DESY 94–123
GPP–UdeM–TH–95–16
CERN–TH/95–30
hep-ph/9504378
February 1995

HIGGS BOSON PRODUCTION AT THE LHC

M. Spira[1], A. Djouadi[2,3], D. Graudenz[4], and P.M. Zerwas[3]

[1] II.Institut für Theoretische Physik, Universität Hamburg,
Luruper Chaussee 149, D–22761 Hamburg, FRG

[2] Groupe de Physique des Particules, Université de Montréal,
Case 6128 A, H3C 3J7 Montréal P.Q., Canada

[3] Deutsches Elektronen–Synchrotron DESY, D-22603 Hamburg, FRG

[4] Theoretical Physics Division, CERN, CH–1211 Geneva 23, Switzerland

Abstract

Gluon fusion is the main production mechanism for Higgs particles at the LHC. We present the QCD corrections to the fusion cross sections for the Higgs boson in the Standard Model, and for the neutral Higgs bosons in the minimal supersymmetric extension of the Standard Model. The QCD corrections are in general large and they increase the cross sections significantly. In two steps preceding the calculation of the production processes, we determine the QCD radiative corrections to Higgs decays into two photons and gluons.

1 Introduction

The Higgs mechanism is a cornerstone in the electroweak sector of the Standard Model [$\mathcal{S}M$]. The fundamental particles, leptons, quarks and gauge particles, acquire the masses through the interaction with a scalar field [1]. To accomodate the well–established electromagnetic and weak phenomena, this mechanism requires the existence of at least one weak–isodoublet scalar field. After absorbing three Goldstone modes to build up the longitudinal $W_L^\pm, Z_L$ states, one degree of freedom is left over which corresponds to a scalar particle. The properties of the Higgs boson, decay widths and production mechanisms, can be predicted if the mass of the particle is fixed [2].

Even though the value of the Higgs mass cannot be predicted in the Standard Model, constraints can nevertheless be derived from internal consistency conditions [3–5]. Upper bounds on the mass can be set by assuming that the Standard Model can be extended up to a scale Λ before perturbation breaks down and new dynamical phenomena emerge. If the Higgs mass is less than 200 GeV, the Standard Model can be extended, with particles interacting weakly, up to the GUT scale of order 10^{16} GeV, a prerequisite to the renormalization of $\sin^2\theta_W$ from the symmetry value 3/8 down to ~ 0.2 at low energies [6]. For Higgs masses of more than about 700 GeV, the theory becomes strongly interacting already at energy scales in the TeV region [7]. For the large top quark mass found experimentally [8–10], the requirement of vacuum stability sets a lower limit on the Higgs mass. For top masses of 150, 175 and 200 GeV, the lower limits on the Higgs mass are 40, 55 and 70 GeV, respectively, if the fields of the Standard Model become strongly interacting at a scale of about 1 TeV. The lower limits are shifted upwards if the Standard Model with weakly interacting fields extends up to energies of the order of the Planck scale. They decrease dramatically, however, if the vacuum is only assumed to be metastable [5].

The most stringent experimental limit on the Higgs mass in the $\mathcal{S}M$ has been set by LEP. A lower bound of 63.9 GeV has been found [11] by exploiting the Bjorken process $Z \to Z^* H$ [12]. The search will be extended to a Higgs mass near 80 to 90 GeV by studying the Higgs–strahlung $e^+e^- \to Z^* \to ZH$ at LEP2 [13, 14]. The detailed exploration of the Higgs sector in e^+e^- collisions for yet higher masses requires the construction of linear colliders [15, 16].

The search for Higgs particles after LEP2 will continue at the pp collider LHC [17–19]. Several mechanisms contribute to the production of $\mathcal{S}M$ Higgs bosons in proton collisions [16]. The dominant mechanism is the gluon fusion process [20]

$$pp \to gg \to H$$

which provides the largest production rate for the entire Higgs mass range of interest. For large Higgs masses, the fusion process $qq \to WW, ZZ \to H$ [21] becomes competitive, while for Higgs particles in the intermediate mass range $M_Z < M_H < 2M_Z$ the Higgs–strahlung off top quarks [22] and W, Z gauge bosons [23] are additional important production processes.

Rare decays to two photons will provide the main signature for the search of $\mathcal{S}M$ Higgs particles in the lower part of the intermediate range for masses below about 130 GeV. To isolate the narrow $\gamma\gamma$ signal in the huge $\gamma\gamma$ continuum background, excellent energy and geometric resolution of the γ detectors is mandatory [18, 19]. Besides, excellent μ–vertex detectors may open the gate to the dominant $b\bar{b}$ decay mode [24] even though the QCD jet background remains very difficult to reject [25]. [At the expense of considerably lower rates the background rejection can be improved for both reactions by selecting Higgs–strahlung events where additional isolated leptons from the associated production of Higgs and top or W bosons reduce the QCD background.] Above this mass range, Higgs decays to two Z bosons – one Z being virtual in the upper part of the intermediate range – will be used to tag the Higgs particle through Z decays into pairs of charged leptons [18, 19]. The background rejection becomes increasingly simpler when the Higgs mass approaches the real–Z decay threshold. At the upper end of the standard Higgs mass range of about 800 GeV the more frequent decays of the Z bosons into neutrino pairs and jets, as well as the WW decays of the Higgs boson, with the W's decaying to leptons and jets, must be exploited to compensate for the quickly dropping production cross section.

Supporting arguments for the supersymmetry extension of the Standard Model are rooted in the Higgs sector. Supersymmetric theories provide a natural mechanism for retaining light Higgs particles in the background of high GUT energy scales [26]. In the minimal supersymmetric extension of the Standard Model [$\mathcal{M}SSM$] two isodoublet scalar fields [27] must be introduced to preserve supersymmetry, leading to two $\mathcal{C}P$–even neutral bosons h^0 and H^0, a $\mathcal{C}P$–odd neutral boson A^0 and a pair of charged Higgs bosons $H^{\pm}$. The observed value of $\sin^2\theta_W$ has been accurately predicted in this theory [28], providing a strong motivation for detailed studies of this theory [29].

The mass of the lightest Higgs boson h^0 is bounded by the Z mass *modulo* radiative corrections of a few tens of GeV [30, 31]. [Triviality bounds similar to the $\mathcal{S}M$ Higgs sector suggest an upper limit of $\sim$ 150 GeV for supersymmetric theories in general [32].] The masses of the heavy neutral and charged Higgs particles are expected to be in the range between the electroweak symmetry breaking scale and the TeV scale.

Apart from radiative corrections the structure of the $\mathcal{M}SSM$ Higgs sector is determined by two parameters, one of the Higgs masses, in general m_{A^0}, and the angle β related to the ratio of the vacuum expectation values of the two neutral Higgs fields, $\mathrm{tg}\beta = v_2/v_1$. While the overall strength of the couplings of the Higgs bosons to the $\mathcal{S}M$ particles is given by the masses, the mixing angles in the Higgs sector modify the hierarchy of the couplings considerably. For example, the coupling of h^0 to bottom quarks is strongly enhanced for large $\mathrm{tg}\beta$ compared with the coupling to the heavier top quarks. Except for a small area in the $[m_{A^0}, \mathrm{tg}\beta]$ parameter space, Z bosons couple predominantly to h^0 while the complementary coupling to the heavy H^0 Higgs boson is suppressed. The pseudoscalar Higgs boson A^0 does not couple to the gauge bosons at the Born level. In addition, the Higgs particles couple to the $\mathcal{S}USY$ particles, with a strength, however, which is essentially set by the gauge couplings.

The couplings determine the decay modes and therefore the signatures of the Higgs particles. Apart from the small region in the parameter space where the heavy Higgs boson H^0 decays into a pair of Z bosons, rare $\gamma\gamma$ and $\tau\tau$ decays must be utilized to search for the neutral Higgs particles [18, 19] if b quark decays cannot be separated sufficiently well from the QCD background. For large Higgs masses, decays into $\mathcal{S}USY$ particles [33, 34] can provide additional experimental opportunities.

The most important production mechanism for $\mathcal{S}USY$ Higgs particles at hadron colliders is the gluon fusion mechanism, similarly to the $\mathcal{S}M$ Higgs boson production,

$$pp \to gg \to h^0, H^0, A^0$$

and the Higgs radiation off top and bottom quarks. Higgs radiation off W/Z bosons and the WW/ZZ fusion of Higgs bosons play minor rôles in the $\mathcal{S}USY$ Higgs sector.

In the present analysis we have studied in detail the gluon fusion of neutral Higgs particles in the Standard Model and its minimal supersymmetric extension. The coupling of gluons to Higgs bosons is mediated primarily by heavy top quark loops, and eventually bottom quark loops in supersymmetric theories. An extensive literature already exists on various aspects of this mechanism.

The fusion mechanism has been proposed in Ref. [20] for the production of $\mathcal{S}M$ Higgs particles at hadron colliders, and has been discussed later in great detail [see Ref. [2, 17, 18, 19] for a set of references]. The phenomenological issues for the production of Higgs particles in the minimal supersymmetric extension of the Standard Model through the gluon fusion mechanism were thoroughly discussed in Refs. [35]. All these analyses, however, were based on lowest–order calculations.

Higher–order QCD corrections have first been carried out in Refs. [36, 37] for the limit of large loop–quark masses in the Standard Model. Later they were extended to the $\mathcal{M}SSM$ Higgs spectrum [38, 39]; for this case, however, areas of the parameter space in which b–quark loops are important, are not covered by the approximation. The higher order QCD corrections of the fusion cross section for the entire Higgs mass range have been given for the Standard Model in Ref. [40] and for its supersymmetric extension in Ref. [41]. As anticipated, the QCD corrections to the fusion processes are important and experimentally significant. Quite generally they are positive and the corresponding K factors run up to values of ~ 2.

Besides the total production cross sections, the QCD corrected transverse momentum spectra of the Higgs particles [42] as well as the cross sections for Higgs + jet final states [43, 44] are of great experimental interest.

The theoretical analysis of QCD corrections to the gluon fusion of Higgs particles involves complicated two–loop calculations; generic Feynman diagrams are depicted in Fig. 1. Therefore they have first been performed for the simpler case of Higgs couplings to two photons, Fig. 2, for which the virtual QCD corrections are a subset of the corrections to the Higgs couplings to gluons, Fig. 3. In the experimentally relevant mass range, the QCD corrections to the $\gamma\gamma$ widths of the $\mathcal{S}M$ and $\mathcal{M}SSM$ Higgs bosons are small, of order α_s [38, 45]. In the $\mathcal{MSSM}$, special attention must be paid to the kinematical range in which the heavy quark–antiquark threshold is nearly mass–degenerate with the pseudoscalar A^0 state so that non–perturbative resonance effects must be controlled [46].

The gluon decay width of the Standard Model Higgs particle has been determined also in next–to–leading order; the QCD corrections are positive and numerically important [36, 47]. The QCD corrections to the rare Higgs boson decay $H \to Z\gamma$ [and to the reverse process $Z \to H\gamma$] have been presented in Ref. [48]; in the mass ranges of experimental interest they are tiny, of order α_s. [The leading electroweak radiative corrections to the Hgg, $H\gamma\gamma$ and $H\gamma Z$ couplings have been evaluated in the heavy top quark limit to $\mathcal{O}(G_F m_t^2)$ [49]; they are very small.]

This paper is divided into two parts. In the first part we will discuss the gluon–gluon fusion cross section of the Higgs particle in the $\mathcal{SM}$ in next–to–leading order QCD. The photonic and gluonic partial decay widths of the particle are included in the first part of the discussion. The calculations of the production cross section and the decay widths have been performed for the entire range of possible Higgs masses. The analytical results are summarized in the Appendix in terms of one–dimensional Feynman integrals. In the limit where the Higgs mass is either small or large compared to the quark–loop masses, the integration can be performed analytically and simple analytical results can be derived for the production cross sections and the decay widths. In the second part of the paper the analysis will be extended to the $\mathcal{CP}$–even and $\mathcal{CP}$–odd neutral $\mathcal{SUSY}$ Higgs bosons. To ensure a coherent presentation of the results, some material published earlier in letter form will be included in the present comprehensive report.

2 The Higgs Particle of the Standard Model

2.1 The Two–Photon Decay Width

The two–photon decay width of the Higgs boson in the Standard Model,

$$H \to \gamma\gamma$$

is of interest for two reasons. In the lower part of the intermediate mass range of the Higgs particle, this rare decay mode provides the signature for the search at hadron colliders [18, 19]. The $\gamma\gamma$ width determines also the cross section for Higgs production in $\gamma\gamma$ collisions [50]. Since the $H\gamma\gamma$ coupling is mediated by triangle loops of all charged particles, the precision measurement of the $\gamma\gamma$ width eventually opens a window to particles with masses much heavier than the Higgs mass. If the masses of these particles are generated through the Higgs mechanism, the couplings to the Higgs boson grow with the masses, balancing the decrease of the triangle amplitude with rising loop mass. As a result, the heavy particles do not decouple. However, if the masses of the particles are generated primarily by different mechanisms [as in supersymmetric theories, for example], their effect on the

$H\gamma\gamma$ coupling is in general small.

The decay process $H \to \gamma\gamma$ proceeds in the Standard Model through W and fermion loops, Fig. 2a,b. Denoting the fermionic amplitude by A_f and the W contribution by A_W, the decay rate is determined by [13, 51]

$$\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 m_H^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c Q_f^2 A_f(\tau_f) + A_W(\tau_W) \right|^2 \tag{1}$$

where N_c is the color factor, Q_f the electric charge of the fermion f. The scaling variables are defined by

$$\tau_f = \frac{m_H^2}{4m_f^2} \qquad \text{and} \qquad \tau_W = \frac{m_H^2}{4m_W^2} \tag{2}$$

The amplitudes A_f and A_W can be expressed as

$$\begin{aligned} A_f(\tau) &= 2[\tau + (\tau - 1)f(\tau)]/\tau^2 \\ A_W(\tau) &= -[2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau)]/\tau^2 \end{aligned} \tag{3}$$

where the function $f(\tau)$ is given by

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau} & \tau \leq 1 \\ -\dfrac{1}{4}\left[\log \dfrac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right]^2 & \tau > 1 \end{cases} \tag{4}$$

If the Higgs mass is smaller than the WW and $f\bar{f}$ pair thresholds, the amplitudes are real; above the thresholds they are complex, Fig. 4. Below the thresholds the W amplitude is always dominant, falling from (-7) for very light Higgs masses to $(-5 - 3\pi^2/4)$ at the WW threshold; for large Higgs masses the W amplitude approaches $A_W \to (-2)$. Quark contributions increase from 4/3 for light Higgs masses (compared with the quark mass) to 2 at the quark–antiquark threshold; far above the fermion threshold, the amplitude vanishes linearly in τ *mod.* logarithmic coefficients, $A_f \to -[\log(4\tau) - i\pi]^2/2\tau$, i.e. proportional to m_f^2/m_H^2. The contribution of the W loop interferes destructively with the quark loop. For Higgs masses of about 600 GeV, the two contributions nearly cancel each other [52].

Since the Hff coupling is proportional to the fermion mass, the contribution of light fermions is negligible so that in the Standard Model with three families, only the top quark and the W gauge boson effectively contribute to the $\gamma\gamma$ width. Since the W and fermion loops interfere destructively, a fourth generation of heavy fermions would reduce the size of the $H\gamma\gamma$ coupling. For small Higgs masses the additional contributions of the heavy quarks and the charged lepton would suppress the decay width by about one order of magnitude.

To fully exploit the potential of the $\gamma\gamma$ decay mode of the Higgs particle and the production in $\gamma\gamma$ collisions, the QCD corrections must be shown to be under proper control. To include the gluonic QCD corrections, twelve two–loop diagrams plus the associated counter terms must be taken into account. Generic examples are depicted in Fig. 2c.

Throughout this analysis we have adopted the on–shell renormalization scheme which is convenient for heavy quarks. In this scheme the quark mass m_Q is defined as the pole of the propagator[1], related in the following way to the running mass

$$m_Q(\mu_Q^2) = m_Q \left[\frac{\alpha_s(\mu_Q^2)}{\alpha_s(m_Q^2)}\right]^{12/(33-2N_F)} \{1+\mathcal{O}(\alpha_s^2)\} \tag{5}$$

at the mass renormalization point μ_Q. It should be noted that this definition of the running mass does *not* coincide with the running $\overline{MS}$ mass. The wave function is renormalized $[Z_2^{1/2}]$ such that the residue at the pole is equal to unity. The photon–quark vertex is renormalized at zero–momentum transfer; the standard QED Ward identity renders the corresponding renormalization factor equal to the renormalization factor of the wave function. Since the fermion masses are generated in the Standard Model by the interaction with the Higgs field, the renormalization factor associated with the Higgs–quark vertex $[Z_{HQQ}]$ is fixed unambiguously by the renormalization factors Z_m for the mass and Z_2 for the wave function. From the Lagrangian

$$\begin{aligned} \mathcal{L}_0 &= -m_0 \bar{Q}_0 Q_0 \frac{H}{v} \\ &= -m_Q \bar{Q} Q \frac{H}{v} + Z_{HQQ} m_Q \bar{Q} Q \frac{H}{v} \end{aligned} \tag{6}$$

we find [53]

$$Z_{HQQ} = 1 - Z_2 Z_m \tag{7}$$

In contrast to the renormalized photon–fermion vertex, the scalar HQQ vertex $\Gamma(p',p)$ is renormalized at zero momentum transfer by a finite amount γ_m of order α_s after subtracting Z_{HQQ} due to the lack of a corresponding Ward identity. The finite renormalization γ_m corresponds to the anomalous mass dimension discussed later.

We have calculated the two–loop amplitudes using dimensional regularization. The five–dimensional Feynman parameter integrals of the amplitudes have been reduced analytically down to one–dimensional integrals over polylogarithms [54] which have been evaluated numerically[2] [see Appendix A]. In the two limits where $m_H^2/4m_Q^2$ is either very small or very large, the amplitudes could be calculated analytically.

The QCD corrections of the quark contribution to the two–photon Higgs decay amplitude can be parameterized as

$$A_Q = A_Q^{LO}\left[1 + C_H \frac{\alpha_s}{\pi}\right] \tag{8}$$

[1]We have chosen $m_t = 174$ GeV for the t pole mass [9] and $m_b = 5$ GeV for the b pole mass.

[2]The scalar integral associated with the gluon correction to the HQQ vertex has also been analyzed by means of analytical [55] and novel approximation methods [56]. The results are in agreement within an accuracy of 10^{-5}.

The coefficient C_H splits into two parts,

$$C_H = c_1 + c_2 \log \frac{\mu_Q^2}{m_Q^2} \tag{9}$$

where the functions c_i depend only on the scaling variable $\tau = m_H^2/4m_Q^2(\mu_Q^2)$. The *same* running quark mass $m_Q(\mu_Q^2)$, evaluated at the renormalization scale μ_Q, enters in the lowest–order triangle amplitude A_Q^{LO}. The scale in α_s is arbitrary to this order; however, in practice it should be defined of order m_H. As a typical renormalization scale we have chosen $\mu_Q = m_H/2$. This choice suggests itself for two reasons. The $Q\overline{Q}$ decay threshold is [perturbatively] defined at the correct position $2m_Q(m_Q) = 2m_Q$. In addition, it turns out *a posteriori* that all relevant large logarithms are effectively absorbed into the running mass for the entire physically interesting range of the scaling variable τ.

