# Momentum Correlations of Charmed Pairs Produced in $\pi^{-}-C u$ Interactions at $230 \mathrm{GeV} / \mathrm{c}$ 

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#### Abstract

We study the production characteristics of 557 pairs of charmed hadrons produced in $\pi^{-}-C u$ interactions at $230 \mathrm{GeV} / \mathrm{c}$ using a momentum estimator for charmed hadrons with missing decay products. We find, the mean value of the transverse momentum squared of the charmed pairs is $\left\langle p_{T, \text { sum }}^{2}\right\rangle=(1.98 \pm 0.11 \pm 0.09)$ $\mathrm{GeV}^{2} / \mathrm{c}^{2}$, the mean rapidity difference is $<\left|y_{d i f f}\right|>=0.54 \pm 0.02 \pm 0.24$, and the mean effective mass is $\left\langle M_{e f f}\right\rangle=(4.45 \pm 0.03 \pm 0.13) \mathrm{GeV} / \mathrm{c}^{2}$. Comparing these results with the next-to-leading order QCD predictions we find an agreement for the $y_{d i f f}$ and $M_{e f f}$, whilst the measured mean value of $p_{T, s u m}^{2}$ is significantly larger than the predicted value.


## 1 Introduction

Experimental results on charm hadronic production are usually compared with the next-to-leading order QCD calculations [1], [2]. Let us recall that the ACCMOR [3] and the E769 [4] collaborations have shown that the charm production cross sections, as well as the $x_{F}$ and $p_{T}$ spectra of charmed particles, do not depend strongly on the nature of the charmed particles $\left.{ }^{2}\right]$ under consideration $\left(D^{0}, D^{+}, D^{*+}, D_{s}^{+}\right.$and $\left.\Lambda_{c}^{+}\right)$. This indicates that the main features of charm production are determined at the parton level with a relatively small dependence on the light quarks forming a charmed hadron. On the other hand, the above characteristics are not very sensitive to the difference between the leading order $\square$ and $O\left(\alpha_{S}^{3}\right) \mathrm{QCD}$ calculations. The charm pair correlations are more sensitive to this difference. $O\left(\alpha_{S}^{2}\right)$ QCD predicts that the heavy quark pair is produced exactly in a back-to-back configuration, corresponding to an azimuthal angle difference of $\Delta \phi=180^{\circ}$ and $p_{T, \text { sum }}^{2}=0$. The third order terms cause the broadening of these distributions. Thus, the distributions of the charmed pairs test the importance of the next-to-leading order terms. On the other hand, disagreement with the perturbative results may show the importance of various nonperturbative effects, which are likely to play an important role at the energy scale set by the charmed quark mass.

There is a relative abundance of data on the azimuthal angle difference distribution $\Delta \phi$ of a charmed pair in hadroproduction as this angle can be determined from the directions of the charmed particles without the momentum measurements, which demand the detection of all decay products. The results of the WA75[8], E653 [6], ACCMOR[7], and to some extent, of the WA92 [8] collaborations show the back-to-back enhancement of the $\Delta \phi$ distribution to be much weaker than the $O\left(\alpha_{s}^{3}\right)$ predictions. There are less experimental results available for distributions of other kinematical variables of a charmed pair because they require a knowledge of the momenta of the charmed particles, while the fully reconstructed decays (no missing decay products) are much less numerous. The E653 [6, 6] collaboration attempted to overcome this difficulty by using a momentum estimator. In this letter we perform a similar analysis of the ACCMOR data with the aim of determining the following characteristics of the pair of charmed particles:
$p_{T, \text { sum }}^{2}$ - square of the vector sum of transverse momenta;
$y_{d i f f}$ - the rapidity difference between particles;
$M_{e f f}$ - effective mass of the pair.
Since the ACCMOR experiment, data processing and selection of double charm events have already been described (see ref.[7] and references therein), we recall here only the essential features of the experiment and the data analysis. Next, we discuss the momentum estimator and its errors. Finally, the results are presented and compared to the $O\left(\alpha_{s}^{3}\right)$ QCD predictions.

