

April, 1995

SU-ITP-95-9
CERN-TH/95-101
hep-ph/9504415

The Supersymmetric Flavor Problem

SAVAS DIMOPOULOS^{a1}, DAVID SUTTER^{b2}^a *Cern Theory Group, 1211 Geneva 23, Switzerland*^b *Physics Department, Stanford University, Stanford, Ca. 94305, USA*

Abstract

The supersymmetric $SU(3) \times SU(2) \times U(1)$ theory with minimal particle content and general soft supersymmetry breaking terms has 110 physical parameters in its flavor sector: 30 masses, 39 real mixing angles and 41 phases. The absence of an experimental indication for the plethora of new parameters places severe constraints on theories possessing Planck or GUT-mass particles and suggests that theories of flavor conflict with naturalness. We illustrate the problem by studying the processes $\mu \rightarrow e + \gamma$ and $K^0 - \bar{K}^0$ mixing which are very sensitive probes of Planckian physics: a single Planck mass particle coupled to the electron or the muon with a Yukawa coupling comparable to the gauge coupling typically leads to a rate for $\mu \rightarrow e + \gamma$ exceeding the present experimental limits. A possible solution is that the messengers which transmit supersymmetry breaking to the ordinary particles are much lighter than M_{Planck} .

HEP-PH-9504415

¹On leave of absence from the Physics Department, Stanford University, Stanford, Ca. 94305, USA. Work supported in part by NSF Grant NSF-PHY-9219345

²Work supported in part by NSF Grant NSF-PHY-9219345 and an NSF Graduate Fellowship. E-mail: sutter@squirrel.stanford.edu

1 Naturalness versus Flavor: A Conflict

Nature is ambivalent about Flavor; Quark masses violate it significantly whereas neutral processes conserve it very accurately. This ambivalence leads to a conflict that has to be resolved in every theory. In the standard model it led to the GIM mechanism. In SUSY-GUTS and the supersymmetric standard model [1] it led to the hypothesis that squarks and sleptons of the same color and charge have the same mass, independent of the generation that they belong to. We call this “horizontal universality”. A stronger version of this hypothesis is that all squarks and sleptons have the same mass at M_{GUT} [1]. This is called “universality” and is a fundamental ingredient of the minimal version of the Supersymmetric Standard Model (MSSM). Universality ensures that the sparticle masses are isotropic in flavor space and thus do not cause any direct flavor violations. Flavor non-conservation in the MSSM originates in the quark masses and is under control.

The hypotheses of universality or horizontal universality are difficult to implement in realistic theories. The reason is simple: the physics that splits particles also splits sparticles. The degree to which this happens is the crucial question; the answer is model dependent. In the MSSM and the minimal SUSY-GUT [1] the interfamily sparticle splittings that are dynamically induced are adequately small. Such minimal theories leave fundamental questions unanswered and are unlikely to be the last word. In more ambitious theories addressing (even small parts of) the problem of flavor, the interfamily sparticle splittings are invariably large and cause unacceptable flavor violations unless the sparticle masses are heavy [2]. However, heavy sparticles spoil naturalness, which was the original reason for low energy SUSY; it implies that parameters related to electroweak symmetry breaking must be tuned to high accuracy. Thus, generic theories addressing the problem of flavor conflict with naturalness. These theories require a new mechanism to suppress flavor violations to solve this conflict.

In this paper, we first present the general supersymmetrized standard model. Next we review sources of flavor dependence in the supersymmetry violating terms and the naturalness criterion that limits the masses of the new supersymmetric partners. We then use the experimental constraints from the flavor changing processes $\mu \rightarrow e + \gamma$ and $K^0 - \bar{K}^0$ mixing to quantify the conflict between sparticle non-universality and naturalness, illustrating the need for a mechanism to suppress flavor changing processes. Finally, we discuss a possible suppression mechanism.

2 Supersymmetric Kobayashi-Maskawa.

2.1 The $U(3)^5$ Flavor Group

Consider the general, renormalizable softly broken supersymmetric $SU(3) \times SU(2) \times U(1)$ theory with minimal particle content at some energy scale significantly smaller than the fundamental scales M_{Planck} or M_{GUT} . The soft SUSY-breaking terms [1, 3] are all taken to be $\sim M_{\text{weak}}$ but are otherwise unconstrained: the sparticle masses are, in general, unrelated to each other and the triscalar couplings are not necessarily proportional to the Yukawa couplings³. This theory can be the low energy manifestation of a SUSY-GUT or a Superstring theory or anything else. The gauge part of the Lagrangian has a $U(3)^5$ global *flavor* symmetry, one $U(3)$ for each of the five species that constitute a family: $q, \bar{d}, \bar{u}, l, \bar{e}$. In this paper we study violations of this flavor symmetry so we will concentrate on the flavor $U(3)^5$ breaking part of the Lagrangian, which is:

$$\mathcal{L}_{U(3)^5}^{\text{break}} = \sum_{A,i,j} m_{Aij}^2 \tilde{A}_i^* \tilde{A}_j + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{triscalar}} \quad (1)$$

where $\tilde{A} = \tilde{q}, \tilde{u}, \tilde{d}, \tilde{l}, \tilde{e}$ labels the five species that constitute a family; the tilde labels a sparticle; and $i, j = 1, 2, 3$ are $U(3)$ flavor labels. The Yukawa part of the Lagrangian includes the corresponding scalar quartic couplings and is derived from the superpotential:

$$W_{\text{Yukawa}} = q \lambda_u \bar{u} H_u + q \lambda_d \bar{d} H_d + l \lambda_e \bar{e} H_d \quad (2)$$

where $\lambda_u, \lambda_d, \lambda_e$ are Yukawa matrices.

The triscalar couplings are given by:

$$\mathcal{L}_{\text{triscalar}} = \tilde{q} A_u \tilde{u} H_u + \tilde{q} A_d \tilde{d} H_d + \tilde{l} A_e \tilde{e} H_d \quad (3)$$

where A_u, A_d, A_e are three by three Yukawa-like matrices with overall magnitude of order $\sim M_{\text{weak}}$.

