String Theory and Cosmology⋄

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ABSTRACT

We discuss the main cosmological implications of considering string-loop effects and a potential for the dilaton in the lowest order string effective action. Our framework is based on the effective model arising from regarding homogeneous and isotropic dilaton, metric and Yang-Mills field configurations. The issues of inflation, entropy crisis and the Polonyi problem as well as the problem of the cosmological constant are discussed.

⋄ This essay was selected for an Honorable Mention by the Gravity Research Foundation, 1995.

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String theory, as a candidate to describe quantum gravity effects and unify all forces of nature is expected to account for particle interactions at energies below the Planck scale and give rise to a consistent cosmological scenario. For achieving these aims, string theory should lead to the standard cosmological scenario and free this model from its weaknesses, such as the existence of the cosmological singularity and initial condition problems. It is hence expected that string low-energy models naturally generate inflation, which is usually achieved through a scalar field endowed with a potential. Furthermore, string models should be free from the Polonyi problem associated with scalar fields that couple only gravitationally as well as providing a mechanism for the vanishing of the cosmological constant.

If, on the one hand, the singularity problem seems to admit a solution in the quantum cosmological framework, i.e. via the vanishing of the wave function for singular metric configurations [1], and a radiation dominated era is shown to emerge from a string cosmological scenario [2], the other difficulties, on the other hand, do not have yet a satisfactory explanation. Indeed, none of the models proposed so far to break supersymmetry and provide the dilaton with a potential, based on the condensation of gauginos, seem to be suitable for inflation [3]; moreover, no workable mechanism has yet been put forward to explain the vanishing of the cosmological constant [4]. In what regards the Polonyi problem, however, some solutions have been presented. For instance, one can mention the suggestion that string-loop effects can satisfactorily drive the dynamics of a massless dilaton [5] and the possibility that the dilaton energy can be efficiently transferred into coherent classical oscillations of Yang-Mills fields [6].

In this essay, we adopt the point of view that supersymmetry is broken by a mechanism other than gaugino condensation and, while not specifying any such mechanism, we assume that a mass term for the dilaton is thus generated and that this term dominates the dilaton potential. We show that inflationary chaotic-type solutions can then be obtained [6, 7]. We also discuss a possible solution to the Polonyi problem [6]. Finally, we present some ideas concerning the problem of the cosmological constant. We stress that the crucial ingredients of our discussion are the presence of a potential for the dilaton and string-loop effects. The former is necessary not only to achieve inflation, but also to break supersymmetry and to compensate the ensuing unbalanced contribution of bosons and fermions to the vacuum energy. In what concerns string-loop effects, it is shown that their inclusion does not lead, on its own, to stable de Sitter solutions [8], but they may be relevant for the Polonyi problem.

The four-dimensional string vacua emerging, for instance, from heterotic string theories, correspond to N=1 non-minimal supergravity and super Yang-Mills models [9]. The bosonic action is, at lowest order in α', given by:

$$S_B = \int d^4x \sqrt{-g} \left\{ -\frac{R}{2k^2} + 2(\partial\phi)^2 - B(\phi)Tr\left(F_{\mu\nu}F^{\mu\nu}\right) + 4V(\phi) \right\}, \qquad (1)$$

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where $k^2 = 8\pi M_P^{-2}$, we have dropped the antisymmetric tensor field $H_{\mu\nu\lambda}$ and introduced a potential, $V(\phi)$, for the dilaton as well as the universal function [5]

$$B(\phi) = e^{-2k\phi} + c_0 + c_1 e^{2k\phi} + c_2 e^{4k\phi} + ...$$ (2)

which expresses the fact that string-loop interactions have an expansion in powers of $g_S \equiv e^{2k\phi}$; the coefficients $c_0, c_1, c_2, ...$ are presently unknown. The field strength $F_{\mu\nu}^a$ in (1) corresponds to the one of a Yang-Mills theory with a gauge group G, which must be a subgroup of $E_8 \times E_8$.

As we are interested in a cosmological setting, we focus on homogeneous and isotropic field configurations on a spatially flat spacetime. The most general metric is given in terms of the lapse function, N(t), and the scale factor, a(t):

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega_3^2 \ .$$ (3)

We consider for simplicity an SO(3) gauge field; our conclusions, however, will be qualitatively independent of this choice. We then use the following homogeneous and isotropic ansatz (up to a gauge transformation) for the gauge potential [10, 11]:

$$A_\mu(t)dx^\mu = \sum_{a,b,c=1}^{3} \frac{\chi_0(t)}{4} T_{ab} \epsilon_{acb} dx^c,$$ (4)

$\chi_0(t)$ being an arbitrary function of time and T_{ab} the generators of SO(3).