The correction factor C_H is displayed in Fig. 5, illustrating the preferred choice $\mu_Q = m_H/2$ for the renormalization scale. The coefficient is real below the quark threshold and complex above. Near the threshold, within a margin of a few GeV, the present perturbative analysis is not valid. The formation of a P–wave 0^{++} resonance, interrupted however by the rapid quark decay [57], modifies the amplitude in this range [46]. The perturbative analysis may nevertheless account for the resonance effects in a dual way. Since $Q\overline{Q}$ pairs cannot form 0^{++} states *at* the threshold, $\Im m\, C_H$ vanishes there. $\Re e C_H$ develops a maximum very close to the threshold.

The QCD–corrected $\gamma\gamma$ decay width of the Higgs boson is shown in Fig. 6a. The correction relative to the lowest order is small in general, Fig. 6b. The corrections are seemingly large only in the area where the destructive W– and Q–loop interference makes the decay amplitude nearly vanish.

The Limit of Large Quark–Loop Mass

In the limit $m_H^2/4m_Q^2 \to 0$, the five–dimensional Feynman parameter integrals can be evaluated analytically. The correction factor for the $H\gamma\gamma$ coupling

$$m_H^2/4m_Q^2 \to 0 : \quad 1 + C_H \frac{\alpha_s}{\pi} \to 1 - \frac{\alpha_s}{\pi} \tag{10}$$

agrees with the result of the numerical integration in this limit.

The $H\gamma\gamma$ coupling can also be derived by means of a general low–energy theorem for amplitudes involving soft Higgs particles [13, 51], $\lim_{p_H \to 0} \mathcal{A}(XH) = (m_0/v)\partial\mathcal{A}(X)/\partial m_0$. The theorem is easy to prove. For zero 4–momentum the kinetic derivative term in the Lagrangian can be neglected and the [space–time independent] Higgs field can be incorporated by adding the potential energy to the bare mass term, $m_0 \to m_0(1 + H/v)$, in the Lagrangian. The expansion of the bare propagators for small values of H/v is then equivalent to inserting a zero–momentum Higgs field in an arbitrary amplitude $\mathcal{A}(X)$,

$$\frac{1}{\not{k} - m_0} \to \frac{1}{\not{k} - m_0} \frac{m_0}{v} \frac{1}{\not{k} - m_0} \tag{11}$$

and generating this way the amplitude $\mathcal{A}(XH)$. Since the bare mass m_0 and the renormalized mass m_Q are related by the anomalous mass dimension, $d\log m_0 = (1+\gamma_m) d\log m_Q$, we find for the final form of the theorem

$$\lim_{p_H\to 0} \mathcal{A}(XH) = \frac{1}{1+\gamma_m} \frac{m_Q}{v} \frac{\partial}{\partial m_Q} \mathcal{A}(X) \tag{12}$$

It is well–known that the theorem can be exploited to derive the $H\gamma\gamma$ coupling in lowest order [13, 51]. However, the theorem is also valid if radiative QCD corrections are taken into account. For large fermion masses, the vacuum polarization of the photon propagator at zero momentum is given by

$$\Pi = -e_Q^2 \frac{\alpha}{\pi} \Gamma(\epsilon) \left(\frac{4\pi\mu^2}{m_Q^2}\right)^\epsilon \left[1 + \frac{\alpha_s}{2\pi}\Gamma(1+\epsilon)\left(\frac{4\pi\mu^2}{m_Q^2}\right)^\epsilon + \mathcal{O}(\epsilon)\right] \tag{13}$$

so that

$$m_Q \frac{\partial \Pi}{\partial m_Q} = 2\frac{\alpha}{\pi}\left(1+\frac{\alpha_s}{\pi}\right) \tag{14}$$

From the anomalous mass dimension to lowest order,

$$\gamma_m = 2\alpha_s/\pi \tag{15}$$

one readily derives the correction C_H of the $H\gamma\gamma$ coupling

$$m_H^2/4m_Q^2 \to 0 : \quad 1 + C_H \frac{\alpha_s}{\pi} \to \frac{1+\alpha_s/\pi}{1+2\alpha_s/\pi} = 1 - \frac{\alpha_s}{\pi} \tag{16}$$

Compared with the radiative QCD correction to the photon propagator, $(1+\alpha_s/\pi)$, just the sign of the correction is reversed, $(1-\alpha_s/\pi)$, for the $H\gamma\gamma$ coupling [45]. In the notation of eq.(9) the correction is attributed to c_1 while $c_2 \sim 1/m_Q^2$ vanishes for large quark masses.

The same result can be derived by exploiting well–known results on the anomaly in the trace of the energy–momentum tensor [58],

$$\Theta_{\mu\mu} = (1+\gamma_m) m_0 \overline{Q}_0 Q_0 + \frac{1}{4}\frac{\beta_\alpha}{\alpha} F_{\mu\nu}F_{\mu\nu} \tag{17}$$

β_α denotes the mixed QED/QCD β function defined by $\partial\alpha(\mu^2)/\partial\log\mu = \beta_\alpha$. Since the matrix element $\langle\gamma\gamma|\Theta_{\mu\mu}|0\rangle$ vanishes for infrared photons, the coupling of the two–photon state to the Higgs source $(m_0/v)\overline{Q}_0 Q_0$ is given by $\beta'_\alpha/[4\alpha(1+\gamma_m)]$, with $\beta'_\alpha = 2e_Q^2\alpha^2/\pi(1+\alpha_s/\pi)$ including *only* the heavy quark contribution to the QED/QCD β function. Thus the $H\gamma\gamma$ coupling is described by the effective Lagrangian

$$\mathcal{L}(H\gamma\gamma) = \frac{e_Q^2\alpha}{2\pi} \left(\sqrt{2}G_F\right)^{1/2} \left[1 - \frac{\alpha_s}{\pi}\right] F_{\mu\nu}F_{\mu\nu}\, H \tag{18}$$

which is apparently equivalent to the previous derivation of the $H\gamma\gamma$ coupling in the limit $m_H^2/4m_Q^2 \to 0$.

The Limit of Small Quark–Loop Masses

In the limit $m_Q(\mu_Q^2) \to 0$ the leading and subleading logarithms of the QCD correction C_H can be evaluated analytically:

$$m_Q(\mu_Q^2) \to 0 \;: \qquad C_H \to -\frac{1}{18}\log^2(-4\tau - i\epsilon) - \frac{2}{3}\log(-4\tau - i\epsilon) + 2\log\frac{\mu_Q^2}{m_Q^2} \tag{19}$$

and, split into real and imaginary parts,

$$\begin{aligned} \Im m C_H &\to \frac{\pi}{3}\left[\frac{1}{3}\log(4\tau) + 2\right] \\ \Re e C_H &\to -\frac{1}{18}\left[\log^2(4\tau) - \pi^2\right] - \frac{2}{3}\log(4\tau) + 2\log\frac{\mu_Q^2}{m_Q^2} \end{aligned} \tag{20}$$

The choice of the renormalization scale μ_Q is crucial for the size of C_H. Choosing the on–shell definition $\mu_Q = m_Q$ leads to very large corrections in the imaginary as well as the real part, as demonstrated in Fig. 5. By contrast, for $\mu_Q = \frac{1}{2}m_H$, the corrections in the real and imaginary part remain small in the entire τ range of interest, $\tau \lesssim$ a few $\times 10^4$ for $m_b \sim 3$ GeV and $m_H \lesssim 1$ TeV. [This coincides with the corresponding observation for the decay $H \to b\bar{b}$ where the running of the b–mass up to the scale $\frac{1}{2}m_H$ absorbs the leading logarithmic coefficients [53].] Only for $\log\tau$ values above the physical range must the leading logarithmic corrections be summed up; such an analysis is beyond the scope of the present investigation.

2.2 The Gluonic Decay Width

Gluonic decays of the Higgs boson

$$H \to gg$$

are of physical interest for arguments similar to the preceding section. However, there are some qualitative differences. Since the particle loops mediating the Hgg coupling carry color charges, the color–neutral W, Z gauge bosons do not contribute. The gluonic branching ratio can only be measured directly at e^+e^- colliders and for Higgs masses presumably less than about 140 GeV [15] since it drops quickly to a level below 10^{-3} for increasing masses. In this range, a fourth generation of fermions would enhance the branching ratio to a level where it becomes competitive with the dominant $b\bar{b}$ decay mode.

The gluonic width determines the production cross section of Higgs bosons in gluon–gluon fusion to leading order at hadron colliders. The cross section, however, is strongly affected by QCD radiative corrections so that the width can be measured in this indirect way only within about 20%.

At the Born level the contribution of heavy quarks to the gluonic width in the Standard Model is given by

$$\Gamma(H \to gg) = \frac{G_F \alpha_s^2}{36\sqrt{2}\pi^3} m_H^3 \left| \frac{3}{4} \sum_Q A_Q(\tau_Q) \right|^2 \tag{21}$$

where A_Q denotes the quark amplitude, already discussed in eq.(3), without the color factor. The top quark contribution is by far dominant in the $\mathcal{SM}$. Any additional heavy quark from a fourth family etc. increases the decay amplitude by a factor 2 in the limit where the Higgs mass is small compared with the $Q\overline{Q}$ threshold energy.

The QCD corrections to the gluonic decay width [36, 47] are large. Several classes of diagrams must be calculated in addition to those familiar from the two–photon decay amplitude. Generic examples are shown in Fig. 3. The virtual corrections involve the non–abelian three–gluon and four–gluon couplings, and the counter terms associated with the renormalization $Z_g - 1 = (Z_1 - 1) - \frac{3}{2}(Z_3 - 1)$ of the QCD coupling. We have defined α_s in the $\overline{MS}$ scheme with five active quark flavors and the heavy top quark decoupled [59]. Besides the virtual corrections, three–gluon and gluon plus quark–antiquark final states must be taken into account,

$$H \to ggg \quad \text{and} \quad gq\bar{q}$$

In the quark channel we will restrict ourselves to the light quark species which we will treat as massless particles[3]. As a consequence of chirality conservation, the gluon decay amplitude does not interfere in this limit with the amplitude in which the $q\overline{q}$ pair is coupled directly to the Higgs boson [$g(Hqq)$ of order m_q, but kept non–zero]. This would be different for top quark decays $H \to t\bar{t}g$ where the decay mechanism of Fig. 3, however, is a higher–order effect, suppressed to $\mathcal{O}(g_s^2)$ already at the amplitude level with respect to the gluon bremsstrahlung correction of the basic $t\bar{t}$ decay amplitude. The light–quark final states in the QCD corrections to the gluonic decays, on the other hand, must be taken into account since they are energy–degenerate with the gluon final states.

The result can be written in the form

$$\Gamma(H \to gg(g),\ gq\bar{q}) = \Gamma_{LO}(H \to gg) \left[1 + E(\tau_Q) \frac{\alpha_s}{\pi} \right] \tag{22}$$

with

$$E(\tau) = \frac{95}{4} - \frac{7}{6} N_F + \frac{33 - 2N_F}{6} \ \log \frac{\mu^2}{m_H^2} + \Delta E \tag{23}$$

[3]We include c, b quarks among the light quarks so that all large logarithms $\log m_H^2/m_{c,b}^2$, associated with final state particle splitting, are removed by virtue of the Kinoshita–Lee–Nauenberg theorem. This assumes that when the theoretical prediction will be compared with data, c and b quark final states in collinear configurations are not subtracted. Note that in Higgs decays to c, b quark pairs plus an additional gluon jet, the heavy quarks are emitted preferentially back–to–back and not in collinear configurations. [A more detailed phenomenological analysis of these final states is in progress.]

The first three terms survive in the limit of large loop masses while ΔE vanishes in this limit. μ is the renormalization point and defines the scale parameter of α_s. It turns out *a posteriori* that the higher order corrections are minimized by choosing the pole mass m_Q for the renormalized quark mass; this is evident from Fig. 7a. The correction ΔE, given explicitely in the Appendix, is displayed in Fig. 7b for the physically relevant mass range. In Fig. 8 we present the gluonic width of the Higgs boson including the QCD radiative corrections[4].

The total decay width and the branching ratios of all decay processes in the Standard Model are shown in Fig. 9 for Higgs boson masses up to 1 TeV. All known QCD and leading electroweak radiative corrections are included.

The size of the QCD radiative corrections depends on the choice of the renormalization scale μ for any fixed order of the perturbative expansion. A transparent prescription is provided by the BLM scheme [60] in which the N_F dependent coefficient of the correction is mapped into the coupling α_s, summing up quark and gluon loops in the gluon propagators. We shall apply this prescription in the large loop–mass limit where the amplitude can be calculated analytically:

$$m_H^2/4m_Q^2 \to 0: \qquad E(\tau) = \frac{95}{4} - \frac{7}{6}N_F + \frac{33-2N_F}{6} \log\frac{\mu^2}{m_H^2} \tag{24}$$

Choosing

$$\mu_{BLM} = e^{-\frac{7}{4}} m_H \approx 0.17 m_H \tag{25}$$

the N_F dependent part drops out of $E(\tau)$ and we are left with

$$\Gamma(H \to gg(g) + gq\bar{q}) = \Gamma_B[\alpha_s(\mu_{BLM})]\left[1 + \frac{9}{2}\frac{\alpha_s}{\pi}\right] \tag{26}$$

A large fraction of the total QCD correction is thus to be attributed to the renormalization of the coupling.

We shall conclude this subsection with a few comments on the effective Hgg Lagrangian [37]. In the same way as for $H\gamma\gamma$, we can derive the effective gluon Lagrangian for quark–loop masses large compared to the Higgs mass by taking the derivative of the gluon propagator with respect to the bare quark mass for $q^2 = 0$. Introducing again the anomalous mass dimension γ_m, one finds for the gauge–invariant Lagrangian,

$$\mathcal{L}_{Hgg} = \frac{1}{4}\,\frac{\beta(\alpha_s)}{1+\gamma_m(\alpha_s)}\; G^a_{\mu\nu}G^a_{\mu\nu}\frac{H}{v} \tag{27}$$

where

$$\beta = \frac{\alpha_s}{3\pi}\left[1 + \frac{19}{4}\frac{\alpha_s}{\pi}\right] \qquad \text{and} \qquad \gamma_m = 2\frac{\alpha_s}{\pi} \tag{28}$$

[4]In all numerical analyses and figures, the contributions of the b quark loops have been included. Even for small Higgs masses these effects remain less than about 10% of the leading t quark contributions.

so that to second order

$$\mathcal{L}_{Hgg} = \frac{\alpha_s}{12\pi} \left(\sqrt{2}G_F\right)^{1/2} \left[1 + \frac{11}{4} \frac{\alpha_s}{\pi}\right] G^a_{\mu\nu} G^a_{\mu\nu} \; H \tag{29}$$

As a consequence of the non–abelian gauge invariance the Lagrangian describes, besides the Hgg coupling, also the $Hggg$ and $Hgggg$ couplings, Fig. 10a.

In contrast to the effective $H\gamma\gamma$ Lagrangian, $\mathcal{L}_{Hgg}$ does not describe the Hgg interaction to second order in α_s in total. This Lagrangian accounts only for the interactions mediated by the heavy quarks directly, but it does not include the quantum effects of the light fields[5]: $\mathcal{L}_{Hgg}$ must be added to the light–quark and gluon part of the basic QCD Lagrangian, and this sum then serves as a new effective Lagrangian for Higgs–gluon–light quark interactions. Physical observables associated with the low–energy Higgs particle are calculated by means of this effective Lagrangian in the standard way, generating gluon self–energies, vertex corrections, gluon–by–gluon scattering, gluon splitting to gluon and light quark pairs, etc. *In summa*, the diagrams displayed in Fig. 10b must be evaluated, taking into account also the corresponding counter terms that renormalize the coupling α_s and the gluon wave function.

The fixed–order program discussed so far can be applied to the mass region where $m_H/2m_Q$ is small [in essence < 1] [36, 37, 61] but $\log m_Q/m_H$ still moderate so that logarithmic terms need not be summed up. This is the kinematical region of physical interest. Based on a careful RG analysis [47], the logarithmic terms have been summed up in the limit where also $\log m_Q/m_H$ is large. This leads to the plausible result that the energy scale in the effective Hgg Lagrangian is set by the heavy–quark mass while the Higgs mass is the scale relevant for the additional light–quantum fluctuations. This can be incorporated by substituting $\alpha_s E(\tau) \to [11/2]\alpha_s(m_Q) + [73/4 - 7/6 N_F]\alpha_s(m_H)$ in eq.(24), leaving us with the light–quantum fluctuations as the main component of the QCD corrections in this mathematical limit. For moderate values of $\log m_Q/m_H$ the splitting is of higher order in the QCD coupling and can be neglected.

2.3 Higgs Boson Production in pp Collisions

Gluon fusion [20]

$$pp \to gg \to H$$

is the main production mechanism of Higgs bosons in high–energy pp collisions throughout the entire Higgs mass range. As discussed before, the gluon coupling to the Higgs boson in the Standard Model is mediated by triangular loops of top quarks. The decreasing form factor with rising loop mass is counterbalanced by the linear growth of the Higgs

[5]Technically, the additional contributions are proportional to a common factor $(\mu^2/m_H^2)^\epsilon$ which vanishes if the Higgs mass is set to zero before ϵ is driven to (-0). [Note that these mass singularities are regularized formally for $\epsilon < 0$.] However, keeping the Higgs mass non–zero but small, the expansion in ϵ gives rise to $\log \mu/m_H$ terms which fix the size of the renormalization scale of the physical process.

coupling with the quark mass. [Heavier quarks still, in a fourth family for instance, would add the same contribution to the production amplitude if their masses were generated through the standard Higgs mechanism.]

To lowest order the parton cross section, Fig. 1a, can be expressed by the gluonic width of the Higgs boson,

$$\begin{aligned} \hat{\sigma}_{LO}(gg \to H) &= \frac{\sigma_0}{m_H^2}\delta(\hat{s} - m_H^2) \\ \sigma_0 &= \frac{8\pi^2}{m_H^3}\Gamma_{LO}(H \to gg) \end{aligned} \tag{30}$$

where $\hat{s}$ is the gg invariant energy squared. Recalling the lowest–order two–gluon decay width of the Higgs boson, we find

$$\sigma_0 = \frac{G_F \alpha_s^2(\mu^2)}{288\sqrt{2}\pi} \left| \frac{3}{4} \sum_q A_Q(\tau_Q) \right|^2 \tag{31}$$

The τ_Q dependence of the form factor has been given in eq.(3). With rising mass, the width of the $\mathcal{SM}$ Higgs boson quickly becomes broader. This effect can be incorporated in the lowest–order approximation by substituting the Breit–Wigner form for the zero–width δ–distribution

$$\delta(\hat{s} - m_H^2) \to \frac{1}{\pi} \frac{\hat{s}\Gamma_H/m_H}{(\hat{s} - m_H^2)^2 + (\hat{s}\Gamma_H/m_H)^2} \tag{32}$$

and changing kinematical factors $m_H^2 \to \hat{s}$ appropriately.