2.2 Counting Parameters

The above Lagrangian contains three fermion Yukawa matrices, three triscalar coupling matrices, and five scalar mass matrices. The Yukawa and triscalar matrices are general 3×3 matrices with nine real magnitudes and nine imaginary phases each. The five scalar mass matrices are 3×3 Hermitean matrices with six real magnitudes and three phases each. This gives a total of 84 real parameters (mass eigenvalues and angles) and 69 phases.

³We also assume R-parity conservation.

Not all of these parameters are physical; some may be eliminated using the $U(3)^5$ flavor symmetry of the gauge sector. (We will only allow superfield rotations in order to maintain the form of the gaugino couplings)⁴. The $SU(3)^5$ subgroup of $U(3)^5$ can be used to remove 15 real angles and 25 phases. The $U(1)^5$ subgroup can be used to remove only three phases. The remaining two phases of $U(1)^5$ can not be used to remove any parameters because the Lagrangian is invariant under these rotations, which correspond to baryon and lepton number⁵.

Subtracting the removeable parameters, we find the theory contains 69 real parameters and 41 phases. Of the 69 real parameters 30 are masses, 9 for fermions and 21 for scalars, and the remaining 39 are mixing angles. Thus, compared to the standard model, there are an additional 21 masses, 36 mixing angles and 40 phases. They all imply new physics. A geometric interpretation of these parameters will become clear in the next section.

2.3 Sparticle Basis

The first term of eq. 1 is quadratic and chirality conserving. A $U(3)^5$ rotation can diagonalize it and take us to the “sparticle” basis⁶ where:

$$m_{Aij}^2 = m_{A(i)}^2 \delta_{ij} \quad (4)$$

Thus, in this basis, these chirality conserving terms of the Lagrangian also conserve a $U(1)^{15}$ flavor subgroup that conserves individual species number for each of the 15 species of quarks and leptons that make up the three families. In the sparticle basis, although the chirality conserving terms in the Lagrangian distinguish the 15 species of sparticles, they do not cause flavor violating transitions between them. This is convenient for tracing flavor violations; they are associated with chirality violations and originate either in the Yukawa superpotential or in the triscalar couplings.

In this basis the Yukawa superpotential has the form:

$$W_{\text{Yukawa}} = qU_q \bar{\lambda}_u U_{\bar{u}} \bar{u} H_u + qU'_q \bar{\lambda}_d U_{\bar{d}} \bar{d} H_d + lU_l \bar{\lambda}_e U_{\bar{e}} \bar{e} H_d \quad (5)$$

where $\bar{\lambda}_u, \bar{\lambda}_d$, and $\bar{\lambda}_e$ are the diagonal Yukawa couplings for the quarks and electrons. $U'_q, U_q, U_{\bar{u}}, U_{\bar{d}}, U_l$, and $U_{\bar{e}}$ are six unitary matrices; $U_q^\dagger U'_q$ is the usual KM matrix, whereas the remaining five are new independent matrices. In general,

⁴Continuous R-Symmetry will be used to render the gaugino masses real. Similarly, Higgs fields will be redefined to make the bilinear soft term real.

⁵In doing a similar counting for the standard model, the Lagrangian is invariant under four $U(1)$ field redefinitions: baryon number and the three individual lepton numbers.

⁶Unless otherwise specified we will always make superfield rotations: sparticles and particles are rotated in parallel. This ensures that the gaugino couplings have their minimal form.

these matrices cannot be rotated away. They have both physical and geometrical significance. Their physical significance is that they cause new flavor violations. Their geometrical significance is that they measure the relative misalignment between sparticle and particle masses in flavor $U(3)^5$ space. The A-terms are of a similar form to the Yukawa couplings. They contain six additional 3×3 unitary matrices with a similar physical interpretation.

2.4 Universality and Proportionality

In minimal supersymmetric theories it is often assumed that, at some fundamental scale $\sim M_{\text{GUT}}$ or M_{string} , each triscalar coupling is proportional to the corresponding Yukawa coupling with a proportionality constant which is the same for each Yukawa matrix. This is sometimes called *proportionality* and reduces the possible 27 complex numbers to one. In addition, again in minimal theories, one of two conditions is also postulated [1]:

- **horizontal universality:**

$$m_{Aij}^2 = m_A^2 \delta_{ij} \tag{6}$$

or, the more restrictive

- **universality:**

$$m_{Aij}^2 = m^2 \delta_{ij} \tag{7}$$

Either version of universality reduces the sparticle masses to spheres in flavor space which preserve the full $U(3)^5$ rotation group. Since a sphere points nowhere, the notion of relative orientation of particle and sparticle masses loses its meaning; the geometric significance of 5 of the 6 matrices $U'_q, U_q, U_{\bar{u}}, U_{\bar{d}}, U_l,$ and $U_{\bar{e}}$ disappears. Only the usual CKM matrix $U_q^\dagger U'_q$ that measures the relative orientation of up and down quark masses continues to have geometrical and physical meaning. In particular, since U_l and $U_{\bar{e}}$ lose their meaning, there are no lepton number violations in theories satisfying horizontal universality. The importance of the hypotheses of proportionality and universality is now clear: They insure that all flavor violations involve the quarks and are proportional to the usual CKM matrix $U_q^\dagger U'_q$; consequently, they are under control.

As we shall review in the next section, the problem with these hypotheses is that they do not seem to emerge from fundamental short distance theories, such as GUTs or strings: Flavor breakings in the fermion sector invariably pollute the soft terms and render them non-universal and non-proportional. This is, in one sense, fortunate because these low energy parameters may serve as a fingerprint of high energy physics that is otherwise beyond the reach of experiment.

Since we wish to do a general analysis of flavor violations we will not assume proportionality or any form of universality.

3 Sources of Non-Universality

All theories have some degree of flavor dependence in the soft SUSY breaking terms. The terms which violate the $U(3)$ flavor symmetry for the fermions will also affect the soft terms; if not at tree level, then at the loop level through the RG equations. The only question is the extent to which the sparticles are non-degenerate between families and misaligned with respect to the fermions. The answer depends on the size of the Yukawa couplings.

In theories that do not address the question of flavor, most of the Yukawa couplings are small so they do not contribute significantly to flavor or CP violations; the top Yukawa is large, so it can induce measurable violations [4]. Recent calculations of $\mu \rightarrow e + \gamma$, as well as electron and neutron electric dipole moments, in the minimal SUSY GUT give results that could be observed soon if sparticles are not too heavy [4]. In this case, the large top Yukawa does not cause a problem because it is sheltered from the first generation by a small mixing angle.

