usual string scenarios these functions can be computed in string perturbation theory without any recourse to strong coupling). $W_\alpha \equiv -\frac{1}{4} \overline{D\overline{D}} (e^{-V} D_\alpha e^V)$ is the usual left-handed chiral spinor superfield which contains the gauge field strengths, that is constructed from the gauge-potential superfield $V = V^a T_a$, with $V^a = \{\lambda_L^a, A_\mu^a, D^a\}$ in the Wess-Zumino gauge. (The generators, T_a , used in V are normalized by the condition $\text{Tr}(T_a T_b) = \delta_{ab}$.) We assume that the matrix elements of the components of W_α are also much smaller than M , and so, in writing eq. (11), we have also neglected all higher powers of this gauge-field-strength superfield.

Notice that the only coupling between the gauge and matter sectors at this point is through the nonminimal gauge-kinetic term, $(\frac{1}{4} f_p(\Sigma_I) \text{Tr} W^\alpha W_\alpha)_F$, and so it is on this term that we now focus. It is convenient to follow string notation and define the gauge coupling $f_p(\Sigma_I)$ as one of the chiral fields of the problem, which we denote hereafter as $S = \{s, \psi_s, f_s\}$, the others being generically referred to as Σ . Provided that $f_p(\Sigma_I)$ is not a constant, this can always be achieved by performing a suitable holomorphic field redefinition amongst the Σ_I 's. To leading order in $1/M$ the implications of the strong gauge dynamics for the light fields, Σ_I , are therefore completely encapsulated in the integral of this term with respect to the gauge configurations, V :

$$\exp\{i\Gamma_V[S]\} = \int \mathcal{D}V \exp\{iI[S, V]\}, \quad (12)$$

⁴ We use the standard notation $(\dots)_F = \int d^2\theta (\dots)$ and $(\dots)_D = \int d^2\theta d^2\overline{\theta} (\dots)$.

with

$$\begin{aligned}
I[S, V] &\equiv \frac{1}{4} \int d^4x [(S \text{Tr } W^\alpha W_\alpha)_F + \text{c.c.}] \\
&= \int d^4x \left\{ \text{Re } s \left[\frac{1}{2} D^a D^a - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{i}{2} \bar{\lambda}^a \not{D} \lambda^a \right] + \frac{1}{4} \text{Im } s F_{\mu\nu}^a \tilde{F}^{a\mu\nu} + \dots \right\}.
\end{aligned} \tag{13}$$

We omit in this equation the terms involving ψ_s and f_s as well as the coupling of $\text{Im } s$ to the derivative of the chiral gaugino current. Notice that for the special case of a spatially constant, supersymmetric, scalar field configuration, $\langle S \rangle \equiv \{\langle s \rangle, 0, 0\}$, the constant $\langle s \rangle$ is related to the gauge coupling constant, g , and vacuum angle, Θ , according to:

$$\langle s \rangle = \frac{1}{g^2} + \frac{i\Theta}{8\pi^2}. \tag{14}$$

It is the purpose of the remainder of this section to determine the result of the integration over the gauge multiplet, V . In order to do so we make one standard, yet crucial, dynamical assumption. We assume that all of the gauge nonsinglet particles become confined into bound states whose masses are of the order of the gauge theory's condensation scale, Λ_c . It is important in what follows that there be no very light bound states with masses that are sufficiently small to appear in the low-energy theory. This assumption is consistent with the symmetries of the pure supersymmetric Yang-Mills system. The same need not be true once strongly-coupled matter multiplets are also considered into the analysis, and so this assumption precludes their inclusion (unless they happen to be free of accidental symmetries which can keep their bound states systematically light). In more general cases we see no difficulty in extending the analysis along the lines of refs. [19], [3], [2], [12].

We follow two approaches to writing down $\Gamma_V[S]$. In the first approach we imagine $\Gamma_V[S]$ to be the result obtained when successively integrating out lower and lower frequency modes of the gauge sector *à la* Wilson. We then recompute the same result using a 2PI effective action for the gaugino bilinear field, $\text{Tr } W^\alpha W_\alpha$. We reproduce in this second way the results of ref. [3], although we are able to do so based on what we think are much more

firmly-grounded assumptions, in which the nature of the approximations being made are more easily seen.

3.1) The Wilson Approach

The total effective action can be written as

$$\Gamma[S, \Sigma] = \Gamma_p[S, \Sigma] + \Gamma_V[S] \quad (15)$$

where

$$\Gamma_p[S, \Sigma] = \int d^4x [(K_p(S, S^*, \Sigma, \Sigma^*))_D + [(W_p(\Sigma))_F + \text{c.c.}]] \quad (16)$$

We wish to constrain the general form that is permitted for $\Gamma_V[S]$, as defined in eq. (12):

$$\exp\{i\Gamma_V[S]\} = \int \mathcal{D}V \exp\left\{i \int d^4x \left[\left(\frac{1}{4} S \text{Tr} W^\alpha W_\alpha \right)_F + \text{c.c.} \right] \right\}.$$

Imagine that we do so by first considering the functional integration of the gauge multiplet only down to a lower scale, M' , from the initial scale, M , at which the original action is as in eq. (13). The result obtained reproduces $\Gamma_V[S]$ once M' is taken to zero. For M' larger than Λ_c , however, the action obtained in this way depends on the gauge potential, V , in addition to the background field, S .

Denote the result of such an integration by $I_W[S, V]$: it is the Wilson action defined at scale M' . It enjoys the following two very important properties: (i) *locality*, and (ii) *supersymmetry*. Imagine expanding the action, $I_W[S, V] = \int d^4x \mathcal{L}_W$, in powers of the supercovariant derivatives, $D_\alpha S$, of the field, S . On dimensional grounds, each power should appear premultiplied by an inverse power of $M^{1/2}$ or $(M')^{1/2}$. Supersymmetry and locality dictate that the lowest-dimension terms may be written:

$$\begin{aligned} \mathcal{L}_W = & [(W_{np}(S))_F + \text{c.c.}] + (K_{np}(S, S^*))_D + (H_{np}(S, S^*) (DDS) + \text{c.c.})_D \\ & + \left[\frac{1}{4} (F(S) \text{Tr} W^\alpha W_\alpha)_F + \text{c.c.} \right] + \dots \end{aligned} \quad (17)$$

and, according to (15), this leads to the total super and Kähler potentials:

$$\begin{aligned} W(S, \Sigma) &= W_p(\Sigma) + W_{np}(S) \\ K(S, S^*, \Sigma, \Sigma^*) &= K_p(S, S^*, \Sigma, \Sigma^*) + K_{np}(S, S^*) \end{aligned} \tag{18}$$

The subscript ‘ np ’ refers now to ‘non-prior’, which again, for the superpotential will also mean non-perturbative given that all its perturbative corrections vanish. As is well known, the same is not true for the Kähler potential, which does receive perturbative contributions, as we discuss below. The ellipses in this expression represent terms involving four or more supercovariant derivatives. Notice that supercovariant derivatives need not be considered in the F -term above, since any such contribution can be rewritten as a D -term using identities like $(S^m [\overline{DD\bar{S}}]^n)_F = -4(S^m \bar{S} [\overline{DD\bar{S}}]^{n-1})_D$. We now turn to a discussion of the form that can be taken by each of these terms.

- *The Superpotential:*

We start with some standard arguments which constrain the superpotential, $W(S)$. We exploit the classical R symmetry of the lagrangian of eq. (13), under which all fermion fields, λ^a and ψ_s , rotate by a common phase $e^{i\beta}$:

$$\begin{aligned} W_\alpha(x, \theta) &\rightarrow e^{i\beta} W_\alpha(x, e^{-i\beta}\theta) \\ S(x, \theta) &\rightarrow S(x, e^{-i\beta}\theta) \end{aligned} \tag{19}$$

Although this symmetry is anomalous, its anomaly can be cancelled by supplementing the transformation law for S of eq. (19) with:

$$S \rightarrow S - \frac{4ib}{3} \beta, \tag{20}$$

where b is a known constant to be calculated below.

Nontrivial information can now be obtained provided that one is prepared to assume that the above transformations, eqs. (19) and (20), are exact symmetries of the Wilson theory. This is equivalent to assuming the Adler-Bardeen theorem for the R -symmetry anomaly, *i.e.* that the anomaly cancellation is automatically exact once it has been enforced

to one loop. If so, then $W_{np}(S)$ must be an R -invariant superpotential, which is to say that $W_{np} \rightarrow e^{2i\beta} W_{np}$ [7], (see also [9]). This implies that the superpotential must be given by:

$$W_{np}(S) = w \exp \left[-\frac{3S}{2b} \right], \quad (21)$$

where w is a constant⁵ which we may take on dimensional grounds to be proportional to M^3 . This expression for the superpotential, $W_{np}(S)$, is familiar from the literature.

- *The Gauge Kinetic Function:*

A similar result constrains the nonminimal gauge kinetic terms in eq. (17) [17]:

$$\frac{1}{4} [(F(S) \text{Tr} W^\alpha W_\alpha)_F + \text{c.c.}], \quad (22)$$

where the new coefficient, $S' \equiv S(M') \equiv F(S)$, is some function of the original quantity, $S = S(M)$, as well as of the two scales, M and M' . This function can be considered to define a scheme for the running of the gauge coupling due to the formula, eq. (14), which relates $\text{Re } s$ to the gauge coupling, $1/g^2$ defined at scale M . According to this scheme, we identify the (Wilson) gauge coupling, g' , at scale M' , using the same relation: $\text{Re } s' = 1/(g')^2$.

The supersymmetry of eq. (22) requires that the expression $S' = S'(S, M/M')$ must be holomorphic in the original variable S . (As is argued in detail in ref. [17], this holomorphy is justified because we are focussing on the couplings of the Wilson action, rather than, say, the 1PI effective action.) But this condition of holomorphy implies that any dependence of S' on $\text{Re } s = 1/g^2$ is intimately related to its dependence on $\text{Im } s = \Theta/(2\pi^2)$, about which we know a great deal. In particular, because we know that no dependence whatsoever on Θ can be generated within perturbation theory, we know that $\text{Re } S'$ must be completely

⁵ A way to derive the nonrenormalization theorem for the superpotential in perturbation theory [20] is to realize that the symmetry (19), (20) leads necessarily to a superpotential as in (21) but with $w=0$ because the gauge coupling is related to $\langle S \rangle$ by $g^2 = (\text{Re } S)^{-1}$. Perturbation theory is an expansion in negative powers of $\langle \text{Re } S \rangle$ and therefore $\exp[-\frac{3S}{2b}]$ can only have nonperturbative origin.

independent of $\text{Im } S$ in perturbation theory. The same conclusion follows as an exact result if $F(S)$ is required to be invariant under the R -transformation of eq. (20).