[^0]
## 2 Experiment, data analysis and acceptance corrections

The second phase of the NA32 experiment was performed at the CERN-SPS using a negative $230 \mathrm{GeV} / \mathrm{c}$ beam ( $96 \%$ pions and $4 \%$ kaons) and a 2.5 mm Cu target. Charm decays were reconstructed with an improved silicon vertex detector and a large acceptance spectrometer. The latter consisted of two magnets, 48 planes of drift chambers and three multicellular Cherenkov counters allowing $\pi, K, p$ identification in a wide momentum range. The vertex detector consisted of a beam telescope (seven microstrip planes) as well as a vertex telescope with two charge-coupled devices (CCDs) and eight microstrip planes. The high precision vertex detector allowed the clean reconstruction of charm decays with very few background events and a purely topological charm search, which was restricted neither to a limited number of decay modes nor to any mass window.

Events with the primary vertex inside the target and at least two secondary tracks not originating from the vertex were selected for further analysis. These tracks were then used as a seed to search for one or more secondary vertices. Events with two secondary vertices were next selected. Since most of them involved unseen neutral decay products, one could not demand the effective mass of charged decay products to be compatible with the mass of a charmed particle. Secondary interactions were excluded by demanding a separation of the secondary vertex of at least two standard deviations from the target edge as well as from the edges of both CCDs. The decays of strange particles were rejected by demanding that the effective mass $m_{v i s}$ of the charged decay products for $\pi^{+} \pi^{-}$vertices had to be larger than the kaon mass. Similarly, all three-prong vertices, which could be interpreted as an accidental overlap of a $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$(or $\Lambda^{0} \rightarrow p \pi^{-}$) decay and a track, were rejected by checking the effective mass of the $\pi^{+} \pi^{-}$(or $p \pi^{-}$) combinations. Additionally, the visible transverse momentum $p_{T}^{v i s}$, with respect to the direction of the parent charmed particle P , was demanded not to exceed the maximum transverse momentum kinematically allowed for any particle in the decay channel under consideration. Finally, decays attributed to short-lived $D^{0}, D_{s}^{+}$or $\Lambda_{c}^{+}$had to occur before the second CCD ( 20 mm from the target).

In ref. 10] such selected secondary vertices were assigned to various decay modes of $D^{0}, D^{+}, D_{s}^{+}$and $\Lambda_{c}^{+}$. This was done with the help of $m_{v i s}$ and of the neutral mass defined as

$$
\begin{equation*}
m_{0}^{2}=m_{P}^{2}+m_{v i s}^{2}-2 m_{P} \sqrt{m_{v i s}^{2}+\left(p_{T}^{v i s}\right)^{2}} \tag{1}
\end{equation*}
$$

For the purpose of this letter the exact assignment is not fully needed since we only want to identify the charmed particle P , as a knowledge of the decay channel is irrelevant here. To use the momentum estimator we need to know the charmed hadron mass, its flight vector and the momenta of visible decay products. As stated in ref. 11] the wrong assignment can amount to about $5 \%$ for $D^{0}$ as well as for $D^{+}$and to about $15 \%$ for $D_{s}^{+}$. In the same paper it was shown that the observed number of $D \bar{D}$ events is consistent with that expected from the charm production cross section, branching fractions, and our acceptance.

The resulting sample of 557 pairs was used for a study of azimuthal angle difference $\Delta \phi$ and pseudorapidity gap distributions [7]. Since the production characteristics depend
only slightly on the nature of the charmed hadrons, we ignore this dependence in the present study.