Denoting the gluon luminosity as

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x}\, g(x, M^2) g(\tau/x, M^2) \tag{33}$$

the lowest–order proton–proton cross section is found in the narrow–width approximation to be

$$\sigma_{LO}(pp \to H) = \sigma_0 \tau_H \frac{d\mathcal{L}^{gg}}{d\tau_H} \tag{34}$$

where the Drell–Yan variable is defined, as usual, by

$$\tau_H = \frac{m_H^2}{s} \tag{35}$$

with s being the invariant pp collider energy squared. The expression $\tau_H d\mathcal{L}^{gg}/d\tau_H$ is only mildly divergent for $\tau_H \to 0$.

The QCD corrections to the fusion process $gg \to H$ [36, 37, 40], Fig. 1b,

$$gg \to H(g) \quad \text{and} \quad gq \to Hq,\ q\overline{q} \to Hg$$

involve the virtual corrections for the $gg \to H$ subprocess and the radiation of gluons in the final state; in addition, Higgs bosons can be produced in gluon–quark collisions and quark–antiquark annihilation. These subprocesses contribute to the Higgs production at the same order of α_s. The virtual corrections modify the lowest–order fusion cross section by a coefficient linear in α_s. Gluon radiation leads to 2–parton final states with invariant energy $\hat{s} \geq m_H^2$ in the gg, gq and $q\overline{q}$ channels. The parton cross sections for the subprocess $i + j \to H + X$ may thus be written

$$\hat{\sigma}_{ij} = \sigma_0 \left\{ \delta_{ig}\delta_{jg} \left[1 + C(\tau_Q)\frac{\alpha_s}{\pi} \right] \delta(1-\hat{\tau}) + D_{ij}(\hat{\tau}, \tau_Q)\frac{\alpha_s}{\pi}\Theta(1-\hat{\tau}) \right\} \tag{36}$$

for $i, j = g, q, \overline{q}$. The new scaling variable $\hat{\tau}$, supplementing the variables $\tau_H = m_H^2/s$ and $\tau_Q = m_H^2/4m_Q^2$ introduced earlier, is defined at the parton level,

$$\hat{\tau} = \frac{m_H^2}{\hat{s}} \tag{37}$$

The quark–loop mass is defined as the pole mass in the scaling variable τ_Q. The coefficients $C(\tau_Q)$ and $D_{ij}(\hat{\tau}, \tau_Q)$ have been determined by means of the same techniques as described for the $H\gamma\gamma$ and Hgg couplings at great detail. The lengthy analytic expressions for arbitrary Higgs boson and quark masses are given in the Appendix in the form of one–dimensional Feynman integrals. The quark–loop mass has been defined in the on–shell renormalization scheme, while the QCD coupling is taken in the $\overline{MS}$ scheme. If all the corrections (36) are added up, ultraviolet and infrared divergences cancel. However collinear singularities are left over. These singularities are absorbed into the renormalization of the parton densities [62]. We have adopted the $\overline{MS}$ factorization scheme for the renormalization of the parton densities. The final result for the pp cross section can be cast into the form

$$\sigma(pp \to H + X) = \sigma_0 \left[1 + C\frac{\alpha_s}{\pi} \right] \tau_H \frac{d\mathcal{L}^{gg}}{d\tau_H} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}} \tag{38}$$

with the renormalization scale in α_s and the factorization scale of the parton densities to be fixed properly.

The coefficient $C(\tau_Q)$ denotes the contributions from the virtual two–loop corrections regularized by the infrared singular part of the cross section for real gluon emission. This coefficient splits into the infrared part π^2, a logarithmic term depending on the renormalization scale μ and a finite τ_Q–dependent piece $c(\tau_Q)$,

$$C(\tau_Q) = \pi^2 + c(\tau_Q) + \frac{33 - 2N_F}{6} \log \frac{\mu^2}{m_H^2} \tag{39}$$

The term $c(\tau_Q)$ can be reduced analytically to a one–dimensional Feynman–parameter integral [see Appendix B] which has been evaluated numerically [40]. In the heavy–quark limit $\tau_Q = m_H^2/4m_Q^2 \ll 1$ and in the light–quark limit $\tau_Q \gg 1$, the integrals could be solved analytically.

The (non–singular) hard contributions from gluon radiation in gg scattering, gq scattering and $q\bar{q}$ annihilation depend on the renormalization scale μ and the factorization scale M of the parton densities [Fig. 1b],

$$
\begin{aligned}
\Delta\sigma_{gg} &= \int_{\tau_H}^{1} d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \times \frac{\alpha_s}{\pi}\sigma_0 \Bigg\{ -\hat{\tau} P_{gg}(\hat{\tau}) \log\frac{M^2}{\hat{s}} + d_{gg}(\hat{\tau},\tau_Q) \\
&\qquad + 12 \left[\left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}} \right)_+ - \hat{\tau}[2-\hat{\tau}(1-\hat{\tau})]\log(1-\hat{\tau}) \right] \Bigg\} \\
\Delta\sigma_{gq} &= \int_{\tau_H}^{1} d\tau \sum_{q,\bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \times \frac{\alpha_s}{\pi}\sigma_0 \left\{ \hat{\tau} P_{gq}(\hat{\tau}) \left[-\frac{1}{2}\log\frac{M^2}{\hat{s}} + \log(1-\hat{\tau}) \right] + d_{gq}(\hat{\tau},\tau_Q) \right\} \\
\Delta\sigma_{q\bar{q}} &= \int_{\tau_H}^{1} d\tau \sum_{q} \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \times \frac{\alpha_s}{\pi}\sigma_0 \; d_{q\bar{q}}(\hat{\tau},\tau_Q) \qquad (40)
\end{aligned}
$$

with $\hat{\tau} = \tau_H/\tau$. The renormalization scale enters through the QCD coupling $\alpha_s(\mu^2)$ in the radiative corrections and the lowest–order parton cross section $\sigma_0[\alpha_s(\mu^2)]$. P_{gg} and P_{gq} are the standard Altarelli–Parisi splitting functions [63],

$$
\begin{aligned}
P_{gg}(\hat{\tau}) &= 6\left\{ \left(\frac{1}{1-\hat{\tau}} \right)_+ + \frac{1}{\hat{\tau}} - 2 + \hat{\tau}(1-\hat{\tau}) \right\} + \frac{33-2N_F}{6}\delta(1-\hat{\tau}) \\
P_{gq}(\hat{\tau}) &= \frac{4}{3}\frac{1+(1-\hat{\tau})^2}{\hat{\tau}} \qquad (41)
\end{aligned}
$$

F_+ denotes the usual + distribution such that $F(\hat{\tau})_+ = F(\hat{\tau}) - \delta(1-\hat{\tau})\int_0^1 d\hat{\tau}' F(\hat{\tau}')$. The coefficients d_{gg}, d_{gq} and $d_{q\bar{q}}$ can be reduced to one–dimensional integrals [Appendix C] which have been evaluated numerically [40] for arbitrary quark masses. They can be solved analytically in the heavy and light–quark limits.

In the heavy–quark limit the coefficients $c(\tau_Q)$ and $d_{ij}(\hat{\tau},\tau_Q)$ reduce to very simple expressions [36, 37],

$$
\tau_Q = m_H^2/4m_Q^2 \ll 1 \; : \qquad
\begin{aligned}
c(\tau_Q) &\to \frac{11}{2} \\
d_{gg}(\hat{\tau},\tau_Q) &\to -\frac{11}{2}(1-\hat{\tau})^3 \\
d_{gq}(\hat{\tau},\tau_Q) &\to -1 + 2\hat{\tau} - \frac{\hat{\tau}^2}{3} \\
d_{q\bar{q}}(\hat{\tau},\tau_Q) &\to \frac{32}{27}(1-\hat{\tau})^3 \qquad (42)
\end{aligned}
$$

The corrections of $\mathcal{O}(\tau_Q)$ in a systematic Taylor expansion have been shown to be very small [61]. In fact, the leading term provides an excellent approximation up to the quark threshold $m_H \sim 2m_Q$.

For the sake of completeness we quote the differential parton cross sections for hard–gluon radiation and quark scattering in the heavy quark–loop limit [36, 37, 43, 44],

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{G_F \alpha_s^3}{288\sqrt{2}\pi^2} \; H(\hat{s}, \hat{t}) \tag{43}$$

with

$$\begin{aligned} H(gg \to Hg) &= \frac{3}{2} \, \frac{\hat{s}^4 + \hat{t}^4 + \hat{u}^4 + m_H^8}{\hat{s}^2 \hat{t} \hat{u}} \\ H(gq \to Hq) &= -\frac{2}{3} \, \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}\hat{t}} \\ H(q\bar{q} \to Hg) &= \frac{16}{9} \, \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \end{aligned}$$

The Mandelstam variables $\hat{t}, \hat{u}$ are the momentum transfer squared from the initial partons $gg, gq, q\bar{q}$, respectively, to the Higgs boson in the final state. [The singularities for $\hat{t}, \hat{u} \to 0$ can be regularized in n dimensions.]

In the opposite limit where the Higgs mass is very large compared with the top mass, a compact analytic result can be derived, too:

$$\tau_Q = m_H^2/4m_Q^2 \gg 1 :$$

$$\begin{aligned} c(\tau_Q) &\to \frac{5}{36}\log^2(-4\tau_Q - i\epsilon) - \frac{4}{3}\log(-4\tau_Q - i\epsilon) \\ d_{gg}(\hat{\tau}, \tau_Q) &\to -\frac{2}{5}\log(4\tau_Q)\left\{7 - 7\hat{\tau} + 5\hat{\tau}^2\right\} - 6\log(1-\hat{\tau})\left\{1 - \hat{\tau} + \hat{\tau}^2\right\} \\ &\quad +2\frac{\log\hat{\tau}}{1-\hat{\tau}}\left\{3 - 6\hat{\tau} - 2\hat{\tau}^2 + 5\hat{\tau}^3 - 6\hat{\tau}^4\right\} \\ d_{gq}(\hat{\tau}, \tau_Q) &\to \frac{2}{3}\left\{\hat{\tau}^2 - \left[1 + (1-\hat{\tau})^2\right]\left[\frac{7}{15}\log(4\tau_Q) + \log\left(\frac{1-\hat{\tau}}{\hat{\tau}}\right)\right]\right\} \\ d_{q\bar{q}}(\hat{\tau}, \tau_Q) &\to 0 \end{aligned} \tag{44}$$

These approximate expressions are valid to leading and subleading logarithmic accuracy.

The final results of our analysis are presented in Fig. 11 and the subsequent figures for the LHC energy $\sqrt{s} = 14$ TeV. [A brief summary is also given for 10 TeV.] They are based on a top–quark mass of 174 GeV [8–10]. If not stated otherwise, we have adopted the GRV parameterizations [64] of the parton densities. These are defined in the $\overline{MS}$ scheme[6]. We have chosen $\alpha_s^{(5)}(m_Z) = 0.117$ of the $\overline{MS}$ scheme in next–to–leading

[6] We may switch to different schemes by adding the appropriate finite shift functions [62] f_{ij} to the integrals $\Delta\sigma_{ij}$, *i.e.* substituting $P_{ij}(\hat{\tau})\log M^2/\hat{s} \to P_{ij}(\hat{\tau})\log M^2/\hat{s} + f_{ij}(\hat{\tau})$ in eqs.(40).

order. This corresponds to the average measured QCD coupling for five quark degrees of freedom [65] with $\Lambda^{(5)}_{\overline{MS}} = 214$ MeV; the standard matching conditions [66] are adopted at $\mu = m_t$: $\alpha_s^{(6)}(\mu = m_t|\Lambda^{(6)}_{\overline{MS}}) = \alpha_s^{(5)}(\mu = m_t|\Lambda^{(5)}_{\overline{MS}})$ with $\Lambda^{(6)}_{\overline{MS}} = 0.413\ \Lambda^{(5)}_{\overline{MS}}$. The GRV fits are based on a somewhat smaller value of α_s. This introduces a slight inconsistency into the numerical evaluation of the cross section which we allow for since, on the other hand, the basic parton cross section is quadratic in α_s and thus depends strongly on the choice of the QCD coupling. In order to correct the difference in the $\Lambda_{\overline{MS}}$ values, the factorization scale M at which the parton densities are evaluated, has been changed to adjust appropriately the ratio $M^2/\Lambda^2_{\overline{MS}}$ which enters in the structure functions[7]. The cross section is sensitive to gluon and quark densities down to x values of order 10^{-2} to 10^{-3}, so that subtle non–linear effects in the evolution at small x need not be taken into account yet.

We introduce K factors in the standard way,

$$K_{tot} = \frac{\sigma_{HO}}{\sigma_{LO}} \tag{45}$$

The cross sections σ_{HO} in next–to–leading order are normalized to the cross sections σ_{LO}, evaluated consistently for parton densities and α_s in leading order; the QCD NLO and LO couplings are taken from the GRV parameterizations of the structure functions. The K factor can be broken down to several characteristic components. K_{virt} accounts for the regularized virtual corrections, corresponding to the coefficient C; K_{AB} $[A, B = g, q, \bar{q}]$ for the real corrections as defined in eqs.(40). These K factors are shown for LHC energies in Fig. 11 as a function of the Higgs boson mass. For both the renormalization and the factorization scales, $\mu = M = m_H$ has been chosen. Apparently K_{virt} and K_{gg} are of the same size and of order 50% while K_{gq} and $K_{q\bar{q}}$ are quite small. [Note that $(K_{virt} + \Sigma K_{AB})$ differs from $(K_{tot} - 1)$ since the cross sections σ_0 are evaluated with different NLO and LO α_s values in the numerator and denominator.] Apart from the threshold region for Higgs decays into $t\bar{t}$ pairs, K_{tot} is insensitive to the Higgs mass.

The absolute magnitude of the correction is positive and large, increasing the cross section for Higgs production at the LHC significantly by a factor of about 1.5 to 1.7. Comparing the exact numerical results with the analytic expressions in the heavy–quark limit, it turns out that these asymptotic solutions provide an excellent approximation even for Higgs masses above the top–decay threshold. For Higgs masses below ~ 700 GeV, the deviations of the QCD corrections from the asymptotic approximation are less than 10%.

There are two sources of uncertainties in the theoretical prediction of the Higgs cross section, the variation of the cross section with different parametrizations of the parton densities and the unknown next–to–next–to–leading corrections. Since all mass scales, the Higgs mass as well as the loop–quark mass, are very large, the notorious uncertainties from higher–twist effects can safely be assumed absent.

[7] The dependence of the cross section on the factorization scale is very small.

One of the main uncertainties in the prediction of the Higgs production cross section is due to the gluon density. This distribution can only indirectly be extracted through order α_s effects from deep–inelastic lepton–nucleon scattering, or through complicated analyses of final states in lepton–nucleon and hadron–hadron scattering. Adopting a set of representative parton distributions [64, 67, 68] which are up–to–date fits to all available experimental data, we find a variation of about 7% between the maximum and minimum values of the cross section for Higgs masses above ~ 100 GeV, Fig. 12a. This uncertainty will be reduced in the near future when the deep–inelastic electron/positron–nucleon scattering experiments at HERA will have reached the anticipated level of accuracy.

The [unphysical] variation of the cross section with the renormalization and factorization scales is reduced by including the next–to–leading order corrections. This is demonstrated in Fig. 13 for two typical values of the Higgs mass, $m_H = 150$ GeV and $m_H = 500$ GeV. The renormalization/factorization scale $\mu = M$ is varied as $\mu = \rho m_H$ for ρ between 1/2 and 2. The ratio of the cross sections is reduced from 1.62 in leading order to 1.32 in next–to–leading order for $m_H = 500$ GeV. While for small Higgs masses the variation with μ for $\rho \sim 1$ is already small at the LO level, the improvement by the NLO corrections is significant at the NLO level for large Higgs masses. However, the figures indicate that further improvements are required since the μ dependence of the cross section is still monotonic in the parameter range set by the scale of order m_H. These uncertainties associated with higher–order corrections appear to be less than about 15% however.

If the total energy is reduced from $\sqrt{s} = 14$ TeV to 10 TeV the production cross section for the $\mathcal{SM}$ Higgs boson decreases by a little less than a factor 2 for small Higgs masses and a little more than 2 for large Higgs masses, Fig. 12b. The K factors agree within less than $\sim 5\%$ for the two energies.

3 The Neutral $\mathcal{SUSY}$ Higgs Particles

3.1 The Basic Set–Up

Supersymmetric theories are very attractive extensions of the Standard Model. At low energies they provide a theoretical framework in which the hierarchy problem in the Higgs sector is solved while retaining Higgs bosons with moderate masses as elementary particles in the context of the high mass scales demanded by grand unification. The minimal supersymmetric extension of the Standard Model ($\mathcal{MSSM}$) [69] may serve as a useful guideline in this domain [70]. This point is underlined by the fact that the model led to a prediction of the electroweak mixing angle [28] that is in striking agreement with present high–precision measurements of $\sin^2\theta_W$ [29]. Although some of the phenomena will be specific to this minimal version, the general pattern will nevertheless be characteristic to more general extensions [32, 71] so that the analyses can be considered as representative for a wide class of $\mathcal{SUSY}$ models.

Supersymmetry requires the existence of at least two isodoublet scalar fields Φ_1 and Φ_2, thus extending the physical spectrum of scalar particles to five [27]. The $\mathcal{M}SSM$ is restricted to this minimal extension. The field Φ_2 [with vacuum expectation value v_2] couples only to up–type quarks while Φ_1 [with vacuum expectation value v_1] couples to down–type quarks and charged leptons. The physical Higgs bosons introduced by this extension are of the following type: two $\mathcal{C}P$–even neutral bosons h^0 and H^0 [where h^0 will be the lightest particle], a $\mathcal{C}P$–odd neutral boson A^0 [usually called pseudoscalar] and two charged Higgs bosons $H^\pm$.

Besides the four masses m_{h^0}, m_{H^0}, m_{A^0} and $m_{H^\pm}$, two additional parameters define the properties of the scalar particles and their interactions with gauge bosons and fermions: the mixing angle β, related to the ratio of the two vacuum expectation values $\mathrm{tg}\beta = v_2/v_1$, and the mixing angle α in the neutral $\mathcal{C}P$–even sector. Supersymmetry gives rise to several relations among these parameters and, in fact, only two of them are independent. These relations impose a strong hierarchical structure on the mass spectrum [$m_{h^0} < m_Z, m_{A^0} < m_{H^0}$ and $m_W < m_{H^\pm}$] which however is broken by radiative corrections [30, 31] due to the large top quark mass. The parameter $\mathrm{tg}\beta$ will in general be assumed in the range $1 < \mathrm{tg}\beta < m_t/m_b$ [$\pi/4 < \beta < \pi/2$], consistent with the restrictions that follow from interpreting the $\mathcal{M}SSM$ as the low energy limit of a supergravity model.

