CERN-TH/95-107
hep-ph/9505238
May 1995

Uncertainties in the Determination of $|V_{cb}|$

Matthias Neubert
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract

I discuss the theoretical uncertainties in the extraction of $|V_{cb}|$ from semileptonic decays of B mesons, taking into account the most recent theoretical developments. The main sources of uncertainty are identified both for the exclusive decay mode $B \to D^* \ell\,\bar\nu$ and for the inclusive channel $B \to X\,\ell\,\bar\nu$. From an analysis of the available experimental data, I obtain $|V_{cb}|_{\rm excl} = 0.041 \pm 0.003_{\rm exp} \pm 0.002_{\rm th}$ from the exclusive mode, and $|V_{cb}|_{\rm incl} = 0.040 \pm 0.001_{\rm exp} \pm 0.005_{\rm th}$ from the inclusive mode. I also give a prediction for the slope of the form factor $\mathcal{F}(w)$ at zero recoil, which is $\widehat{\varrho}^2 = 0.8 \pm 0.3$.

to appear in the Proceedings of the XXXth Rencontres de Moriond
"Electroweak Interactions and Unified Theories"
Les Arcs, France, March 1995

CERN-TH/95-107
May 1995

1 Introduction

Semileptonic decays of B mesons have received a lot of attention in recent years. The decay channel $B \to D^* \ell\, \bar\nu$ has the largest branching fraction of all B-meson decay modes, and large data samples have been collected by various experimental groups. From the theoretical point of view, semileptonic decays are simple enough to allow for a reliable, quantitative description. Yet, the analysis of these decays provides much information about the strong forces that bind the quarks and gluons into hadrons. Schematically, a semileptonic decay process is shown in Fig. 1. The strength of the $b \to c$ transition vertex is governed by the element V_{cb} of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. The parameters of this matrix are fundamental parameters of the Standard Model. A primary goal of the study of semileptonic decays of B mesons is to extract with high precision the values of V_{cb} and V_{ub}. The problem is that the Standard Model Lagrangian describing these transitions is formulated in terms of quark and gluon fields, whereas the physical hadrons are bound states of these degrees of freedom. Hence, an understanding of the transition from the quark to the hadron world is necessary before the fundamental parameters can be extracted from experimental data.

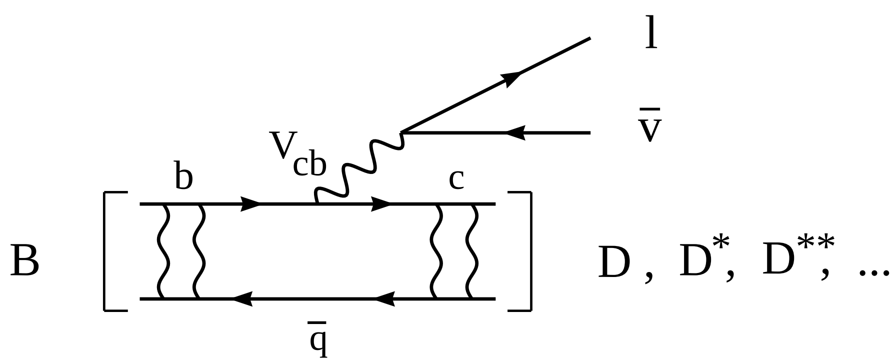

Figure 1: Semileptonic decay of a B meson.

Today, our knowledge of the elements of the CKM matrix, as extracted from direct measurements of flavour-changing transitions, is as follows: The best known entries are V_{ud} and V_{us}, which have an uncertainty of 0.1% and 1%, respectively. The next well known entry is already related to the b-quark; V_{cb} is now determined to an accuracy of 7%. Then follow V_{cd}, V_{cs}, and V_{ub}, with 10%, 20%, and 30% uncertainties, respectively. No direct measurements exist for the matrix elements related to the top quark.

In this talk I discuss the status of the theoretical developments underlying the determination of V_{cb}, both from exclusive and from inclusive semileptonic decays of B mesons.