In theories that do address the question of flavor [5] we expect that there are no small parameters and that all non-vanishing Yukawa couplings are of the same order as the gauge coupling at some high scale $\sim M_{\text{PL}}$ (or $\sim M_{\text{GUT}}$), which we call the flavor scale. These Yukawas couple the three ordinary families to superheavy multiplets residing at the flavor scale. As we shall demonstrate, they can create large splittings among the ordinary squarks and sleptons. These subsequently lead to dangerous flavor violating interactions. Even if there is a flavor symmetry protecting the soft terms, threshold corrections will occur when the symmetry is spontaneously broken, resulting once again in dangerous contributions.

As an example, let us examine the $SU(5)$ superpotential term of equation 8 which involves one light matter multiplet and two heavy fields. The RG equation describing the evolution of the soft mass term for the matter multiplet is given in equation 9.

$$W = \lambda \bar{5}_{\text{family}} \bar{5}_{\text{heavy}} 10_{\text{heavy}} \quad (8)$$

$$\frac{dm_{\text{family}}^2}{dt} = \frac{1}{8\pi^2} 4\lambda^2 (m_{\text{family}}^2 + m_{\bar{5}\text{heavy}}^2 + m_{10\text{heavy}}^2 + A^2) \quad (9)$$

Let us first examine the running of m_{family} due to evolution from the string scale, 5×10^{17} GeV, to the GUT scale, 2×10^{16} GeV. Assuming the Yukawa coupling is equal to one, and the tri-linear mass A is equal to the scalar masses m , we obtain $\delta m^2 = .65m^2$ using the linear approximation. Clearly the linear approximation breaks down, but we do expect fractional splittings of O(100%) if there are large Yukawa couplings over a broad range of energies.

If we redo the calculation, now assuming only a factor of two between the masses over which we integrate the RG equation instead of the factor of 25 from

the string scale to the GUT scale, the answer is $\delta m^2 = .14m^2$. This is a calculation typical of a threshold correction estimate from a broken symmetry, and the answer is large, as we shall see from flavor changing calculations. Because there is a logarithmic dependence on the ratio of mass scales, even a small integration interval gives a significant mass correction.

In any theory of soft SUSY breaking terms, there must be a violation of the flavor symmetry communicated through loop effects from the flavor breaking Yukawa sector. Small Yukawa couplings are not dangerous, but in the presence of large Yukawa couplings, as expected near the flavor scale, there will be large flavor violations in the SUSY breaking sector.

4 Naturalness

The naturalness criterion measures the sensitivity of the weak scale to variations of the SUSY parameters at a fundamental scale, for example the GUT scale. In this section we will review a simplified form of the analysis of Barbieri and Giudice [6].

If the conditions for symmetry breaking are met, the minimum of the Higgs potential at tree level can be written in terms of two equations, one for $\tan\beta$ and one for M_z^2 . The latter reads as:

$$M_z^2 = 2 \frac{(m_{H_d}^2 + \mu^2) - (m_{H_u}^2 + \mu^2) \tan^2\beta}{\tan^2\beta - 1} \quad (10)$$

Here $m_{H_d}^2$ and $m_{H_u}^2$ are the soft scalar masses of the down and up Higgs respectively, and μ is the Higgsino mass. All parameters in the above equation are evaluated at M_z .

The next step is to write this equation in terms of parameters at the GUT scale, for which one loop RG equations are sufficient. For clarity, we will keep $\tan\beta$, evaluated at the weak scale, in the equation as a fundamental parameter. This simplifies the resulting equation, making M_z^2 linear in the GUT scale parameters, without changing the numerical results significantly. In addition, we will keep the μ parameter evaluated at M_{weak} . This does not effect the results because μ is renormalized only by a multiplicative constant.

Equation 11 gives M_z in terms of the parameters of interest.

$$M_z^2 = c_\mu \mu^2 + c_{H_d} m_{H_{d0}}^2 + c_{H_u} m_{H_{u0}}^2 + c_t m_{t_0}^2 + c_{\bar{t}} m_{\bar{t}_0}^2 + c_M M_0^2 + c_{AM} A_{t_0} M_0 + c_A A_{t_0}^2 \quad (11)$$

A subscript 0 refers to a parameter evaluated at the GUT scale⁷. M is the gaugino mass (unification is assumed), and m^2 is a soft scalar mass. The c

⁷We will keep this convention throughout the paper.

coefficients are functions of $\tan\beta$ and constants of $O(1)$ from RG solutions. We have assumed the top Yukawa is the only contributing Yukawa coupling.

There is no *a priori* relation among the c coefficients, so it is unlikely that a large cancellation between separate terms of equation 11 will occur. We define the fine tuning of a given term as the fraction by which M_z^2 is smaller than that term. For example, the fine tuning of the term associated with the parameter μ , which we label f_μ , is given in equation 12.

$$f_\mu = \frac{M_z^2}{c_\mu \mu^2} \tag{12}$$

Unless there is some cancellation mechanism, the limit to a reasonable cancellation is usually placed at a fine tuning of $f = .1$. This is the 10% naturalness criterion.

This analysis gives especially tight constraints on the parameters μ and M_0 . Independent of $\tan\beta$ and the renormalization group, the coefficient $c_\mu = 2$. The minimum value of c_M is ≈ 6 and occurs for $\tan\beta \gg 1$. There is only a weak dependence on the size of the top Yukawa because we are near the fixed point. With these c values, the 10% fine tuning criterion gives the following upper mass limits:

$$\begin{aligned} M_0 &= 117 \text{ GeV} \\ \mu &= 203 \text{ GeV} \end{aligned}$$

If one wants to allow for a larger fine tuning, the square of the masses can be scaled up by the factor by which the fine tuning is increased. For example, a 1% fine tuning gives an upper limit on M_0 of 370 GeV.

5 Flavor Changing Processes

We will now calculate the constraints placed on sparticle non-universality and SUSY masses by the processes $\mu \rightarrow e + \gamma$ and $K^0 - \bar{K}^0$ mixing. To do this, we will make two simplifying approximations, valid in a large class of theories: we will neglect the contributions of the A terms and the third family. Including these contributions will improve the bounds and will make our results stronger.