These conditions have as their only solution

$$\begin{aligned} S' &= S + B\left(\frac{M}{M'}\right) \\ &= S - b \log\left(\frac{M^2}{M'^2}\right) + O\left(\frac{M'}{M}\right), \end{aligned} \tag{23}$$

where we have used the fact that the *a priori* arbitrary function $B(M/M')$ must arise independently of S , and so can be computed purely at one loop. Eq. (23) shows that the running of the gauge coupling, g , as defined by the evolution of the supersymmetric Wilson action, only gets a contribution at one loop [17]:

$$\frac{1}{g'^2} = \frac{1}{g^2} - b \log\left(\frac{M^2}{M'^2}\right). \tag{24}$$

The constant b that appears here is the same as the constant which appears in eq. (20), and is recognized as the one-loop beta-function coefficient for a supersymmetric theory. It is therefore explicitly given by:

$$b = \frac{1}{16\pi^2} \left[3C(G) - \sum_I T(R_I) \right], \tag{25}$$

where the quantity $T(R)$ is the index for the representation R of the gauge group G , $C(G)$ denotes its quadratic Casimir [$C(G) = T(\text{Adj } G)$] and R_I represents the representation of whatever matter multiplets the theory may contain.

For future purposes, the exact running of g given in eq. (24) permits the definition of a renormalization group (RG)-invariant scale, Λ , defined by:

$$\Lambda = M \exp\left(-\frac{1}{2bg^2}\right) = M \exp\left(-\frac{(s + s^*)}{4b}\right). \tag{26}$$

We expect the condensation scale, Λ_c , to be of the same order of magnitude as Λ .

- *The Kähler Potential:*

We next turn to the general constraints on the form for the Kähler potential, $K_{np}(S, S^*)$. The anomaly-free combination of R and shift symmetries used above implies the exact result that K_{np} cannot depend on $\text{Im } S$: $K_{np} = K_{np}(S + S^*)$. (This same result alternatively follows to all orders in perturbation theory from its independence from Θ in the perturbative approximation.)

The unknown function, $K_{np}(S + S^*)$, must satisfy one further constraint, which expresses the independence of all physical results on the floating scale M . To implement this condition it is important to remember how M drops out of low-energy results. There are two ways in which this happens. Some of the M -dependence simply cancels the explicit M dependence which is already present in K_p . K_p depends on M as well as on the various physical mass parameters (call them m_i) of the high-energy theory. For string theory, for example, the m_i would denote the masses of the various multiplets that are heavier than M and so have been integrated out to obtain the Wilson action at the floating scale M . After the cancellation of the M dependence in K_p , the physical content of the higher-energy physics that K_p encodes is set by the physical masses, m_i . For the present purposes we regard this higher-energy physics to be known, and so we focus here on the contribution of the strongly-coupled gauge sector at lower energies.

The second way for M to cancel out of physical results is for explicit M dependence to cancel the M -dependence that is implicit in the coupling, $S + S^* = 1/g^2$. This M -dependence of the coupling is as is required by the RG equation, as expressed by eq. (24). We therefore write:

$$K_{np} = K_{np\text{-ct}}(S + S^*, M) + K_{np\text{-inv}}(S + S^*, M, M'), \quad (27)$$

where $K_{np\text{-ct}}$ is defined by the condition

$$K_p(S + S^*, M, m_i) + K_{np\text{-ct}}(S + S^*, M) = \text{independent of } M. \quad (28)$$

$K_{np\text{-inv}}$, on the other hand, contains the contribution due to the strongly-coupled gauge physics, and must be RG invariant. That is, on dimensional grounds $K_{np\text{-inv}}$ may always

be written:

$$K_{np\text{-inv}}(S + S^*) = M'^2 \mathcal{G} \left(S + S^*, \frac{M}{M'} \right), \quad (29)$$

where RG invariance implies that the unknown dimensionless function, $\mathcal{G}(x, y)$, satisfies:

$$\frac{d\mathcal{G}}{dM} = 0 = \frac{d(S + S^*)}{dM} \frac{\partial \mathcal{G}}{\partial x} + \frac{1}{M'} \frac{\partial \mathcal{G}}{\partial y}. \quad (30)$$

This last equation has as its general solution

$$K_{np\text{-inv}} = M'^2 k \left(\frac{\Lambda}{M'} \right). \quad (31)$$

Λ here denotes the RG-invariant scale as defined by eq. (26). Thus general considerations determine the running Kähler potential, $K_{np\text{-inv}}$, in terms of a function, $k(z)$, of the single variable, $z = \Lambda/M'$.

More can be inferred by considering various limiting cases. The perturbative limit is given by $M' \gg \Lambda$, or $z \ll 1$. In this limit we expect $K_{np\text{-inv}}$ to admit an expansion in powers of g^2 or g'^2 :

$$\begin{aligned} k(z) &= \sum_n k_n g'^{2n} \\ &= \sum_n k_n \left(\frac{-1}{2b \log z} \right)^n. \end{aligned} \quad (32)$$

The other limit of interest is $z \rightarrow \infty$, since this corresponds to $M' \rightarrow 0$, with Λ fixed. This is the Kähler potential which results when the gauge sector has been integrated out completely. In this case the limiting form follows from the assumption that the spectrum of the gauge sector contains no very light states. As a result the Wilson action, and $K_{np\text{-inv}}$ in particular, should not become singular as $M' \rightarrow 0$. This implies that the asymptotic form for $k(z)$ for large z must be $k \sim k_\infty z^2 + (\text{subleading terms})$, where k_∞ is a (possibly zero) constant. The resulting Kähler potential then is:

$$\begin{aligned} K_{np\text{-inv}}(S, S^*) &= \lim_{M' \rightarrow 0} M'^2 k \left(\frac{\Lambda}{M'} \right) \\ &= k_\infty \Lambda^2 \\ &= k_\infty M^2 \exp \left[- \frac{(S + S^*)}{2b} \right]; \end{aligned} \quad (33)$$

a result which is in any case clear on dimensional grounds.

This last expression gives the non-perturbative part of the Kähler potential that is produced by integrating out the strongly-coupled gauge physics at the condensation scale.

- *Higher-Derivative Terms:*

The unknown function, $H_{np}(S, S^*)$, which premultiplies the leading higher-derivative term can be determined using the same arguments as were used above to fix K_{np} . As before we can separate and discard those M -dependent pieces — *i.e.* H_{np-ct} — which serve to cancel the M -dependence of any higher-derivative terms appearing in the higher-energy theory. We focus the remainder of this section on determining the remaining term, H_{np-inv} . In this case, since the R symmetry rotates the derivatives according to $DDS \rightarrow e^{2i\beta} DDS$, we find:

$$H_{np-inv}(S, S^*) = M' \exp \left[\frac{3(S - S^*)}{4b} \right] h \left(\frac{\Lambda}{M'} \right). \quad (34)$$

$h(z)$ is an unspecified function of the RG-invariant variable $z = \Lambda/M'$, which is itself a function of the R -invariant quantity $S + S^*$ [see eq. (26)].

A nonsingular result as $M' \rightarrow 0$ in this case implies the asymptotic form $h(z) \sim h_\infty z + (\text{subleading terms})$, and the result for H_{np-inv} in this limit therefore is:

$$\begin{aligned} H_{np-inv}(S, S^*) &= h_\infty \Lambda \exp \left[\frac{3(S - S^*)}{4b} \right] \\ &= h_\infty M \exp \left[\frac{1}{2b}(S - 2S^*) \right]. \end{aligned} \quad (35)$$

Notice that, as expected, these higher-derivative terms are suppressed by powers of the condensation scale, $\Lambda_c \sim \Lambda$, rather than by M . This makes it legitimate to work with these higher-derivative corrections while continuing to ignore the higher-derivative terms in the initial action, \mathcal{I} , at scale M . Higher-derivative corrections to the effective lagrangian, such as these, appear not to have been considered before in the literature. Clearly their neglect is justified to the extent that the supersymmetry-breaking vev 's, such as the auxiliary field f_s , are much smaller than the condensation scale.

These results can be combined to obtain the Wilson action for the full low-energy theory — neglecting powers of $1/M$ but not necessarily powers of $1/\Lambda_c$ — by (i) adding the nonperturbative piece just found (for $M' \rightarrow 0$) to $K_p(S, S^*)$ and $W_p(S)$ of the high-energy action, and (ii) integrating the result over the high-frequency part of S . That is, if $K_{\text{tot}} = K_p + K_{np} + O(\frac{1}{M})$ and $W_{\text{tot}} = W_p + W_{np} + O(\frac{1}{M})$, then

$$\exp\left\{iI_W[\Sigma_I, \mu]\right\} = \int \mathcal{D}(\Sigma_I)_h \exp\left\{i \int d^4x \left[(K_{\text{tot}})_D + (W_{\text{tot}})_F\right] + \dots\right\}, \quad (36)$$

where the ellipses denote terms involving higher supercovariant derivatives, such as in eq. (17).

3.2) Using the 2PI Action

To obtain more detailed information as to how the strong gauge dynamics generates the quantity $\Gamma_V[S]$ just found, we next use the 2PI action to compute the expectation value of the gaugino bilinear, $\langle \text{Tr } \lambda^\alpha \lambda_\alpha \rangle$. We wish to show that this approach reproduces our earlier result for $\Gamma_V[S]$. We also wish to make contact with, and improve on, the results of earlier workers, starting with ref. [3].

We start by coupling an external chiral supermultiplet of currents, J , to the chiral scalar superfield, $\text{Tr } W^\alpha W_\alpha$, which contains as its lowest component the gaugino bilinear, $\text{Tr } \lambda^\alpha \lambda_\alpha$:

$$\exp\left\{i\mathcal{W}_{np}[J, S]\right\} = \int \mathcal{D}V \exp\left\{i \int d^4x \frac{1}{4} \left[(S \text{Tr } W^\alpha W_\alpha)_F + (J \text{Tr } W^\alpha W_\alpha)_F + \text{c.c.}\right]\right\}. \quad (37)$$

In terms of the chiral Legendre transformed superfield:⁶

$$U \equiv 4 \frac{\delta \mathcal{W}_{np}}{\delta J} = \langle \text{Tr } W^\alpha W_\alpha \rangle_J, \quad (38)$$

⁶ The definition of the functional derivative with respect to a chiral superfield ($\frac{\delta \Phi(x')}{\delta \Phi(x)} = \overline{DD} \delta(x-x')$) is given, for instance, in ref. [21].

the corresponding 2PI effective action for U becomes:

$$\Gamma_{np}[U, S] \equiv \mathcal{W}_{np}[J, S] - \frac{1}{4} \int d^4x [(U J)_F + \text{c.c.}]. \quad (39)$$

Using now (16), the total effective action can be decomposed as:

$$\Gamma[S, U, \Sigma] = \Gamma_p[S, \Sigma] + \Gamma_{np}[S, U] \quad (40)$$

and, accordingly, the total super and Kähler potentials as:

$$\begin{aligned} W(S, U, \Sigma) &= W_p(\Sigma) + W_{np}(S, U) \\ K(S, S^*, U, U^*, \Sigma, \Sigma^*) &= K_p(S, S^*, \Sigma, \Sigma^*) + K_{np}(S, S^*, U, U^*) \end{aligned} \quad (41)$$

As usual, the variables J and U , satisfy the relation:

$$\frac{\delta}{\delta U} \Gamma_{np}[U, S] + \frac{1}{4} J = 0, \quad (42)$$

and so the physical case of vanishing external current corresponds to choosing U to lie at a stationary point of $\Gamma_{np}[U, S]$.

