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NUMERICAL SIMULATIONS OF HEAD-TAIL INSTABILITIES IN THE HERA PROTON RING*

FRANCESCA GALLUCCIO

Istituto Nazionale di Fisica Nucleare — Sezione di Napoli, Mostra d'Oltremare, Pad. 20, I-80125 Napoli, Italy

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A transverse instability with a clear dependence on chromaticity has been observed in the proton ring of the HERA collider at DESY since spring 1992. In order to try to understand some of the features of this phenomenon, a simulation study, involving multi-particle tracking in the presence of the realistic wake-field of an accelerating cavity, has been performed. The results of this study are shown here, and are compared with what can be expected from Sacherer's theory of head-tail instabilities.

KEY WORDS: Chromaticity, head-tail, HERA, instability, particle tracking

1 INTRODUCTION

HERA (Hadron Electron Ring Anlage) is a unique particle collider, 6335 m long, in which protons and electrons circulating in separate rings are brought into collision at the kinetic energies of 820 and 27 GeV, respectively.

Some of the machine parameters have not attained their design value yet, but they are approaching it closer and closer year after year. In particular, the present number of particles per bunch, N_b, in the proton ring is still a factor 2.2 below the design value (see Table 1); nevertheless, already in May 1992 a transverse coherent excitation of the proton beam at the injection energy of 40 GeV was observed for the first time, affecting the motion in both the horizontal and the vertical plane, indifferently. Since then, much effort has been dedicated to the understanding of this phenomenon.[1]

Soon it was found that the instability can be artificially stimulated by decreasing the Landau damping in the machine, either by scraping the beam transversely by means of collimators, or compressing it longitudinally by means of extra RF voltage; so, several systematic experimental studies could be pursued. All possible sources of external excitation were examined, and excluded. The instability was also proved to be independent of the choice of betatron tunes. It was shown that, although extra multi-bunch effects can also be present, it is mainly a single bunch effect. Finally a clear chromaticity dependence could

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be evidenced, the instability appearing for negative ξ values; however, quite unexpectedly in the beginning, its strength was decreasing when moving towards larger negative values ($\xi = \Delta Q / \frac{\Delta p}{p} < -6$), which made someone doubt that it could be the case of an head-tail instability.

Although, as just said, the machine can be safely operated at large negative chromaticity, it is obviously essential, also in view of the future upgrades of the machine, to achieve the best possible understanding of the phenomena observed.

For this purpose, computer simulations can be very helpful, allowing different effects to be disentangled and to be analyzed separately.

The numerical simulations of the motion of a bunch of protons in the presence of the wake-field induced by a single RF cavity will be considered in the following.

2 NUMERICAL SIMULATIONS

In order to explore the mechanism leading to emittance growth, to clarify the role played by chromaticity in the instability at HERA-p, and to check if the experimental observations could be consistent with what foreseen by the standard theory of head-tail instabilities[2] (a short reminder of the theory, including the relevant formulae and definitions, is given in the Appendix), a particle tracking simulation study has been performed.

2.1 *The tracking code*

A dedicated multi-particle tracking computer code has been written for this study.

Very much stress has been put in the modeling of the interaction between the beam and the machine impedance in order to make it as realistic as possible, whilst a very simple model of the machine has been adopted, including linearized synchrotron motion, and betatron motion in only one transverse direction.

TABLE 1: Some of the HERA proton ring parameters at injection time.

L	$=$	6336 m			
E_{inj}	$=$	40 GeV			
I_{achieved}	$\simeq$	60 mA	$(I_{\text{design}}$	$=$	160 mA)
$N_{b\,\text{achieved}}$	$\simeq$	4.5×10^{10}	$(N_{b\,\text{design}}$	$=$	$10^{11})$
σ_z	$\simeq$	0.3 m			
$\Delta p/p$	$\simeq$	0.2×10^{-3}			
$\varepsilon_N(2\sigma)$	$\simeq$	$10\times10^{-6}\pi$ m rad			
f_{rev}	$=$	47.304 KHz			
f_{RF_1}	$=$	52 MHz	$(h$	$=$	1100)
f_{RF_2}	$=$	208 MHz	$(h$	$=$	4400)
Q_s	$\simeq$	0.56×10^{-3}			
η	$=$	0.74×10^{-3}			

Thus, the motion in the longitudinal phase-space $(z - \Delta p/p)$ is assumed purely harmonic, with phase advance from turn to turn simply given by:

$$\Delta\phi_T = -2\pi Q_s \tag{1}$$

Q_s being the synchrotron tune.