The $\mathcal{M}SSM$ Higgs sector is generally parameterized by the mass m_{A^0} of the pseudoscalar Higgs boson and $\mathrm{tg}\beta$. Once these two parameters [as well as the top quark mass and the associated squark masses which enter through radiative corrections] are specified, all other masses and the mixing angle α can be predicted. To discuss the radiative corrections we shall neglect, for the sake of simplicity, non–leading effects due to non–zero values of the supersymmetric Higgs mass parameter μ and of the parameters A_t and A_b in the soft symmetry breaking interaction. The radiative corrections are then determined by the parameter ϵ which grows as the fourth power of the top quark mass m_t and logarithmically with the squark mass M_S,

$$\epsilon = \frac{3\alpha}{2\pi} \frac{1}{s_W^2 c_W^2} \frac{1}{\sin^2\beta} \frac{m_t^4}{m_Z^2} \log\left(1 + \frac{M_S^2}{m_t^2}\right) \tag{46}$$

with $s_W^2 = 1 - c_W^2 = \sin^2\theta_W$. [The main part of the two–loop effects can be incorporated by using the running $\overline{MS}$ top mass evaluated at the pole mass [72].]

These corrections are positive and they shift the mass of the light neutral Higgs boson h^0 upward with increasing top mass. The variation of the upper limit on m_{h^0} with the top quark mass is shown in Fig. 14a for $M_S = 1$ TeV and two representative values of $\mathrm{tg}\beta = 1.5$ and 30. While the dashed lines correspond to the leading radiative corrections in eq.(46),

$$m_{h^0}^2 \leq m_Z^2 \cos^2 2\beta + \epsilon \sin^2\beta \tag{47}$$

the solid lines correspond to the Higgs mass parameter $\mu = -200, 0, +200$ GeV and the Yukawa parameters $A_t = A_b = 1$ TeV. The upper bound on m_{h^0} is shifted from the tree–level value m_Z up to ~ 140 GeV for $m_t = 174$ GeV.

Taking m_{A^0} and tgβ as the base parameters, the mass of the lightest scalar state h^0 is given to leading order by

$$m_{h^0}^2 = \frac{1}{2}\Big[m_{A^0}^2 + m_Z^2 + \epsilon - \sqrt{(m_{A^0}^2 + m_Z^2 + \epsilon)^2 - 4m_{A^0}^2 m_Z^2 \cos^2 2\beta - 4\epsilon(m_{A^0}^2 \sin^2\beta + m_Z^2\cos^2\beta)}\Big] \quad (48)$$

The masses of the heavy neutral and charged Higgs bosons follow from the sum rules

$$\begin{aligned} m_{H^0}^2 &= m_{A^0}^2 + m_Z^2 - m_{h^0}^2 + \epsilon \\ m_{H^\pm}^2 &= m_{A^0}^2 + m_W^2 \end{aligned} \quad (49)$$

In the subsequent discussion we will assume for definiteness that $m_t = 174$ GeV, $M_S =$ 1 TeV and $\mu = A_t = A_b = 0$. For the two representative values of tgβ introduced above, the masses m_{h^0}, m_{H^0} and $m_{H^\pm}$ are displayed in Figs. 14b–d as a function of m_{A^0}. [The dependence of the masses on the parameters μ, A_t, A_b is weak and the mass shifts are limited by a few GeV [34].]

The mixing parameter α is determined by tgβ and the Higgs mass m_{A^0},

$$\mathrm{tg}2\alpha = \mathrm{tg}2\beta \frac{m_{A^0}^2 + m_Z^2}{m_{A^0}^2 - m_Z^2 + \epsilon/\cos 2\beta} \quad \text{with} \quad -\frac{\pi}{2} < \alpha < 0 \quad (50)$$

The couplings of the various neutral Higgs bosons to fermions and gauge bosons depend on the angles α and β. Normalized to the $\mathcal{SM}$ Higgs couplings, they are summarized in Table 1. The pseudoscalar particle A^0 has no tree level couplings to gauge bosons, and its couplings to down (up)–type fermions are (inversely) proportional to tgβ.

Φ		$g_{\Phi\bar{u}u}$	$g_{\Phi\bar{d}d}$	$g_{\Phi VV}$
$\mathcal{SM}$	H	1	1	1
$\mathcal{MSSM}$	h^0	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H^0	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A^0	$1/\mathrm{tg}\beta$	$\mathrm{tg}\beta$	0

Table 1: Higgs couplings in the $\mathcal{MSSM}$ to fermions and gauge bosons relative to $\mathcal{SM}$ couplings.

Typical numerical values of these couplings are shown in Fig. 15 as a function of m_{A^0} and for two values of tgβ. The dependence on the parameters μ and A_t, A_b is very weak and the leading radiative corrections provide an excellent approximation [34]. There is in general a strong dependence on the input parameters tgβ and m_{A^0}. The couplings to down (up)–type fermions are enhanced (suppressed) compared to the $\mathcal{SM}$ Higgs couplings. If m_{A^0} is large, the couplings of h^0 to fermions and gauge bosons are $\mathcal{SM}$ like. It is therefore very difficult to distinguish the Higgs sector of the $\mathcal{MSSM}$ from that of the $\mathcal{SM}$, if all Higgs bosons, except the lightest neutral Higgs boson, are very heavy.

Apart from cascade decays in some corners of the $\mathcal{SUSY}$ parameter space, the main decay modes of the neutral Higgs particles are in general $b\bar{b}$ decays [$\sim$ 90%] and $\tau^+\tau^-$ decays [$\sim$ 10%], and top decays above threshold. The branching ratios for all the dominant decay modes are shown in Fig. 16. The gold–plated ZZ decays of the $\mathcal{SM}$ Higgs particle above 140 GeV play only a minor rôle in the $\mathcal{SUSY}$ Higgs sector — and in large parts of the parameter space their rôle is even negligible. The total widths of the states remain small, $\mathcal{O}$(1 GeV), anywhere in the intermediate mass range and they do not exceed a few tens of GeV even for Higgs masses of the order of 1 TeV, Fig. 17.

In addition to the conventional decays into $\mathcal{SM}$ particles, the Higgs particles may also decay into chargino and neutralino pairs [33, 34]. Depending on the details of the $\mathcal{SUSY}$ parameters, the branching ratios for decays into these channels can add up to a few tens of percent; invisible LSP (lightest neutralino) decays, in particular, can even dominate in some domains of the $\mathcal{MSSM}$ parameter space. When kinematically allowed, the Higgs particles also decay into squarks and sleptons, with generally small branching ratios, though. For the present experimental bounds on non–colored and colored supersymmetric particles, see Refs. [73] and [74], respectively.

The neutral Higgs particles will be searched for mainly in the decay channels $\tau^+\tau^-$ and $\gamma\gamma$ at the LHC [18, 19]. Large QCD backgrounds render the analysis of the dominating $b\bar{b}$ final states very difficult. Nonetheless, detailed feasibility studies have demonstrated that the $b\bar{b}$ decay channel [75] may be accessible in associated Wh^0 and $t\bar{t}/b\bar{b}h^0$ events if a set of strong detector requirements is met [25].

3.2 The Two–Photon Decay Widths

Similarly to the Standard Model Higgs boson, the precise prediction of the $\gamma\gamma$ widths of the $\mathcal{SUSY}$ Higgs particles is motivated by several points. This rare decay mode provides the most important signature for the search of the light Higgs bosons at hadron colliders. The values of the coupling constants are affected by the charged particle loops of the entire $\mathcal{SUSY}$ spectrum with masses far exceeding the light Higgs mass. The effect however is small in general for heavy $\mathcal{SUSY}$ particles since the main component of their masses is not generated by the Higgs mechanism so that these particles decouple asymptotically.

The $\gamma\gamma$ coupling to Higgs bosons in supersymmetric theories is mediated by charged heavy particle loops built up by W bosons, standard fermions f, charged Higgs bosons $H^\pm$, charginos $\tilde{c}$ and sfermions $\tilde{f}$ in the scalar cases h^0, H^0, and standard fermions and charginos [in the absence of sfermion mixing] in the pseudoscalar case A^0. Denoting the amplitudes by A_f etc., the $\gamma\gamma$ decay rates are given[8] by

$$\Gamma(\mathcal{H}^0 \to \gamma\gamma) = \frac{G_F \alpha^2 m_{\mathcal{H}}^3}{128\sqrt{2}\pi^3} \left| \sum_f N_c e_f^2 g_f^{\mathcal{H}} A_f^{\mathcal{H}} + g_W^{\mathcal{H}} A_W^{\mathcal{H}} + g_{H^\pm}^{\mathcal{H}} A_{H^\pm}^{\mathcal{H}} + \sum_{\tilde{c}} g_{\tilde{c}}^{\mathcal{H}} A_{\tilde{c}}^{\mathcal{H}} + \sum_{\tilde{f}} N_c e_{\tilde{f}}^2 g_{\tilde{f}}^{\mathcal{H}} A_{\tilde{f}}^{\mathcal{H}} \right|^2 \qquad (51)$$

and

$$\Gamma(A^0 \to \gamma\gamma) = \frac{G_F \alpha^2 m_A^3}{32\sqrt{2}\pi^3} \left| \sum_f N_c e_f^2 g_f^A A_f^A + \sum_{\tilde{c}} g_{\tilde{c}}^A A_{\tilde{c}}^A \right|^2 \qquad (52)$$

The spin 1, spin 1/2 and spin 0 amplitudes read to lowest order for the scalar Higgs bosons

$$\begin{aligned} A_1^{\mathcal{H}} &= -[2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau)]/\tau^2 \\ A_{1/2}^{\mathcal{H}} &= 2[\tau + (\tau - 1)f(\tau)]/\tau^2 \\ A_0^{\mathcal{H}} &= -[\tau - f(\tau)]/\tau^2 \end{aligned} \qquad (53)$$

and for the pseudoscalar Higgs boson

$$A_{1/2}^A = f(\tau)/\tau \qquad (54)$$

As usual, the scaling variable is defined as $\tau = m_\Phi^2/4m_i^2$ with m_i denoting the loop mass. The universal scaling function $f(\tau)$ is the same as in eq.(4). The coefficients g_i^Φ denote the couplings of the Higgs bosons to W bosons, top and bottom quarks given in Table 1 and the couplings to sfermions and charginos which are recollected for the sake of convenience in Table 2 in the absence of sfermion mixing. [Including mixing effects in the scalar squark sector due to the soft parameters A_t, A_b and μ does not change the production cross sections and photonic decay widths of the $\mathcal{SUSY}$ Higgs bosons in most of the parameter space, except in small regions where they play a significant rôle and lead to an enhancement of the signal [76].]

Since the contributions of the squark loops are strongly suppressed compared to t, b loops, we shall restrict the discussion of the QCD corrections to the standard quark loops. These corrections will be parameterized again as

$$A_Q = A_Q^{LO}\left[1 + C\frac{\alpha_s}{\pi}\right] \qquad (55)$$

The coefficient C depends on $\tau = m_\Phi^2/4m_Q^2(\mu_Q^2)$, where the running mass $m_Q(\mu_Q^2)$ is defined at the renormalization point μ_Q,

$$C = c_1[m_Q(\mu_Q^2)] + c_2[m_Q(\mu_Q^2)] \log\frac{\mu_Q^2}{m_Q^2} \qquad (56)$$

[8]The scalar particles h^0, H^0 will generically be denoted by $\mathcal{H}^0$, and all the neutral Higgs particles by Φ.

Φ		H^c	$\tilde{c}_i$
$\mathcal{S}M$	H	0	0
$\mathcal{M}SSM$	h^0	$\frac{m_W^2}{m_{H^c}^2}\left[\sin(\beta-\alpha)+\frac{\cos 2\beta\sin(\beta+\alpha)}{2\cos^2\theta_W}\right]$	$2\frac{m_W}{m_{\tilde{c}_i}}(S_{ii}\cos\alpha-Q_{ii}\sin\alpha)$
	H^0	$\frac{m_W^2}{m_{H^c}^2}\left[\cos(\beta-\alpha)-\frac{\cos 2\beta\cos(\beta+\alpha)}{2\cos^2\theta_W}\right]$	$2\frac{m_W}{m_{\tilde{c}_i}}(S_{ii}\sin\alpha+Q_{ii}\cos\alpha)$
	A^0	0	$2\frac{m_W}{m_{\tilde{c}_i}}(-S_{ii}\cos\beta-Q_{ii}\sin\beta)$

Φ		$\tilde{f}_{L,R}$
$\mathcal{S}M$	H	0
$\mathcal{M}SSM$	h^0	$\frac{m_f^2}{m_{\tilde{f}}^2}g_f^h \mp \frac{m_Z^2}{m_{\tilde{f}}^2}(I_3^f-e_f\sin^2\theta_W)\sin(\alpha+\beta)$
	H^0	$\frac{m_f^2}{m_{\tilde{f}}^2}g_f^H \pm \frac{m_Z^2}{m_{\tilde{f}}^2}(I_3^f-e_f\sin^2\theta_W)\cos(\alpha+\beta)$
	A^0	0

Table 2: Higgs couplings in the $\mathcal{M}SSM$ to charged Higgs bosons, charginos and sfermions relative to $\mathcal{S}M$ couplings. Q_{ii} and S_{ii} $(i=1,2)$ are related to the mixing angles between the charginos $\tilde{c}_1$ and $\tilde{c}_2$, Ref.[70].

The renormalization point is taken to be $\mu_Q = m_\Phi/2$; this value is related to the pole mass by the QCD formula noted in eq.(5). The lowest order amplitude A_Q^{LO} must be evaluated for the same mass value $m_Q(\mu_Q^2 = [m_\Phi/2]^2)$. The choice $\mu_Q = m_\Phi/2$ of the renormalization point ensures, *a posteriori*, a behavior of the $\gamma\gamma$ couplings which is well controlled for Higgs masses much larger than the quark mass. The QCD coupling α_s is evaluated at $\mu = m_\Phi$ for $\Lambda^{(5)}_{\overline{MS}} = 214$ MeV.

To regularize the pseudoscalar amplitude involving the γ_5 coupling, we have adopted the 't Hooft–Veltman prescription [77]. A technical remark ought to be added on a subtle problem related to this implementation of γ_5 which reproduces the axial–vector anomaly to lowest order automatically. The multiplicative renormalization factor of the scalar $(Q\overline{Q})$ current is given by $Z_{\mathcal{H}QQ} = 1 - Z_2 Z_m$ where Z_2, Z_m are the wave–function and mass renormalization factors, respectively. To ensure the chiral–symmetry relation $\Gamma_5(p',p) \to \gamma_5\Gamma(p',p)$ in the limit $m_Q \to 0$ for the fermionic matrix element of the pseudoscalar and scalar currents, the renormalization factor of the pseudoscalar current has to be chosen [78] as

$$Z_{AQQ} = Z_{\mathcal{H}QQ} + \frac{8}{3}\,\frac{\alpha_s}{\pi} \tag{57}$$

The additional term, supplementing the naive expectation, is due to spurious anomalous contributions that must be subtracted by hand.

The Limit of Large Loop Masses

For large m_Q, the coefficient c_2 in eq.(56) is of order $1/m_Q^2$ and approaches zero for the scalar and pseudoscalar Higgs bosons. It has been shown before that c_1 approaches (-1) for scalar Higgs bosons; for the pseudoscalar Higgs particle c_1 vanishes asymptotically, i.e.

$$
\begin{aligned}
m_Q \to \infty : \quad & c_1^{\mathcal{H}} \;\to\; -1 \\
& c_1^{A} \;\to\; 0
\end{aligned}
\tag{58}
$$

This result for the pseudoscalar Higgs boson can also be derived from the non–renormalization of the anomaly of the axial–vector current. In the same way in which the $\mathcal{H}\gamma\gamma$ coupling in the local limit can be related to the anomaly of the trace of the energy–momentum tensor, we can derive the $A^0\gamma\gamma$ coupling from the anomaly of the axial–vector current [79],

$$
\partial_\mu j_\mu^5 = 2m_Q \overline{Q} i\gamma_5 Q + N_c e_Q^2 \frac{\alpha}{4\pi} F_{\mu\nu}\widetilde{F}_{\mu\nu} \tag{59}
$$

with $\widetilde{F}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}$ denoting the dual field strength tensor. Since, as familiar from the Sutherland–Veltman paradox, the matrix element $\langle\gamma\gamma|\partial_\mu j_5^\mu|0\rangle$ of the divergence of the axial-vector current vanishes for zero photon energy, the matrix element $\langle\gamma\gamma|m_Q\overline{Q}i\gamma_5 Q|0\rangle$ of the Higgs source can be linked directly to the anomalous term in eq.(59). It is well–known that the anomaly is not renormalized if the QCD strong interactions are switched on [79]. As a result, the effective $A^0\gamma\gamma$ Lagrangian

$$
\mathcal{L}_{eff}(A^0\gamma\gamma) = N_c e_Q^2 \frac{\alpha}{8\pi}\left(\sqrt{2}G_F\right)^{1/2} F_{\mu\nu}\widetilde{F}_{\mu\nu}A^0 \tag{60}
$$

is valid to all orders of perturbation theory in α_s in the limit $m_{A^0}^2 \ll 4m_Q^2$.

The Limit of Small Loop Masses

Also in the opposite limit of small quark–loop masses compared with the Higgs masses, the $\mathcal{H}^0\gamma\gamma$ and $A^0\gamma\gamma$ couplings can be calculated analytically. This limit is useful in practice for large tgβ values where the b quark coupling to the heavy Higgs bosons H^0 and A^0 is strongly enhanced. As anticipated theoretically, the leading and subleading logarithmic terms are chirally invariant and we obtain the same QCD correction in this limit for the scalar and pseudoscalar couplings,

$$
m_Q(\mu_Q^2) \to 0 : \quad C^{\mathcal{H},A} \to -\frac{1}{18}\log^2(-4\tau - i\epsilon) - \frac{2}{3}\log(-4\tau - i\epsilon) + 2\log\frac{\mu_Q^2}{m_Q^2} \tag{61}
$$

The finite non–logarithmic contributions to C may be different in the scalar and pseudoscalar cases.

The amplitudes $C^{\mathcal{H}}$ for scalar loops and C^A for pseudoscalar loops are shown in Fig. 18 as a function of τ [38]. The coefficients are real below the quark threshold $\tau < 1$, and complex above. Very close to the threshold, within a margin of a few GeV, the present perturbative analysis[9] can not be applied. [It may account to some extent for resonance effects in a global way.] Since $Q\overline{Q}$ pairs cannot form 0^{++} states at the threshold, $\Im m C_{\mathcal{H}}$ vanishes there; $\Re e C_{\mathcal{H}}$ develops a maximum very close to the threshold. By contrast, since $Q\overline{Q}$ pairs do form 0^{-+} states, the imaginary part $\Im m C_A$ develops a step that is built up by the Coulombic gluon exchange [familiar from the Sommerfeld singularity of the QCD correction to $Q\overline{Q}$ production in e^+e^- annihilation]; $\Re e C_A$ is singular at the threshold.