An important property of $\Gamma_{np}[U, S]$ follows from the fact that the generating functional, $\mathcal{W}_{np}[J, S]$, only depends on its two arguments, J and S , through their sum: $\mathcal{W}_{np}[J, S] = \mathcal{W}_{np}[J + S]$, as may be seen from eq. (37). This turns out to imply the following *exact* property for the 2PI function, $\Gamma_{np}[U, S]$:

$$\frac{\delta}{\delta S} \Gamma_{np}[U, S] = \frac{\delta}{\delta J} \mathcal{W}_{np}[J, S] = \frac{1}{4} U. \quad (43)$$

for all S and U . As a consequence, the S -dependence of $\Gamma_{np}[U, S]$ is completely determined to be

$$\Gamma_{np}[U, S] = \Xi[U] + \frac{1}{4} \int d^4x [(U S)_F + \text{c.c.}]. \quad (44)$$

In principle, since the functional $\Xi[U]$ represents an effective (as opposed to Wilson) action, it need not be a local functional of U . However, it is here that our dynamical

assumption — that the spectrum of the gauge sector involve only bound states of masses equal to or larger than the condensation scale, Λ_c — plays a role. So long as we consider only configurations, U , which vary appreciably over very long distances compared to $1/\Lambda_c$, $\Xi[U]$ can be taken to be a local expansion in powers of the supercovariant derivatives D^α of the fields U ⁷ and S . This is the domain of interest for determining the vacuum expectation value for U . Notice also that since $\Xi[U]$ does not depend on S , the scale against which its higher-derivative dependence must be compared is M rather than Λ_c . This is because no factors of $\text{Re } S$ can arise to convert powers of M into powers of Λ_c . Since we work only to leading order in $1/M$, we therefore neglect all supercovariant derivatives in $\Xi[U]$ in what follows. $\Xi[U]$ can therefore be written

$$\Xi[U] = \int d^4x [(K_{np}(U, U^*))_D + ((F(U))_F + \text{c.c.})]. \quad (45)$$

We next determine the implications of the accidental R symmetry for $\Xi[U]$. In terms of the variables U and S , the symmetry of eqs. (19) and (20) of the path integral becomes

$$U \rightarrow e^{2i\beta}U \quad \text{and} \quad S \rightarrow S - \frac{4ib}{3}\beta, \quad (46)$$

together with the corresponding rotation of the coordinates, θ_L . This symmetry dictates that $K_{np}(U, U^*)$ must be a function only of the invariant combination U^*U , and that $F(U)$ must take the form:

$$F(U) = \frac{b}{6}U \log\left(\frac{\zeta U}{M^3}\right), \quad (47)$$

where ζ is an arbitrary dimensionless constant, which we expect to be $O(1)$. This form for the superpotential was first obtained in ref. [3].

We pause to remark at this point that this symmetry argument leading to the superpotential (47) is actually a statement of anomaly cancellation. To see this notice that the

⁷ See Ref. [14] for a discussion on higher supercovariant derivatives of U .

chiral superfield U satisfies a constraint which follows from the equation

$$\text{Tr } W^\alpha W_\alpha = \overline{DD}\Omega, \quad \text{Tr } \overline{W}_{\dot{\alpha}} \overline{W}^{\dot{\alpha}} = DD\Omega, \quad (48)$$

which relates the chiral superfield $\text{Tr } W^\alpha W_\alpha$ to the Chern-Simons superfield Ω , a real vector superfield. It then follows from the definition of U [see eqs. (37) and (38)] that there exists a real vector superfield V such that

$$U = \overline{DD}V, \quad \overline{U} = DDV, \quad (49)$$

V being defined up to the addition of a linear multiplet [22].⁸ In terms of components, eq. (49) has a unique consequence: the imaginary part of the highest component of U is a total derivative,

$$\text{Im } f_u = \partial_\mu v^\mu. \quad (50)$$

Now, under the symmetry of eq. (46), the term $\frac{1}{4}(US)_F + \text{c.c.}$ appearing in $\Gamma_{np}[U, S]$ transforms into a total derivative: $\frac{2b}{3}\beta(\partial^\mu v_\mu)$, and it is this anomaly of the term $\frac{1}{4}(US)_F$ that is cancelled by the transformation of $\Xi[U]$.

Notice that the entire nonperturbative effective superpotential W_{np} can be combined into the form

$$\begin{aligned} W_{np}(U, S) &= F(U) + \frac{1}{4}U S = \frac{b}{6}U \log \left(\frac{\zeta U e^{3S/2b}}{M^3} \right) \\ &\equiv \frac{b}{6}U \left[\log \left(\frac{U}{\Lambda_c^3} \right) - 1 \right], \end{aligned} \quad (51)$$

where we use this last equality to precisely define the condensation scale: $\Lambda_c(S) = \left(M/\zeta^{\frac{1}{3}} \right) \exp \left[-(S/2b) - \frac{1}{3} \right]$.

More can also be said about the Kähler potential, $K_{np}(U, U^*)$, which appears in eq. (45). *A-priori*, R -invariance and dimensional analysis permit an arbitrary function of the invariant dimensionless combination U^*U/M^6 :

$$K_{np}(U, U^*) = M^2 K_{np} \left(\frac{U^*U}{M^6} \right). \quad (52)$$

⁸ This fact has also been observed in Ref. [23].

The key feature of this last equation is that the absence of a dependence on S implies that the scale over which the undetermined function, K_{np} , can vary appreciably is set by $U \sim M^3$, rather than $U \sim \Lambda_c^3$. Provided the region $U \sim \Lambda_c^3$ is the one that is of interest — as is the case for gaugino condensation — we need only consider the region which satisfies $U^*U \ll M^6$. But the result for K_{np} should not become singular in this limit, once we have performed the subtractions which renormalize the composite operator $\text{Tr } W^\alpha W_\alpha$. Neglecting all inverse powers of M , and keeping in mind the vanishing anomalous dimension of the composite field $\text{Tr } W^\alpha W_\alpha$ [2], [12], the only possible M -independent expression for K_{np} becomes

$$K_{np}(U, U^*) = a(U^*U)^{\frac{1}{3}}, \quad (53)$$

with a a constant.

Eqs. (51) and (53) contain the complete result for $\Gamma_{np}[U, S]$, up to $O(1/M)$ corrections.

3.3) Reproducing $\Gamma_v[S]$ from $\Gamma_{np}[U, S]$

From the general discussion of section 2, we know that $\Gamma_v[S]$ is obtained by minimizing $\Gamma_{np}[U, S]$ with respect to U . Notice that this is an exact relationship, and it is *not* to be regarded as a ‘tree approximation’ to performing a path integral over U . We will first perform the minimization in components and later on in terms of superfields to obtain more compact expressions.

In order to see what is implied by this minimization, it is useful to restrict $\Gamma_{np}[U, S]$ briefly to constant configurations of its scalar components: s, f_s, u and f_u . The Lagrangian density for $\Gamma_{np}[U, S]$ in this case reduces to:

$$\mathcal{L}_{aux} = K_{np,uu^*} f_u f_u^* - \left[f_u (W_{np})_{,u} + f_s (W_{np})_{,s} + \text{c.c.} \right], \quad (54)$$

where K_{np,uu^*} and $(W_{np})_{,u}$ respectively denote $\partial^2 K_{np} / \partial u \partial u^*$ and $\partial W_{np} / \partial u$, *etc.*. We must eliminate u and f_u from this lagrangian using their equations of motion. We can easily solve for f_u , to get

$$f_u^* = K_{np,uu^*}^{-1} (W_{np})_{,u} \equiv f_u^*(s, u, u^*). \quad (55)$$

Substituting this back into the lagrangian then gives

$$\mathcal{L}_{aux} = -K_{np,uu^*}^{-1} |(W_{np})_{,u}|^2 - (f_s(W_{np})_{,s} + \text{c.c.}) \quad (56)$$

This expression must now be extremized with respect to variations of u .

Notice that if we simply neglect the term $f_s(W_{np})_{,s}$, the minimum is found by solving $(W_{np})_{,u} = 0$. This is the procedure which has been commonly followed in the literature.⁹ The rationale for doing so has been that the condition $(W_{np})_{,u} = 0$ ensures that the auxiliary field f_u vanishes, which is the condition that the strongly-coupled sector does not itself break supersymmetry. We see from above that this condition actually cannot be imposed, since, in general, it does not minimize the 2PI action. The best one can do is to look for solutions in the neighbourhood of $f_s = 0$, for which f_s is small, and so minimize eq. (56) perturbatively in powers of f_s . This gives as its solution a minimum, $u = \tilde{u}(s, f_s) = \tilde{u}_0(1 + \tilde{u}_1 + \dots)$, where $(W_{np})_{,u}(u = \tilde{u}_0) = 0$ and so on. The solution so obtained is:

$$\begin{aligned} \tilde{u}_0 &= \Lambda_c^3 = \frac{M^3}{\zeta e} e^{-3s/2b}, \\ \tilde{u}_0 \tilde{u}_1 &= \frac{a}{4b^2} (\tilde{u}_0^* \tilde{u}_0)^{\frac{1}{3}} f_s^*, \quad \text{etc} \dots \end{aligned} \quad (57)$$

This expression for \tilde{u}_0 represents one of the main results of the 2PI approach, giving as it does the leading approximation to $\langle \text{Tr } \lambda^\alpha \lambda_\alpha \rangle$.

We may now return to our original goal, which was to eliminate the entire supermultiplet, U , to obtain $\Gamma_V[S] = \Gamma_{np}[U = \tilde{U}(S), S]$. Given the explicit form for $\Gamma[U, S]$ found earlier, the equation of motion of the superfield U is:

$$\frac{a}{3} U^{-\frac{2}{3}} \overline{DD} (U^*)^{\frac{1}{3}} = \frac{\partial W_{np}}{\partial U}. \quad (58)$$

With our analysis of the scalar potential in mind, we look for solutions to this equation in powers of $D_\alpha U$ about the zeroeth-order solution, for which $(W_{np})_{,u} = 0$. That is, we take

⁹ See, for example, [24], [25], [12] for a recent discussion.

the solution to eq. (58) of the form $U = \tilde{U} \equiv \tilde{U}_0(1 + \tilde{U}_1 + \tilde{U}_2 + \dots)$, where:

$$\begin{aligned}\tilde{U}_0 &= \frac{M^3}{\zeta e} e^{-3S/2b}, \\ \tilde{U}_1 &= \frac{a}{2bM} (\zeta e)^{1/3} X, \\ \tilde{U}_2 &= \frac{a^2}{12b^2 M^2} (\zeta e)^{2/3} \left[\overline{DD}(Y X^*) - \frac{1}{2} X^2 \right],\end{aligned}\tag{59}$$

where $Y \equiv e^{(2S-S^*)/2b}$ and $X = \overline{DD}Y$. The appearance of supersymmetric derivatives in $\tilde{U}_1, \tilde{U}_2, \dots$ indicates that nonzero constant values of these chiral superfields break supersymmetry.