For each particle, the one turn phase advance in the transverse phase-space is, instead, given by:

$$\Delta\mu_T = 2\pi Q = 2\pi(Q_o + \xi\frac{\Delta p}{p} + DA^2) \tag{2}$$

where Q_o is the nominal unperturbed betatron tune, $\xi\frac{\Delta p}{p}$ is the momentum-dependent tune-shift, which changes from turn to turn, depending on the current single-particle momentum in longitudinal phase-space, A is the single-particle amplitude normalized to the beta function $\beta = 1$, and D is the detuning coefficient, set to zero in the calculations presented here, to take into account the amplitude-dependent detuning in the accelerator due to non-linear elements such as octupoles.

In order to describe the interaction of the particles with the transverse impedance, instead, any wake potential can be given in form of numerical input as a function of the longitudinal coordinate, over the appropriate range, and with the desired resolution. At each turn this potential will be associated with each particle, and the wake field, depending on its current transverse position, will be recalculated; finally, to each particle, the transverse kick corresponding to the field of all preceeding particles will be applied.

An optional graphical display in the tracking code allows the evolution of the bunch to be followed turn by turn both in phase-space and in real space.

2.2 *Simulation environment*

The parameters of the HERA proton machine which are relevant for this study are listed in Table 1.

There are two accelerating systems in the HERA proton ring. Being amongst the most critical elements, the four radio-frequency cavities of the 208 MHz system were carefully measured and optimized;[3] their electromagnetic parameters were also computed numerically,[4] with results in good agreement with measurements. In order to analyze the influence of the 208 MHz cavity on the beam, the data from calculations have been used, and the first 15 transverse modes of one cavity have been considered to calculate the transverse impedance and the wake potential for the simulations: they are shown in Figure 1.

2.3 *Chromaticity scans*

As already said, single bunch effects seem to be dominating in HERA-p, so the case of a single bunch has been treated with the highest priority.

Scans in chromaticity, similar to those made during the experimental studies, have been produced, recording the corresponding amplitude growth rates.

Three different cases have been treated quite thoroughly, with the bunch and machine parameters of Table 2. In all these cases the chromatic frequency shift of the bunch spectrum,

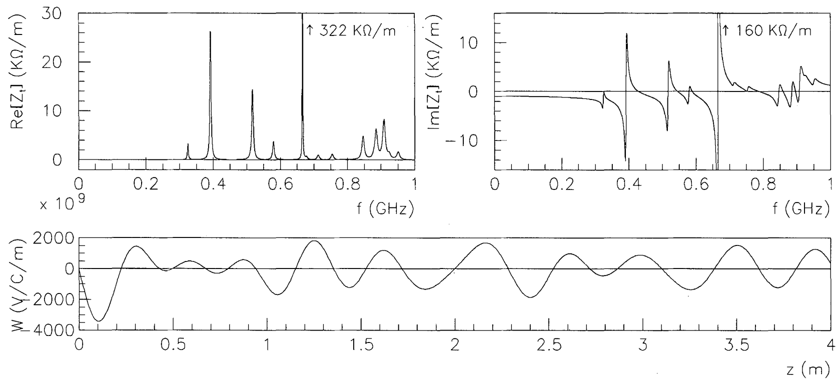

FIGURE 1: Transverse impedance Z_t and wake potential W of one HERA-p 208 MHz cavity.

$f_\xi = \frac{\xi}{\eta} f_{\text{rev}}$, is comparable to the one of the HERA proton ring at injection, and so it is also the related frequency range.

The starting transverse emittance has been kept fixed at a large enough but realistic value. In order to increase artificially the instability growth time, and to magnify the effect of the wake-field with the aim of reducing the CPU time needed for tracking, a bunch charge 100 times the design value has been used; on the other hand, only one cavity has been considered instead of four. Therefore, the final results will have to be scaled by a factor 62 roughly to be compared with the present situation in HERA-p, and by a factor 25 to give predictions for the machine with the final bunch population.