We start by dimensionally reducing the system (1), allowing only for homogeneous and isotropic field configurations. This procedure allows one to treat the contribution of the Yang-Mills fields on the same footing as the remaining fields, as opposed to the usual treatment of radiation as a fluid.

Introducing the ansätze (3) and (4) into the action (1) yields, after integrating over R^3 and dividing by the infinite volume of its orbits:

$$S_{eff} = -\int_{t_1}^{t_2} dt \left\{ -\frac{3\dot{a}^2 a}{k^2 N} + \frac{3a}{N} B(\phi) \left[\frac{\dot{\chi}_0^2}{2} - \frac{N^2}{a^2} \frac{\chi_0^4}{8} \right] + \frac{2a^3}{N} \dot{\phi}^2 - 4a^3 N V(\phi) \right\},$$ (4)

where the dots denote time derivatives. The equations of motion in the N=1 gauge are the following:

$$2\frac{\ddot{a}}{a} + H^2 + \frac{k^2}{3} B(\phi)\rho_{\chi_0} + 2k^2[\dot{\phi}^2 - 2V(\phi)] = 0,$$ (5)

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{4}B'(\phi)\zeta_{\chi_o} + \frac{\partial V(\phi)}{\partial \phi} = 0,$$ (6)

$$\ddot{\chi}_0 + [H + \frac{B'(\phi)}{B(\phi)}\dot{\phi}]\dot{\chi}_0 + \frac{\chi_0^3}{2a^2} = 0,$$ (7)

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where the primes denote derivatives with respect to ϕ, $H = \dot{a}/a$, $\rho_{\chi_\circ} = 3 \left[\frac{\dot{\chi}_0^2}{2a^2} + \frac{\chi_0^4}{8a^4} \right]$ and $\zeta_{\chi_0} = 3 \left[\frac{\dot{\chi}_0^2}{2a^2} - \frac{\chi_0^4}{8a^4} \right]$.

Furthermore, extremizing action (4) with respect to N, yields the Friedmann equation:

$$H^2 = \frac{k^2}{3} \left[4\rho_\phi + B(\phi)\rho_{\chi_\circ} \right] \tag{8}$$

with $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$.

Using our field treatment of radiation, we now discuss the way inflationary solutions can be obtained from the dynamical system arising from eqs. (5)–(8). We follow the discussion of ref. [7], where one obtains chaotic inflation driven by the dilaton potential $V(\phi) = \frac{1}{2}m^2(\phi - \phi_0)^2$, with $10^{-8}M_P < m < 10^{-6}M_P$ and $\phi_0 \sim M_P$, a result which remains valid if we add a quartic term to the potential. Furthermore, in this situation $H \gg \frac{B'(\phi)}{B(\phi)}\dot{\phi}$, which allows us to solve eq. (7) in the conformal time $d\xi \equiv a^{-1}(t)dt$, the solution being given implicitly in terms of an elliptic function [12]. Moreover, we find that $\rho_{\chi_0} = \frac{C}{a^4(t)}$, where C is an integration constant. Substituting these results into eqs. (5) and (6), we obtain, after introducing the dimensionless variables $x \equiv \frac{m}{\mu}(\phi - \phi_0)$, $y \equiv \frac{1}{\mu}\dot{\phi}$, $z \equiv \frac{1}{m}H$ and $\eta \equiv mt$, where $\mu^2 = 3m^2/2k^2$, the following non-autonomous three-dimensional dynamical system

$$x_\eta = y, \tag{9 - a}$$

$$y_\eta = -x - 3yz + C_1\zeta_{\chi_0}(t)B'(x), \tag{9 - b}$$

$$z_\eta = 2x^2 - y^2 - 2z^2, \tag{9 - c}$$

where the index η denotes derivative with respect to η and $C_1 = k^2/6m^2$. The phase space of the system is the region in R^3 characterized by the constraint equation (8):

$$z^2 - x^2 - y^2 = \frac{k^2}{3m^2}\frac{C}{a^4}B(x) . \tag{10}$$

As argued in refs. [7, 11], during inflation, terms proportional to $a^{-4}(t)$ in (10) and (9-b), where $\zeta_{\chi_0} \sim a^{-4}$, become much smaller than the remaining ones. Clearly this does not hold if $x \ll 0$ and $x \gg 0$, hence we disregard these regions of phase space. Dropping these terms, the resulting dynamical system has in the finite region of variation of x, y, z, a critical point at the origin, which for expanding models ($z > 0$), is an asymptotically stable focus. Hence, inflationary solutions do exist and inflation with more than 65 e-foldings requires that the initial value of the ϕ field is such that $\phi_i \gtrsim 4M_P$ [7, 11].