2 $|\,V_{cb}|$ from Exclusive Decays

With the discovery of heavy-quark symmetry (for a review see Ref. [1] and references therein), it has become clear that the study of the exclusive semileptonic decay mode $\bar{B} \to D^*\ell\,\bar{\nu}$ allows for a reliable determination of $|\,V_{cb}|$, which is free, to a large extent, of hadronic uncertainties [2]–[5]. Model dependence enters this analysis only at the level of power corrections of order $(\Lambda_{\rm QCD}/m_Q)^2$.[1] These corrections can be investigated in a systematic way, using the heavy-quark effective theory [6]. They are found to be small, of order a few per cent.

The analysis consists in measuring the recoil spectrum in the decay $B \to D^*\ell\,\bar{\nu}$. One introduces the kinematic variable

$$w = v_B \cdot v_{D^*} = \frac{E_{D^*}}{m_{D^*}} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}}\,, \qquad (1)$$

which is the product of the four-velocities of the mesons. Here E_{D^*} denotes the recoil energy of the D^* meson in the parent rest frame, and $q^2 = (p_B - p_{D^*})^2$ is the invariant momentum transfer. The differential decay rate is given by [4, 5]

$$\begin{aligned} \frac{\mathrm{d}\Gamma}{\mathrm{d}w} &= \frac{G_F^2}{48\pi^3}\,(m_B - m_{D^*})^2\, m_{D^*}^3 \sqrt{w^2-1}\,(w+1)^2 \\ &\quad \times \left[1 + \frac{4w}{w+1}\,\frac{m_B^2 - 2w\, m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2} \right] |\,V_{cb}|^2\, \mathcal{F}^2(w)\,. \end{aligned} \qquad (2)$$

The function $\mathcal{F}(w)$ denotes the (suitably defined) hadronic form factor for this decay. It is conventional to factorize it in the form $\mathcal{F}(w) = \eta_A\,\widehat{\xi}(w)$, where η_A is a short-distance coefficient, and the function $\widehat{\xi}(w)$ contains the long-distance hadronic dynamics. Apart from corrections of order $\Lambda_{\rm QCD}/m_Q$, this function coincides with the Isgur–Wise form factor [3, 7]. In analogy to the case of light-quark SU(3) flavour symmetry, in which the Ademollo–Gatto theorem protects the $K \to \pi$ transition form factor against first-order symmetry-breaking corrections at $q^2 = 0$ [8], there is a theorem which protects the function $\widehat{\xi}(w)$ against first-order $\Lambda_{\rm QCD}/m_Q$ corrections at the kinematic point of zero recoil ($w = 1$). This is Luke's theorem [9], which determines the normalization of $\widehat{\xi}(w)$ at $w = 1$ up to corrections of order $(\Lambda_{\rm QCD}/m_Q)^2$, i.e. $\widehat{\xi}(1) = 1 + \delta_{1/m^2}$.

The strategy for a precise determination of $|\,V_{cb}|$ is thus to extract the product $|\,V_{cb}|\,\mathcal{F}(w)$ from a measurement of the differential decay rate, and to extrapolate it to $w = 1$ to measure

$$|\,V_{cb}|\,\mathcal{F}(1) = |\,V_{cb}|\,\eta_A\,(1 + \delta_{1/m^2})\,. \qquad (3)$$

The task of theorists is to provide a reliable calculation of the quantities η_A and δ_{1/m^2} in order to turn this measurement into a precise determination of $|\,V_{cb}|$. I will now discuss the status of these calculations.

[1] I shall use $\Lambda_{\rm QCD} \sim 0.25$ GeV as a characteristic low-energy scale of the strong interactions, and m_Q as a generic notation for m_c or m_b.

2.1 Perturbative corrections

The short-distance coefficient η_A takes into account the finite renormalization of the axial vector current arising from virtual gluon exchange. It can be calculated in perturbation theory. At the one-loop order, one finds [10, 2, 11]

$$\eta_A = 1 + \frac{\alpha_s(M)}{\pi} \left(\frac{m_b + m_c}{m_b - m_c} \ln \frac{m_b}{m_c} - \frac{8}{3} \right) . \qquad (4)$$

The scale M of the running coupling constant is not determined at this order. A reasonable choice is to take the average virtuality of the gluon in the one-loop diagrams [12]. In the case of η_A, this so-called BLM scale has been calculated to be $M = 0.51\sqrt{m_b m_c}$ (in the $\overline{\rm MS}$ scheme) [13]. Taking then $m_b = 4.8$ GeV, $m_c/m_b = 0.30 \pm 0.05$, and $\Lambda_{\rm QCD} = 0.25$ GeV for the scale parameter in the two-loop expression for the running coupling constant, one obtains values in the range $\eta_A = 0.950$–0.965.