5.1 $\mu \rightarrow e + \gamma$

We will use the $\mu \rightarrow e + \gamma$ branching ratio calculation of reference [7], which calculates all leading one loop contributions. Previous analyses omitted several significant contributions, often even the largest ones.

Neglecting the A terms and the third family, the calculation includes only three 2×2 mass matrices: the Yukawa matrix, the lepton doublet scalar mass matrix, and the electron singlet scalar mass matrix. The associated physical parameters are the two Yukawa eigenvalues; two scalar mass eigenvalues for both the lepton doublet and electron singlet; and a mixing angle for both the lepton doublet, θ_l , and electron singlet, $\theta_{\bar{e}}$, that describes the rotation between the sparticle and particle mass eigenbases.

Because the scalar mass splittings are required to be small, we will parametrize the doublet and singlet scalar mass eigenvalues by the average masses, m_l^2 and $m_{\bar{e}}^2$, and the mass splittings, $\delta\tilde{m}_l^2$ and $\delta\tilde{m}_{\bar{e}}^2$. We will also keep only the leading contribution in both the mass splittings and the mixing angles. Equation 13 gives the branching ratio for the process $\mu \rightarrow e + \gamma$. The functions X_l and $X_{\bar{e}}$ are given in appendix A.

$$BR(\mu \rightarrow e + \gamma) = \frac{3e^2}{2\pi^2} \left\{ \theta_l^2 \left(\frac{M_w}{m_{\bar{l}}} \right)^4 (X_l)^2 \left(\frac{\delta\tilde{m}_l^2}{m_{\bar{l}}^2} \right)^2 + \theta_{\bar{e}}^2 \left(\frac{M_w}{m_{\bar{e}}} \right)^4 (X_{\bar{e}})^2 \left(\frac{\delta\tilde{m}_{\bar{e}}^2}{m_{\bar{e}}^2} \right)^2 \right\} \quad (13)$$

5.2 $K^0 - \bar{K}^0$

We will use the $K^0 - \bar{K}^0$ mixing calculation of reference [8] which computed the dominant supersymmetric contribution, the gluino box diagrams. Because there are no charged currents, the weak singlet up quark does not appear in our calculation. The parameters in this calculation are the same as in the $\mu \rightarrow e + \gamma$ calculation with the quark doublet and down quark singlet replacing the lepton doublet and electron singlet. We will therefore use a parallel notation. The q subscript refers to the quark doublet, and \bar{d} refers to the down quark singlet.

The kaon mass splitting is given in equation 14. The definitions of f_1 and f_2 are given in appendix A.

$$\Delta M_K = \frac{\alpha_s^2}{216m_{\tilde{q}}^2} \left(\frac{2}{3} f_K^2 m_K \right) \left\{ \theta_{dl}^2 \left(\frac{\delta\tilde{m}_{\tilde{q}}^2}{m_{\tilde{q}}^2} \right)^2 f_1 \left(\frac{M_{\tilde{g}}^2}{m_{\tilde{q}}^2} \right) + \theta_q \theta_{\bar{d}} \left(\frac{\delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} \right) \left(\frac{\delta\tilde{m}_{\tilde{d}}^2}{m_{\tilde{d}}^2} \right) f_2 \left(\frac{M_{\tilde{g}}^2}{m_{\tilde{q}}^2} \right) + \theta_{\bar{d}}^2 \left(\frac{\delta\tilde{m}_{\tilde{d}}^2}{m_{\tilde{d}}^2} \right)^2 f_1 \left(\frac{M_{\tilde{g}}^2}{m_{\tilde{d}}^2} \right) \right\} \quad (14)$$

5.3 Experimental Constraints

We will take equations 13 and 14 and solve them for the fractional scalar mass splitting $\delta m^2/m^2$. We then use the one loop RG equations to relate the low energy result to the fundamental scale, which we assume is M_{gut} for the graphs

of figures 1-4. Because we ignore the contribution of the two lightest generation Yukawa couplings, the only source of mass splitting between the first and second family will be the boundary conditions at the GUT scale.

For our graphs we choose $\tan \beta = 3$ and, as stated above, $A = 0$. The mass splitting constraint gets stricter for larger values of $\tan \beta$, but does not change much as $\tan \beta$ gets smaller. For simplicity of presentation, we assume the singlet and doublet mixing angles and mass splittings are the same. Furthermore, we assume each mixing angle is equal to the square root of the masses of the two particles it relates: for the leptons, $\theta_l = \theta_{\bar{e}} = \sqrt{e/\mu}$, and for the quarks, $\theta_q = \theta_{\bar{d}} = \sqrt{d/s}$. The result holds, at least approximately, in most unified theories of fermion masses [5] and is a consequence of quark-lepton unification and the the successful relation: $\theta_{\text{Cabibbo}} = \sqrt{d/s}$.

Figures 1 through 4 are contour plots of the upper limits on the fractional scalar mass splittings, evaluated at the GUT scale, as a function of SUSY parameter space. We show four graphs, one for each of four values of the scalar mass evaluated at the GUT scale, m_0 . The axes of the graphs are the Higgs mixing parameter evaluated at the weak scale, μ , and the gaugino mass evaluated at the GUT scale, M_0 . The solid contours are the upper limit of the fractional mass splitting of the sleptons from $\mu \rightarrow e + \gamma$. The dashed lines, which are labeled in parentheses, are the upper limit of the fractional mass splitting of the down and strange squarks from $K^0 - \bar{K}^0$ mixing. We have also included a bold line at $M_0 = 120$ GeV which is the maximum value of M_0 based on the 10% naturalness criterion [6]. The shaded region is the experimentally excluded region where the lightest chargino is less than 45 GeV.

Accompanying each contour plot, we have included two graphs which give the associated physical masses of the three sleptons and two down squarks as a function of M_0 .