Let us now consider the entropy crisis and Polonyi problems. The former difficulty concerns the dilution of the baryon asymmetry generated prior to ϕ conversion into radiation. The entropy crisis problem can be solved either by regenerating in string-inspired models

the baryon asymmetry after ϕ decay [13] or by considering the Affleck-Dine mechanism to generate an $\mathcal{O}(1)$ baryon asymmetry and then allowing for its dilution via ϕ decay [14]. In models where the dilaton mass is very small, such that its lifetime is greater than the age of the Universe, one may encounter the Polonyi problem if ρ_ϕ dominates the energy density of the Universe at present. These problems afflict various N=1 supergravity [15, 16] as well as string models [17] and dynamical supersymmetry breaking scenarios [18].

In what concerns avoiding the Polonyi problem, a necessary requirement is that, at the time when ϕ becomes non-relativistic, i.e. $H(t_{NR}) = m$, the ratio of its energy density to the one of radiation satisfies [16]

$$\epsilon \equiv \frac{\rho_\phi(t_{NR})}{\rho_{\chi_0}(t_{NR})} \lesssim 10^{-8} . \tag{11}$$

Our field treatment of radiation reveals an energy exchange mechanism that may be relevant in this context. Working out eqs. (6)–(8), one obtains the energy exchange equations:

$$\dot{\rho}_\phi = -3H\dot{\phi}^2 + \frac{1}{4}B'(\phi)\zeta_{\chi_o}\dot{\phi}, \tag{12}$$

$$\dot{\rho}_{\chi_0} = -4H\rho_{\chi_0} - 3\frac{B'(\phi)}{B(\phi)}\frac{\dot{\chi}_0^2}{a^2}\dot{\phi} . \tag{13}$$

The new feature in eqs. (12), (13) are the terms proportional to $\dot{\phi}$. Clearly, these terms do not play any role in the reheating process, which is due to ϕ decay and conversion into radiation as it quickly oscillates around the minimum of its potential.

Notice that the condition, $\Gamma_\phi^{-1} \geq t_U \approx 10^{60}\ M_P^{-1}$, implies from the dilaton decay width $\Gamma_\phi \simeq \frac{m^3}{M_P^2}$ [14] that $m \leq 10^{-20}M_P$, which falls outside the mass interval for which inflation takes place; thus, we have to assume that, in models where this problem occurs, some field other than the dilaton, eg. moduli, E_6 singlets or scalars of the hidden E_8 sector, will drive inflation and be responsible for reheating. In the absence of the terms proportional to $\dot{\phi}$ in eqs. (12), (13) and until ϕ becomes non-relativistic, $\rho_\phi \simeq \rho_{\chi_0} \simeq \frac{1}{2}m^2\phi_*^2$, implying that $\epsilon = \mathcal{O}(1)$. Hence, any mechanism for draining ϕ energy into radiation has to be fairly efficient to avoid the Polonyi problem. However, assuming that after inflation the term proportional to $\dot{\phi}$ in eq. (12) can effectively drain the dilaton energy into radiation over the period $(t_i,\ t_{NR})$, during which $H \simeq \frac{B'(\phi)}{B(\phi)}\dot{\phi}$ (cf. eq. (7)), and that $\phi_* \simeq \phi(t_{RN}) \approx M_P$, $\zeta_{\chi_0} \sim a^{-4}$ and $a(t) = a_R\left(\frac{t}{t_R}\right)^{1/2}$, where the index R refers to the inflaton decay time, it then follows [6]

$$t_i \simeq \frac{1}{mM_P}\frac{t_R}{a_R^2} . \tag{14}$$

For typical values of the relevant parameters, e.g. $t_i \simeq 10^{10}M_P^{-1}$, $t_R \gtrsim 10^{30}M_P^{-1}$ and $a_R \gtrsim 10^{30}M_P^{-1}$, and we see that the dilaton mass is required to be exceedingly small,

$m \leq 10^{-40} M_P$. It is clear that an effective transfer of the dilaton energy into radiation can be achieved if the conditions described above can be maintained over a sufficiently long time interval and actually via terms in $B(\phi)$ with negative coefficients. Other contributions to this process would be present if we had chosen a larger gauge group as, besides $\chi_0(t)$, a multiplet of scalar fields would appear in the effective action leading to extra energy exchange terms [10, 11].