Several (partial) higher-order calculations have been performed to improve this result. Using renormalization-group techniques, logarithms of the type $(\alpha_s \ln z)^n$, $\alpha_s(\alpha_s \ln z)^n$, and $(m_c/m_b)(\alpha_s \ln z)^n$, where $z = m_c/m_b$, have been resummed to all orders in perturbation theory [14]–[18]. This leads to the somewhat larger value $\eta_A \simeq 0.985$. Another class of higher-order corrections consists of the so-called renormalon chain contributions, which are terms of order $\beta_0^{n-1}\alpha_s^n$ in the perturbative series for η_A. Resumming these terms to all orders gives the lower value $\eta_A \simeq 0.945$ [19].

The main virtue of these partial higher-order calculations is to provide an estimate of the theoretical uncertainty in the value of η_A. Thus, I quote the final result as

$$\eta_A = 0.965 \pm 0.020 \, . \qquad (5)$$

2.2 Power corrections

Hadronic uncertainties enter the determination of $|\,V_{cb}|$ at the level of second-order power corrections, which are expected to be of order $(\Lambda_{\rm QCD}/m_c)^2 \sim 3\%$. For a precision measurement, it is important to understand the structure of these corrections in detail. This is the most complicated aspect of the theoretical analysis, which unavoidably introduces some amount of model dependence. However, since the goal is to estimate an effect which by itself is very small, even a large relative error in δ_{1/m^2} is acceptable.

Three approaches have been suggested to estimate these corrections. The idea of the "exclusive" approach of Falk and myself is to classify all $1/m_Q^2$ operators in the heavy-quark effective theory and to estimate their matrix elements between meson states [20]. This last step is model-dependent. A typical result obtained in this way is $\delta_{1/m^2} = -(3 \pm 2)\%$. In Ref. [1], the error has been increased to

$\pm 4\%$ in order to account for the model dependence and unknown higher-order corrections. A similar result, $-5\% < \delta_{1/m^2} < 0$, has been obtained by Mannel [21].

The idea of the "inclusive" approach of Shifman et al. is to apply the operator product expansion to the B-meson matrix element of the time-ordered product of two flavour-changing currents, and to equate the resulting theoretical expression to a phenomenological expression obtained by saturating the matrix element with physical intermediate states [22]. This leads to sum rules that imply inequalities for the $B \to D^*$ transition form factors. In particular, one obtains the bound $\delta_{1/m^2} < -\frac{1}{8}\,(m_{B^*}^2 - m_B^2)/m_c^2 \simeq -3\%$. The authors of Ref. [22] make an "educated guess" that the value of δ_{1/m^2} is actually much larger, namely $-(9 \pm 3)\%$.

It is possible to combine the above predictions in a "hybrid" approach, which uses the sum rules to put bounds on the hadronic parameters that enter the "exclusive" analysis [5]. One then finds that for all reasonable choices of parameters the results are in the range $-8\% < \delta_{1/m^2} < -3\%$, which is consistent with all previous estimates at the 1σ level. Thus, I quote the final result as

$$\delta_{1/m^2} = -(5.5 \pm 2.5)\% \,. \tag{6}$$

2.3 Determination of $|\,V_{cb}|$

Combining the above results, I obtain for the normalization of the hadronic form factor at zero recoil

$$\mathcal{F}(1) = \eta_A\,(1 + \delta_{1/m^2}) = 0.91 \pm 0.04 \,. \tag{7}$$

To be conservative, I have added the theoretical errors linearly. Three experiments have recently presented new measurements of the product $|\,V_{cb}|\,\mathcal{F}(1)$. When rescaled using the new lifetime values $\tau_{B^0} = (1.61 \pm 0.08)$ ps and $\tau_{B^+} = (1.65 \pm 0.07)$ ps [23], the results are