An ensemble of particles with gaussian distributions in both transverse and longitudinal planes has been tracked for many turns in the simple machine described above, letting the particles interact with each other through the wake-field induced by the 208 MHz cavity. The σ_x of the bunch transverse distribution has been recorded turn by turn, and its growth rate has been estimated by means of an exponential fit.

The results of the scan in chromaticity for Case No. 1 are summarized in Figure 2b, where the continuous line represents the amplitude growth rate obtained from the numerical simulations, the dotted lines represent the theoretical growth rates of the oscillation modes from $l = 0$ to $l = 5$ order, according to Eq. 7 in the Appendix, and the points marked with a star will be referred to in Section 2.4; the frequency shift corresponding to chromaticity is given on the top axis, while the growth rate is given in units of both inverse number of turns (left axis) and inverse seconds (right axis).

TABLE 2: Machine and bunch parameters used in simulations.

Case No.	No. of Cavities	N_b (10^{11})	ε_N (π mm mrad)	σ_z (m)	$\Delta p/p$ (10^{-3})	Q_s (10^{-3})	η (10^{-3})
1	1	100	65	0.1	1.	5.6	0.56
2	1	100	65	0.268	0.2	0.56	0.744
3	1	100	65	1.34	1.	0.56	0.744

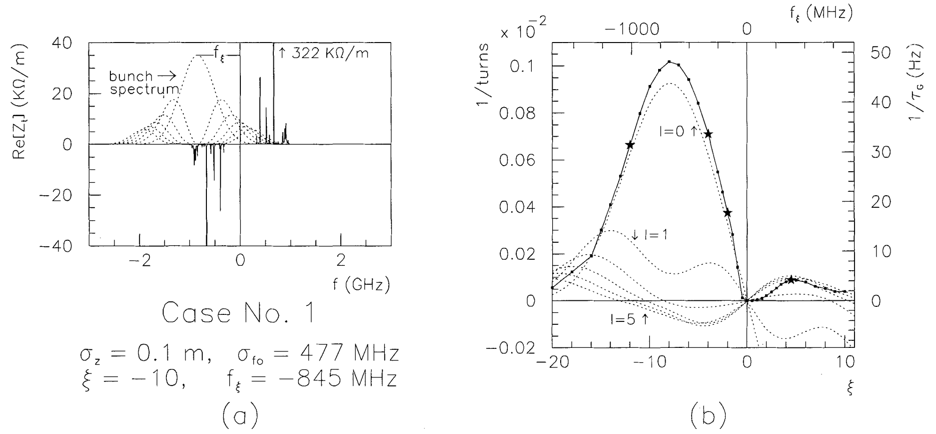

FIGURE 2: (a) Real part of the impedance, compared with the first six modes of the bunch power spectrum for Case No. 1. (b) Amplitude growth rate versus chromaticity from numerical simulations (continuous line), and from theory (dotted lines, computed mode by mode).

As a reference, in Figure 2a the same modes of the bunch power spectrum are superimposed to the real part of the impedance for a chromaticity $\xi = -10$, corresponding to a chromatic frequency shift $f_\xi = 845$ MHz (the bunch modes are shown in arbitrary units and their magnitude is scaled as $1/1{+}l$). The bunch spectrum is quite broad, $\sigma_{f_o} = 477$ MHz, and the 0^{th} order mode overlaps with all the negative peaks of the impedance for most of the range of negative chromaticities we are interested in, resulting in a broad-band-like effective impedance $Z_{\perp\text{eff}}(f_\xi)$ (Eq. 5 in the Appendix); the maximum superposition, i.e. the maximum effective impedance, occurs for $\xi \simeq -8$, which corresponds to $f_\xi \simeq 670$ MHz.

The simulation results are in quite good agreement with what one might expect from theory. For negative chromaticities the 'experimental' curve follows rather well the theoretical one for the 0^{th} order oscillation mode, which has the fastest growth, and the slight discrepancy can be attributed to the contribution of the higher order modes; whilst for positive chromaticities the amplitude growth from simulations follows closely the theoretical curves for those modes having the largest growth rate.

Analogous pictures are shown in Figure 3 for Case No. 2, whose parameters are the closest ones to the present HERA-p injection parameters.

Here the bunch spectrum is a bit narrower, thus the strong superposition of the 0^{th} order mode with the impedance is limited to a smaller range in chromaticity/frequency, therefore the overall amplitude growth is somewhat slower than in Case No. 1.