The final expression for $\Gamma_V[S]$ follows from substituting these results into $\Gamma_{np}[U, S]$. The leading (\tilde{U}_0) term gives the superpotential contribution to $\Gamma_V[S]$ as:

$$W_{np}(S, U)|_{\tilde{U}_0(S)} = -\frac{b}{6} \tilde{U}_0(S) = -b \frac{M^3}{6\zeta e} e^{-3S/2b},\tag{60}$$

in agreement with our previous expression, eq. (21), and the leading term in the Kähler potential for $\Gamma_V[S]$:

$$K_{np} = a(\tilde{U}_0^* \tilde{U}_0)^{\frac{1}{3}},\tag{61}$$

which again agrees with our previous result, eq. (33), with $k_\infty = a(M^3/\zeta e)^{\frac{2}{3}}$. Continuing to include \tilde{U}_1 and beyond generates the higher-supercovariant derivative terms in $\Gamma_V[S]$. In particular one can easily check that the function $H_{np}(S, S^*)$ in (35) is given by $H_{np}(S, S^*) = h_\infty M Y^*$, where the field Y is that used in the definition of \tilde{U}_n ($n \geq 1$) in (59).

The above approach should be contrasted with the standard procedure that has often been used in the literature, for which U is either considered as a quantum field to be integrated out, or is eliminated using the supersymmetry-preserving condition $(W_{np})_{,u} = 0$. By recognizing U as a classical field in the 2PI construction, we avoid the conceptual problems of trying to interpret $\Gamma_{np}[U, S]$ as a *bona fide* low-energy (Wilson) effective action. Similarly, although the condition $(W_{np})_{,u} = 0$ suffices to determine the superpotential of $\Gamma_V[S]$, it does not correctly reproduce the Kähler potential, or the higher-derivative

corrections. Since neither of these has been discussed in earlier work, this discrepancy has not yet arisen in practice.

Notice also that it is perfectly consistent not to impose $(W_{np})_{,u} = 0$ and yet still claim that the strongly-coupled sector does not itself break supersymmetry. That is, even though it is true that the deviations from $(W_{np})_{,u} = 0$ permit the auxiliary field, f_u to become nonzero, these deviations only make f_u proportional to f_s , and so they leave the burden of supersymmetry breakdown with the low-energy auxiliary field, f_s . Of course, the actual minimum of the exponential scalar potential corresponding to the superpotential of eq. (60) (or eq. (21)) occurs for $s \rightarrow \infty$, and so $\tilde{u} = 0$. This is the well known runaway weak-coupling, supersymmetric solution, and gives the result which is required from general considerations based on the Witten index [1].

3.4) More Than One Condensate

It is very easy to generalize the above considerations to the case of several independent gauge groups, $G = \prod_n (\times G_n)$, each of which may condense separately. This case has attracted much attention in the string literature because it provides a possible way of generating a potential for the dilaton field which could fix the dilaton at a finite value and break supersymmetry [26], [25]. In this case the classical lagrangian is:

$$\mathcal{L} = \sum_n \left[\frac{1}{4} (f_n S \text{Tr} W_n^\alpha W_{\alpha n})_F + \text{c.c.} \right], \quad (62)$$

where $W_{n\alpha}$ is the field strength multiplet for the n 'th gauge group factor, and f_n are independent constants (or functions of other (moduli) fields besides S).

The direct symmetry argument for this lagrangian is more complicated because the anomaly in all of the independent classical R symmetries — one for each group factor — cannot in general be cancelled by a single shift in $\text{Im} S$. The analogue of the anomaly-free symmetry of the previous sections is, in this case, now only an approximate symmetry of the theory.

We therefore analyze this theory by first considering the more general case where each gauge factor has its own coupling-constant field, S_n , and write:

$$\mathcal{L}' = \sum_n \left[\frac{1}{4} (S_n \text{Tr} W_n^\alpha W_{\alpha n})_F + \text{c.c.} \right], \quad (63)$$

with the limit $S_n \rightarrow f_n S$ taken at the end of the discussion.

We may proceed either by directly writing down the expression for $\Gamma_V[S_n]$ that is permitted by the symmetries, or by constructing the 2PI action and eliminating the condensate fields, U_n . We follow here this second approach, since it contains more information than does the first. The generating functional for this theory is then obtained by coupling an independent current, J_n , to each of the bilinears, $\text{Tr} W_n^\alpha W_{\alpha n}$:

$$\begin{aligned} \exp\{i\mathcal{W}_{np}[J_1, S_1, \dots]\} &= \int \prod_n \mathcal{D}V_n \exp\left\{i \int d^4x \left[\sum_n \frac{1}{4} ((S_n + J_n) \text{Tr} W_n^\alpha W_{\alpha n})_F + \text{c.c.} \right]\right\} \\ &= \exp\left\{ \sum_n i\mathcal{W}_{np}[J_n, S_n] \right\}. \end{aligned} \quad (64)$$

Clearly the total generating functional is simply the sum of the same simple-gauge-group contribution for each factor of the gauge group. As a result the same is true for the effective action: $\Gamma_{np}[U_1, S_1, \dots] = \sum_n \Gamma_{np}[U_n, S_n]$. The results of the previous sections may then be directly used for the expression for each of the factors, $\Gamma_{np}[U_n, S_n]$, giving

$$\begin{aligned} \Gamma_{np}[U_1, \dots, U_N, S] &= \sum_n \int d^4x \left[\left(a_n (U_n^* U_n)^{1/3} \right)_D \right. \\ &\quad \left. + \left[\frac{b_n}{6} \left(U_n \log \left(\frac{\zeta_n U_n e^{3f_n S/2b_n}}{M^3} \right) \right)_F + \text{c.c.} \right] \right]. \end{aligned} \quad (65)$$

The stationary point of this potential for the field \tilde{U}_n needs again to be written as $\tilde{U}_n = \tilde{U}_{n0}(S)(1 + \tilde{U}_{n1}(S) + \dots)$ which we can find perturbatively in the supercovariant derivatives. The first term is therefore

$$\tilde{U}_{n0}(S) = \frac{M^3}{\zeta_n e} \exp\left(-\frac{3f_n S}{2b_n}\right), \quad (66)$$

and so the low-energy superpotential and Kähler potential for S become:

$$\begin{aligned}
W_{np}(S) &= -\frac{1}{6} \sum_n b_n \tilde{U}_{n0}(S) = -\frac{1}{6} \sum_n \left(\frac{M^3}{\zeta_n e} \right) b_n \exp \left[-\frac{3f_n S}{2b_n} \right]. \\
K_{np}(S, S^*) &= \sum_n a_n (\tilde{U}_{n0}^* \tilde{U}_{n0})^{\frac{1}{3}} = \sum_n a_n \left(\frac{M^3}{\zeta_n e} \right)^{\frac{2}{3}} \exp \left[-\frac{f_n S}{2b_n} (S + S^*) \right].
\end{aligned} \tag{67}$$

Notice that even if the superpotential (67) has been previously used [26], to our knowledge this is the first time it actually has been derived.

4. Supergravity

In this section we extend the previous results to the case of local supersymmetry, as is necessary if the scale M should be as large as M_P , such as for string theory. For our purposes, we believe that the most convenient formulation of supergravity coupled to matter takes advantage of the simplicity of the superconformal tensor calculus. Poincaré supergravity is then obtained from superconformal gravity by imposing symmetry-fixing conditions on certain components of a supermultiplet used as a compensator. We use the simplest choice of compensator, a chiral multiplet S_0 with components $\{z_0, \psi_0, f_0\}$. It also provides the most general supergravity–matter couplings [27].

The most general action for chiral matter supermultiplets Σ coupled to supergravity can then be written as [28]:

$$\mathcal{I} = \int d^4x \left\{ -\frac{3}{2} \left(S_0 S_0^* e^{-K_p(\Sigma, \Sigma^*)/3} \right)_D + \left[(S_0^3 W_p(\Sigma))_F + \left(\frac{1}{4} f_{ab}(\Sigma) W^{\alpha a} W_\alpha^b \right)_F + \text{c.c.} \right] \right\}. \tag{68}$$

As usual, the chiral and Weyl weights of the matter chiral multiplets Σ and of the gauge vector multiplet V vanish, while S_0 has unit weights. As in the previous section, we neglect terms with higher powers of $S_0^{-3} W^{\alpha a} W_\alpha^b$, as well as higher derivative terms. For future use, the bosonic part of the component expansion of theory (68) is given in the appendix, following refs. [28] and [29], with a discussion on the gauge-fixing of dilatation symmetry which leads to Poincaré supergravity and introduces Newton’s constant. In

(68), the function K_p is the Kähler potential of the scalar sigma-model if the compensating multiplet is chosen to normalize canonically the Einstein term.

Notice that the action (68) has an automatic symmetry under Kähler transformations:

$$\begin{aligned} K_p &\longrightarrow K_p + \varphi(\Sigma) + \varphi^*(\Sigma^*) \\ W_p &\longrightarrow e^{-\varphi(\Sigma)} W_p. \end{aligned} \tag{69}$$

since any such a transformation can be absorbed by redefining S_0 : $S_0 \longrightarrow e^{\varphi/3} S_0$. Kähler invariance indicates that the action only depends on the invariant functions $\mathcal{G}_p = K_p + \log(W_p W_p^*)$ and f_{ab} . In conformal supergravity, this information exhausts the content of Kähler symmetry, which is not to be confused with ‘active’ symmetries like Weyl transformations.