The simulation of the geometrical acceptance of our apparatus and of the selection criteria requires a complex Monte Carlo program. This program generates an uncorrelated pair of charmed particles $P_{i}$ and $P_{j}$. The particles decay into the observed channels, and we calculate the acceptance $A_{i j}$ for each combination of decays, each decay being generated from phase-space distribution. The distributions of the $x_{F}$ and of the transverse momenta [3] of the $P_{i}$ and $P_{j}$, as well as the lifetime of $\Lambda_{c}^{+}$[12] are taken from this experiment, and the lifetimes of the charmed mesons are taken from RPP [13]. The generated pair of charmed particles is mixed with tracks from an event randomly chosen from an interaction-trigger sample recorded with our apparatus, thus faking a real event. All tracks from such an event are subsequently traced through the apparatus and subjected to the same cuts as those made in our analysis. The acceptance $A_{i j}$ vanishes in the backward centre-of-mass hemisphere, otherwise it amounts to ( $1-5$ )\% depending on the decay channel. The acceptance depends only slightly on $p_{T}$, with a small decrease for increasing $p_{T}$. All results presented in this letter have been corrected for the acceptance.

## 3 Momentum estimator

The momentum estimator, used in order to account for the unseen decay products, was first applied to study differential cross sections [9] and the correlations of charmed hadrons [6] in the E653 experiment, where the charm decay vertices were observed in nuclear emulsion. The NA32 system of the beam hodoscope and the silicon microstrip detectors supplemented with two CCDs yielded a comparable precision of measuring the primary and the decay vertex, thus allowing us to find the direction of flight of the charmed hadron with a high level of accuracy. The visible decay products were identified and their momenta were measured with a high level of accuracy by the NA32 large-angle spectrometer. The estimation of the charmed hadron momentum is based on the assumption that in its rest frame the invisible system momentum is perpendicular to the charmed hadron laboratory momentum. This assumption, the angle between the charmed hadron flight vector, and the visible momentum vector, uniquely fix the boost from the charmed hadron rest frame to the laboratory frame, thus allowing calculation of the laboratory momentum of the charmed hadron. The momentum estimator was obviously not used for fully reconstructed decays in our data sample.

The systematic error of the momentum estimator has been determined using a special procedure [14]. The charmed hadrons have been generated according to the distributions measured in the NA32 experiment[3]. Then, the hadron undergoes the multibody isotropic decay with momenta chosen according to the phase-space distribution ${ }^{(1)}$ Next, the momentum of the visible decay products and the charmed hadron flight vector are found, and the momentum estimator is applied. Then, various kinematical variables for the charmed pair are calculated, using the estimated momentum, and finally they are

[^1]Table 1

| $D^{+} / D^{0}$ decay <br> mode | $\Delta p_{\text {lab }} / p_{\text {lab }}$ <br> $[\%$ | $\Delta p_{T, \text { sum }}^{2}$ <br> $\left[\mathrm{GeV}^{2} / \mathrm{c}^{2}\right]$ | $\Delta y_{\text {diff }}$ <br> $[1]$ | $\Delta M_{\text {eff }}$ <br> $\left[\mathrm{GeV} / \mathrm{c}^{2}\right]$ |
| :--- | :---: | :---: | :---: | :---: |
| $K^{-} \pi^{+}\left(\pi^{0}\right)$ | 15.1 | 0.086 | 0.229 | 0.124 |
| $K^{-} \pi^{+} \pi^{-}\left(\pi^{0}\right)$ | 10.0 | 0.078 | 0.157 | 0.099 |
| $\left(K^{0}\right) \pi^{+} \pi^{-}$ | 18.7 | 0.094 | 0.276 | 0.143 |
| $\left(K^{0}\right) \pi^{+} \pi^{+} \pi^{-}$ | 14.2 | 0.087 | 0.222 | 0.123 |
| $K^{-} \pi^{+}\left(\pi^{0} \pi^{0}\right)$ | 15.8 | 0.089 | 0.244 | 0.137 |
| $\left(K^{0} \pi^{0}\right) \pi^{+} \pi^{-}$ | 19.6 | 0.095 | 0.292 | 0.148 |
| $\left(K^{0} \pi^{0}\right) \pi^{+} \pi^{+} \pi^{-}$ | 14.9 | 0.090 | 0.230 | 0.129 |
| Average | 15.6 | 0.089 | 0.238 | 0.130 |