$$|\,V_{cb}|\,\mathcal{F}(1) = \left\{ \begin{array}{ll} 0.0351 \pm 0.0019 \pm 0.0020\,; & \text{Ref. [24]}, \\ 0.0364 \pm 0.0042 \pm 0.0031\,; & \text{Ref. [25]}, \\ 0.0388 \pm 0.0043 \pm 0.0025\,; & \text{Ref. [26]}, \end{array} \right. \tag{8}$$

where the first error is statistical and the second one systematic. I will follow the suggestion of Ref. [27] and add 0.001 ± 0.001 to these values to account for a small positive curvature[2] of the function $\mathcal{F}(w)$. Taking the weighted average of the experimental results, which is $|\,V_{cb}|\,\mathcal{F}(1) = 0.0370 \pm 0.0025$, and using the theoretical prediction (7), I then obtain

$$|\,V_{cb}| = 0.0407 \pm 0.0027_{\rm exp} \pm 0.0016_{\rm th} \,, \tag{9}$$

[2]The fit results are obtained assuming a linear form of $\mathcal{F}(w)$.

which corresponds to a measurement of $|\,V_{cb}|$ with 7% accuracy.

2.4 Prediction for the slope parameter $\widehat{\varrho}^2$

In the extrapolation of the differential decay rate (2) to zero recoil, the slope of the function $\mathcal{F}(w)$ close to $w = 1$ plays an important role. One defines a parameter $\widehat{\varrho}^2$ by

$$\mathcal{F}(w) = \mathcal{F}(1)\left\{1 - \widehat{\varrho}^2\,(w-1) + \dots\right\}. \tag{10}$$

It is important to distinguish $\widehat{\varrho}^2$ from the slope parameter ϱ^2 of the Isgur–Wise function. They differ by corrections that break the heavy-quark symmetry. Whereas the slope of the Isgur–Wise function is a universal, mass-independent parameter, the slope of the physical form factor depends on logarithms and inverse powers of the heavy-quark masses. The relation between the two parameters is [5]

$$\widehat{\varrho}^2 = \varrho^2 + (0.16 \pm 0.02) + O(1/m_Q)\,. \tag{11}$$

An estimate of the $\Lambda_{\rm QCD}/m_Q$ corrections to this relation is model-dependent and thus has a large theoretical uncertainty. I shall not attempt it here.

The slope parameter of the Isgur–Wise function, ϱ^2, is constrained by the Bjorken [28, 29] and Voloshin [30] sum rules. At the tree level, it was known for a long time that $1/4 < \varrho^2 <\approx 0.75$. However, only recently Grozin and Korchemsky have shown how to include perturbative and nonperturbative corrections to these bounds [31, 32]. The results are shown in Fig. 2. Here, the scale parameter μ has to be chosen large enough for the operator product expansion to be well defined, but it is otherwise arbitrary. Assuming that values $\mu > 0.8$ GeV are sufficiently large, one finds that ϱ^2 is constrained to be very close to 0.6. This value is in good agreement with earlier predictions obtained from QCD sum rules, which gave $\varrho^2 = 0.7 \pm 0.1$ [1, 33, 34].

From Fig. 2, and using (11), I conclude that

$$\varrho^2 = 0.65 \pm 0.15\,, \qquad \widehat{\varrho}^2 = 0.8 \pm 0.3\,. \tag{12}$$

This prediction compares well with the average value observed experimentally, which is $\widehat{\varrho}^2 = 0.87 \pm 0.12$ [24]–[26].

3 $|\,V_{cb}|$ from Inclusive Decays

Complementary to the analysis of exclusive decays is the study of the inclusive semileptonic decay rate for $B \to X\,\ell\,\bar\nu$. Since $|\,V_{ub}/V_{cb}|^2 < 1\%$, one can to very good approximation neglect the contribution of charmless final states and consider X to be a hadronic state containing a charm particle. An obvious advantage of inclusive decays is the existence of high-statistics data samples. From the