Two symmetries of the superconformal algebra have a particular importance for us: Weyl and chiral $U(1)$ transformations. These two symmetries which are not included in the super-Poincaré algebra do not commute with (Poincaré) supersymmetry. The chiral $U(1)$ group is at the origin of the R-symmetry of Poincaré theories. Its gauge field A_μ is an auxiliary field of minimal Poincaré supergravity, as is the highest component f_0 of the compensating multiplet. Weyl and chiral transformations with parameters λ (Weyl) and θ (chiral) act on component fields with a factor

$$e^{w_j \lambda + i n_j \theta / 2},$$

w_j and n_j being the Weyl and chiral weights of the component field. For a left-handed chiral multiplet (z, ψ, f) , one finds the following weights:

$$\begin{aligned} z : & \quad w, & \quad n = w & \quad (w : \text{arbitrary}), \\ \psi : & \quad w + \frac{1}{2}, & \quad n - \frac{3}{2}, \\ f : & \quad w + 1, & \quad n - 3. \end{aligned} \tag{70}$$

Since we are considering chiral matter multiplets Σ with $w = n = 0$ and since the chiral multiplet of gauge field strength W^a has $w = n = 3/2$, one deduces that the $U(1)$

transformations of (left-handed) gauginos and chiral fermions are

$$\lambda^a \longrightarrow e^{3i\theta/4}\lambda^a, \quad \psi \longrightarrow e^{-3i\theta/4}\psi.$$

These transformations generate a gauge-chiral $U(1)$ mixed anomaly clearly controlled by the coefficient

$$c = \frac{3}{16\pi^2} \left[T(G) - \sum_I T(R_I) \right], \quad (71)$$

as already discussed in refs. [30], [31] and [12]. It is crucial that these superconformal symmetries must really be quantum symmetries, and so any anomalies in them must be cancelled by an appropriate ‘Green-Schwarz’ counterterm. Such a counterterm can easily be constructed using the chiral compensating multiplet S_0 , with $w = n = 1$ [31], [32], [12]:

$$\Delta\mathcal{I} = -2c \left\{ \int d^4x \left(\frac{1}{4} \text{Tr} W^\alpha W_\alpha \log S_0 \right)_F + \text{c.c.} \right\}. \quad (72)$$

This term is claimed to cancel the anomaly to all orders in perturbation theory [12]. Notice also that the constant c coincides with the coefficient b defined in eq. (25) for the case of no charged matter, that we are considering. This counterterm plays an important role in what follows.

We now apply the symmetry argument of the global-supersymmetry case. This argument must be modified in two ways. First, the R and scale invariances of the global case are now both already contained in the superconformal invariance, so their consequences follow automatically from the expression for the superconformal action. Secondly, due to the presence of the S_0 field, and of the anomaly-cancelling Green-Schwarz term, we need not postulate a non-trivial transformation rule for S to cancel the scale and R anomalies.

4.1) Symmetry Arguments

We start with the Wilson action, for the case of a single condensing gauge group. Combining eqs. (68) and (72), the total effective action can be written as

$$\Gamma[S_0, S, \Sigma] = \Gamma_p[S_0, S, \Sigma] + \Gamma_v[S_0, S] \quad (73)$$

where

$$\Gamma_p = \int d^4x \left\{ -\frac{3}{2} \left(S_0 S_0^* \exp \left(-\frac{K_p(S, S^*, \Sigma, \Sigma^*)}{3} \right) \right)_D + [(S_0^3 W_p(\Sigma))_F + \text{c.c.}] \right\} \quad (74)$$

and Γ_v is given (compare with eq. (12)) by

$$\exp\{i\Gamma_v[S, S_0]\} = \int \mathcal{D}V \exp \left\{ i \int d^4x \frac{1}{4} [((S - 2c \log S_0) \text{Tr} W^\alpha W_\alpha)_F + \text{c.c.}] \right\}. \quad (75)$$

This expression automatically incorporates the cancellation of the local Weyl and R anomalies, and so contains all of the information that was used in the globally-supersymmetric case. Since the result is therefore superconformally invariant, and local, its leading contribution can also be put into the form of eq. (68):

$$\Gamma[S, \Sigma, S_0] = \int d^4x \left\{ -\frac{3}{2} \left(S_0 S_0^* \exp \left[-\frac{K(S, S^*, \Sigma, \Sigma^*)}{3} \right] \right)_D + [(S_0^3 W(S, \Sigma))_F + \text{c.c.}] \right\}. \quad (76)$$

Now comes the main argument for the supergravity case. Inspection of eq. (75) shows that $\Gamma_v[S, S_0]$ depends on its two arguments only through the combination $e^{-S/2c} S_0$. Since eq. (76) completely fixes the result's S_0 -dependence, we can immediately read off the nonperturbative super- and Kähler-potentials:

$$\begin{aligned} W(S, \Sigma) &= W_p(\Sigma) + W_{np}(S), \\ W_{np}(S) &= w \exp \left[-\frac{3S}{2c} \right], \\ \exp \left(-\frac{K(S, S^*, \Sigma, \Sigma^*)}{3} \right) &= \exp \left(-\frac{K_p}{3} \right) - k \exp \left(-\frac{S + S^*}{2c} \right), \end{aligned} \quad (77)$$

with k and w arbitrary constants.

With these terms, the effective action becomes

$$\begin{aligned} \Gamma[S, \Sigma, S_0] &= \int d^4x \left\{ -\frac{3}{2} \left(S_0 S_0^* \left[\exp \left(-\frac{K_p(S, S^*, \Sigma, \Sigma^*)}{3} \right) - k \exp \left(-\frac{S + S^*}{2c} \right) \right] \right)_D \right. \\ &\quad \left. + \left[(S_0^3 (W_p(\Sigma) + w e^{-\frac{3S}{2c}}))_F + \text{c.c.} \right] \right\}. \end{aligned} \quad (78)$$

The extension of these results to the case with several condensing factors in a gauge group is straightforward. The results are a simple sum of the corresponding results for a single condensing gauge group:

$$\begin{aligned} W &= W_p + \sum_n w_n e^{-(3f_n S/2c_n)}, \\ e^{-K/3} &= e^{-K_p/3} - \sum_n k_n e^{-(f_n/2c_n)(S+S*)}. \end{aligned} \quad (79)$$

These equations neglect all subleading terms in powers of supercovariant derivatives.

The neglected higher supercovariant-derivative terms involve the superconformal generalization of the superfield $\overline{DD}S^*$ of global supersymmetry, which uses the ‘kinetic multiplet’ $T(S_0^*S^*)$ [33], [34]. The construction of this multiplet is as follows. Consider a chiral multiplet ϕ with weights $w = n = 1$, and the action

$$\int d^4x [\phi\phi^*]_D = \int d^4x e[2ff^* - 2(D_\mu^c z)(D_c^\mu z^*) + \frac{1}{3}zz^*R] + \dots, \quad (80)$$

omitting fermionic contributions. Define then the kinetic multiplet by the identity

$$\int d^4x [\phi\phi^*]_D = \int d^4x [\phi T(\phi^*)]_F = \int d^4x e[zz_T + fz_T] + \dots, \quad (81)$$

where the chiral $T(\phi^*)$ has as components: (z_T, ψ_T, f_T) . After a partial integration, the comparison of (80) and (81) leads to $z_T = 2f^*$ and $f_T = 2\Box_c z^* = 2\Box z^* + \frac{1}{3}Rz^* + \dots$. The appearance of $T(\phi^*)$ in the F-density formula (81) indicates that the kinetic multiplet has weights $w = n = 2$. In our case, we have to consider $T(S_0^*S^*)$, the kinetic multiplet of $S_0^*S^*$ with $w = -n = 1$, and $\Gamma_V[S, S_0]$ will generically depend on the weight-zero chiral multiplet

$$S_0^{-2} T(S_0^*S^*), \quad (82)$$

or, more generally, on $S_0^{-2}T(S_0^*g(S^*))$, with an arbitrary function g . The factor S_0^{-2} indicates that the components of this multiplet are suppressed in the super-Poincaré theory by one power of the gauge-fixing scale of z_0 , which is the Planck scale.

Therefore, neglected terms involve the dependence of the D-density on the invariant chiral multiplets with zero weight

$$S_0^{-2} e^{S/c} T(S_0^* e^{-S^*/2c}),$$

which are suppressed by an additional power of S_0 .

In order to show the equivalence of the globally supersymmetric limit of (78) with the results obtained in the preceding section, we first need to discuss the gauge-fixing procedure applied to the compensating multiplet which leads to the Poincaré theory and introduces the Planck scale. This will be done at the end of this section.

4.2) The 2PI Effective Action

We may similarly reproduce the 2PI analysis using local supersymmetry. We start with a single gauge-group factor, and write the expression for the generating functional for correlations of gaugino bilinears:

$$\begin{aligned} \exp\{i\mathcal{W}_{np}[J, S, S_0]\} = \int \mathcal{D}V \exp\left\{i \int d^4x \frac{1}{4} [(S - 2c \log S_0) \text{Tr} W^\alpha W_\alpha]_F \right. \\ \left. + (J \text{Tr} W^\alpha W_\alpha)_F + \text{c.c.} \right\}. \end{aligned} \quad (83)$$

This expression includes the anomaly-cancelling term (72). Writing the Legendre transform variable as $\widehat{U} = \langle \text{Tr} W^\alpha W_\alpha \rangle_J$ — where the ‘caret’ is introduced for later notational convenience — we may construct the 2PI effective action, $\Gamma_{np}[\widehat{U}, S, S_0]$. Since \mathcal{W}_{np} depends on its three arguments only through the one combination $J + S - 2c \log S_0$, it follows that

$$\frac{\delta \Gamma_{np}[\widehat{U}, S, S_0]}{\delta(S - 2c \log S_0)} = \frac{1}{4} \widehat{U},$$

and so the 2PI action can be written as

$$\Gamma_{np}[\widehat{U}, S, S_0] = \Xi[\widehat{U}] + \frac{1}{4} \int d^4x \left[\left(\widehat{U} (S - 2c \log S_0) \right)_F + \text{c.c.} \right]. \quad (84)$$

Since the second term has anomalous scale and chiral transformations, these must be cancelled by $\Xi[\widehat{U}]$, which must therefore include an anomaly-cancelling term constructed using only the chiral multiplet \widehat{U} , with weights $w = n = 3$. One can then write

$$\begin{aligned} \Xi[\widehat{U}] = & -\frac{3}{2} \int d^4x \left(-a(\widehat{U}\widehat{U}^*)^{1/3} \right)_D + \zeta' \int d^4x \left[\left(\widehat{U} \right)_F + \text{c.c.} \right] \\ & + \frac{1}{4} \int d^4x \left[\left(\frac{2c}{3} \widehat{U} \log \widehat{U} \right)_F + \text{c.c.} \right], \end{aligned} \quad (85)$$

with constants ζ' and a . The first two terms are the unique invariant D and F densities one can write with \widehat{U} only, and the last term cancels the anomaly. In other words,

$$\begin{aligned} \Gamma_{np}[\widehat{U}, S, S_0] = & -\frac{3}{2} \int d^4x \left(-a(\widehat{U}\widehat{U}^*)^{1/3} \right)_D \\ & + \frac{1}{4} \int d^4x \left[\left(\widehat{U} \left\{ S + \frac{2c}{3} \log(\zeta \widehat{U} S_0^{-3}) \right\} \right)_F + \text{c.c.} \right], \end{aligned} \quad (86)$$

the constant ζ replacing ζ' . To derive the Poincaré theory and obtain its Kähler potential, it is useful and convenient to work with zero-weight chiral matter. We then define a new chiral multiplet, $U \equiv \widehat{U} S_0^{-3}$, with $w = n = 0$, and rewrite

$$\begin{aligned} \Gamma_{np}[U, S, S_0] = & -\frac{3}{2} \int d^4x \left(-aS_0 S_0^* (UU^*)^{1/3} \right)_D \\ & + \frac{1}{4} \int d^4x \left[\left(S_0^3 U \left\{ S + \frac{2c}{3} \log(\zeta U) \right\} \right)_F + \text{c.c.} \right]. \end{aligned} \quad (87)$$

Comparing with eq. (68) and keeping in mind that $\Gamma[U, S, S_0] = \Gamma_p + \Gamma_{np}$, we can read the total Kähler¹⁰ and superpotentials,

$$\begin{aligned} \exp \left(-\frac{K(U, U^*, S, S^*, \Sigma, \Sigma^*)}{3} \right) &= \exp \left(-\frac{K_p}{3} \right) - a(U^*U)^{1/3}, \\ W(S, U, \Sigma) &= W_p + W_{np}; \quad W_{np} = \frac{1}{4} U \left[S + \frac{2c}{3} \log(\zeta U) \right]. \end{aligned} \quad (88)$$

¹⁰ Notice that different expressions for the U -field Kähler potential have been used in the literature for the supergravity case, without any real derivation. Here we have proved the uniqueness of expression (85), which agrees with the one given in ref. [35].