These constraints can easily be adapted for new values of the mixing angles or the branching ratio. The resulting fractional mass splittings may be read from the contour graphs using equations 15 and 16.

$$\left(\frac{\delta m_0^2}{m_0^2}\right)_{lepton} = \left(\frac{\delta m_0^2}{m_0^2} \text{ from graph}\right)_{\mu \rightarrow e + \gamma} \left\{ \frac{BR(\mu \rightarrow e + \gamma)}{4.9 \times 10^{-11}} \right\}^{\frac{1}{2}} \left\{ \frac{.07}{\theta_{lepton}} \right\} \quad (15)$$

$$\left(\frac{\delta m_0^2}{m_0^2}\right)_{down} = \left(\frac{\delta m_0^2}{m_0^2} \text{ from graph}\right)_{K^0 - \bar{K}^0} \left\{ \frac{.22}{\theta_{down}} \right\} \quad (16)$$

If we take the upper limit of $M_0 = 120$ GeV from the 10% fine tuning criterion, we see that the upper limit to the slepton fractional mass splitting is about .01 on all the graphs. There is an exception to this is for large values of m_0 (400 GeV) and a negative value for μ . The amplitude for the decay to a final state left handed electron passes through zero here, leaving only the

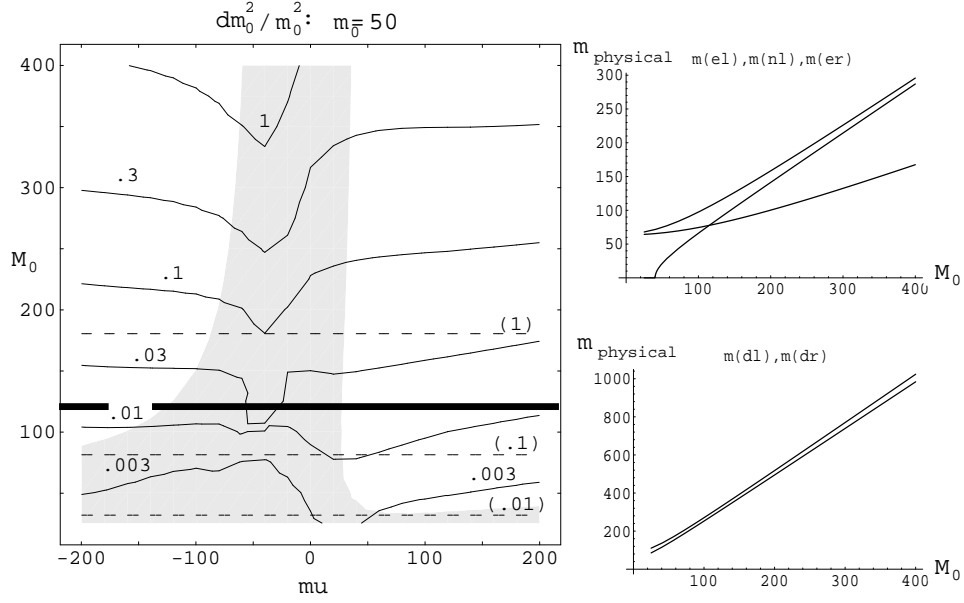


Figure 1: $m_0 = 50$ GeV: Fractional mass splittings (*left*) and physical masses of sleptons (*top right*) and down squarks (*bottom right*).

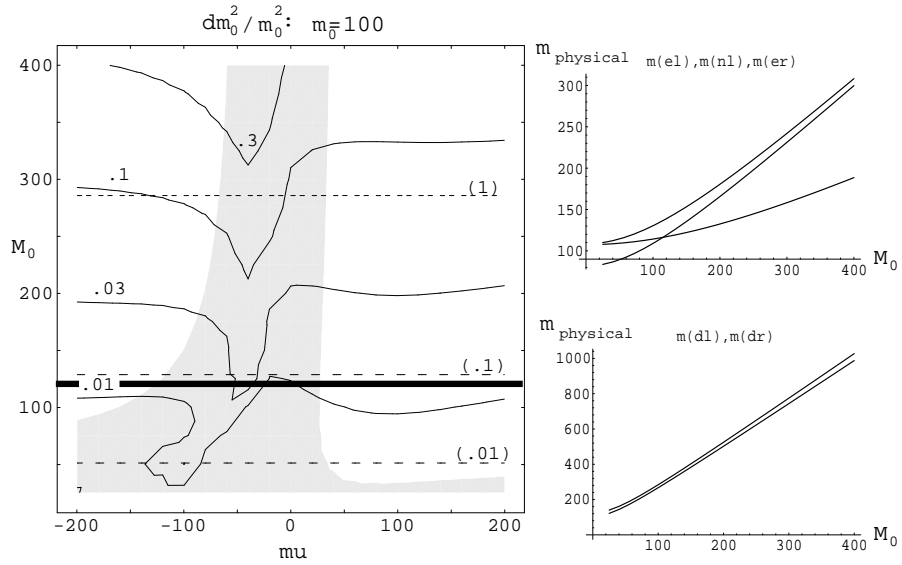


Figure 2: $m_0 = 100$ GeV: Fractional mass splittings (*left*) and physical masses of sleptons (*top right*) and down squarks (*bottom right*).

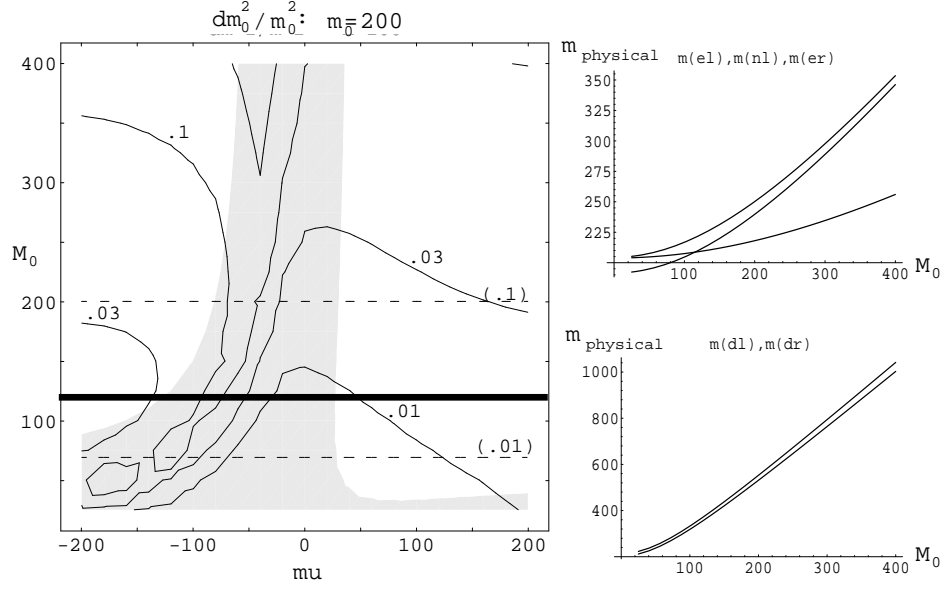


Figure 3: $m_0 = 200$ GeV: Fractional mass splittings (*left*) and physical masses of sleptons (*top right*) and down squarks (*bottom right*).