Also in this case the simulation results are consistent with the theoretical expectations: for negative chromaticities the contribution of the higher order modes to the amplitude growth is more pronounced than in Case No. 1, and it can be possibly recognized in the small distortions of the simulation results from a smooth curve around their maximum; for positive chromaticities we see that modes at least up to order 5^{th} are damped, and the tiny growth rates recorded are to be due to even higher order oscillation modes.

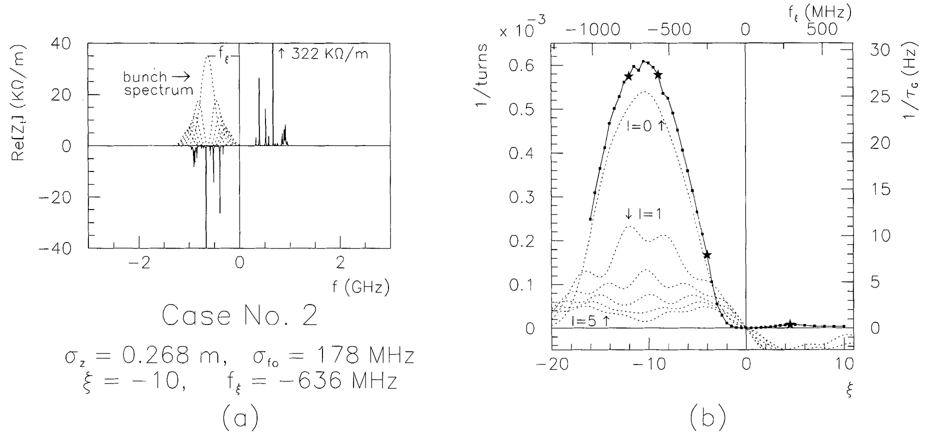

FIGURE 3: (a) Real part of the impedance, compared with the first six modes of the bunch power spectrum for Case No. 2. (b) Amplitude growth rate versus chromaticity from numerical simulations (continuous line), and from theory (dotted lines).

Both from Case No. 1 and from Case No. 2, we can see that, although it might not be very common in other particle accelerators due either to the allowed range of chromaticity or to the chromatic frequency shift involved, a decrease of the instability strength towards very large negative values of chromaticity, as the one observed in HERA, is to be expected, and should not be surprising at all.

After scaling the results from Case No. 2 to the correct number of cavities and to the present bunch population, if the 208 MHz cavities were dominating in the global HERA-p transverse impedance, in the real machine we should expect a maximum amplitude growth time of the order of some seconds, or even more due to the presence in the machine of sources of Landau damping which have not been included in these simulations. This time is considerably longer than the typical growth times measured in the machine,[5] which suggests that there are other elements in the accelerator, whose interaction with the beam is stronger than the cavity interaction.

A completely different situation is found in Case No. 3, which is shown in Figure 4. The bunch is much longer now, and its frequency spectrum is even narrower than the distance between the strongest impedance resonances, as it is shown in Figure 4a, where, for the sake of clearness, only the first two modes are displayed. This allows the bunch spectrum to superpose with one big peak of the impedance at a time, and therefore the effective impedance to exhibit its original resonating characteristics. Varying the chromaticity in the negative range, the bunch spectrum is displaced in frequency, and only at frequencies around the impedance resonances a significant, although smaller than in Cases No. 1 and 2, amplitude growth, and therefore an instability, can be detected. The three narrow peaks at $\xi = -6, -8$ and -10.5 correspond to the three strongest resonances of the transverse impedance at 391, 517 and 666 MHz respectively, while in the broader peak around $\xi = -14$ the weaker resonances at 846, 885 and 908 MHz are hidden, because the bunch spectrum width