The singular behavior of the $A^0\gamma\gamma$ coupling demands a more careful analysis at the quark threshold [46]. The form factor is given to lowest order near the threshold by

$$A_Q^{A,LO}(\tau_Q) = f(\tau_Q)/\tau_Q \to \frac{\pi^2}{4} + i\pi\beta \quad \text{for } \tau_Q \to 1 \tag{62}$$

Where $\beta = \sqrt{1-\tau_Q^{-1}}$ is the quark velocity above the threshold. The QCD corrections to the imaginary part can be found by attaching the Sommerfeld rescattering correction [80]

$$C_{Coul} = \frac{Z}{1-e^{-Z}} \approx 1 + \frac{1}{2}Z \quad \text{for } Z = \frac{4\pi\alpha_s}{3\beta} \tag{63}$$

which corresponds to the exchange of a ladder of Coulombic gluon quanta between the slowly moving quarks. The QCD corrected imaginary part of the $A^0\gamma\gamma$ coupling may thus be written

$$\Im m\ A_Q^A = \pi\beta C_{Coul} = \pi\beta + \frac{2}{3}\pi^2\alpha_s \tag{64}$$

approaching a non–zero value at threshold. The real part can be derived from a once–subtracted dispersion relation so that near the threshold

$$A_Q^A \to A_Q^{A,LO} + \frac{2\pi\alpha_s}{3}\left[-\log(\tau_Q - 1) + i\pi + const\right] \tag{65}$$

The smooth constant term needs not be fixed if we analyze only the singular behavior. For the QCD correction C^A near the threshold we therefore obtain the simple relations,

$$\begin{aligned} \tau_Q \to 1: \quad \Re e\ C^A &\to -\frac{8}{3}\log(\tau_Q - 1) + const \\ \Im m\ C^A &\to +\frac{8}{3}\pi \approx 8.38 \end{aligned} \tag{66}$$

The absolute size of the imaginary part and the logarithmic singularity of the real part are in agreement with the numerical analysis presented in Fig. 18b.

In Figs. 19a,b the QCD corrected $\gamma\gamma$ widths for the h^0, H^0, A^0 Higgs bosons are displayed, taking into account only quark and W boson loops for two values $\mathrm{tg}\beta = 1.5$ and

[9] By choosing the renormalization point $\mu_Q = m_\Phi/2$ the perturbative threshold $E_{th} = 2m_Q(m_Q^2)$ coincides with the on–mass shell value proper. A shift between $m_\Phi/2$ and m_Φ, for instance, affects the widths very little away from the threshold.

$\text{tg}\beta = 30$. While in the first case top loops give a significant contribution, bottom loops are the dominant component for large $\text{tg}\beta$. The overall QCD corrections are shown in Figs. 19c,d. The corrections to the widths are small, $\sim \mathcal{O}(\alpha_s/\pi)$ everywhere. [Artificially large values of δ occur only for specific large Higgs masses when the lowest order amplitudes vanish accidentally as a consequence of the destructive interference between W and quark–loop amplitudes, see also [52].] Thus, the QCD corrections are well under control across the physically interesting mass range if the running of the quark masses is properly taken into account.

3.3 The Gluonic Decay Widths

The gluonic decays of the Higgs bosons

$$h^0, H^0, A^0 \to gg$$

are mediated by quark and squark triangle loops. In the same notation as in the preceding section we find for the widths in lowest order

$$\begin{aligned}
\Gamma(h^0 \to gg) &= \frac{G_F \alpha_s^2}{36\sqrt{2}\pi^3} m_h^3 \left| \frac{3}{4} \sum_Q g_Q^h A_Q^h + \frac{3}{4} \sum_{\widetilde{Q}} g_{\widetilde{Q}}^h A_{\widetilde{Q}}^h \right|^2 \\
\Gamma(H^0 \to gg) &= \frac{G_F \alpha_s^2}{36\sqrt{2}\pi^3} m_H^3 \left| \frac{3}{4} \sum_Q g_Q^H A_Q^H + \frac{3}{4} \sum_{\widetilde{Q}} g_{\widetilde{Q}}^H A_{\widetilde{Q}}^H \right|^2 \\
\Gamma(A^0 \to gg) &= \frac{G_F \alpha_s^2}{16\sqrt{2}\pi^3} m_A^3 \left| \sum_Q g_Q^A A_Q^A \right|^2
\end{aligned} \quad (67)$$

Since the contribution of heavy squark loops is small, we will neglect these effects in the following discussion and we will focus on the dominant quark contributions.

The QCD corrections to the gluonic decay widths are large. Besides the virtual corrections, the widths are affected by three–gluon and gluon plus light quark–antiquark final states,

$$h^0, H^0, A^0 \to ggg \text{ and } gq\bar{q} \quad (68)$$

Proceeding in the same way as for the Standard Model, the result can be written in the form $[\Phi = h^0, H^0, A^0]$

$$\Gamma(\Phi \to gg(g),\ gq\bar{q}) = \Gamma_{LO}(\Phi \to gg) \left[1 + E_\Phi(\tau_Q) \frac{\alpha_s}{\pi} \right] \quad (69)$$

with

$$\begin{aligned}
E_{\mathcal{H}}(\tau_Q) &= \frac{95}{4} - \frac{7}{6} N_F + \frac{33 - 2N_F}{6} \log \frac{\mu^2}{m_{\mathcal{H}}^2} + \Delta E_{\mathcal{H}} \\
E_A(\tau_Q) &= \frac{97}{4} - \frac{7}{6} N_F + \frac{33 - 2N_F}{6} \log \frac{\mu^2}{m_A^2} + \Delta E_A
\end{aligned} \quad (70)$$

In the limit of large loop masses, a contribution 11/2 to the coefficients for scalar states is related to the effective Lagrangian after the heavy quarks are integrated out; the remaining part is associated with the rescattering and splitting corrections. As a result of the non–renormalization of the axial anomaly, the coefficient for the pseudoscalar state is entirely due to the rescattering and splitting corrections. The corrections ΔE_Φ are displayed in Figs. 20a,b as functions of the corresponding Higgs masses within their relevant mass ranges for $\mathrm{tg}\beta = 1.5$ and 30. Due to the bottom contribution, the deviations from the heavy quark–loop limit are significantly larger than in the $\mathcal{SM}$ case, thus rendering this limit useful only for $\mathrm{tg}\beta$ close to unity. In Fig. 21 the gluonic decay widths including the QCD radiative corrections are presented for $\mathrm{tg}\beta = 1.5$ and 30. They are enhanced by about 50% to 70% as a result of the large QCD corrections. In a margin of a few GeV near the threshold $[m_A \approx 2m_t]$ the perturbative result of the pseudoscalar decay width is not valid due to the Coulomb singularity in analogy to the photonic decay $A^0 \to \gamma\gamma$. The final branching ratios of all decay processes in the $\mathcal{M}SSM$ are shown in Fig. 16. For the light Higgs particle h^0 the gluonic decay mode is significant only for h^0 masses close to the maximal value, where h^0 has $\mathcal{SM}$ like couplings. For H^0 the gluon decay mode is significant only slightly below the top–antitop threshold and for small values of $\mathrm{tg}\beta$ where the coupling to top quarks is sufficiently large. For the pseudoscalar Higgs boson A^0, the gluonic decay mode is important for small values of $\mathrm{tg}\beta$ and below the top–antitop threshold, where it can reach a branching fraction of $\sim 20\%$.

In the limit of large quark masses, the Higgs–gluon–gluon coupling can be described by gauge–invariant effective Lagrangians,

$$
\begin{aligned}
\mathcal{L}_{\mathcal{H}^0 gg} &= \frac{1}{4}\left(\sqrt{2}G_F\right)^{1/2} \frac{\beta(\alpha_s)}{1+\gamma_m(\alpha_s)}\, G^a_{\mu\nu} G^a_{\mu\nu} \mathcal{H}^0 \\
\mathcal{L}_{A^0 gg} &= \frac{\alpha_s}{8\pi}\left(\sqrt{2}G_F\right)^{1/2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} A^0
\end{aligned}
\tag{71}
$$

with β and γ_m defined previously. They take account of the local interactions of the particles involved and serve as kernels for the standard gluon and light–quark corrections.

3.4 Higgs Production in pp Collisions

The production of $\mathcal{SUSY}$ Higgs particles at hadron colliders has received much attention in recent years after the pioneering investigations in Refs. [35]. The situation is critical since the first analyses could not ensure that the entire $\mathcal{M}SSM$ Higgs parameter space could be covered at the LHC. Yet, high statistics analyses appear to solve this problem if the decays to $\mathcal{SM}$ particles are dominant [19]. A second similarly severe problem has arisen from the difficulty to detect the heavy Higgs particles for masses above a few hundred GeV and moderate values of $\mathrm{tg}\beta$ where the production rates in the experimentally clear $\tau^+\tau^-$ channel are too small to be exploited in practice. However, no final picture has emerged yet, since the detailed conclusions depend strongly on the detector design. Additional h^0 decay and production channels, based on the tagging of heavy quarks, may also help close the hole in the parameter space [75].

The dominant production process for $\mathcal{SUSY}$ Higgs particles at the LHC is the gluon fusion mechanism. Besides the virtual corrections, the bremsstrahlung of additional gluons, the inelastic Compton process and quark–antiquark annihilation,

$$gg \to h^0/H^0/A^0(g) \qquad \text{and} \qquad gq \to h^0/H^0/A^0 q, \quad q\bar{q} \to h^0/H^0/A^0 g$$

contribute to the Higgs production. The diagrams relevant to the various subprocesses are the same as for the Standard Model in Fig. 1b. The parton cross sections may thus be written

$$\hat{\sigma}_{ij} = \sigma_0 \left\{ \delta_{ig}\delta_{jg} \left[1 + C(\tau_Q)\frac{\alpha_s}{\pi} \right] \delta(1-\hat{\tau}) + D_{ij}(\hat{\tau},\tau_Q)\frac{\alpha_s}{\pi}\Theta(1-\hat{\tau}) \right\} \tag{72}$$

for $i,j = g,q,\bar{q}$ and $\hat{\tau} = m_\Phi^2/\hat{s}$. The final result for the pp cross sections can be cast into the compact form $[\Phi = h^0, H^0, A^0]$

$$\sigma(pp \to h^0/H^0/A^0 + X) = \sigma_0 \left[1 + C\frac{\alpha_s}{\pi} \right] \tau_\Phi \frac{d\mathcal{L}^{gg}}{d\tau_\Phi} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}} \tag{73}$$

with

$$\begin{aligned} \sigma_0^{h,H} &= \frac{G_F \alpha_s^2}{288\sqrt{2}\pi} \left| \frac{3}{4} \sum_Q g_Q^{\mathcal{H}} A_Q^{\mathcal{H}} \right|^2 \\ \sigma_0^{A} &= \frac{G_F \alpha_s^2}{128\sqrt{2}\pi} \left| \sum_Q g_Q^{A} A_Q^{A} \right|^2 \end{aligned} \tag{74}$$

after folding the parton cross sections with the $\overline{MS}$ renormalized quark and gluon densities $[\tau_Q = m_\Phi^2/4m_Q^2$ and $\tau_\Phi = m_\Phi^2/s]$. The virtual/IR and hard corrections have the same generic form as before, eqs. (39–41). As a result of the factorization theorem, the parity and the specific couplings of the Higgs bosons are not relevant for the infrared/collinear form of the cross sections, related to interactions at large distances. The specific properties of the Higgs bosons affect only the non–singular coefficients c and d in eqs.(39–41).

In the limit of large quark–loop masses compared with the Higgs masses, only the coefficients c depend on the parity of the Higgs particle,

$$\begin{aligned} \tau_Q = m_\Phi^2/4m_Q^2 \to 0: \quad c^{h^0/H^0} &\to \frac{11}{2} \\ c^{A^0} &\to 6 \end{aligned} \tag{75}$$

The coefficients d are universal. The next–to–leading term in the expansion for the scalar Higgs bosons has also been calculated analytically [61]. The form of the cross sections for the parton subprocesses in the heavy quark–loop limit if the final states are analyzed, is given by the same expressions as eq. (43). In the opposite limit of small quark–loop masses, chiral symmetry is restored for the leading and subleading logarithmic contributions to the coefficients, which are given by the same expressions as eq. (44).

The final results of the pp cross sections are predicted in the subsequent figures for the LHC energy $\sqrt{s} = 14$ TeV. [A brief summary is appended for $\sqrt{s} = 10$ TeV.] Again, the two representative values $\mathrm{tg}\beta = 1.5$ and 30 are chosen and the top mass is fixed to $m_t = 174$ GeV. If not stated otherwise, we have adopted the GRV parameterizations of the quark and gluon densities. For the QCD coupling we have chosen the average value $\alpha_s^{(5)}(m_Z) = 0.117$ in the final cross sections while the discussion of the K factors is carried out consistently in the GRV frame. [The GRV NLO coupling is close to the lower 1σ boundary of the global α_s fit.]

The K factors, $K_{tot} = \sigma_{HO}/\sigma_{LO}$, are defined by the ratios of the HO cross sections to the LO cross sections. They are shown for LHC energies in Fig. 22. They vary little with the masses of the scalar and pseudoscalar Higgs bosons in general, yet they depend strongly on $\mathrm{tg}\beta$ as shown in Fig. 23. For small $\mathrm{tg}\beta$, their size is about the same as in the $\mathcal{SM}$, varying between 1.5 and 1.7; for large $\mathrm{tg}\beta$ however they are in general close to unity, except when h^0 approaches the $\mathcal{SM}$ domain. The cross sections are shown in Fig. 24. Apart from exceptional cases, they vary in the range between 100 and 10 pb for Higgs masses up to several hundred GeV. Beyond ~ 300 GeV they drop quickly to a level below 10^{-1} pb. Similarly to the $\mathcal{SM}$, a factor of about 2 is lost if the pp collider energies is reduced to 10 TeV, Fig. 25.

The variation of the cross sections with the renormalization/factorization scale is reduced by including the next–to–leading order corrections. The dependence of the cross sections for low masses, Fig. 26, is of order 15%; the μ dependence remains monotonic. Thus the next–to–leading order corrections stabilize the theoretical predictions for the Higgs particles in the intermediate to large mass range, yet further improvements must be envisaged in the future.

It is apparent from the previous figures that the next–to–leading order corrections increase the production cross sections for the $SUSY$ Higgs particles, in some areas of the parameter space even strongly.

4 Summary

We have presented a complete next–to–leading order calculation for the production of Higgs particles at the LHC in the Standard Model of the electroweak interactions as well as in its minimal supersymmetric extension. These corrections stabilize the theoretical predictions compared with the (ill–defined) leading–order predictions. The QCD radiative increase the production cross sections significantly so that experimental opportunities to discover and detect these fundamental particles increase.

Acknowledgements

In carrying out this analysis we have benefited from discussions with many colleagues, in particular S. Dawson and J. van der Bij. The work of M.S. is supported by Bundesministerium für Bildung und Forschung (BMBF), Bonn, Germany, under Contract 05 6 HH 93P (5) and by EEC Program *Human Capital and Mobility* through Network *Physics at High Energy Colliders* under Contract CHRX–CT93–0357 (DG12 COMA). A.D. would like to thank the DESY Theory Group for the hospitality during the final stage of this work.

APPENDIX A: The $\mathcal{H}\gamma\gamma$ and $A\gamma\gamma$ Couplings

In this Appendix, we summarize the complete analytical result for the QCD corrected $\mathcal{CP}$–even $\mathcal{H}\gamma\gamma$ and $\mathcal{CP}$–odd $A\gamma\gamma$ vertex form–factors, in the case of arbitrary Higgs boson and quark masses.

As discussed in sections 2.1 and 3.1, the radiative QCD corrections to the quark contribution to the two–photon Higgs boson decay amplitudes can be written as

$$A_Q = A_Q^{LO}\left[1 + C_\Phi \frac{\alpha_s}{\pi}\right] \tag{A.1}$$

where the coefficients C_Φ split into two parts,

$$C_\Phi = C_1^\Phi + C_2^\Phi \log \frac{\mu_Q^2}{m_Q^2} \tag{A.2}$$

with the two functions C_1^Φ and C_2^Φ depending only on the scaling variable τ[10],

$$\tau \;\equiv\; m_\Phi^2/(4m_Q^2) \;\equiv\; \rho_\Phi/4 \tag{A.3}$$

For the $\mathcal{CP}$–even and $\mathcal{CP}$–odd Higgs bosons, the coefficients C_1^Φ and C_2^Φ are given by

$$\begin{aligned}
C_1^H &= -\left[2\tau^{-1}F_0^H(\tau)\right]^{-1} \Big\{2\tau^{-2}(8\tau^{-1}-17) + 4\tau^{-2}(3\tau^{-1}-4)f(\tau)g(\tau) \\
&\quad +4\tau^{-2}(\tau^{-2}-2\tau^{-1}+3)f(\tau) + 24\tau^{-2}(-\tau^{-1}+2)g(\tau) + 2\tau^{-2}(-5\tau^{-1}+6)l(\tau) \\
&\quad +\tau^{-2}(-2\tau^{-2}+\tau^{-1}+3)k(\tau) + 6\tau^{-2}(\tau^{-1}-1)h(\tau) - 8(\tau^{-2}-3\tau^{-1}+2)I_1 \\
&\quad +4\tau^{-1}(2\tau^{-1}-3)[I_2 - I_3 + I_4] + 16\tau^{-1}(\tau^{-1}-1)I_5 \;\Big\} \\
C_2^H &= -3\tau^{-1}\left[F_0^H(\tau)\right]^{-1} \Big\{(2\tau^{-1}-1)f(\tau) - g(\tau) - 1\Big\} \\
C_1^A &= \left[3\tau^{-1}F_0^A(\tau)\right]^{-1} \Big\{24\tau^{-2} + 16\tau^{-2}f(\tau)g(\tau) + 6\tau^{-2}(\tau^{-1}-2)f(\tau) - 36\tau^{-2}g(\tau) \\
&\quad -12\tau^{-2}l(\tau) - 3\tau^{-2}(\tau^{-1}+1)k(\tau) + 6\tau^{-2}h(\tau) + 8(-\tau^{-1}+2)I_1 \\
&\quad +12\tau^{-1}[I_2 - I_3 + I_4] + 16\tau^{-1}I_5 \;\Big\} \\
C_2^A &= 2\tau^{-1}\left[F_0^A(\tau)\right]^{-1} \Big\{f(\tau) - g(\tau)/(\tau^{-1}-1)\Big\}
\end{aligned} \tag{A.4}$$

with

$$F_0^H(\tau) = \frac{3}{2}\tau^{-1}\left[1 + (1-\tau^{-1})f(\tau)\right] \quad \text{and} \quad F_0^A(\tau) = \tau^{-1}f(\tau) \tag{A.5}$$