Let us now turn to the problem of the cosmological constant. Our mechanism is inspired on the Atkin-Lehner symmetry known to hold at 1-loop order. We start adding to action (1) the contribution of bosons and fermions to obtain S_T. We then assume that fermionic fields can be redefined such that they do not interact with the dilaton [5] whereas bosons do interact with the dilaton via the universal $B(\phi)$ function. This universality is related with the string S-duality which implies that $B(\phi) = B(-\phi)$. Furthermore, we shall assume that the contributions to the vacuum energy of bosons and fermions depend on the value of the dilaton field. Hence, at the minimum $\phi = \phi_0$:

$$\frac{\delta S_T}{\delta \phi} = \sqrt{-g} \left[B'(\phi_0) V_B(\phi_0) + V'(\phi_0) \right] = 0 \ , \tag{15}$$

and, for the trace of the energy-momentum tensor, we have:

$$\Lambda(\phi_0) \equiv \sqrt{-g} \left[B(\phi_0) V_B(\phi_0) + V_F(\phi_0) + V(\phi_0) \right] . \tag{16}$$

Vanishing of $\Lambda(\phi_0)$, for the value $\phi = \phi_0$ that satisfies eq. (15), implies that space-time is flat and that the cosmological constant vanishes. However, as discussed in ref. [4], adjusting mechanisms that aim to dynamically solve the cosmological constant problem are unable to satisfy both (15) and (16) since one cannot usually preserve symmetries and achieve equilibrium simultaneously. However, if one assumes that $\Lambda(\phi) = \Lambda(-\phi)$ and that at the ground-state potentials do not respect this duality, i.e. $V(-\phi) = -V(\phi)$, then one gets a vanishing cosmological constant due to S-duality. Notice here the "awareness" that boson and fermion vacuum contributions must have of the value of the dilaton field at its minimum indicating that, in order to fully implement string symmetries into the corresponding field theory, non-local effects must be introduced. We stress that string theory is rich in discrete and duality-type symmetries and it would not be at all surprising that they would play a role in solving the cosmological constant problem.

References

[1] M.C. Bento and O. Bertolami, "Scale Factor Duality: A Quantum Cosmological Approach", Preprint CERN-TH.7488/94, DFTT 43/94.

[2] A.A. Tseytlin and C. Vafa, Nucl. Phys. B372 (1992) 443.

[3] P. Binetruy and M.K. Gaillard, Phys. Rev. D34 (1986) 3069;
R. Brustein and P.J. Steinhardt, Phys. Lett. B302 (1993) 196.

[4] S. Weinberg, Rev. Mod. Phys. 61 (1989) 1.

[5] T. Damour and A.M. Polyakov, Nucl. Phys. B423 (1994) 532.

[6] M.C. Bento and O. Bertolami, Phys. Lett. B336 (1994) 6.

[7] M.C. Bento, O. Bertolami and P.M. Sá, Phys. Lett. B262 (1991) 11.

[8] M.C. Bento and O. Bertolami, "Cosmological Solutions of Higher–Curvature String Effective Theories with Dilatons", Preprint CERN-TH/95-63 , DFTT 19/95.

[9] E. Witten, Phys. Lett. B155 (1985) 151; Nucl. Phys. B268 (1986) 79.

[10] O. Bertolami, J.M. Mourão, R.F. Picken and I.P. Volobujev, Int. J. Mod. Phys. A6 (1991) 4149.

[11] P.V. Moniz, J.M. Mourão and P.M. Sá, Class. Quantum Gravity 10 (1993) 517.

[12] M.C. Bento, O. Bertolami, P.V. Moniz, J.M. Mourão and P.M. Sá, Class. Quantum Gravity 10 (1993) 285.

[13] K. Yamamoto, Phys. Lett. B168 (1986) 341;
O. Bertolami and G.G. Ross, Phys. Lett. B183 (1987) 163.

[14] M.C. Bento, O. Bertolami and P.M. Sá, Mod. Phys. Lett. A7 (1992) 911.

[15] G.D. Coughlan, W. Fischler, E.W. Kolb, S. Raby and G.G. Ross, Phys. Lett. B131 (1983) 59;
G.D. Coughlan, R. Holman, P. Ramond and G.G. Ross, Phys. Lett. B140 (1984) 44;
A.S. Goncharov, A.D. Linde and M.I. Vysotsky, Phys. Lett. B147 (1984) 279;
O. Bertolami and G.G. Ross, Phys. Lett. B171 (1986) 46.

[16] O. Bertolami, Phys. Lett. B209 (1988) 277.

[17] B. de Carlos, J.A. Casas, F. Quevedo and E. Roulet, Phys. Lett. B318 (1993) 447.

[19] T. Banks, D.B. Kaplan and A.E. Nelson, Phys. Rev. D49 (1994) 779.