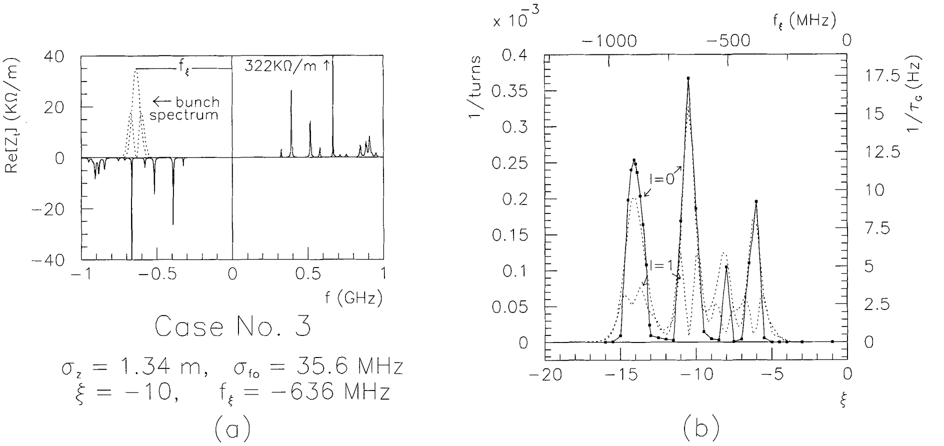

FIGURE 4: (a) Real part of the impedance, compared with the first two modes of the bunch power spectrum for Case No. 3. (b) Amplitude growth rate versus chromaticity from numerical simulations (continuous line), and from theory (dotted lines).

of $\sigma_{f_o} = 36$ MHz is still too large to resolve among them. For positive chromaticities all modes are damped, and, as expected, no amplitude growth was seen.

Measuring the head-tail instability growth rate as a function of chromaticity is one of the most common methods to evaluate experimentally the overall transverse machine impedance. However, from the comparison between the results of Case No. 3 and those of the other two cases, we can see that the same device can interact very differently with the bunch, depending on the length of the bunch itself, sometimes even showing a misleading behaviour. We have found out, in fact, that by means of a short bunch with a wide frequency spectrum one can only evidence a broad-band-like behaviour of the impedance, all resonances closer than the bunch spectrum width remaining hidden. On the other hand, while a longer bunch allows to sample the impedance with more resolution, the growth rates involved are smaller, and therefore much more difficult to detect, considering also the effects of Landau damping that are always present in an accelerator.

The effective resolving power of the bunch spectrum should always be kept in mind when trying to extract information on impedance from instability growth rate measurements.

2.4 *Bunch profiles*

The bunch profile in the $(z - x)$ plane changes from turn to turn under the effect of its own wake-field. It is interesting, and it can be quite instructive, to observe the wiggling of the bunch profile in time, and to compare it with what should be expected from the theory of head-tail instabilities.

In each of the pictures in Figure 5 the transverse position of the center of charge as a function of the longitudinal position along the bunch is recorded, at the same azimuthal

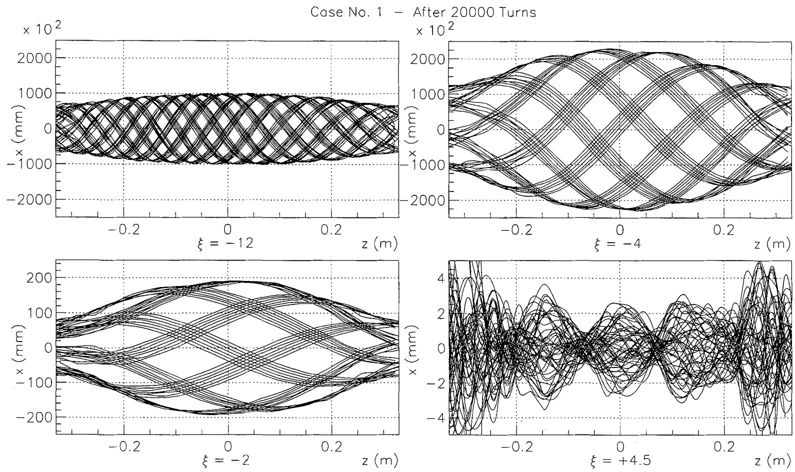

FIGURE 5: Chromaticity dependence of z-x bunch profile in simulation Case No. 1, after 20000 turns and for 50 successive turns.

position in the machine, for 50 successive turns for simulated Case No. 1. The bunch profiles for the four values of chromaticity marked with a star in Figure 2b are displayed. The resulting picture is very similar to the image of the bunch profile, as it could be detected over many turns in a real machine by means of a difference pick-up (see, for example, the measurements taken at the CERN PS Booster[6]). Remembering that the order of the oscillation mode is equal to the number of nodes found in this kind of picture, one can easily say that, although with very different amplitudes (notice the vertical scales), in the three cases with $\xi < 0$ a 0^{th} order mode is excited: note also that, after 20000 turns and without external damping, most of the beam would be lost in all of these cases. In the case with $\xi > 0$, instead, a mixture of high order modes can be seen, among which a mode of the 4^{th} order seems to be developing.