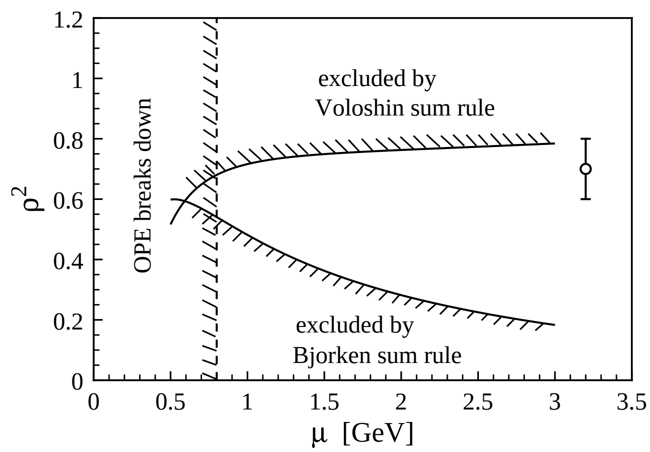

Figure 2: Bounds for the slope parameter ϱ^2 following from the Bjorken and Voloshin sum rules, from Ref. [32]. The point with the error bar shows the QCD sum rule prediction.

theoretical point of view, summing over many final states eliminates part of the hadronic uncertainty.

As in the exclusive case, the framework for the theoretical description of inclusive decays is provided by the heavy-quark expansion. It could be shown that the leading term in this expansion reproduces the free-quark decay model, while the nonperturbative corrections to this model can be systematically included in an expansion in powers of $1/m_b$ [35]–[41]. The total semileptonic decay rate can be written as

$$\Gamma = \frac{G_F^2}{192\pi^3}\,|\,V_{cb}|^2\,m_b^5\,\left\{\,\left(1+\frac{\lambda_1+3\lambda_2}{2m_b^2}\right)f(m_c/m_b)-\frac{6\lambda_2}{m_b^2}\left(1-\frac{m_c^2}{m_b^2}\right)^4 + \frac{\alpha_s(M)}{\pi}\,g(m_c/m_b)+\dots\right\}, \tag{13}$$

where the ellipsis represents terms of higher order in $1/m_b$ or α_s. In this expression, m_b and m_c denote the pole masses (defined to the appropriate order in perturbation theory) of the heavy quarks, $f(m_c/m_b)\simeq 0.52$ and $g(m_c/m_b)\simeq -0.87$ are kinematic functions, and λ_1 and λ_2 are nonperturbative hadronic parameters. I will now discuss the theoretical uncertainties in the evaluation of (13).

3.1 Perturbative corrections

Let me first discuss the uncertainty due to unknown higher-order perturbative corrections. Only the correction of order α_s is known exactly [42]. However, recently Luke et al. have computed the part of the next-order term that depends on the number of light-quark flavours [43]. In the $\overline{\rm MS}$ scheme, the result is

$$\frac{\Gamma}{\Gamma_0} = 1 - 1.67\,\frac{\alpha_s(m_b)}{\pi} - (1.68\,\beta_0 + \ldots)\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 + \ldots = 1 - 0.11 - 0.06 - \ldots\,, \quad (14)$$

where Γ_0 is the decay rate at the tree level, and $\beta_0 = 11 - \frac{2}{3}\,n_f$ is the first coefficient of the β-function. If one uses this partial calculation to estimate the uncertainty, which for an asymptotic series is given by the size of the last term to be included, one finds $(\delta\Gamma/\Gamma)_{\rm pert} \simeq 6\%$, which is in good agreement with an estimate of the renormalization-scale and -scheme dependence by Ball and Nierste [44]. Recently, Ball et al. have performed an all-order resummation of the terms of order $\beta_0^{n-1}\alpha_s^n$ for the above series [45]. They find that the effect of higher-order terms is important and leads to $\Gamma/\Gamma_0 = 0.77 \pm 0.05$. Note that this corresponds to an effective scale $M \sim 1$ GeV in (13), which is rather low. The corresponding value of the coupling constant is $\alpha_s(M) \simeq 0.43$. It is difficult to derive a reliable error estimate from these analyses, but I think a reasonable number is

$$\left(\frac{\delta\Gamma}{\Gamma}\right)_{\rm pert} \simeq 10\%\,. \quad (15)$$

3.2 Power corrections

The leading power corrections in the expression for the inclusive decay rate appear at order $1/m_b^2$. They are proportional to two hadronic parameters with a simple physical interpretation: λ_1 is related to the average momentum of the b-quark inside a B meson at rest, and λ_2 is proportional to the vector–pseudoscalar mass splitting. I shall use

$$\begin{aligned} \lambda_1 &= -\langle\,\vec p_b^{\,2}\rangle = -(0.4 \pm 0.2)~\text{GeV}^2\,, \\ \lambda_2 &= \frac{m_{B^*}^2 - m_B^2}{4} = 0.12~\text{GeV}^2\,. \end{aligned} \quad (16)$$

The value of λ_1 is a compromise between the theoretical estimates obtained in Refs. [46, 47].