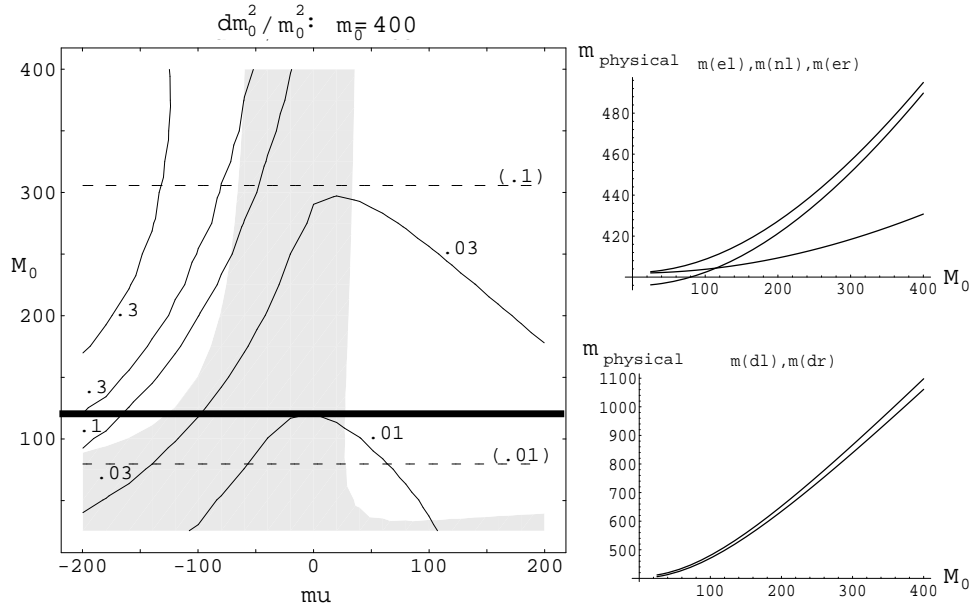


Figure 4: $m_0 = 400$ GeV: Fractional mass splittings (*left*) and physical masses of sleptons (*top right*) and down squarks (*bottom right*).

less important right handed electron contribution and making the limit not as strong. However, the $K^0 - \bar{K}^0$ mixing constraint is important in this range of parameter space, and we still obtain a mass splitting limit near .01, this time for the down-type quarks. In light of the mass splitting induced by a threshold correction at the flavor scale (section 3), this mass splitting is unnaturally small.

We can obtain more reasonable mass splitting limits if we relax the $M_0 = 120$ GeV constraint. If we allow $M_0 \approx 300$ GeV or 400 GeV, the fractional mass splitting constraints are weakened to $\approx .1$ to $.3$. However, this requires fine tuning of 1% from the naturalness criterion. In other words, the apparently unrelated terms in the equation for electroweak breaking, equation 11, sum to give an answer 100 times smaller than the individual terms. This is difficult to swallow unless a cancellation mechanism exists.

We can not simultaneously satisfy constraints from naturalness and flavor differentiation. This implies that there must be a mechanism that suppresses the supersymmetric contribution to flavor changing processes.

6 A Case for Light Messengers

The present paper has focused on the conflict between the following statements:

- 1) Naturalness implies light sparticles;
- 2) Theories attempting to explain (even small parts of) the problem of flavor predict large sparticle splittings;
- 3) Suppression of rare processes implies that sparticles are either very heavy or highly degenerate.

One way to resolve the conflict is to decouple the physics of fermionic flavour from that of the soft terms and thus evade statement (2). Consider, for example, a theory in which the soft terms shut off above a scale $\Lambda \ll M_{\text{PL}}$ (or M_{GUT}). In such a theory the soft terms would not be distorted by the flavour physics that takes place at $\sim M_{\text{PL}}$ (or M_{GUT}) and gives rise to the ordinary quark and lepton masses. If the soft terms are generated at the scale $\Lambda \ll M_{\text{PL}}$ and satisfy universality and proportionality then they will not cause any large flavour violations near the weak scale. The deviations from universality and proportionality that arise between the scales Λ and M_{W} are caused by the ordinary Yukawa couplings and are harmless.

An interesting class of such theories are those with dynamically broken supersymmetry near the weak scale [9]. Another class are (scaled down versions of) the geometric hierarchy type theories[10]. These are theories in which SUSY breaking originates in a hidden sector (H) and is communicated to the particles

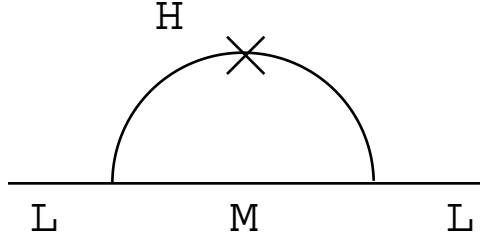


Figure 5: Schematic diagram illustrating how SUSY breaking is communicated from the hidden sector H via a messenger M to $SU_3 \times SU_2 \times U_1$ carrying sparticles L . L can be light $\sim M_W$ or heavy $\sim M_{\text{GUT}}$.

carrying $SU_3 \times SU_2 \times U_1$ quantum numbers (L) via messengers (M) as pictured in Fig. 5. The particles L carry $SU_3 \times SU_2 \times U_1$ quantum numbers and can be light $\sim M_{\text{weak}}$ or heavy $\sim M_{\text{GUT}}$.