The general 2PI effective action for the case of several condensates is similarly found to be

$$\Gamma[U_n, S_0, S, \Sigma] = \int d^4x \left\{ \left(S_0 S_0^* \left(\exp \left(-\frac{K_p(S, S^*, \Sigma, \Sigma^*)}{3} \right) - \sum_n a_n (U_n U_n^*)^{1/3} \right) \right)_D + \left[\left(S_0^3 \left(W_p + \frac{1}{4} \sum_n \frac{2c_n}{3} U_n \log \left(\zeta_n U_n e^{3f_n S/2c_n} \right) \right) \right)_F + \text{c.c.} \right] \right\}, \quad (89)$$

neglecting terms of higher order in S_0^{-1} which involve in particular the kinetic multiplet.

4.3) *Eliminating the Field U*

We may now proceed to eliminate U from the 2PI action, in order to retrieve our previous result for $\Gamma_v[S, S_0]$. To do so, we must solve the equations of motion for the components of the *classical* multiplet U , and substitute the result back into $\Gamma_{np}[U, S, S_0]$.

In the supergravity case, we prefer to begin with a treatment in terms of components, by using the scalar part of the 2PI action. The component expansion of the bosonic part of the 2PI effective lagrangian can be derived from the general expressions given in the appendix, or from refs. [28], [29] and [36], among others. Since we are interested in eliminating the multiplet U , it suffices to focus on the nonperturbative part of the action, in which U appears. With eq. (87), the terms which depend on f_0 , f_u or u are given by

$$e^{-1} \mathcal{L}_{aux} = \frac{a}{3} (z_0 z_0^*) (uu^*)^{-2/3} f_u f_u^* - 3\Phi f_0 f_0^* + a (uu^*)^{-2/3} [z_0 u^* f_0^* f_u + z_0^* u f_0 f_u^*] + [z_0^3 f_u (W_{np})_{,u} + z_0^3 f_s (W_{np})_{,s} + 3z_0^2 W f_0] + \text{c.c.}, \quad (90)$$

where $\Phi = e^{-K_p/3} - a (uu^*)^{1/3}$, the superpotential is $W = W_p + W_{np}(s, u)$, and

$$(W_{np})_{,u} = \frac{\partial}{\partial u} W_{np}(u, s) = \frac{1}{4} \left[s + \frac{2c}{3} \{1 + \log(\zeta u)\} \right], \quad (91)$$

$$(W_{np})_{,s} = \frac{\partial}{\partial s} W_{np}(u, s) = \frac{1}{4} u.$$

Extremizing with respect to f_u , we obtain

$$z_0^* f_u^* = -\frac{3}{a} (uu^*)^{2/3} z_0^2 (W_{np})_{,u} - 3u^* f_0^*, \quad (92)$$

which when put back into eq. (90) gives

$$\begin{aligned} \mathcal{L}_{aux} = & -\frac{3}{a} \left| z_0^2 u^{2/3} (W_{np})_{,u} \right|^2 + \frac{1}{4} z_0^2 u (z_0 f_s - 2c f_0) + \text{c.c.} \\ & - 3e^{-K_p/3} f_0 f_0^*. \end{aligned} \quad (93)$$

The last term does not depend on u . As in the global case, one cannot solve for the scalar field u (even for constant configurations) without first eliminating the auxiliary fields f_s and f_0 , which depend on K_p and W_p . We then proceed as in the global case: we solve for $u = \tilde{u}(s, f_s, z_0, f_0)$ as a power series in the auxiliary fields, with the result this time turning out to be a series in the combination $\xi \equiv z_0 f_s - 2c f_0$. The first three terms in the expansion, $\tilde{u} = \tilde{u}_0 (1 + \sum_{k=1}^{\infty} \tilde{u}_k)$, which are the local version of eq. (59), are

$$\begin{aligned} \tilde{u}_0 &= \frac{1}{\zeta} e^{-3s/2c} = -\frac{3}{2c} W_{np}(s), \\ z_0^2 \tilde{u}_0 \tilde{u}_1 &= -\frac{3a}{4c^2} (\tilde{u}_0^* \tilde{u}_0)^{1/3} \xi^* \\ (\tilde{u}_0^* \tilde{u}_0^2) \tilde{u}_2 &= -\frac{1}{6} \tilde{u}_0 \tilde{u}_1^* \tilde{u}_1^* - \frac{1}{3} \tilde{u}_0^* \tilde{u}_1 \tilde{u}_1^*, \end{aligned} \quad (94)$$

where $W_{np}(s)$ is defined in eq. (77). The first term in the expansion, \tilde{u}_0 , corresponds to $\xi = 0$ (i.e. it corresponds to minimizing the potential to $\mathcal{O}(\xi^0)$), and is the solution to $(W_{np})_{,u} = 0$. The second term, \tilde{u}_1 , corresponds to minimizing the potential to linear order in ξ , and so on. Keeping now \tilde{u}_0 and \tilde{u}_1 from eq. (94) we obtain in \mathcal{L}_{aux} all terms quadratic in the auxiliary fields ξ . This reproduces the Wilson action up to higher-derivative terms. Notice, however, that in this case the leading condition, $(W_{np})_{,u} = 0$, is *not* equivalent to the statement that the strongly-coupled sector does not break supersymmetry, which was the standard argument used in Refs. [24],[25],[12], see also Ref. [13] for a recent discussion. This is because for local supersymmetry, it is the Kähler derivative, $D_u W_{np} \equiv (W_{np})_{,u} + K_{,u} W_{np} = W_{np} \mathcal{G}_u$, rather than $(W_{np})_{,u}$, that is the order parameter for supersymmetry breaking. Nevertheless, it is the solution to $(W_{np})_{,u} = 0$ that provides the leading stationary point for the 2PI action, and which also reproduces the Wilson action. This emphasizes the fallacy of using supersymmetry preservation of the strongly-coupled sector as the argument for determining how to eliminate U .

To eliminate U at the supermultiplet level, use the kinetic multiplet to rewrite

$$\Gamma_{np}[U, S, S_0] = \frac{3}{2}a \int d^4x \left(S_0 U^{1/3} T(S_0^* U^{*1/3}) \right)_F + \left[\int d^4x \left(S_0^3 W_{np}(S, U) \right)_F + \text{c.c.} \right].$$

The equation determining U is then

$$\frac{a}{2} S_0^{-2} U^{-2/3} T(S_0^* U^{*1/3}) = -\frac{\partial}{\partial U} W_{np}(S, U) = -\frac{1}{4} \left[S + \frac{2c}{3} \{1 + \log(\zeta U)\} \right], \quad (95)$$

an equation to be interpreted with the superconformal tensor calculus, and applied to all components of the chiral multiplets appearing in it. This equation of motion is the local, superconformal generalization of eq. (58) derived in the global case. Since the lowest component of $T(S_0^* U^{*1/3})$ is $2(u^{1/3} f_0 + \frac{1}{3} z_0 u^{-2/3} f_u)^*$, the lowest component of this equation of motion is again eq. (92), the equation for the auxiliary field f_u . As in the global case, one could iteratively derive a solution to eq. (95) of the form $U = \tilde{U} = \tilde{U}_0(1 + \tilde{U}_1 + \tilde{U}_2 + \dots)$, with $\frac{\partial}{\partial U} W_{np}|_{U=\tilde{U}_0} = 0$, or

$$\tilde{U}_0 = \frac{1}{\zeta} e^{-3S/2c}, \quad (96)$$

as in the first eq. (59), which is the supermultiplet extension of the first eq. (94). It is however important to keep in mind that equation of motion (95) is derived at the superconformal level. We are interested in the super-Poincaré theory in which the compensator S_0 is not an independent propagating multiplet.

4.4) The Poincaré theory and its global limit

One of the gauge-fixing conditions imposed to reduce superconformal symmetry down to super-Poincaré is applied on the scalar component z_0 of the compensating chiral multiplet S_0 . The microscopic theory with action Γ_p contains Einstein terms of the form

$$-\frac{1}{2} \left(z_0 z_0^* e^{-K_p/3} \right) eR,$$

and a natural choice is

$$z_0 z_0^* = \frac{1}{\kappa^2} e^{K_p/3}, \quad (97)$$

(where κ is defined in eq. (A8)) leading to a canonically normalized gravitational lagrangian, with a field-independent Newton's constant. With this choice, obviously, the z_0 contributions in the 2PI effective action $\Gamma_p + \Gamma_{np}$ do not depend on u . The same holds for the fermionic component ψ_0 of S_0 , which is constrained by a gauge-fixing condition for special supersymmetry. The analysis of the elimination of the components of U given in the preceding paragraph applies then to both Poincaré and conformal supergravities.