The order of the excited modes, as deduced from the observation of the wiggling of the bunch profile, are in quite good agreement with the theoretical expectations (see the dotted curves in Figure 2b): indeed, for negative chromaticities the mode of order 0^{th} grows much faster than the other modes, and, therefore, it is dominating in the long term; in the relevant range of positive chromaticities, instead, the 0^{th} and the 1^{st} order modes are strongly damped, the 2^{nd} order mode grows very slowly, and, amongst the other higher order modes (it could be seen by zooming on the picture) the 4^{th} order one has slightly faster growth at $\xi = 4.5$.

Similar bunch profile sequences are shown in Figure 6 for Case No. 2; also here the results from particle tracking are in agreement with what is expected theoretically. In the three cases with negative chromaticity we find, with more or less clear signature (see below), a 0^{th} order mode of oscillation. In the case with positive chromaticity, instead, only an unresolved mixture of modes is found: this is consistent with the fact that all modes up to order 5 or

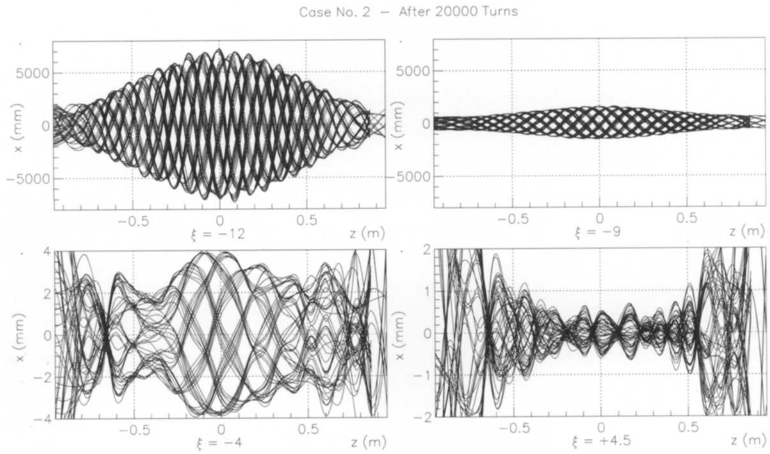

FIGURE 6: Chromaticity dependence of z-x bunch profile in simulation Case No. 2, after 20000 turns and for 50 successive turns.

more should be damped for positive chromaticity (Figure 3b), and only modes with rather high order and very small growth rate could be possibly excited.

The case with $\xi = -4$ displayed in Figure 6 is an interesting one, because it allows us to follow the build up of the dominating mode of oscillation. In Figure 7 the 50 consecutive bunch profiles have been recorded at the start-up of the instability (turn 0), and then every 10000 turns: we can see how, among the mixture of modes, the 0^{th} order one, whose growth rate is larger, develops, emerging over the other modes until it finally prevails, driving the amplitude growth.

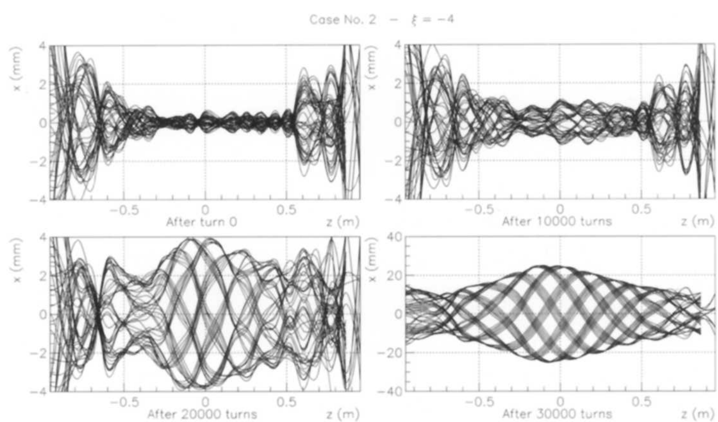

FIGURE 7: Bunch profiles for Case No. 2, ξ=−4, at different times: emerging of the 0^{th} order head-tail mode.