The power corrections reduce the total decay rate by $-(4.2\pm0.5)\%$, which is a rather small effect. The uncertainty in the value of λ_1 introduces an uncertainty on the value of Γ of 0.6%, which is almost negligible. I increase this value in order to account for higher-order power corrections, which I expect to be of order $(\Lambda_{\rm QCD}/m_c)^3 \sim 0.5\%$, and quote

$$\left(\frac{\delta\Gamma}{\Gamma}\right)_{\rm power} \simeq 1\%\,. \quad (17)$$

This is the smallest contribution to the total theoretical uncertainty.

3.3 Dependence on quark masses

Another source of nonperturbative uncertainty results from the appearance of the heavy-quark masses in the expression for the inclusive decay rate. The pole masses of the bottom and charm quarks have an uncertainty of at least several hundred MeV. Since the rate is proportional to m_b^5, this seems to be a severe limitation. However, it has been pointed out that the actual uncertainty is lower due to a strong correlation between the values of the two heavy-quark masses [22]. In fact, one should consider the decay rate as a function of m_b and of the difference $\Delta m = m_b - m_c$. I shall assume that m_b has an uncertainty of 300 MeV. However, the mass difference is known to much higher precision. Using the heavy-quark expansion, one can derive that [20]

$$\Delta m = m_b - m_c = (\overline{m}_B - \overline{m}_D) \left\{ 1 - \frac{\lambda_1}{2\overline{m}_B \overline{m}_D} + O(1/m_Q^3) \right\} , \qquad (18)$$

where $\overline{m}_B = \frac{1}{4}(m_B + 3m_{B^*}) = 5.31$ GeV and $\overline{m}_D = \frac{1}{4}(m_D + 3m_{D^*}) = 1.97$ GeV denote the spin-averaged meson masses. This relation leads to

$$\Delta m = (3.40 \pm 0.03 \pm 0.03) \text{ GeV} , \qquad (19)$$

where the first error reflects the uncertainty in the value of λ_1, and the second one takes into account unknown higher-order corrections. Hereafter, I shall assume an uncertainty of 60 MeV in the value of Δm.

In Fig. 3, I show the dependence of the decay rate on these two parameters, using the value $\alpha_s(M) = 0.4$ for the strong coupling constant in (13). Clearly, the variation with Δm is much stronger than the variation with m_b. For $m_b = 4.8$ GeV and $\Delta m = 3.4$ GeV, I find the partial derivatives $\delta\Gamma/\Gamma \simeq 5.73\,\delta(\Delta m)/\Delta m$ and $\delta\Gamma/\Gamma \simeq -0.55\,\delta m_b/m_b$. Since the errors in m_b and Δm are essentially uncorrelated, this leads to

$$\left(\frac{\delta\Gamma}{\Gamma} \right)_{\text{masses}} = \sqrt{\left(0.101\,\frac{\delta(\Delta m)}{60 \text{ MeV}} \right)^2 + \left(0.034\,\frac{\delta m_b}{300 \text{ MeV}} \right)^2} \simeq 11\% . \qquad (20)$$

Note that this is dominated by the (rather small) uncertainty in the mass difference Δm.