The soft masses induced by Fig. 5 are, for example, of the form

$$\tilde{m}_L \simeq \frac{M_H^2}{M_M} \sim M_{\text{weak}} \quad (17)$$

where M_M is the messenger mass and M_H is a SUSY breaking mass. In the geometric hierarchy models [10], $M_M \sim M_{\text{GUT}}$ and $M_H \sim 3 \times 10^9$ GeV. In models where the messenger is gravity [11] $M_M \sim M_{\text{PL}}$ and $M_H \sim 3 \times 10^{10}$ GeV. It is not difficult to consider geometric hierarchy type models where the messenger mass M_M is lighter than M_{GUT} and $M_H \sim \sqrt{M_M M_{\text{weak}}}$ is proportionally lighter. What does one gain by this? At high momenta p the soft term \tilde{m}_L of the above equation behaves as:

$$\tilde{m}_L(p) \simeq \frac{M_H^2}{p} \quad (18)$$

It shuts off at $p \gg M_M$, and does not feel any of the flavor physics happening near M_{PL} (or M_{GUT}). Consequently, the sparticle splittings and rare processes coming from Planckian (or GUT) physics are suppressed by powers of M_M/M_{PL} (or M_M/M_{GUT}) relative to their values in models where the messenger is supergravity. The phenomenology of such models is quite different from the canonical supersymmetric theories where $M_M \sim M_{\text{GUT}}$ or M_{PL} . In particular, if $M_M \ll M_{\text{GUT}}$, the sparticle masses are more degenerate and deviations from universality or proportionality are smaller.

7 Conclusion

The minimal supersymmetric standard model with universality and proportionality provides a means of preserving the light weak scale and it is consistent with the observed flavor changing data. However, it does not seem to arise from a fundamental theory explaining flavor. Models which do explain the fermionic flavor hierarchy typically contain flavor dependence in the sparticle masses which contributes to flavor changing interactions. The same is true in any theory that contains Planck or GUT- mass particles that couple asymmetrically to the families with strength comparable to the gauge couplings. One solution to this problem is to increase the mass of the sparticles. However, this implies severe fine tuning of the parameters of the theory.

In this paper, we have reviewed the origin of flavor dependence in the scalar mass matrices and the fine tuning constraints (naturalness criterion). We then graphed the constraints from flavor changing processes for a general class of theories to quantify the conflict that exists between the physics of flavor and naturalness in electroweak breaking. We have found that it is not possible to simultaneously satisfy the constraints from both flavor and naturalness, suggesting that there is some mechanism which accounts for the observed smallness in flavor changing processes. One possibility is to have supersymmetry breaking transmitted to the observed particles by messengers much lighter than the Planck scale.

Acknowledgments

It is a pleasure to thank Uri Sarid for many valuable discussions and for reading the manuscript. D.S. would like to thank the CERN Theory Group for its hospitality.

A Special Function Definitions

A.1 $\mu \rightarrow e + \gamma$ Loop Functions

Equations 19 through 22 give the loop functions for $\mu \rightarrow e + \gamma$. Complete details of the calculation are in reference [7].

$$f(r) = \frac{1}{12(1-r)^4} (2r^3 + 3r^2 - 6r + 1 - 6r^2 \log r) \quad (19)$$

$$g(r) = \frac{1}{12(1-r)^4} (r^3 - 6r^2 + 3r + 2 + 6r \log r) \quad (20)$$

$$h(r) = \frac{1}{2(1-r)^3} (-r^2 + 1 + 2r \log r) \quad (21)$$

$$j(r) = \frac{1}{2(1-r)^3} (r^2 - 4r + 3 + 2 \log r) \quad (22)$$

A.2 $\mu \rightarrow e + \gamma$ Amplitude Functions

In section 5.1, we calculated the transition amplitude in the context of a particular theory of lepton masses. In this appendix we give the necessary functions for equation 13. We use modified loop functions which are defined below. The argument of these loop function is $r_{pk} = M_k^2/m_p^2$ where k represents the chargino or neutralino, and p represents the slepton.

The U matrices rotate the gaugino/higgsino interaction basis into the neutralino/chargino mass basis. U^0 is for the neutralinos; U^+ is for the charginos \tilde{W}^+ and \tilde{H}_u^+ ; and U^- is for the charginos \tilde{W}^- and \tilde{H}_d^- .

$$X_l = X_{lf} + X_{lh} + X_{lg} + X_{lj} \quad (23)$$

$$X_r = X_{rf} + X_{rh} \quad (24)$$

$$X_{lf} = \frac{1}{2} \left(U_{Wk}^0 + \frac{g_1}{g_2} U_{Bk}^0 \right)^2 f_g(r_{ek}) \quad (25)$$

$$X_{rf} = -2 \left(\frac{g_1}{g_2} U_{Bk}^0 \right)^2 f_g(r_{\bar{e}k}) \quad (26)$$

$$\begin{aligned} X_{lh} = & \frac{(A + \mu \tan \beta)M}{m_{\tilde{e}}^2} \left(\frac{g_1}{g_2} U_{Bk}^0 \right) \left(U_{Wk}^0 + \frac{g_1}{g_2} U_{Bk}^0 \right) h_k(r_{ek}, r_{\bar{e}k}) \\ & - \frac{M}{\sqrt{2}g_2v_1} (U_{Hk}^0) \left(U_{Wk}^0 + \frac{g_1}{g_2} U_{Bk}^0 \right) h_g(r_{ek}) \\ & + \left(\frac{m_{\tilde{e}}^4}{\delta m_{\tilde{e}}^2} \right) \frac{\delta A_{\mu e} M}{m_{\tilde{e}}^2 - m_{\tilde{e}}^2} \left(\frac{g_1}{g_2} U_{Bk}^0 \right) \left(U_{Wk}^0 + \frac{g_1}{g_2} U_{Bk}^0 \right) \left[\frac{h(r_{ek})}{m_{\tilde{e}}^2} - \frac{h(r_{\bar{e}k})}{m_{\tilde{e}}^2} \right] \end{aligned} \quad (27)$$

$$\begin{aligned} X_{rh} = & \frac{(A + \mu \tan \beta)M}{m_{\tilde{e}}^2} \left(U_{Wk}^0 + \frac{g_1}{g_2} U_{Bk}^0 \right) \left(\frac{g_1}{g_2} U_{Bk}^0 \right) h_k(r_{\bar{e}k}, r_{ek}) \\ & + \frac{\sqrt{2}M}{g_2v_1} (U_{Hk}^0) \left(\frac{g_1}{g_2} U_{Bk}^0 \right)^2 h_g(r_{\bar{e}k}) \\ & + \left(\frac{m_{\tilde{e}}^4}{\delta m_{\tilde{e}}^2} \right) \frac{\delta A_{\mu \bar{e}} M}{m_{\tilde{e}}^2 - m_{\tilde{e}}^2} \left(U_{Wk}^0 + \frac{g_1}{g_2} U_{Bk}^0 \right) \left(\frac{g_1}{g_2} U_{Bk}^0 \right) \left[\frac{h(r_{\bar{e}k})}{m_{\tilde{e}}^2} - \frac{h(r_{ek})}{m_{\tilde{e}}^2} \right] \end{aligned} \quad (28)$$