In terms of the auxiliary variables

$$\alpha_\pm = (1 \pm \sqrt{1-\tau^{-1}})/2 \tag{A.6}$$

[10]Singularities are fixed by attributing to the quark–loop mass a small imaginary part: $m_Q^2 \to m_Q^2 - i\epsilon$.

the functions f, g, l, k and h are defined by

$$
\begin{aligned}
f(\tau) &= \begin{cases} \arcsin^2 \sqrt{\tau} & \tau \leq 1 \\ -\frac{1}{4}\left[\log \frac{\alpha_+}{\alpha_-} - i\pi\right]^2 & \tau > 1 \end{cases} \\
g(\tau) &= \begin{cases} \sqrt{\tau^{-1} - 1} \arcsin \sqrt{\tau} & \tau \leq 1 \\ \frac{\sqrt{1 - \tau^{-1}}}{2}\left[\log \frac{\alpha_+}{\alpha_-} - i\pi\right] & \tau > 1 \end{cases} \\
l(\tau) &= Li_3(1/\alpha_+) + Li_3(1/\alpha_-) \\
k(\tau) &= S_{1,2}(1/\alpha_+) + S_{1,2}(1/\alpha_-) \\
h(\tau) &= 4\left[S_{1,2}\left(\frac{\alpha_+}{\alpha_-}\right) + S_{1,2}\left(\frac{\alpha_-}{\alpha_+}\right)\right] + 2\left[Li_3\left(\frac{\alpha_+}{\alpha_-}\right) + Li_3\left(\frac{\alpha_-}{\alpha_+}\right)\right] + 2\zeta(3) \quad \text{(A.7)}
\end{aligned}
$$

Here, Li_2, Li_3 and $S_{1,2}$ are polylogarithms, defined [77] as

$$
\begin{aligned}
Li_2(x) &= \int_0^1 \frac{dy}{y} \log(1 - xy) \\
Li_3(x) &= \int_0^1 \frac{dy}{y} \log y \log(1 - xy) \\
S_{1,2}(x) &= \frac{1}{2} \int_0^1 \frac{dy}{y} \log^2(1 - xy) \quad \text{(A.8)}
\end{aligned}
$$

The expressions of $I_{1,\ldots,5}$, which have been reduced from four– and five–dimensional down to one–dimensional Feynman integrals, are much more involved:

$$
\begin{aligned}
I_1 = & \int_0^1 \frac{dx}{\rho} \frac{\log[1 - \rho x(1-x)]}{1 - \rho x(1-x)} \left\{ Li_2(\rho x) - Li_2[\rho x(1-x)] + Li_2\left(\frac{-\rho x^2}{1 - \rho x}\right) \right. \\
& \qquad - Li_2\left(\frac{-\rho x}{1 - \rho x}\right) - log[x(1-x)] \log[1 - \rho x] \\
& + Li_2\left(\frac{\rho x^2}{1 - \rho x(1-x)}\right) - Li_2\left(\frac{-\rho x(1-x)}{1 - \rho x(1-x)}\right) \\
& + Li_2\left(\frac{1 - \rho x}{1 - \frac{x}{\alpha_+}}\right) + Li_2\left(\frac{1 - \rho x}{1 - \frac{x}{\alpha_-}}\right) - Li_2\left(\frac{1}{1 - \frac{x}{\alpha_+}}\right) - Li_2\left(\frac{1}{1 - \frac{x}{\alpha_-}}\right) \\
& \left. + \frac{\log^2(1 - \rho x)}{2} + \log(1 - \rho x) \log\left(\frac{\rho x^2}{1 - \rho x(1-x)}\right) \right\} \\
+ & \int_0^1 \frac{dx}{\rho[1 - \rho x(1-x)]} \left\{ Li_3[\rho x(1-x)] + Li_3\left(\frac{1 - \rho x}{-\rho x}\right) - Li_3\left(\frac{1 - \rho x}{-\rho x^2}\right) \right.
\end{aligned}
$$

$$
\begin{aligned}
&+S_{1,2}(1-x) - S_{1,2}\left(\frac{1-x}{-x}(1-\rho x)\right)\\
&-Li_2\left(\frac{1-\rho x}{-\rho x}\right)\log x + \frac{\log^2 x}{2}\log(\rho x)\\
&+Li_3\left(\frac{-x}{1-x}\right) - Li_3\left(\frac{-x}{(1-x)(1-\rho x)}\right)\\
&+S_{1,2}[\rho x(1-x)] + S_{1,2}(\rho x) - S_{1,2}\left(\frac{-\rho x^2}{1-\rho x}\right)\\
&-\log(1-\rho x)Li_2\left(\frac{-x}{1-x}\right) + \frac{\log(1-x)}{2}\log^2(1-\rho x)\Bigg\} \qquad \text{(A.9)}
\end{aligned}
$$

$$
\begin{aligned}
I_2 \;=\; & \int_0^1 \frac{dx}{\rho^2 x^2}\Bigg\{\rho x\left[6 - \log\left(1-\frac{x}{\alpha_+}\right)\log\left(1-\frac{x}{\alpha_-}\right)\right]\\
& -\rho x(1-x)\left[Li_2(\rho x(1-x)) - 2Li_2\left(\frac{x}{\alpha_+}\right) - 2Li_2\left(\frac{x}{\alpha_-}\right)\right]\\
& +2\left[2\sqrt{\rho x(1-x)}\left(\log\left(1-\sqrt{\rho x(1-x)}\right) - \log\left(1+\sqrt{\rho x(1-x)}\right)\right)\right.\\
& \left.+\rho x(\alpha_+ - \alpha_-)\left(\log\left(1-\frac{x}{\alpha_+}\right) - \log\left(1-\frac{x}{\alpha_-}\right)\right)\right]\\
& +[2-\rho x(1-x)]\left[S_{1,2}(\sqrt{\rho x(1-x)}) + S_{1,2}(-\sqrt{\rho x(1-x)})\right.\\
& +S_{1,2}\left(\frac{-\sqrt{\rho x(1-x)}}{1-\sqrt{\rho x(1-x)}}\right) + S_{1,2}\left(\frac{\sqrt{\rho x(1-x)}}{1+\sqrt{\rho x(1-x)}}\right) - S_{1,2}\left(\frac{x}{\alpha_+}\right) - S_{1,2}\left(\frac{x}{\alpha_-}\right)\\
& \left.\left. -S_{1,2}\left(\frac{-x}{\alpha_+ - x}\right) - S_{1,2}\left(\frac{-x}{\alpha_- - x}\right)\right]\right\}\\
+ & \int_0^1 dx\Bigg\{\frac{1-x}{\rho}\left[K_9\left(-(1-x), \frac{\rho x(1-x)}{1-\rho x(1-x)}\right) - K_6\left(\frac{\rho x(1-x)}{1-\rho x(1-x)}, -(1-x)\right)\right.\\
& \qquad\left. -\log(1-x)K_{11}\left(\frac{\rho x(1-x)}{1-\rho x(1-x)}, -(1-x)\right)\right]\\
& -(1-x)\frac{1+\rho x}{\rho^2 x^2}\left[K_3\left(1, -(1-x), 1, \frac{\rho x(1-x)}{1-\rho x(1-x)}\right)\right.\\
& \qquad -K_1\left(1, \frac{\rho x(1-x)}{1-\rho x(1-x)}, 1, -(1-x)\right)\\
& \qquad\left. -\log(1-x)K_2\left(1, \frac{\rho x(1-x)}{1-\rho x(1-x)}, 1, -(1-x)\right)\right]\\
& +\frac{1-x}{\rho^2 x^2(1-\rho x)}\frac{1-\rho x(1-2x)}{1-\rho x(1-x)}\left[K_4\left(\frac{\rho x(1-x)}{1-\rho x(1-x)}, -(1-x), -\frac{(1-x)(1-\rho x)}{1-\rho x(1-x)}\right)\right.
\end{aligned}
$$

$$
\begin{aligned}
& -K_1\left(1,\frac{\rho x(1-x)}{1-\rho x(1-x)},1,-(1-x),-\frac{(1-x)(1-\rho x)}{1-\rho x(1-x)}\right) \\
& -\log(1-x)K_2\left(1,\frac{\rho x(1-x)}{1-\rho x(1-x)},1,-\frac{(1-x)(1-\rho x)}{1-\rho x(1-x)}\right)\Bigg] \\
& -2\frac{1-x}{\rho x(1-\rho x)}\left[K_{10}\left(\frac{\rho x(1-x)}{1-\rho x(1-x)},-(1-x)\right)-K_7\left(1,\frac{\rho x(1-x)}{1-\rho x(1-x)}\right)\right. \\
& \left.\left.+\log(1-x)\left(1+\frac{\log[1-\rho x(1-x)]}{\rho x(1-x)}\right)\right]\right\} \\
- \; & 2\int_0^1\frac{dx(1-x)}{\rho x}\left\{\log[1-\rho x(1-x)]\left[\log\left(\rho(1-x)^2\right)-\frac{1}{2}\log[1-\rho x(1-x)]\right]\right. \\
& +\log\left[1+\sqrt{\rho x(1-x)}\right]\log\left[1-\sqrt{\rho x(1-x)}\right]+Li_2(\rho x(1-x))+Li_2\left(1-\frac{1-x}{\alpha_+}\right) \\
& \left.+Li_2\left(1-\frac{1-x}{\alpha_-}\right)-Li_2\left(\frac{1}{1-\frac{1-x}{\alpha_+}}\right)-Li_2\left(\frac{1}{1-\frac{1-x}{\alpha_-}}\right)\right\} \qquad (A.10)
\end{aligned}
$$

$$
\begin{aligned}
I_3 \;=\; & \int_0^1 dx\left\{\frac{\log[1-\rho x(1-x)]}{\rho x}+\frac{x(1-2x)}{2[1-\rho x(1-x)]}\log^2[1-\rho x(1-x)]\right. \\
& +\frac{2-\rho x-\rho^2x^2(1-x)(1-2x)}{\rho^2x(1-x)[1-\rho x(1-x)]}\log\left[1+\sqrt{\rho x(1-x)}\right]\log\left[1-\sqrt{\rho x(1-x)}\right] \\
& +\frac{2}{\rho\sqrt{\rho x(1-x)}}\left[\log\left(1+\sqrt{\rho x(1-x)}\right)-\log\left(1-\sqrt{\rho x(1-x)}\right)\right] \\
& +\frac{(2-\rho x)(1-\rho x)}{\rho^2x^2[1-\rho x(1-x)]}\log\left(1-\frac{x}{\alpha_+}\right)\log\left(1-\frac{x}{\alpha_-}\right) \\
& +\frac{\alpha_+-\alpha_-}{\rho x}\left[\log\left(1-\frac{x}{\alpha_+}\right)-\log\left(1-\frac{x}{\alpha_-}\right)\right]-\frac{3x(1-2x)}{2[1-\rho x(1-x)]}Li_2[\rho x(1-x)] \\
& +\frac{x(1-2x)}{1-\rho x(1-x)}\left[Li_2\left(\frac{x}{\alpha_+}\right)+Li_2\left(\frac{x}{\alpha_-}\right)-\log[1-\rho x(1-x)]\log[\rho(1-x)^2]\right. \\
& \left.\left.+Li_2\left(\frac{-\alpha_+}{\alpha_--x}\right)+Li_2\left(\frac{-\alpha_-}{\alpha_+-x}\right)-Li_2\left(\frac{\alpha_--x}{-\alpha_+}\right)-Li_2\left(\frac{\alpha_+-x}{-\alpha_-}\right)\right]\right\} \qquad (A.11)
\end{aligned}
$$

$$
\begin{aligned}
I_4 \;=\; & \int_0^1 dx\left\{-\frac{1}{\rho x}\left[Li_3\left(\frac{-\rho x(1-x)}{1-\rho x(1-x)}\right)+S_{1,2}\left(\frac{-\rho x(1-x)}{1-\rho x(1-x)}\right)\right]+(1-2x)\log x\right. \\
& \left.\times\left[\frac{\log\left(1+\sqrt{\rho x(1-x)}\right)\log\left(1-\sqrt{\rho x(1-x)}\right)}{\rho x(1-x)}-\frac{Li_2[\rho x(1-x)]}{2[1-\rho x(1-x)]}\right]\right\} \qquad (A.12)
\end{aligned}
$$

$$
I_5=\int_0^1\frac{dx}{1-\rho x}\left\{\alpha_+\log\left(1-\frac{x}{\alpha_+}\right)+\alpha_-\log\left(1-\frac{x}{\alpha_-}\right)\right\}\log\left(\frac{1-\rho x(1-x)}{x}\right) \qquad (A.13)
$$

In terms of the variables $\alpha = (ad - bc)/d$ and $\beta = b/d$, and the functions

$$F_1(a,b) = S_{1,2}(-a) + S_{1,2}(-b) - S_{1,2}\left(\frac{b-a}{1+b}\right) - Li_3\left(\frac{b}{a}\frac{1+a}{1+b}\right) + Li_3\left(\frac{b}{a}\right) + \log\left(\frac{1+a}{1+b}\right) Li_2\left(\frac{b}{a}\frac{1+a}{1+b}\right) + \frac{1}{2}\log^2\left(\frac{1+a}{1+b}\right)\log\left(\frac{a-b}{a(1+b)}\right) \quad \text{(A.14)}$$

$$F_2(a,b,c,d) = \frac{1}{d}\left\{\log^2\alpha\log\left(\frac{c+d}{c}\right) + 2\log\alpha\left[Li_2\left(-\frac{\beta c}{\alpha}\right) - Li_2\left(-\frac{\beta}{\alpha}(c+d)\right)\right] + 2S_{1,2}\left(-\frac{\beta}{\alpha}(c+d)\right) - 2S_{1,2}\left(-\frac{\beta c}{\alpha}\right)\right\} \quad \text{(A.15)}$$

the expressions K_i, which appear in the integral I_2, are given by [K_5 and K_8 will be used later on]

$$\begin{aligned}
K_1(a,b,c,d) &= \frac{1}{d}\left\{\log\alpha\log\left(-\frac{c}{d}\right)\log\left(\frac{c+d}{c}\right) + \log\alpha\left[\zeta(2) - Li_2\left(\frac{c+d}{c}\right)\right]\right.\\
&\quad + \log\left(-\frac{c}{d}\right)\left[Li_2\left(-\frac{\beta c}{\alpha}\right) - Li_2\left(-\frac{\beta}{\alpha}(c+d)\right)\right]\\
&\quad \left. - Li_3\left(-\frac{\beta c}{\alpha}\right) - S_{1,2}\left(-\frac{\beta c}{\alpha}\right) + F_1\left[-\frac{c+d}{c}, \frac{\beta}{\alpha}(c+d)\right]\right\}\\
K_2(a,b,c,d) &= \frac{1}{d}\left\{\log\alpha\log\left(\frac{c+d}{c}\right) + Li_2\left(-\frac{\beta c}{\alpha}\right) - Li_2\left(-\frac{\beta}{\alpha}(c+d)\right)\right\}\\
K_3(a,b,c,d) &= \frac{1}{2b}\left\{\log^2(a+b)\log(c+d) - \log^2 a\log c - dF_2(a,b,c,d)\right\}\\
K_4(a,b,c) &= \frac{1}{c}\left\{\log\left(\frac{c-a}{c}\right)\log\left(\frac{c-b}{c}\right)\log(1+c)\right.\\
&\quad + \log\left(\frac{c-a}{c}\right)\left[Li_2\left(\frac{-b}{c-b}\right) - Li_2\left(-b\frac{1+c}{c-b}\right)\right] + F_1\left(a\frac{1+c}{c-a}, b\frac{1+c}{c-b}\right)\\
&\quad \left. + \log\left(\frac{c-b}{c}\right)\left[Li_2\left(\frac{-a}{c-a}\right) - Li_2\left(-a\frac{1+c}{c-a}\right)\right] - F_1\left(\frac{a}{c-a}, \frac{b}{c-b}\right)\right\}\\
K_5(a) &= a\left\{1 - Li_2(-a)\right\} - (1+a)\log(1+a)\\
K_6(a,b) &= \frac{1}{a-b}\left\{Li_2(-a) - \frac{a}{b}Li_2(-b)\right\} - K_2(1,a,1,b)\\
K_7(a,b) &= 2 + \frac{a}{b}\left\{\log a + Li_2\left(-\frac{b}{a}\right)\right\} - \frac{a+b}{b}\log(a+b)\\
K_8(a,b) &= -\log(1+a)\log(1+b) - aLi_2(-b)\\
&\quad - a^2K_2(1,b,1,a) - b^2K_2(1,a,1,b)\\
K_9(a,b) &= \frac{1}{a}\left\{\frac{b}{a-b}\left[\log\left(\frac{1+a}{1+b}\right) + \frac{1}{2}\log^2(1+a) - bK_2(1,a,1,b)\right]\right.
\end{aligned}$$

$$
\begin{aligned}
& -\log(1+b)\frac{1+\log(1+a)}{1+a}\Bigg\} \\
K_{10}(a,b) &= 2+\frac{1+a}{a}\log(1+a)\left\{\log(1+b)-1\right\}-\frac{1+b}{b}\log(1+b) \\
& +\frac{a-b}{ab}\left\{\log\left(\frac{b-a}{b}\right)\log(1+b)+Li_2\left(\frac{-a}{b-a}\right)-Li_2\left(-a\frac{1+b}{b-a}\right)\right\} \\
K_{11}(a,b) &= \frac{1}{b-a}\left\{\frac{a}{b}\log(1+b)-\frac{1+a}{1+b}\log(1+a)\right\} \qquad \text{(A.16)}
\end{aligned}
$$

APPENDIX B: The Infrared Regularized Virtual Corrections for the $\mathcal{H}gg$ and Agg Couplings

The complete analytical expressions for the virtual, infrared regularized, QCD radiative corrections to the $\mathcal{H}gg$ and Agg couplings are summarized in this appendix. As discussed in section 2.3 and 3.3, the virtual corrections split into an infrared part π^2, a logarithmic part depending on the renormalization scale μ and a finite piece depending on τ

$$
C = \pi^2 + \frac{33-2N_F}{6}\ \log\frac{\mu^2}{m_\Phi^2} + c_\Phi(\tau) \qquad \text{(B.1)}
$$