3.4 Determination of $|V_{cb}|$

Adding the above errors linearly and taking the square root, I conclude that the theoretical uncertainty in the extraction of $|V_{cb}|$ from inclusive decays is

$$\frac{\delta|V_{cb}|}{|V_{cb}|} \simeq 11\% . \qquad (21)$$

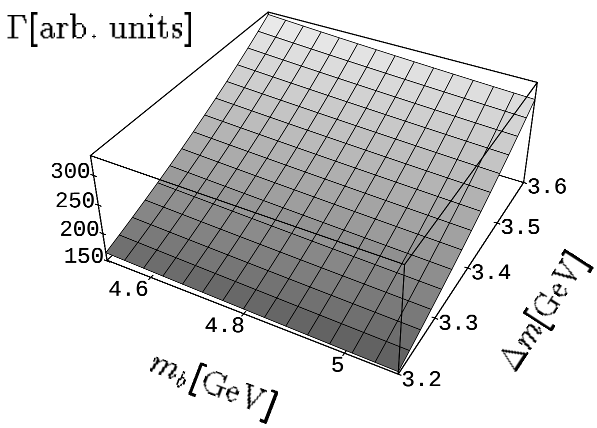

Figure 3: Dependence of the inclusive semileptonic decay rate Γ on the parameters m_b and $\Delta m = m_b - m_c$.

From an analysis of the experimental data, one then obtains [27]

$$|\,V_{cb}| = \begin{cases} 0.039 \pm 0.001_{\rm exp} \pm 0.005_{\rm th}\ ; & \text{measurements at } \Upsilon(4s), \\ 0.042 \pm 0.002_{\rm exp} \pm 0.005_{\rm th}\ ; & \text{measurements at } Z^0. \end{cases} \tag{22}$$

The theoretical uncertainty in these numbers is somewhat larger than in the case of the exclusive analysis; however, the experimental errors are smaller.

4 Summary

The most precise measurements of the element V_{cb} of the CKM matrix come from the analysis of semileptonic decays of B mesons. From the measurement of the recoil spectrum in the exclusive channel $B \to D^*\ell\,\bar\nu$, one obtains

$$|\,V_{cb}|_{\rm excl} = 0.041 \pm 0.003_{\rm exp} \pm 0.002_{\rm th}\,, \tag{23}$$

where the theoretical error is dominated by the uncertainty in the calculation of nonperturbative power corrections of order $(\Lambda_{\rm QCD}/m_Q)^2$. From the measurement of the total inclusive decay rate, on the other hand, one finds

$$|\,V_{cb}|_{\rm incl} = 0.040 \pm 0.001_{\rm exp} \pm 0.005_{\rm th}\,. \tag{24}$$

In this case, the main theoretical uncertainty comes from the uncertainty in the value of the quark mass difference $m_b - m_c$, as well as from higher-order perturbative corrections.

Given the fact that both methods are very different both from the experimental and from the theoretical point of view, it is most satisfying that the results are in perfect agreement. Combining them, I obtain the final value

$$|\, V_{cb}| = 0.041 \pm 0.003\,. \tag{25}$$

References

[1] M. Neubert, Phys. Rep. **245**, 259 (1994); in: *The Building Blocks of Creation*, Proc. of the 1993 Theor. Adv. Study Inst. in Elementary Particle Physics, Boulder, Colorado, 1993, eds. S. Raby and T. Walker (World Scientific, Singapore, 1994), p. 125.

[2] M.B. Voloshin and M.A. Shifman, Yad. Fiz. **47**, 801 (1988) [Sov. J. Nucl. Phys. **47**, 511 (1988)].

[3] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); *ibid.* **237**, 527 (1990).

[4] M. Neubert, Phys. Lett. B **264**, 455 (1991).

[5] M. Neubert, Phys. Lett. B **338**, 84 (1994).

[6] H. Georgi, Phys. Lett. B **240**, 447 (1990).

[7] A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. B **343**, 1 (1990).

[8] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13**, 264 (1964).

[9] M.E. Luke, Phys. Lett. B **252**, 447 (1990).

[10] J.E. Paschalis and G.J. Gounaris, Nucl. Phys. B **222**, 473 (1983); F.E. Close, G.J. Gounaris and J.E. Paschalis, Phys. Lett. B **149**, 209 (1984).

[11] M. Neubert, Nucl. Phys. B **371**, 149 (1992).

[12] S.J. Brodsky, G.P. Lepage and P.B. Mackenzie, Phys. Rev. D **28**, 228 (1983); G.P. Lepage and P.B. Mackenzie, Phys. Rev. D **48**, 2250 (1993).

[13] M. Neubert, Phys. Lett. B **341**, 367 (1995).