$$X_{lg} = - \left(\frac{m_{\tilde{e}}^4}{m_{\tilde{\nu}}^4} \right) (U_{Wk}^+)^2 g_g(r_{\bar{\nu}k}) \quad (29)$$

$$X_{lj} = \left(\frac{m_{\tilde{e}}^4}{m_{\tilde{\nu}}^4} \right) \frac{M}{g_2v_1} (U_{Hk}^-) (U_{Wk}^+) j_g(r_{\bar{e}k}) \quad (30)$$

Although we set $A = 0$ in the main text, we include the A dependence here in the appendix. Equations 31 and 32 give definitions for the nonuniversality of the A terms used above.

$$\delta A_{\bar{\mu}e} = A_{\bar{\mu}e} - A_{\bar{\mu}\mu} \quad (31)$$

$$\delta A_{\bar{e}\mu} = A_{\bar{e}\mu} - A_{\bar{\mu}\mu} \quad (32)$$

For our original functions f , g , h , and j , we have two modifications that result from our expansion in the inter-family mass difference. Equation 33 defines the g subscript, and equation 34 defines the k subscript. Z represents any of the four functions f , g , h , or j .

$$Z_g\left(\frac{M^2}{m^2}\right) \equiv m^4 \frac{d}{dm^2} \left\{ \frac{1}{m^2} Z\left(\frac{M^2}{m^2}\right) \right\} \quad (33)$$

$$Z_k\left(\frac{M^2}{m_a^2}, \frac{M^2}{m_b^2}\right) \equiv m_a^6 \frac{d}{dm_a^2} \left\{ \frac{1}{m_a^2 - m_b^2} \left[\frac{1}{m_a^2} Z\left(\frac{M^2}{m_a^2}\right) - \frac{1}{m_b^2} Z\left(\frac{M^2}{m_b^2}\right) \right] \right\} \quad (34)$$

A.3 $K^0 - \bar{K}^0$ Functions

Equations 35 and 36 are the loop functions from the text in terms of the functions f_6 and \tilde{f}_6 of Hagelin et. al., shown in equations 37 and 38.

$$f_1(r) = -66\tilde{f}_6(r) - 24rf_6(r) \quad (35)$$

$$f_2(r) = \left\{ -36 - 24 \left(\frac{m_K}{M_s + m_d} \right)^2 \right\} \tilde{f}_6(r) + \left\{ -72 + 384 \left(\frac{m_K}{m_s + m_d} \right)^2 \right\} r f_6(r) \quad (36)$$

$$f_6(r) = \frac{1}{6(1-r)^5} \left(-r^3 + 9r^2 + 9r - 17 - 18r \log r - 6r \log r \right) \quad (37)$$

$$\tilde{f}_6(r) = \frac{1}{3(1-r)^5} \left(r^3 + 9r^2 - 9r - 1 - 6r^2 \log r - 6r \log r \right) \quad (38)$$

References

- [1] S. Dimopoulos and H. Georgi, “*Supersymmetric GUTs*”, p 285, Second Workshop on Grand Unification, University of Michigan, Ann Arbor, April 24-26, 1981, eds. J.Leveille, L.Sulak, D.Unger; Birkhauser, 1981; S. Dimopoulos and H. Georgi, *Nucl. Phys.***B 193** 150 (1981).
- [2] L.J. Hall, V. A. Kostelecky and S. Raby, *Nucl. Phys.* **B267** 415 (1986); H.Georgi, *Phys. Lett.***169B** 231 (1986); N. Polonsky and A. Pomarol, UPR-0616T, hep-ph/9406224; H. Muryama, LBL-36962, hep-ph/9503392.
- [3] L. Girardello, M.T. Grisaru, *Nucl.Phys.*B194:65,1982.
- [4] R. Barbieri and L.J. Hall, *Phys. Lett.***338B** 212 (1994); S.Dimopoulos and L.J.Hall, *Phys. Lett.***344B** 185 (1994) R. Barbieri, L.J. Hall and A. Strumia, Pisa preprint, February 95.
- [5] See, for example, G. Anderson, S. Dimopoulos, L.J. Hall, S. Raby and G.D. Starkman, *Phys. Rev.* **D49** (1994) 3660; S. Dimopoulos, *Phys. Lett.* **B129** (1983) 417.
- [6] J.Ellis, K.Enqvist, D.V.Nanopoulos, F.Zwirner, *Mod.Phys.Lett.* A1, 57, 1986; R. Barbieri , G.F. Giudice, *Nucl.Phys.*B306:63,1988.
- [7] D. Sutter, PhD Thesis, Stanford University, and references cited therein.
- [8] F. Gabbiani and A. Masiero, *Nucl. Phys.* **B322** (1989) 235; J.S. Hagelin, S. Kelley and T. Tanaka, *Nucl. Phys.* **B415** (1994) 293.
- [9] M. Dine, A. Nelson and Y. Shirman, hep-ph/9408384, and references cited therein.
- [10] M.Dine,W.Fischler,*Nucl.Phys.*B204:346,1982; S. Dimopoulos and S. Raby, *Nucl.Phys.*B219,479(1982); J.Polchinski and L.Susskind,*Phys.Rev.*D26,3661,1982.
- [11] E.Cremmer,S.Ferrara,L.Girardello,A.Van.Proyen,*Phys.Lett.*116B, 231,1982; A. Chamseddine, R. Arnowitt, and P. Nath, *Phys. Rev. Lett.* **49** 970 (1982); R. Barbieri, S. Ferrara, and C. Savoy, *Phys. Lett.* **B110** 343 (1982); L. J. Hall, J. Lykken, and S. Weinberg, *Phys. Rev.* **D27** 2359 (1983).