Table 1. The systematic errors of the momentum estimator for various decay modes (with unseen decay products in brackets). The numbers are RMS values of the distributions of the estimated minus the generated kinematical variable of the charmed pairs. The average errors are calculated using weights proportional to the abundance of the decay channel in our double tag sample.
compared to the generated variables. The errors for various decay modes are collected in Table 1. The systematic errors of various kinematical variables are determined for seven decay modes, which constitute a fair majority of events in our sample. The distributions of the estimated minus the generated kinematical variables of the charmed pairs are symmetric and centered at zero. An example of such distributions of the differences are plotted in Fig. 1 for the $D^{0} \rightarrow K^{-} \pi^{+}\left(\pi^{0} \pi^{0}\right)$ mode with unseen $\pi^{0}$ 's. This decay channel is almost twice as abundant in our double charm sample than any other decay mode listed in Table 1. In the second column of Table 1 we also quote the estimation error of the laboratory momentum of the charmed meson, relative to the generated one. As expected, the error increases with the increasing fraction of energy carried by invisible decay products. Calculating the weighted average we obtain $15.6 \%$ accuracy for our estimate of the laboratory momentum. It should be noted that this error has little impact on the reconstructed kinematical variables of the charmed pairs, especially on the $p_{T, \text { sum }}^{2}$ for which there is very little dependence on the decay channel. We use the RMS of the distribution of the difference between the estimated and the generated variable as the measure of the systematic error on each kinematical variable in Table 1, checking that the mean value of the difference distribution is negligible in each case. The overall errors are calculated using the weights based on the numbers of such double charmed events observed in the ACCMOR experiment 10]. We assume these errors to be valid also for the remaining, less abundant, decay modes.

## 4 Results

Using the laboratory momenta estimated in the previous section we calculate the characteristics of charmed pairs in our sample. The acceptance corrected $p_{T, \text { sum }}^{2}$ distribution

Table 2

| Experiment <br> Beam | $\sqrt{s}$ <br> $[\mathrm{GeV}]$ | No of <br> pairs | $b_{p}$ <br> $\left[(\mathrm{GeV} / \mathrm{c})^{-2}\right]$ | $\sigma_{y}$ <br> $[1]$ | $\alpha_{M}$ <br> $\left[\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{-1}\right]$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| E653 $p-$ emul. <br> $800 \mathrm{GeV} / \mathrm{c}[6]$ | 38.7 | 35 | - | $1.85_{-0.41}^{+0.45}$ | $0.53_{-0.10}^{+0.14}$ |
| $\mathrm{WA} 75 \pi^{-}-$emul. <br> $350 \mathrm{GeV} / \mathrm{c}[15]$ | 25.6 | 177 | $0.50 \pm 0.10$ | $1.00 \pm 0.06^{a)}$ | $1.17_{-0.17}^{+0.13}$ |
| $\mathrm{NA} 32 \pi^{-}-C u$ <br> $230 \mathrm{GeV} / \mathrm{c}$ <br> $\chi^{2} / \mathrm{NDF}$ | 20.8 | 557 | $0.50 \pm 0.02$ | $0.65 \pm 0.02$ | $1.39 \pm 0.06$ |

${ }^{\text {a) }}$ calculated from the measured mean value of the $\left|y_{\text {diff }}\right|$ distribution.
Table 2. The results of the maximum-likelihood fits to the NA32 data and the corresponding results of other experiments.
is shown in Fig. 2. We have fitted the $p_{T, \text { sum }}^{2}$ distribution with

$$
\begin{equation*}
\frac{d \sigma}{d p_{T, \text { sum }}^{2}} \sim e^{-b_{p} p_{T, \text { sum }}^{2}} \tag{2}
\end{equation*}
$$

using the maximum-likelihood method. We have done a similar fit for the distribution of the rapidity difference of the charmed pairs

$$
\begin{equation*}
\frac{d \sigma}{d y_{d i f f}} \sim e^{-\frac{y_{d i f f}^{2}}{2 \sigma_{y}^{2}}} \tag{3}
\end{equation*}
$$