3 CONCLUSION

Multi-particle tracking simulations with a realistic wake-field are in quite remarkable agreement with the predictions of the theory of transverse bunched-beam instabilities in the presence of the corresponding impedance. In addition to these predictions computer simulations can provide valuable insight of the global motion of the bunch, also allowing us to estimate the overall growth rates, all modes included.

The single-bunch transverse excitation of the beam seen in the HERA proton ring at injection can be understood in terms of head-tail instability. The chromaticity dependence of the observed amplitude growth rate, including the weakening of the instability at large negative ξ, is fully consistent with what should be expected according to both theory and numerical simulations.

It remains now to investigate about the multi-bunch effects which have been detected recently.

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APPENDIX A: SACHERER'S THEORY ABOUT TRANSVERSE INSTABILITIES IN BUNCHED BEAMS

The oscillation frequency of the l^{th} azimuthal mode of a single bunch is given by:

$$\omega_{nl} = (n + Q)\omega_{\text{rev}} + l\omega_s + \omega_\xi \qquad l = 0, \pm 1, \pm 2, \ldots \tag{3}$$

where:

Q	is the betatron tune,
$\omega_{\text{rev}} = 2\pi f_{\text{rev}}$	is the revolution frequency,
$\omega_\xi = \frac{\xi}{\eta}\omega_{\text{rev}}$	is the so-called chromatic frequency.

The interaction of the beam with the transverse impedance $Z_\perp(\omega)$ of the machine induces a complex shift of the oscillation frequencies of these modes given by:[2]

$$\Delta\omega_l = -i \, \frac{1}{1+l} \, \frac{e\beta_o I_o}{2\omega_\beta \gamma m_o L} \, Z_{\perp\text{eff}}(\omega_\xi) \tag{4}$$

where a time dependence $e^{-i\omega t}$ is assumed, and where:

e	is the unit charge,
β_o and $\gamma = \frac{E}{m_o c^2}$	are the relativistic factors,
$I_o = N_b e f_{\mathrm{rev}}$	is the bunch current,
$\omega_\beta = Q\ \omega_{\mathrm{rev}}$	is the betatron frequency,
m_o	is the particle rest mass,
L	is the bunch full length,
$Z_{\perp\mathrm{eff}}(\omega_\xi)$	is the effective transverse impedance:

$$Z_{\perp\mathrm{eff}}(\omega_\xi) = \frac{\int_{-\infty}^{+\infty} h_l(\omega - \omega_\xi)\, Z_\perp(\omega)\, d\omega}{\int_{-\infty}^{+\infty} h_l(\omega)\, d\omega} \tag{5}$$

where $h_l(\omega)$ is the bunch power spectrum for mode l. For a gaussian bunch:

$$h_l(\omega) = \left(\frac{\omega\sigma_z}{c}\right)^{2l} \exp - \left(\frac{\omega\sigma_z}{c}\right)^2 \tag{6}$$

The imaginary frequency shift is proportional to the opposite of the real part of the transverse impedance,

$$\frac{1}{\tau_l} = \Im[\Delta\omega_l] = -\frac{1}{1+l}\,\frac{\beta_o c^2 I_o}{4\pi f_{\mathrm{rev}} Q \frac{E}{e} L}\,\Re[Z_{\perp\mathrm{eff}}(\omega_\xi)] \tag{7}$$

and yields amplitude growth if $\Re[Z_{\perp\mathrm{eff}}] < 0$, or amplitude damping if $\Re[Z_{\perp\mathrm{eff}}] > 0$.

REFERENCES

1. E. Gianfelice *HERA-p Progress and Performance in 1993* Proc. HERA Seminar '94, (Härz, DE, 1994), DESY–HERA 94–3, p. 25.
2. F.J. Sacherer *Transverse bunched beam instabilities — Theory*, Proc. 9th Int. Conf. on High Energy Accelerators (Stanford, CA, 1974), p. 347.
3. A. Gamp *Private communication* (DESY 1993).
4. R. Wanzenberg *HERA-p Impedanz* Proc. HERA Seminar '94, (Härz, DE, 1994), DESY–HERA 94–3, p. 392.
5. F. Galluccio *First attempt to measure the HERA-p impedance with beam* In preparation.
6. J. Gareyte, F. Sacherer *Head-tail type instabilities in the CERN PS and Booster*, Proc. 9th Int. Conf. on High Energy Accelerators (Stanford, CA, 1974), p. 341.