Again the coefficient c_Φ can be split into two parts

$$
c_\Phi = \Re e\left\{\frac{\sum_Q F_0^\Phi(\tau)\left(B_1^\Phi + B_2^\Phi \log\frac{\mu_Q^2}{m_Q^2}\right)}{\sum_Q F_0^\Phi(\tau)}\right\} \qquad \text{(B.2)}
$$

where μ_Q is the scale at which the quark mass is defined; the coefficients B_1, B_2 read in the case of $\mathcal{C}P$–even and $\mathcal{C}P$–odd Higgs bosons,

$$
\begin{aligned}
B_1^H &= -\left[16\tau^{-1}F_0^H(\tau)\right]^{-1}\ \left\{32\tau^{-2}(-3\tau^{-1}+1)f(\tau)g(\tau)\right. \\
&+40\tau^{-2}(-2\tau^{-2}+4\tau^{-1}+3)f(\tau)+192\tau^{-2}(4\tau^{-1}-5)g(\tau) \\
&+216\tau^{-2}(-2\tau^{-1}+3)r(\tau)+36\tau^{-2}(-2\tau^{-1}+3)p(\tau) \\
&+4\tau^{-2}(18\tau^{-2}-49\tau^{-1}+30)l(\tau)+2\tau^{-2}(38\tau^{-2}-199\tau^{-1}+186)k(\tau) \\
&+3\tau^{-2}(-18\tau^{-2}+65\tau^{-1}-53)h(\tau)+144(\tau^{-1}-1)[2I_6-\tau I_8] \\
&+16\tau^{-2}(-29\tau^{-1}+20)+128(-\tau^{-2}+3\tau^{-1}-2)I_1 \\
&\left.-8(2\tau^{-1}-3)[\tau^{-1}I_2-10\tau^{-1}I_3+\tau^{-1}I_4+18I_7]-32\tau^{-1}(\tau^{-1}-1)I_5\ \right\} \\
B_2^H &= -6\tau^{-1}\ \left[F_0^H(\tau)\right]^{-1}\left\{(2\tau^{-1}-1)f(\tau)-g(\tau)-1\right\} \\
B_1^A &= \left[24\tau^{-1}F_0^A(\tau)\right]^{-1}\ \{-32\tau^{-2}f(\tau)g(\tau)-120\tau^{-2}(\tau^{-1}+1)f(\tau)+1152\tau^{-2}g(\tau) \\
&-648\tau^{-2}r(\tau)-108\tau^{-2}p(\tau)+12\tau^{-2}(9\tau^{-1}-10)l(\tau)+6\tau^{-2}(19\tau^{-1}-62)k(\tau) \\
&+3\tau^{-2}(53-27\tau^{-1})h(\tau)+128(2-\tau^{-1})I_1-24[\tau^{-1}I_2-10\tau^{-1}I_3+\tau^{-1}I_4+18I_7] \\
&-32\tau^{-1}I_5+144[2I_6-\tau^{-1}I_8]-696\tau^{-2}\ \} \\
B_2^A &= 4\tau^{-1}\left[F_0^A(\tau)\right]^{-1}\ \left\{f(\tau)-g(\tau)/(\tau^{-1}-1)\right\} \qquad \text{(B.3)}
\end{aligned}
$$

The functions F_0^H, F_0^A are given in eq. (A.5), while the functions f, g, l, k and h are given in eq. (A.7); the two remaining functions r and p read

$$
\begin{aligned}
r(\tau) &= -\frac{1}{2}\sqrt{1-\tau^{-1}}\left[Li_2\left(\frac{\alpha_+}{\alpha_+-\alpha_-}\right) - Li_2\left(\frac{\alpha_-}{\alpha_--\alpha_+}\right)\right] + \frac{g(\tau)}{2}\log\left[4(1-\tau)\right] \\
p(\tau) &= \sqrt{1-\tau^{-1}}\left[Li_2(1/\alpha_+) - Li_2(1/\alpha_-)\right] \qquad \text{(B.4)}
\end{aligned}
$$

The integrals I_1 to I_5 were presented in eqs. (A.9–A.13); the integrals I_6, I_7 and I_8 are given by

$$
\begin{aligned}
I_6 = {} & \frac{2}{\rho}\int_0^1 \frac{dx}{\rho x(1-x)}\left\{\frac{8}{3}\log^3[1-\rho x(1-x)] - 2\log(1-x)\log^2[1-\rho x(1-x)]\right. \\
& +3\log[1-\rho x(1-x)]Li_2(\rho x(1-x)) + 8S_{1,2}(\rho x(1-x)) \\
& +2\frac{\rho x^2}{1-\rho x}\left[K_4\left(-\rho x(1-x), -x, \frac{\rho x^2}{1-\rho x}\right) - K_{17}\left(1, -\rho x(1-x), 1, \frac{\rho x^2}{1-\rho x}\right)\right] \\
& \left. +4\rho x(1-x)K_3\left[1, -\rho x(1-x), 1-\rho x, \rho x^2\right]\right\} \qquad \text{(B.5)}
\end{aligned}
$$

$$
\begin{aligned}
I_7 = {} & \frac{2}{\rho}\int_0^1 \frac{dx}{\rho}\left\{\frac{8}{3}\log^3[1-\rho x(1-x)] - 2\log(1-x)\log^2[1-\rho x(1-x)]\right. \\
& +3\log[1-\rho x(1-x)]Li_2(\rho x(1-x)) + 8S_{1,2}(\rho x(1-x)) \\
& +2\frac{\rho x^2}{1-\rho x}\left[K_4\left(-\rho x(1-x), -x, \frac{\rho x^2}{1-\rho x}\right) - K_{17}\left(1, -\rho x(1-x), 1, \frac{\rho x^2}{1-\rho x}\right)\right] \\
& \left. +4\rho x(1-x)K_3\left[1, -\rho x(1-x), 1-\rho x, \rho x^2\right]\right\} \\
+ {} & \frac{2}{\rho}\int_0^1 dx\ x(1-x)\left\{\frac{2}{\rho x(1-x)}\left[\log[1-\rho x(1-x)]Li_2\left(\frac{-\rho x(1-x)}{1-\rho x(1-x)}\right)\right.\right. \\
& \left. -S_{1,2}\left(\frac{-\rho x(1-x)}{1-\rho x(1-x)}\right)\right] \\
& +F_2[1, -x, 1, -\rho x(1-x)] + F_2\left[1, -\rho x(1-x), 1-\rho x, \rho x^2\right] \\
& -F_2\left[1, -x, 1-\rho x, \rho x^2\right] + K_1[1, -x, 1, -\rho x(1-x)] \\
& -K_1\left[1, -\rho x(1-x), 1-\rho x, \rho x^2\right] - K_1\left[1, -x, 1-\rho x, \rho x^2\right] \\
& -2K_3[1, -\rho x(1-x), 1, -x] - K_{17}[1, -x, 1, -\rho x(1-x)] \\
& +K_{17}\left[1, -x, 1-\rho x, \rho x^2\right] - K_{17}\left[1, -\rho x(1-x), 1-\rho x, \rho x^2\right] \\
& \left. -K_{18}[1, -\rho x(1-x)] + K_{18}\left[1-\rho x, \rho x^2\right]\right\} \\
+ {} & \frac{2}{\rho}\int_0^1 dx\ x\left\{\log[1-\rho x(1-x)]\left[-\frac{2}{\rho x} - \frac{2}{1-\rho x}\right]\right.
\end{aligned}
$$

$$
\begin{aligned}
&+\log^2[1-\rho x(1-x)]\left[\frac{3}{2\rho x}+\frac{1}{\rho(1-x)}+\frac{3}{2(1-\rho x)}\right]\\
&+\log\left(\frac{1-\rho x(1-x)}{1-\rho x}\right)\left[-2\frac{2-x}{\rho x^2}+2\frac{1-x}{x}\right]+\frac{1-x}{2x}\frac{1-\rho x}{\rho x}\log^2\left(\frac{1-\rho x(1-x)}{1-\rho x}\right)\\
&+\frac{2}{\rho(1-x)}Li_2(\rho x(1-x))+2\frac{1-x}{\rho x^2}Li_2\left(\frac{\rho x^2}{1-\rho x(1-x)}\right)+2\frac{\rho x}{1-\rho x}\log x\\
&+\frac{1}{2}\frac{\rho x}{1-\rho x}\log^2 x-F_2\left[1,-\rho x(1-x),1-\rho x,\rho x^2\right]+F_2\left[1,-x,1-\rho x,\rho x^2\right]\\
&+K_1[1,-x,1-\rho x,\rho x^2]+K_1[1,-\rho x(1-x),1-\rho x,\rho x^2]\\
&+2(1-x)K_2[1,-\rho x(1-x),1-\rho x,\rho x^2]+2K_2[1,-x,1,-\rho x(1-x)]\\
&+2\rho x(1-x)K_2[1,-x,1-\rho x,\rho x^2]\\
&+2\frac{\rho x(1-x)}{1-\rho x}\left[K_2[1,-(1-x),1,-\rho x(1-x)]+K_2[1,-\rho x(1-x),1,-(1-x)]\right]\\
&+2(1-x)K_2[1,-\rho x(1-x),1,-x]\\
&-\frac{1-\rho x(1-x)}{(1-\rho x)^2}\left[K_6\left(-x,\frac{\rho x^2}{1-\rho x}\right)+K_6\left(-\rho x(1-x),\frac{\rho x^2}{1-\rho x}\right)\right]\\
&-2\frac{1-\rho x(1-x)}{(1-\rho x)^2}\left[K_{11}\left(-x,\frac{\rho x^2}{1-\rho x}\right)+K_{11}\left(-\rho x(1-x),\frac{\rho x^2}{1-\rho x}\right)\right]\\
&-\frac{1-\rho x(1-x)}{(1-\rho x)^2}\left[K_{12}\left(-x,\frac{\rho x^2}{1-\rho x}\right)-K_{12}\left(-\rho x(1-x),\frac{\rho x^2}{1-\rho x}\right)\right]\\
&-K_{17}[1,-x,1-\rho x,\rho x^2]+K_{17}[1,-\rho x(1-x),1-\rho x,\rho x^2]\\
&-K_{18}[1-\rho x,\rho x^2]+[1-\rho x(1-x)]K_{19}[1-\rho x,\rho x^2]\\
&-[1-\rho x(1-x)]\left[K_{20}[1,-\rho x(1-x),1-\rho x,\rho x^2]-K_{20}[1,-x,1-\rho x,\rho x^2]\right]\Big\} \qquad \text{(B.6)}
\end{aligned}
$$

$$
\begin{aligned}
I_8 \;=\; &\frac{2}{\rho}\int_0^1\frac{dx}{x}\left\{-\frac{1}{2}\log(1-x)\log^2[1-\rho x(1-x)]+3S_{1,2}(\rho x(1-x))\right.\\
&-2\log[1-\rho x(1-x)]Li_2\left(\frac{-\rho x(1-x)}{1-\rho x(1-x)}\right)\\
&+F_1\left[-\rho x(1-x),\frac{1-x}{x}(1-\rho x)\right]-F_1\left[-\rho x(1-x),\frac{1-x}{x}\right]\\
&\left.+\rho x(1-x)K_3\left[1,-\rho x(1-x),x,(1-x)(1-\rho x)\right]\right\} \qquad \text{(B.7)}
\end{aligned}
$$

While the functions K_1–K_{11} are defined in eqs.(A.16), the remaining functions follow from

$$
\begin{aligned}
K_{12}(a,b) &= \frac{1}{a-b}\left\{\frac{1+a}{1+b}\log^2(1+a)-2aK_2(1,a,1,b)\right\}\\
K_{13}(a,b,c) &= -\frac{\log(1+a)\log(1+b)}{c(1+c)}+\frac{1}{c}\left\{\frac{a^2}{a-c}K_2(1,b,1,a)\right.\\
&\left.-\frac{ac}{a-c}K_2(1,b,1,c)+\frac{b^2}{b-c}K_2(1,a,1,b)-\frac{bc}{b-c}K_2(1,a,1,c)\right\}
\end{aligned}
$$

$$
\begin{aligned}
K_{14}(a) &= a-(1+a)\log(1+a)\\
K_{15}(a) &= -(1+a)\log^2(1+a)-2aLi_2(-a)\\
K_{16}(a,b) &= K_7(a+b,-b)\\
K_{17}(a,b,c,d) &= K_1(a+b,-b,c+d,-d)\\
K_{18}(a,b) &= \frac{1}{b}\left\{\log\left(\frac{a+b}{a}\right)\left[Li_2\left(\frac{-b}{a}\right)-\zeta(2)\right]+S_{1,2}\left(\frac{-b}{a}\right)-2Li_3\left(\frac{-b}{a}\right)\right\}\\
K_{19}(a,b) &= \frac{1}{ab}Li_2\left(\frac{b}{a+b}\right)-\frac{1}{b(a+b)}Li_2\left(-\frac{b}{a}\right)-\frac{\zeta(2)}{a(a+b)}\\
K_{20}(a,b,c,d) &= \frac{1}{d}\left\{\frac{b}{\alpha d}\left[Li_2\left(\frac{b}{a+b}\right)-Li_2\left(\frac{d}{c+d}\right)\right]\right.\\
&\quad\left.+\frac{1}{c+d}Li_2\left(\frac{b}{a+b}\right)-\frac{d}{c+d}K_2(a,b,c,d)\right\}
\end{aligned}
\tag{B.8}
$$

APPENDIX C: The Real Corrections for pp $\to \mathcal{H}, A$ and $\mathcal{H}, A \to$ gg

Finally, we give here the complete analytical expressions for the real corrections to the processes $pp \to \mathcal{H}/A$ and $\mathcal{H}/A \to gg$. We start with the corrections to the production process and define the variables

$$
\rho=\frac{m_H^2}{m_Q^2}\,,\qquad S=\frac{\hat{s}}{m_Q^2}\,,\qquad T=\frac{\hat{t}}{m_Q^2}\,,\qquad U=\frac{\hat{u}}{m_Q^2}
\tag{C.1}
$$

$$
\hat{s}=\frac{m_H^2}{\hat{\tau}}\,,\qquad \hat{t}=-\hat{s}(1-\hat{\tau})v\,,\qquad \hat{u}=-\hat{s}(1-\hat{\tau})(1-v)
\tag{C.2}
$$

$$
\tau_Q=\frac{\rho}{4}\,,\qquad \tau_s=\frac{S}{4}\,,\qquad \tau_t=\frac{T}{4}\,,\qquad \tau_u=\frac{U}{4}
\tag{C.3}
$$

The coefficients $d_{q\bar{q}}, d_{gq}$ and d_{gg} which appear in the real QCD corrections, eq. (40), for the Higgs production, can be cast into the form

$$
\begin{aligned}
d_{q\bar{q}}(\hat{\tau},\tau_Q) &= \frac{2}{3\left|\sum_Q F_0^\Phi(\tau_Q)\right|^2}(1-\hat{\tau})^3\left|\sum_Q \mathcal{A}^\Phi_{qqg}(S)\right|^2\\
d_{gq}(\hat{\tau},\tau_Q) &= \frac{2}{3}\hat{\tau}^2+\frac{2}{3}\hat{\tau}^2\int_0^1\frac{dv}{v}\left\{-1-2\frac{1-\hat{\tau}}{\hat{\tau}^2}\right.\\
&\quad\left.+\frac{1+(1-\hat{\tau})^2(1-v)^2}{\hat{\tau}^2}\left|\frac{3}{2\sum_Q F_0^\Phi(\tau_Q)}\sum_Q \mathcal{A}^\Phi_{qqg}(T)\right|^2\right\}\\
d_{gg}(\hat{\tau},\tau_Q) &= \frac{3}{1-\hat{\tau}}\int_0^1\frac{dv}{v}\left\{\hat{\tau}^4\frac{\mathcal{A}^\Phi_{ggg}(S,T,U)}{\left|\sum_Q F_0^\Phi(\tau_Q)\right|^2}-1-\hat{\tau}^4-(1-\hat{\tau})^4\right\}
\end{aligned}
\tag{C.4}
$$

with F_0^Φ given in eq. (A.5). With the functions f and g given in eq. (A.7), A_{qqg}^Φ and A_{ggg}^Φ can be represented as

$$\begin{aligned}
\mathcal{A}_{qqg}^H(S) &= \frac{8}{S-\rho}\left\{1 - 2S\;\frac{g(\tau_s)-g(\tau_Q)}{S-\rho} - \left(1+\frac{4}{S-\rho}\right)[f(\tau_s)-f(\tau_Q)]\right\} \\
\mathcal{A}_{qqg}^A(S) &= \frac{16}{3(S-\rho)}\,[f(\tau_s)-f(\tau_Q)] \\
\mathcal{A}_{ggg}^\Phi(S,T,U) &= |C_1^\Phi|^2 + |C_2^\Phi|^2 + |C_3^\Phi|^2 + |C_4^\Phi|^2 \qquad \text{(C.5)}
\end{aligned}$$

The functions C_i^Φ follow from

$$C_2^\Phi(S,T,U) = -C_1^\Phi(T,S,U) \qquad\qquad C_3^\Phi(S,T,U) = C_1^\Phi(U,T,S) \qquad \text{(C.6)}$$

$$C_i^\Phi = \sum_Q \frac{1}{2\rho^2}\;\sum_{j=1}^{12} P_{ij}^\Phi T_j \qquad \text{(C.7)}$$

The coefficients T_i read

$$\begin{aligned}
T_1 &= 1 \;\;,\quad T_2 = 2f(\tau_Q) \;\;,\quad T_3 = 2f(\tau_s) \;\;,\quad T_4 = 2f(\tau_t) \qquad \text{(C.8)} \\
T_5 &= 2f(\tau_u) \;\;,\quad T_6 = 2[1-g(\tau_Q)] \;\;,\quad T_7 = 2[1-g(\tau_s)] \;\;,\quad T_8 = 2[1-g(\tau_t)] \\
T_9 &= 2[1-g(\tau_u)] \;,\; T_{10} = J(S,T,U) \;,\; T_{11} = J(S,U,T) \;,\; T_{12} = J(T,S,U)
\end{aligned}$$

with

$$\begin{aligned}
J(S,T,U) &= I_3(S,T,U,S) + I_3(S,T,U,U) - I_3(S,T,U,\rho) \\
I_3(S,T,U,X) &= \frac{1}{SU}\frac{2}{\beta_+ - \beta_-}\left\{Li_2\left(\frac{\beta_-}{\beta_- - \alpha_-}\right) - Li_2\left(\frac{\beta_+}{\beta_+ - \alpha_+}\right)\right. \qquad \text{(C.9)} \\
&+ \left. Li_2\left(\frac{\beta_-}{\beta_- - \alpha_+}\right) - Li_2\left(\frac{\beta_+}{\beta_+ - \alpha_-}\right) + \log\left(-\frac{\beta_+}{\beta_-}\right)\log\left(1+\frac{XT}{SU}\right)\right\}
\end{aligned}$$

and

$$\alpha_\pm = \frac{1}{2}\left(1 \pm \sqrt{1-\frac{4}{X}}\right) \quad , \quad \text{and} \quad \beta_\pm = \frac{1}{2}\left(1 \pm \sqrt{1+\frac{4T}{SU}}\right) \qquad \text{(C.10)}$$