In Fig. 3 we plot the absolute value of the rapidity difference $\left|y_{d i f f}\right|$. Finally, the distribution of the effective mass $M_{\text {eff }}$ of charmed particles is fitted to

$$
\begin{equation*}
\frac{d \sigma}{d M_{e f f}} \sim e^{-\alpha_{M}\left(M_{e f f}-M_{0}\right)}, \tag{4}
\end{equation*}
$$

where $M_{0}=2 m_{D}$. The $M_{e f f}$ distribution is shown in Fig. 4. The fitted parameters, their statistical errors, the $\chi^{2}$ values, and the NDF (number of degrees of freedom) are collected in Table 2. The results are compared with the results of other charm hadroproduction experiments. As can be seen, the $y_{\text {diff }}$ and $M_{\text {eff }}$ distributions become broader with increasing $\sqrt{s}$, in accordance with QCD predictions.

Table 3

| Result | $<p_{T, \text { sum }}^{2}>$ <br> $\left[\mathrm{GeV}^{2} / \mathrm{c}^{2}\right]$ | $<\left\|y_{\text {diff }}\right\|>$ <br> $[1]$ | $<M_{\text {eff }}>$ <br> $\left[\mathrm{GeV} / \mathrm{c}^{2}\right]$ |
| :---: | :---: | :---: | :---: |
| $\alpha_{s}^{3}$ | 0.434 | 0.631 | 3.851 |
| $\alpha_{s}^{3}+$ h.f. | 0.396 | 0.559 | 4.467 |
| NA32 | $1.98 \pm 0.11 \pm 0.09$ | $0.54 \pm 0.02 \pm 0.24$ | $4.45 \pm 0.03 \pm 0.13$ |

Table 3. The $O\left(\alpha_{s}^{3}\right)$ QCD predictions (first row) supplemented by the hard charmed quark fragmentation (second row) compared to the NA32 experimental results (bottom row). The first error is the statistical error of the mean value of the distribution. The second error is the systematic error due to the uncertainties of the momentum estimator.

## 5 Comparison with $O\left(\alpha_{s}^{3}\right)$ QCD predictions

The next-to-leading order QCD predictions for the charmed pairs production, calculated using the program from ref. [2], are collected in Table 3. The mean values of the distributions were calculated using HMRSB 16] parametrization of the nucleon and the central set SMRS [17] parametrization of the pion structure functions. The default value of $\Lambda_{\overline{M S}}=122 \mathrm{MeV}$ was used, and the $\mu_{R}$ and $\mu_{F}$ scales were chosen to be $m_{c}$ and $2 m_{c}$, respectively, where $m_{c}=1.5 \mathrm{GeV} / \mathrm{c}^{2}$. The first row in Table 3 represents the results for bare charmed quarks with $x_{F}>0$. As the fragmentation generally leads to the softening of the distributions, we use the hard fragmentation function

$$
\begin{equation*}
f(z) \sim \delta(z-1) \tag{5}
\end{equation*}
$$

where $z=E_{D} / E_{c}$, is the fraction of charmed quark energy transferred to the charmed hadron. The results ( $x_{F}>0$ for both charmed hadrons) are presented in the second row of Table 3. Our results, this time determined as the acceptance weighted mean values of each distribution, are shown in the last row of Table 3. First errors are the statistical errors of the mean values. The systematic errors are taken from the bottom row of Table 1.

For the $M_{e f f}$ and $y_{d i f f}$ distributions there is a good agreement with the $O\left(\alpha_{s}^{3}\right)$ QCD supplemented by the hard hadronization of the charmed quark. On the other

[^2]hand, the experimental value of $p_{T, \text { sum }}^{2}$ is much larger than the predicted value. Let us recall that this result depends weakly on the uncertainties associated with the momentum estimator (cf. Table 1). It should also be mentioned that $\left\langle p_{T, \text { sum }}^{2}\right\rangle=(1.52 \pm 0.34) \mathrm{GeV}^{2} / \mathrm{c}^{2}$ for 20 pairs of fully reconstructed charmed particles in this sample [7]. Moreover, the WA75 collaboration [15] measured $<p_{T, \text { sum }}^{2}>=\left(2.00_{-0.33}^{+0.50}\right) \mathrm{GeV}^{2} / \mathrm{c}^{2}$. All these results are qualitatively consistent with the relatively flat $\Delta \phi$ distributions described previously.