Using the compensator fixing (97) in the 2PI effective action has a drawback: nonperturbative contributions in Γ_{np} also include gravitational contributions so that the complete Einstein term, which is

$$-\frac{1}{2\kappa^2} \left(1 - a(uu^*)^{1/3} e^{K_p/3}\right) eR, \quad (98)$$

is not canonical. As explained in the appendix, a by-product of non-canonical Einstein terms is the fact that scalar fields are not in a Kähler basis. Scalar kinetic terms have an additional contribution of the form

$$\frac{3}{4}\kappa^{-2} \left(1 - a(uu^*)^{1/3} e^{K_p/3}\right) \left(\partial_\mu \log [1 - a(uu^*)^{1/3} e^{K_p/3}]\right)^2. \quad (99)$$

One can return to the Kähler basis by performing a rescaling of the vierbein of the form [see the appendix]:

$$e_{m\mu} \quad \longrightarrow \quad [1 - a(uu^*)^{1/3} e^{K_p/3}] e_{m\mu},$$

which also redefines the field-dependent Newton constant present in expression (98). The resulting theory can be directly obtained by choosing instead of (97) the compensator in such a way that the Einstein term in the 2PI effective action is canonical. In the superconformal 2PI action, this term is

$$-\frac{1}{2}z_0 z_0^* \left[e^{-K_p/3} - a(uu^*)^{1/3}\right] eR.$$

Taking then

$$z_0 = z_0^* = \frac{1}{\kappa} \left[e^{-K_p/3} - a(uu^*)^{1/3}\right]^{-1/2} \quad (100)$$

leads to a canonical gravity lagrangian. The ‘effective’ Kähler potential is given by

$$K = -3 \log \left[e^{-K_p/3} - a(uu^*)^{1/3} \right] = K_p - 3 \log \left[1 - a(uu^*)^{1/3} e^{K_p/3} \right]. \quad (101)$$

To discuss the global supersymmetry limit of the Poincaré supergravity defined by the Kähler potential (101) and the superpotential $W = W_p + W_{np}$, the first step is to introduce the physical dimensions of the scalar fields in the theory. The appropriate substitutions are

$$\begin{aligned} K, K_p &\longrightarrow \kappa^2 K, \kappa^2 K_p \\ W &\longrightarrow \kappa^3 W, \\ u &\longrightarrow \kappa^3 u. \end{aligned}$$

It is at this point useful to reintroduce the physical dimension of the scalar field u , which describes the gaugino bilinear condensate. With these rescalings, eq. (101) becomes

$$\begin{aligned} \kappa^2 K &= \kappa^2 K_p - 3 \log \left[1 - \kappa^2 a(uu^*)^{1/3} e^{\kappa^2 K_p/3} \right] \\ &= \kappa^2 \left[K_p + 3a(uu^*)^{1/3} \right] + \mathcal{O}(\kappa^4), \end{aligned}$$

in the flat limit $\kappa \rightarrow 0$. This result indicates that the global supersymmetry limit of the effective 2PI Poincaré supergravity has Kähler potential

$$K_{flat} = K_p + 3a(uu^*)^{1/3},$$

as already demonstrated in the previous section.

5. Moduli couplings and duality anomalies

The main target of application for the above expressions is to the low-energy limit of (2,2) compactifications of the heterotic string. For this particular case, there are typically many supermultiplets, T_A , whose scalar components parameterize the moduli space of the string vacuum being considered, in addition to the model-independent dilaton supermultiplet, S . An important feature of these compactifications are the target-space symmetries

which they exhibit. For many compactifications the fields T_A transform nontrivially under a target space duality symmetry group \mathcal{G} . This transformation is a symmetry in the sense that it changes the Kähler potential and superpotential, but it does so only by a Kähler transformation, as in eq. (69). These ‘duality’ symmetries are subject to anomalies which can also be cancelled by local counterterms [31], [37]. Once string loop effects are included, the moduli fields also modify the gauge couplings due to threshold corrections [38]. These corrections can change the tree-level nonminimal gauge kinetic function, $f_{\text{tree}} = S$, by adding to it a moduli-dependent one-loop contribution: $f = S + \Delta(T)$. In this case the role played by S in the previous sections, is instead played by this full gauge kinetic function $f(S, T)$ [10].

For ‘realistic’ scenarios, for which several factors of a hidden sector gauge group condense, it has not been clear how to formally derive an expression for the low-energy theory which manifestly displays these symmetries after supersymmetry breaking. The purpose of this section is to provide explicit expressions for the low-energy theory, and to show in particular how the symmetries are all realized in the result. All this as an application of the discussion of the previous sections.

Let us, for simplicity, discuss the case of an overall modulus field T , with a target-space duality group $\mathcal{G} = SL(2, \mathbf{Z})$ acting as

$$T \longrightarrow \frac{\alpha T - i\beta}{i\gamma T + \delta}; \quad \text{with} \quad \alpha, \beta, \gamma, \delta \in \mathbf{Z}, \quad \text{and} \quad \alpha\delta - \beta\gamma = 1. \quad (102)$$

At string tree level, the perturbative part of the low-energy action takes the form [39]:

$$K_p^{\text{tree}} = -3 \log(T + T^*) - \log(S + S^*), \quad W_p = 0. \quad (103)$$

This is invariant with respect to the $SL(2, \mathbf{Z})$ transformations of eq. (102), with K_p transforming into itself up to a Kähler transformation

$$\begin{aligned} K &\longrightarrow K + \varphi(T) + \varphi^*(T^*) \\ W &\longrightarrow e^{-\varphi(T)} W, \\ S_0 &\longrightarrow e^{\frac{\varphi(T)}{3}} S_0 \end{aligned} \quad (104)$$

having $\varphi(T) = 3 \log(i \gamma T + \delta)$ as its parameter. Notice that the dilaton field, S , is invariant under this transformation.

After one-loop string corrections are included, S cannot remain invariant, however. This has been observed in ref. [31], where it was found that the Kähler potential acquires loop corrections given by

$$K_p^{1\text{-loop}} = -3 \log(T + T^*) - \log[S + S^* + 3\delta_{GS} \log(T + T^*)], \quad (105)$$

while the gauge kinetic terms become modified from eq. (75) to

$$\sum_n \left((f_n S - 2c_n \log S_0 + (2c_n - 3f_n \delta_{GS}) \log \eta^2(T)) \text{Tr} W_n^\alpha W_{\alpha n} \right)_F + \text{c.c.} \quad (106)$$

Here $\eta(T)$ is the Dedekind η function and the coefficients δ_{GS} are explicitly known for the Z_N orbifolds [31]. These interactions are only invariant under Kähler transformations if the field S transforms in the following way:

$$S \longrightarrow S + 3\delta_{GS} \varphi(T). \quad (107)$$

With this information, we can apply our previous analysis to find the 2PI effective action, $\Gamma[U_n, S, S_0]$, and Wilson action, $\Gamma_V[S, S_0]$, that are induced by gaugino condensation. For the 2PI effective action we find the following results for the total Kähler- and super-potentials:

$$K(S, S^*, T, T^*, U_n, U_n^*) = -3 \log \left[(S + S^* + 3\delta_{GS} \log(T + T^*))^{\frac{1}{3}} (T + T^*) + e^{-K_2/3} - \sum_n a_n (U_n U_n^*)^{\frac{1}{3}} \right], \quad (108)$$

where $K_2(S, S^*, T, T^*)$ stands for the (at present unknown) higher-loop corrections to the Kählerpotential, and

$$W_{np}(S, T, U) = \frac{1}{4} \sum_n U_n \left(f_n S + \frac{2c_n}{3} \log(\xi_n U_n) + h_n(T) \right), \quad (109)$$

with

$$h_n(T) \equiv (2c_n - 3f_n \delta_{GS}) \log \eta^2(T). \quad (110)$$

These expressions should be the starting point for the discussion of gaugino condensation. It is remarkable that our ignorance here lies with the perturbative rather than the nonperturbative part of the Kählerpotential! Quantitative results therefore become possible for supersymmetry breaking in any model for which these perturbative contributions can be computed. The important point here is that the nonperturbative corrections to $e^{-K/3}$ due to gaugino condensation, are only functions of U , and these are completely under control since they are independent of the particular vacuum. The correct procedure to see if gauginos actually condense, and if they break supersymmetry, should be using the 2PI action defined by (108), (109) and (110). This, as far as we know has not been pursued yet, in part because there was no confidence on what Kähler potential should be considered for the field U (which we are providing) and also because it is simpler just to work with the Wilson action approach, which we consider next.

We can obtain the Wilson action below all condensation scales by eliminating the superfields U_n . We can follow a procedure similar to that of section 4.3. The extremal for f_{U_n} yields,

$$z_0^* f_{U_n}^* = 3z_0^2 |u_n|^{4/3} (W_{np})_{,u_n} - 3u_n^* f_0^*. \quad (111)$$

The stationary condition for u_n can be solved by expanding as before $u_n = \tilde{u}_{n0} (1 + \tilde{u}_{n1} + \dots)$.

One obtains,

$$\begin{aligned} \tilde{u}_{n0} &= -\frac{3}{2c_n} w_n \exp \left\{ -\frac{3(f_n s + h_n(T))}{2c_n} \right\}, \\ z_0^2 \tilde{u}_{n0} \tilde{u}_{n1} &= -\frac{3a_n}{4c_n^2} (\tilde{u}_{n0}^* \tilde{u}_{n0})^{1/3} \xi_n^*, \\ (\tilde{u}_{n0}^* \tilde{u}_{n0}^2) \tilde{u}_{n2} &= -\frac{1}{6} \tilde{u}_{n0} \tilde{u}_{n1}^* \tilde{u}_{n1} - \frac{1}{3} \tilde{u}_{n0}^* \tilde{u}_{n1} \tilde{u}_{n1}^*, \end{aligned} \quad (112)$$

where the auxiliary field around which the expansion is done now reads

$$\xi_n = z_0^2 (f_s + h_n(T))_{,T} f_T - 2c_n f_0. \quad (113)$$

From the terms \tilde{u}_{n0} and \tilde{u}_{n1} , we obtain the Wilson action with superpotential

$$W_{np}(S, T) = \sum_n w_n \exp \left\{ -\frac{3}{2c_n} (f_n S + h_n(T)) \right\}, \quad (114)$$

and Kähler potential

$$K(S, S^*, T, T^*) = -3 \log \left[(S + S^* + 3\delta_{GS} \log(T + T^*))^{1/3} (T + T^*) + e^{-K_2/3} - \sum_n k_n \exp \left\{ -\frac{1}{2c_n} (f_n (S + S^*) + h_n(T) + h_n^*(T^*)) \right\} \right]. \quad (115)$$

Notice that the effective action is invariant under Kähler transformations, and so also under $SL(2, \mathbf{Z})$ transformations, provided that the field U transforms under duality as its definition would suggest: $U \rightarrow e^{-\varphi(T)} U$ and the unknown perturbative corrections $e^{-K_2/3}$ transform properly.

6. Conclusions

We have presented here a systematic treatment of the process of gaugino condensation in supersymmetric $N = 1$ Yang-Mills theories coupled to neutral scalar fields. Our analysis has accomplished several things:

- *1*: It has put some previous approaches on a firmer basis by showing how they can be better interpreted in terms of the 2PI effective and Wilson actions. In particular, our modification of the VY approach solves a minor puzzle as to why the heavy degree of freedom corresponding to a *quantum* field, U , can consistently appear in the low energy theory below the condensation scale. In our approach the problem does not arise because U is a classical field corresponding to a composite operator whose expectation we wish to study. As a result, in order to retrieve the Wilson action from the effective action for U , U must simply be eliminated using its field equations. This is *not* to be regarded as the tree approximation to ‘integrating it out’ as a quantum field. We regard our treatment to also shed light on the more recent treatments in which U is regarded as a quantum field which is ‘integrated in’ [40].