[14] H.D. Politzer and M.B. Wise, Phys. Lett. B **206**, 681 (1988); *ibid.* **208**, 504 (1988).

[15] X. Ji and M.J. Musolf, Phys. Lett. B **257**, 409 (1991).

[16] D.J. Broadhurst and A.G. Grozin, Phys. Lett. B **267**, 105 (1991).

[17] A.F. Falk and B. Grinstein, Phys. Lett. B **247**, 406 (1990).

[18] M. Neubert, Phys. Rev. D **46**, 2212 (1992).

[19] M. Neubert, CERN-TH.7487/94 (1994) [hep-ph/9412265], to appear in Phys. Rev. D.

[20] A.F. Falk and M. Neubert, Phys. Rev. D **47**, 2965 and 2982 (1993).

[21] T. Mannel, Phys. Rev. D **50**, 428 (1994).

[22] M. Shifman, N.G. Uraltsev and A. Vainshtein, Phys. Rev. D **51**, 2217 (1995).

[23] F. DeJongh, in: Proc. of the XXVII Int. Conf. on High Energy Physics, Glasgow, Scotland, July 1994, eds. P.J. Bussey and I.G. Knowles (IOP Publ. Ltd., Bristol, 1995), Vol. 2, p. 451.

[24] T.E. Browder (CLEO Collaboration), *ibid.* Vol. 2, p. 1117.

[25] I.J. Scott (ALEPH Collaboration), *ibid.* Vol. 2, p. 1121.

[26] H. Albrecht et al. (ARGUS Collaboration), Z. Phys. C **57**, 533 (1993).

[27] R. Patterson, in: Proc. of the XXVII Int. Conf. on High Energy Physics, Glasgow, Scotland, July 1994, eds. P.J. Bussey and I.G. Knowles (IOP Publ. Ltd., Bristol, 1995), Vol. 1, p. 149.

[28] J.D. Bjorken, in: *Results and Perspectives in Particle Physics*, Proc. of the 4th Rencontres de Physique de la Vallé d'Aoste, La Thuile, Italy, 1990, ed. M. Greco (Editions Frontières, Gif-sur-Yvette, 1990), p. 583.

[29] N. Isgur and M.B. Wise, Phys. Rev. D **43**, 819 (1991).

[30] M.B. Voloshin, Phys. Rev. D **46**, 3062 (1992).

[31] A.G. Grozin and G.P. Korchemsky, OUT-4102-53 (1994) [hep-ph/9411323].

[32] G.P. Korchemsky and M. Neubert, CERN-TH/95-108 (1995), in preparation.

[33] E. Bagan, P. Ball and P. Gosdzinsky, Phys. Lett. B **301**, 249 (1993).

[34] M. Neubert, Phys. Rev. D **47**, 4063 (1993).

[35] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. B **247**, 399 (1990).

[36] I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B **293**, 430 (1992); I.I. Bigi, M.A. Shifman, N.G. Uraltsev and A.I. Vainshtein, Phys. Rev. Lett. **71**, 496 (1993).

[37] B. Blok, L. Koyrakh, M.A. Shifman and A.I. Vainshtein, Phys. Rev. D **49**, 3356 (1994).

[38] A.V. Manohar and M.B. Wise, Phys. Rev. D **49**, 1310 (1994).

[39] A.F. Falk, M. Luke and M.J. Savage, Phys. Rev. D **49**, 3367 (1994).

[40] T. Mannel, Nucl. Phys. B **413**, 396 (1994).

[41] M. Neubert, Phys. Rev. D **49**, 3392 and 4623 (1994).

[42] Y. Nir, Phys. Lett. B **221**, 184 (1989).

[43] M. Luke, M.J. Savage and M.B. Wise, Phys. Lett. B **345**, 301 (1995).

[44] P. Ball and U. Nierste, Phys. Rev. D **50**, 5841 (1994).

[45] P. Ball, M. Beneke and V.M. Braun, CERN-TH/95-26 (1995) [hep-ph/9502300].

[46] P. Ball and V.M. Braun, Phys. Rev. D **49**, 2472 (1994).

[47] M. Neubert, Phys. Lett. B **322**, 419 (1994).