To conclude, the perturbative next-to-leading order QCD calculation, whilst quite successful in determining total cross sections, single particle spectra and "longitudinal" correlation distributions of charmed pairs ( $y_{d i f f}$ or $M_{e f f}$ ), is not able to reproduce the "transverse" correlation distributions ( $p_{T, \text { sum }}^{2}$ and $\Delta \phi$ ). As discussed in refs. [7, 18] a fairly large intrinsic (or "primordial") transverse momentum of the colliding quarks, comparable with the measured mean value of $p_{T, s u m}^{2}$, is needed to reproduce the experimental results.

## Acknowledgements

The authors are much indebted to the ACCMOR collaboration for the kind permission to use their data.

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## Figure captions:

Fig.1. Plots of the estimated minus the generated kinematic variables of charmed pairs like: the $p_{T, \text { sum }}^{2}(\mathbf{b})$; the $y_{\text {diff }}(\mathbf{c})$; and the $M_{\text {eff }}(\mathbf{d})$; with one decay vertex being $D^{0} \rightarrow K^{-} \pi^{+}\left(\pi^{0} \pi^{0}\right)$ mode ( $\pi^{0} \pi^{0}$ system unseen). The same difference is plotted for the laboratory momentum (a) of a single $D^{0}$ decaying as above.

Fig.2. The acceptance corrected distribution of the vector sum of transverse momenta squared of the charmed hadron pairs $p_{T, \text { sum }}^{2}$. The points are experimental data, the solid line is the result of the maximum-likelihood fit, with $b_{p}=(0.50 \pm 0.02)(\mathrm{GeV} / \mathrm{c})^{-2}$.

Fig.3. The acceptance corrected distribution of the charmed hadron pairs rapidity difference $\left|y_{\text {diff }}\right|$. The points are experimental data, the solid line is the result of the maximum-likelihood fit, with $\sigma_{y}=0.65 \pm 0.02$.

Fig.4. The acceptance corrected distribution of the charmed hadron pairs invariant mass $M_{\text {eff }}$. The points are experimental data, the solid line is the result of the maximumlikelihood fit, with $\alpha_{M}=(1.39 \pm 0.06)\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{-1}$.


[^0]:    ${ }^{1}$ Hereafter called $O\left(\alpha_{s}^{3}\right)$.
    ${ }^{2}$ Throughout the paper a particle symbol stands for particle and antiparticle, i.e., any reference to a specific state implies the charge-conjugate state as well. Thus, e.g. $D^{+} \overline{D^{0}}$ stands for $D^{-} D^{0}$ as well.
    ${ }^{3}$ Hereafter called $O\left(\alpha_{S}^{2}\right)$.

[^1]:    ${ }^{4}$ We belive this is a good approximation in our case since, the majority of our charmed hadrons are pseudoscalar mesons, and the main source of error of the momentum estimator arises from the angular distribution of the decay products, rather than a particular choice of their momenta.

[^2]:    ${ }^{5}$ The $\alpha_{s}^{3}$ predictions displayed in Table 3 are the mean values calculated using the program from ref. [2], and its (signed) weights. These mean values are consistent with the mean values determined from the histograms produced by the program, with the maximum number of bins allowed.
    ${ }^{6}$ It should be noted that the results are not very sensitive to the structure functions, e.g., taking other (SMRS) pion structure functions results in $\pm 0.01 \mathrm{GeV}^{2} / \mathrm{c}^{2}, \pm 0.02$, and $\pm 0.02 \mathrm{GeV} / \mathrm{c}^{2}$ differences for the mean values of the $p_{T, s u m}^{2}, y_{d i f f}$ and $M_{e f f}$ distributions of the charmed hadron pairs, respectively.