Another interesting discussion of gaugino condensation uses the Nambu-Jona-Lasinio approach [11]. In this case U is also treated as a quantum field and Coleman-Weinberg corrections to its potential are considered, potentially leading to modifications of the standard results of [3], [7]. Although such corrections can and do arise in a theory containing a quantum field U , they are irrelevant in our approach, since U is classical. Our treatment shows how the general results of [3], [7] (such as the impossibility of fixing the vev of S with a single condensate) are more robust than would be expected for an approximate treatment of a theory of a quantum field U .¹¹

- 2: We have further shown, both by general arguments, and by explicit calculation, how the approaches based on the 2PI effective action and the Wilson action give equivalent results for the low-energy theory. Furthermore, since the important issue here is not only to find the effective action below condensation, but also to trace the condensation process, we can say that the 2PI action formalism is more suitable for the discussion than the Wilson approach, since using it we can learn if the condensate actually forms or not. Moreover, to be able to use the Wilson approach we have to know the light degrees of freedom beforehand. We have previously encountered a case [22] for which it is not possible to use the Wilson approach directly, since the degrees of freedom change after the condensation process. In that case a massless two-index tensor of the underlying theory is replaced by a massive three-index tensor in the effective theory below condensation. This we discovered by using the 2PI effective action approach, of course after that identification and the corresponding elimination of U we can find the Wilson action below condensation. In this sense we see the 2PI approach as more fundamental.

- 3: We have obtained new results concerning the Kähler potential of the low energy effective theory below the condensation scale, in addition to reproducing earlier work for

¹¹ There are other differences of detail as well. In [11], the field U is identified as a Goldstone boson for the R -symmetry. In our approach the corresponding Goldstone boson is the axion field $\text{Im } S$, and is not related to U (which is, after all, only classical). In the case of several condensates, there the R -symmetry is only approximate and one expects at best to have pseudo-Goldstone bosons, since the axion gets a mass even if supersymmetry is not broken. (We thank Graham Ross for helping us clarify these points.)

the superpotential, which we put into a firmer basis, especially the several condensates case. We found calculable nonperturbative corrections to the Kähler potential, as well as many other terms involving higher numbers of supercovariant derivatives of the light fields. These higher-derivative terms give corrections only to the Kähler potential, and are systematically suppressed by inverse powers of the condensation scale, $1/\Lambda_c$.

- *4*: Our work could shed some light on the question of supersymmetry breaking due to gaugino condensates in superstring theories. Besides showing how to write effects of the condensation in a way that manifestly respects the duality symmetries, our determination of the Kähler potential may require a reanalysis of the previous phenomenological studies of the low energy scalar potential, and the soft supersymmetry-breaking terms. However, presently unknown perturbative corrections to the Kähler potential will be more important, in the weak coupling regime, than the non-perturbative ones we have found. Further progress must wait for these corrections to be computed in particular models.

Finally, we mention in passing that our techniques are generalizable to the case where the condensing system also contains charged matter. Such models have played an important role in globally supersymmetric theories, and can arise in the hidden sector of string theories, where it permits more freedom to obtain minima with supersymmetry broken at a phenomenologically interesting scale.

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Appendix

In section 4, we have formulated supergravity actions in the framework of superconformal tensor calculus. This appendix enumerates some results which are useful in component expansions of tensor calculus expressions.

We consider the general supergravity lagrangian for a set of chiral multiplets Σ^i , with zero chiral and Weyl weights, and a chiral multiplet S_0 , with unit weights, used as compensator. We will restrict ourselves to the contributions of the bosonic components z^i, f^i and z_0, f_0 of Σ^i and S_0 respectively.

The general conformal supergravity for these multiplets is defined by the expression

$$\mathcal{L} = -\frac{3}{2} [S_0 S_0^* \Phi(\Sigma^i, \Sigma_i^*)]_D + [S_0^3 w(\Sigma^i)]_F + \text{c.c.} \quad (A1)$$

The real function Φ and the superpotential w are arbitrary.

The bosonic part of this lagrangian density also depends on the bosonic gauge fields of the superconformal algebra, some of them being algebraic (like the spin connection) or gauge-fixed to reduce the symmetry and obtain the Poincaré theory (this is the case of the gauge field of scale transformation). Only the metric tensor and the chiral $U(1)$ gauge field A_μ (which is auxiliary) will participate in the bosonic terms of the Poincaré theory.

After the elimination of all auxiliary fields, a convenient expression for the bosonic part of this theory is

$$e^{-1} \mathcal{L}_{bos} = -\frac{1}{2} (z_0 z_0^* \Phi) R + \frac{3}{4} (z_0 z_0^* \Phi) [\partial_\mu \log(z_0 z_0^* \Phi)]^2 - (z_0 z_0^* \Phi) K_j^i (\partial_\mu z^j) (\partial^\mu z_i^*) - V_0, \quad (A2)$$

with $\Phi = \Phi(z^i, z_i^*)$, $w = w(z^i)$, and with the definitions

$$K = -3 \log \Phi, \quad \Phi = e^{-K/3}, \quad K_j^i = \frac{\partial^2}{\partial z^j \partial z_i^*} K. \quad (A3)$$

These scalar kinetic terms are obtained after solving for the auxiliary A_μ . Defining further

$$\mathcal{G} = K + \log(w w^*), \quad \mathcal{G}_i = \frac{\partial}{\partial z^i} \mathcal{G}, \quad \mathcal{G}^i = \frac{\partial}{\partial z_i^*} \mathcal{G}, \quad (A4)$$

the scalar potential reads

$$V_0 = (z_0 z_0^* \Phi)^2 e^{\mathcal{G}} [(K^{-1})_j^i \mathcal{G}^j \mathcal{G}_i - 3]. \quad (A5)$$

It is generated by the auxiliary scalar field lagrangian

$$\begin{aligned} e^{-1} \mathcal{L}_{aux} &= -3z_0 z_0^* \Phi_j^i f_i^* f^j - 3\Phi f_0^* f_0 - 3z_0 \Phi_i f_0^* f^i - 3z_0^* \Phi^i f_i^* f_0 \\ &\quad + [3z_0^2 w f_0 + z_0^3 w_i f^i] + \text{c.c.} \\ &= (z_0 z_0^* \Phi) K_j^i f_i^* f^j - 3\Phi \tilde{f}^* \tilde{f} + [3z_0^2 w \tilde{f} + z_0^3 w f^i \mathcal{G}_i] + \text{c.c.}, \end{aligned} \quad (A6)$$

with $\tilde{f} = f_0 + \Phi^{-1} z_0 \Phi_i f^i$, which implies

$$f^i = -(z_0 z_0^* \Phi)^{-1} z_0^3 w^* (K^{-1})_j^i \mathcal{G}^j, \quad \tilde{f} = \Phi^{-1} z_0^2 w^*. \quad (A7)$$

The super-Poincaré invariant theory is obtained by gauge-fixing of the unwanted superconformal symmetries. This procedure includes conditions of the form

$$z_0 = z_0^* = \kappa^{-1} h(z^i, z_i^*), \quad \kappa^{-1} = \frac{M_P}{\sqrt{8\pi}} \simeq 2.4 \times 10^{18} \text{ GeV}, \quad (A8)$$

which fixes scale and chiral $U(1)$ symmetries and eliminate the scalar component of the compensator.

The most natural fixing condition is obtained when imposing that the Einstein term in the supergravity lagrangian is canonically normalized:

$$z_0 z_0^* \Phi = \kappa^{-2},$$

or $h = \Phi^{-1/2}$. The bosonic lagrangian is then

$$\begin{aligned} e^{-1} \mathcal{L}_{Poin} &= -\frac{1}{2} \kappa^{-2} R - \kappa^{-2} K_j^i (\partial_\mu z^j) (\partial^\mu z_i^*) - V, \\ V &= \kappa^{-4} e^{\mathcal{G}} [(K^{-1})_j^i \mathcal{G}^j \mathcal{G}_i - 3]. \end{aligned} \quad (A9)$$

With this choice of compensator, the function K is the Kähler potential of the Poincaré theory, and the lagrangian only depends on the combination $\mathcal{G} = K + \log(ww^*)$ and on its derivatives.

On the other hand, choosing a general gauge-fixing condition (A 8) leads to

$$\begin{aligned} e^{-1}\mathcal{L}'_{Poin} &= -\frac{1}{2}\kappa^{-2}(h^2\Phi)R + \frac{3}{4}\kappa^{-2}(h^2\Phi)^{-1}[\partial_\mu(h^2\Phi)]^2 - \kappa^{-2}(h^2\Phi)K_j^i(\partial_\mu z^j)(\partial^\mu z_i^*) - V', \\ V' &= \kappa^{-4}(h^2\Phi)^2 e^{\mathcal{G}} [(K^{-1})^i_j \mathcal{G}^j \mathcal{G}_i - 3]. \end{aligned} \quad (\text{A10})$$

Since the two lagrangians \mathcal{L}_{Poin} and \mathcal{L}'_{Poin} differ in the fixing of dilatation symmetry, they should be related by a simple rescaling of the vierbein. If:

$$e_{m\mu} \longrightarrow (h^2\phi)^{-1/2}e_{m\mu}, \quad e \longrightarrow (h^2\phi)^{-2}e,$$

then:

$$\begin{aligned} -\frac{1}{2}(h^2\Phi)eR &\longrightarrow -\frac{1}{2}eR - \frac{3}{4}e[\partial_\mu \log(h^2\phi)]^2, \\ \frac{3}{4}e(h^2\Phi)^{-1}[\partial_\mu(h^2\Phi)]^2 &\longrightarrow \frac{3}{4}e(h^2\Phi)^{-2}[\partial_\mu(h^2\Phi)]^2 = \frac{3}{4}e[\partial_\mu \log(h^2\phi)]^2, \\ e(h^2\Phi)K_j^i(\partial_\mu z^j)(\partial^\mu z_i^*) &\longrightarrow eK_j^i(\partial_\mu z^j)(\partial^\mu z_i^*), \\ eV' &\longrightarrow e\kappa^{-4}e^{\mathcal{G}} [(K^{-1})^i_j \mathcal{G}^j \mathcal{G}_i - 3] = eV, \end{aligned}$$

and, finally,

$$\mathcal{L}'_{Poin} \longrightarrow \mathcal{L}_{Poin}.$$

In the Poincaré theory (A 9), all fields are dimensionless. Formally, their canonical dimension can be restored by the rescalings

$$\begin{aligned} K &\longrightarrow \kappa^2 K, \\ W &\longrightarrow \kappa^3 W. \end{aligned}$$

The bosonic lagrangian becomes

$$\begin{aligned} e^{-1}\mathcal{L}_{Poin} &= -\frac{1}{2}\kappa^{-2}R - K_j^i(\partial_\mu z^j)(\partial^\mu z_i^*) - V, \\ V &= e^{\kappa^2 K} \left[(K^{-1})^i_j (W_i + \kappa^2 W K_i)(W^{*j} + \kappa^2 W^* K^j) - 3\kappa^2 W W^* \right]. \end{aligned} \quad (\text{A11})$$

The flat limit, obtained with $\kappa \rightarrow 0$, is then

$$\mathcal{L}_{flat} = -K_j^i (\partial_\mu z^j) (\partial^\mu z_i^*) - (K^{-1})_j^i W_i W^{*j},$$

with flat Kähler potential K and superpotential W .

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