The coefficients P_{ij} for $\mathcal{CP}$–even neutral Higgs bosons are given [44] by

$$\begin{aligned}
P_{1,1} &= -12\;S\;\frac{UT-S^2}{(U+S)(T+S)} \\
P_{1,2} &= 3\left\{4U^3T^3 + 8U^3T^2S + 4U^3TS^2 + 8U^2T^3S + 15U^2T^2S^2 + 4U^2T^2S\right. \\
&\quad +8U^2TS^3 + 8U^2TS^2 + U^2S^4 - 4U^2S^3 + 4UT^3S^2 + 8UT^2S^3
\end{aligned}$$

$$+8UT^2S^2+8UTS^4+16UTS^3+4US^5-8US^4+T^2S^4-4T^2S^3$$
$$+4TS^5-8TS^4+3S^6-12S^5\Big\} \times \frac{1}{S(U+S)^2(T+S)^2}$$

$$P_{1,3} = -3\ (S-4)$$

$$P_{1,4} = -3\ \frac{4U^3T+8U^2TS-U^2S^2+4U^2S+4UTS^2+8US^2+S^4-4S^3}{S(U+S)^2}$$

$$P_{1,5} = -3\ \frac{4UT^3+8UT^2S+4UTS^2-T^2S^2+4T^2S+8TS^2+S^4-4S^3}{S(T+S)^2}$$

$$P_{1,6} = -12\ UT\ \frac{U^2T+2U^2S+UT^2+4UTS+5US^2+2T^2S+5TS^2+4S^3}{(U+S)^2(T+S)^2}$$

$$
\begin{array}{lll}
P_{1,7} = 0 & & P_{1,8} = 12\ UT\ (U+2S)(U+S)^{-2} \\
P_{1,9} = 12\ UT\ (T+2S)(T+S)^{-2} & & P_{1,10} = 3\ US\ (4-S)/2 \\
P_{1,11} = 3\ TS\ (4-S)/2 & & P_{1,12} = -3\ UTS^{-1}(4UT-S^2+12S)/2 \\
P_{4,1} = 12\ \rho & & P_{4,2} = -9\ (4-\rho) \\
P_{4,3} = 3\ (4-\rho) & & P_{4,4} = 3\ (4-\rho) \\
P_{4,5} = 3\ (4-\rho) & & P_{4,6} = 0 \\
P_{4,7} = 0 & & P_{4,8} = 0 \\
P_{4,9} = 0 & & P_{4,10} = 3\ US\ (4-\rho)/2 \\
P_{4,11} = 3\ TS\ (4-\rho)/2 & & P_{4,12} = 3\ UT\ (4-\rho)/2
\end{array}
$$

Similarly for $\mathcal{C}P$–odd neutral Higgs bosons,

$$
\begin{array}{lll}
P_{1,1} = 0 & & P_{1,2} = -2S(UT-US-TS-3S^2)(U+S)^{-1}(T+S)^{-1} \\
P_{1,3} = -2S & & P_{1,4} = 2S\ (U-S)(U+S)^{-1} \\
P_{1,6} = 0 & & P_{1,5} = 2S(T-S)(T+S)^{-1} \\
P_{1,7} = 0 & & P_{1,8} = 0 \\
P_{1,9} = 0 & & P_{1,10} = -US^2 \\
P_{1,11} = -TS^2 & & P_{1,12} = STU \\
P_{4,1} = 0 & & P_{4,2} = 6\rho \\
P_{4,3} = -2\rho & & P_{4,4} = -2\rho \\
P_{4,5} = -2\rho & & P_{4,6} = 0 \\
P_{4,7} = 0 & & P_{4,8} = 0 \\
P_{4,9} = 0 & & P_{4,10} = -\rho US \\
P_{4,11} = -\rho TS & & P_{4,12} = -\rho UT
\end{array}
$$

For the QCD corrections to the gluonic decays of the Higgs bosons the correction factors ΔE_Φ defined in eqs. (23) and (70) can be written as

$$\Delta E_\Phi = \Delta E^\Phi_{virt} + \Delta E^\Phi_{ggg} + N_F \Delta E^\Phi_{gq\bar{q}} \quad \text{(C.11)}$$

$$\Delta E^{\mathcal{H}}_{virt} = c_{\mathcal{H}}(\tau_Q) - \frac{11}{2} \quad \text{(C.12)}$$

$$\Delta E^{A}_{virt} = c_{A}(\tau_Q) - 6 \quad \text{(C.13)}$$

$$\Delta E^\Phi_{ggg} = \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \left\{ \frac{m_\Phi^6}{stu} \frac{\mathcal{A}^\Phi_{ggg}(S,T,U)}{\left|\sum_Q F_0^\Phi(\tau_Q)\right|^2} - \frac{m_\Phi^8 + s^4 + t^4 + u^4}{2stu\; m_\Phi^2} \right\} \quad \text{(C.14)}$$

$$\Delta E^\Phi_{gq\bar{q}} = \int_0^1 dx_1 \int_{1-x_1}^1 dx_2 \frac{s^2+u^2}{t\; m_\Phi^2} \left\{ \frac{m_\Phi^2}{s} \left| \frac{\sum_Q \mathcal{A}^\Phi_{qqg}(T)}{\sum_Q F_0^\Phi(\tau_Q)} \right|^2 - 1 \right\} \quad \text{(C.15)}$$

with $c_{\mathcal{H}}$ and c_A given in eqs. (B.2, B.3); the kinematical variables are defined as

$$s = m_H^2(1-x_3)\;, \quad t = m_H^2(1-x_2)\;, \quad u = m_H^2(1-x_1)\;, \quad x_1+x_2+x_3 = 2 \quad \text{(C.16)}$$

and

$$S = \frac{s}{m_Q^2}\;, \qquad T = \frac{t}{m_Q^2}\;, \qquad U = \frac{u}{m_Q^2} \quad \text{(C.17)}$$

for the Mandelstam variables normalized by the quark mass.

References

[1] P. W. Higgs, Phys. Rev. Lett. 12 (1964) 132 and Phys. Rev. 145 (1966) 1156;
F. Englert and R. Brout, Phys. Rev. Lett. 13 (1964) 321;
G. S. Guralnik, C. R. Hagen and T. W. Kibble, Phys. Rev. Lett. 13 (1964) 585.

[2] For a review on the Higgs sector in the Standard Model and in its supersymmetric extensions, see J. Gunion, H. Haber, G. Kane and S. Dawson, "The Higgs Hunter's Guide", Addison–Wesley, Reading, Mass., 1990.

[3] N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B158 (1979) 295;
M. Chanowitz, M. Furman and I. Hinchliffe, Phys. Lett. B78 (1978) 285;
R. A. Flores and M. Sher, Phys. Rev. D27 (1983) 1679;
M. Lindner, Z. Phys. C31 (1986) 295;
M. Sher, Phys. Rep. 179 (1989) 273.

[4] M. Sher, Phys. Lett. B317 (1993) 159 and addendum B331 (1994) 448;
G. Altarelli and G. Isidori, Phys. Lett. B337 (1994) 141;
J. Casas, J. Espinosa and M. Quiros, Phys. Lett. B342 (1995) 171.

[5] J. Espinosa and M. Quiros, Report DESY 95–039.

[6] H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.

[7] A. Hasenfratz, K. Jansen, C. Lang, T. Neuhaus and H. Yoneyama, Phys. Lett. 199B (1987) 531;
J. Kuti, L. Liu and Y. Shen, Phys. Rev. Lett. 61 (1988) 678;
M. Lüscher and P. Weisz, Nucl. Phys. B318 (1989) 705.

[8] D. Schaile, Report CERN–PPE–94–162, Proceedings of the International Conference on High Energy Physics, Glasgow 1994.

[9] F. Abe et al., CDF Collaboration, Phys. Rev. Lett. 73 (1994) 225 and Report FERMILAB–PUB–95/022–E.

[10] S. Abachi et al., D0 Collaboration, Report FERMILAB–PUB–95/028–E.

[11] P. Janot, Report LAL–94–59.

[12] J.D. Bjorken, Proceedings of the Summer Institute on Particle Physics, SLAC Report 198 (1976).

[13] J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B106 (1976) 292.

[14] B.W. Lee, C. Quigg and H.B. Thacker, Phys. Rev. D16 (1977) 1519;
B.L. Ioffe and V.A. Khoze, Sov. J. Part. Nucl. 9 (1978) 50.

[15] Proceedings of the Workshop "e^+e^- Collisions at 500 GeV: The Physics Potential", DESY Reports 92–123A and 93–123C (P. Zerwas, ed).

[16] For a recent review on Higgs phenomenology at future e^+e^- and pp colliders see, A. Djouadi, Int. J. Mod. Phys. A10 (1995) 1.

[17] D. Froidevaux, Z. Kunszt and J. Stirling [conv.] et al. in the Proceedings of the Large Hadron Collider Workshop, Aachen 1990, CERN Report 90–10, Vol. II;
see also the Rapporteurs talks by G. Altarelli and D. Denegri *ibid.*, Vol. I.

[18] CMS Collaboration, Technical Proposal, Report CERN–LHCC 94–38 (December 1994).

[19] ATLAS Collaboration, Technical Proposal, Report CERN–LHCC 94–43 (December 1994).

[20] H. Georgi, S. Glashow, M. Machacek and D. Nanopoulos, Phys. Rev. Lett. 40 (1978) 692.

[21] R.N. Cahn and S. Dawson, Phys. Lett. 136B (1984) 196;
K. Hikasa, Phys. Lett. 164B (1985) 341;
G. Altarelli, B. Mele and F. Pitolli, Nucl. Phys. B287 (1987) 205;
T. Han, G. Valencia and S. Willenbrock, Phys. Rev. Lett. 69 (1992) 3274.

[22] Z. Kunszt, Nucl. Phys. B247 (1984) 339;
J. F. Gunion, Phys. Lett. B253 (1991) 269;
W. J. Marciano and F. E. Paige, Phys. Rev. Lett. 66 (1991) 2433.

[23] S. L. Glashow, D. V. Nanopoulos and A. Yildiz, Phys. Rev. D18 (1978) 1724;
Z. Kunszt, Z. Trocsanyi and W. J. Stirling, Phys. Lett. B271 (1991) 247;
T. Han and S. Willenbrock, Phys. Lett. B273 (1991) 167.

[24] J. Dai, J.F.Gunion and R. Vega, Phys. Rev. Lett. 71 (1993) 2699.

[25] D. Froidevaux and E. Richter–Was, Report CERN–TH–7459/94.

[26] E. Witten, Phys. Lett. B105 (1981) 267.

[27] S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;
N. Sakai, Z. Phys. C11 (1981) 153;
K. Inoue, A. Komatsu and S. Takeshita, Prog. Theor. Phys. 67 (1982) 927; (E) 70 (1983) 330; 71 (1984) 413;
E. Witten, Nucl. Phys. B231 (1984) 419.

[28] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681;
L.E. Ibáñez and G.G. Ross, Phys. Lett. B105 (1981) 439.

[29] J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. 260B (1991) 131;
U. Amaldi, W. de Boer and H. Fürstenau, Phys. Lett. 260B (1991) 447;
P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817.

[30] J. Gunion and A. Turski, Phys. Rev. D39 (1989) 2701 and D40 (1990) 2333;
M. Berger, Phys. Rev. D41 (1990) 225;
Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85 (1991) 1;
H. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815;
J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. 257B (1991) 83;
R. Barbieri, F. Caravaglios and M. Frigeni, Phys. Lett. 258B (1991) 167;
A. Yamada, Phys. Lett. 263B (1991) 233;
A. Brignole, J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B271 (1991) 123;
P. H. Chankowski, S. Pokorski and J. Rosiek, Phys. Lett. B274 (1992) 191;
M. Drees and M. M. Nojiri, Phys. Rev. D45 (1992) 2482.

[31] J. R. Espinosa and M. Quiros, Phys. Lett. 266B (1991) 389;
R. Hempfling and A. Hoang, Phys. Lett. B331 (1994) 99;
J. Espinosa and M. Quiros, Report CERN–TH–7334–94.

[32] T. Elliot, S. King and P. White, Phys. Lett. B305 (1993) 71;
G. Kane, C. Kolda and J. Wells, Phys. Rev. Lett. 70 (1993) 2686;
U. Ellwanger, M. Rausch de Traubenberg and C. Savoy, Report LPTHE Orsay 95–04.

[33] J. F. Gunion and H. E. Haber, Nucl. Phys. B307 (1988) 445;
K. Griest and H. Haber, Phys. Rev. D37 (1988) 719;
H. Baer, M. Bisset, D.Dicus, C. Kao and X. Tata, Phys. Rev. D47 (1993) 1062;
S. G. Frederiksen, N. P. Johnson, G. L. Kane and J. Reid, Phys. Rev. D50 (1994) R4244.

[34] A. Djouadi, J. Kalinowski and P. M. Zerwas, Z. Phys. C57 (1993) 565.

[35] Z. Kunszt and F. Zwirner, Nucl. Phys. B385 (1992) 3;
V. Barger, M. Berger, S. Stange and R. Phillips, Phys. Rev. D45 (1992) 4128;
H. Baer, M. Bisset, C. Kao and X. Tata, Phys. Rev. D46 (1992) 1067;
J. Gunion and L. Orr, Phys. Rev. D46 (1992) 2052;
J. F. Gunion, H. E. Haber and C. Kao, Phys. Rev. D46 (1992) 2907.

[36] A. Djouadi, M. Spira and P.M. Zerwas, Phys. Lett. B264 (1991) 440.

[37] S. Dawson, Nucl. Phys. B359 (1991) 283.

[38] A. Djouadi, M. Spira and P. M. Zerwas, Phys. Lett. B311 (1993) 255.

[39] R.P. Kauffman and W. Schaffer, Phys. Rev. D49 (1994) 551.

[40] D. Graudenz, M. Spira and P. M. Zerwas, Phys. Rev. Lett. 70 (1993) 1372.

[41] M. Spira, A. Djouadi, D. Graudenz and P.M. Zerwas, Phys. Lett. B318 (1993) 347.

[42] R. P. Kauffman, Phys. Rev. D44 (1991) 1415 and D45 (1992) 1512;
C. Kao, Phys. Lett. B328 (1994) 420.

[43] I. Hinchliffe and S. Novaes, Phys. Rev. D38 (1988) 3475.

[44] R. K. Ellis, I. Hinchliffe, M. Soldate and J.J. van der Bij, Nucl. Phys. B297 (1987) 221.

[45] H. Zheng and D. Wu, Phys. Rev. D42 (1990) 3760;
A. Djouadi, M. Spira, J. van der Bij and P. Zerwas, Phys. Lett. B257 (1991) 187;
S. Dawson and R. P. Kauffman, Phys. Rev. D47 (1993) 1264;
K. Melnikov and O. Yakovlev, Phys. Lett. B312 (1993) 179;
M. Inoue, R. Najima, T. Oka and J. Saito, Mod. Phys. Lett. A9 (1994) 1189.

[46] K. Melnikov, M. Spira and O. Yakovlev, Z. Phys. C64 (1994) 401.

[47] T. Inami, T. Kubota and Y. Okada, Z. Phys. C18 (1983) 69.

[48] M. Spira, A. Djouadi and P.M. Zerwas, Phys. Lett. B276 (1992) 350.

[49] A. Djouadi and P. Gambino, Phys. Rev. Lett. 73 (1994) 2523.

[50] H. Haber and J. Gunion, Phys. Rev. D48 (1993) 5109;
I. F. Ginzburg, Novosibirsk Preprint TF–28–182 (1990);
D. L. Borden, D. A. Bauer and D. O. Caldwell, Phys. Rev. D48 (1993) 4018;
M. Krämer, J. Kühn, M. L. Stong and P. M. Zerwas, Z. Phys. C64 (1994) 21;
D. Borden, V.A. Khoze, W.J. Stirling and J. Ohnemus, Phys. Rev. D50 (1994) 4499.

[51] A. I. Vainshtein, M. B. Voloshin, V. I. Sakharov and M. A. Shifman, Sov. J. Nucl. Phys. 30 (1979) 711.

[52] M. Spira, Ph. D. Thesis, RWTH Aachen, 1992;
K. Melnikov and O. Yakovlev in [45].

[53] E. Braaten and J.P. Leveille, Phys. Rev. D22 (1980) 715;
N. Sakai, Phys. Rev. D22 (1980) 2220;
T. Inami and T. Kubota, Nucl. Phys. B179 (1981) 171;
M. Drees and K. Hikasa, Phys. Lett. B240 (1990) 455;
A. Djouadi and P. Gambino, Phys. Rev. D51 (1995) 218.

[54] G. 't Hooft and M. Veltman, Nucl. Phys. B15 (1979) 365;
A. Devoto and D. W. Duke, Riv. Nuovo. Cim. 7 (1984) 1.

[55] A. Czarnecki, U. Kilian and D. Kreimer, Nucl. Phys. B433 (1995) 259.

[56] J. Fleischer and O.V. Tarasov, Z. Phys. C64 (1994) 413.

[57] I. Bigi, Y.L. Dokshitser, V.Khoze, J. Kühn and P.M. Zerwas, Phys. Lett. B181 (1986) 157.

[58] S. L. Adler, J. C. Collins and A. Duncan, Phys. Rev. D15 (1977) 1712.

[59] P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B327 (1989) 49.

[60] S. Brodsky, G. P. Lepage and P. B. Mackenzie, Phys. Rev. D28 (1983) 228.

[61] S. Dawson and R. P. Kauffman, Phys. Rev. D49 (1993) 2298.

[62] G. Altarelli, R.K. Ellis and G. Martinelli, Nucl. Phys. B157 (1979) 461;
W. Furmanski and R. Petronzio, Z. Phys. C11 (1982) 293.

[63] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.

[64] M. Glück, E. Reya and A. Vogt, Z. Phys. C53 (1992) 127.

[65] S. Bethke, Proceedings QCD 94, Montpellier 1994, Report PITHA–94–30.

[66] W. Bernreuther and W. Wetzel, Phys. Lett. B132 (1983) 382;
W. Marciano, Phys. Rev. D29 (1984) 580.

[67] A. Martin, R. Roberts and W. Stirling, Phys. Rev. D50 (1994) 6734.

[68] H.L. Lai, J. Botts, J. Huston, J.G. Morfin, J.F. Owens, J.W. Qiu, W.K. Tung and H. Weerts, CTEQ–Collaboration, Report MSU–HEP–41024.

[69] For reviews on supersymmetric theories, see:
P. Fayet and S. Ferarra, Phys. Rep. 32 (1977) 249;
H. P. Nilles, Phys. Rep. 110 (1984) 1;
H. Haber and G. Kane, Phys. Rep. 117 (1985) 75;
R. Barbieri, Riv. Nuovo Cimento 11 (1988) 1.

[70] J. Gunion and H. Haber, Nucl. Phys. B272 (1986) 1 and B278 (1986) 449.

[71] H. Haber and M. Sher, Phys. Rev. D35 (1987) 2206;
J. L. Hewett and T. G. Rizzo, Phys. Rep. 183 (1989) 194;
J. Ellis, J.F. Gunion, H. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D39 (1989) 844;
M. Drees, Int. J. Mod. Phys. A4 (1989) 3635.

[72] M. Carena and C. Wagner, private communication.

[73] M. Davier, Proceedings of the Lepton–Photon Conference, Geneva 1991;
M. Pohl, Proceedings of the International Conference on High Energy Physics, Glasgow 1994.

[74] F. Abe et al., CDF Collaboration, Phys. Rev. Lett. 69 (1992) 3439.

[75] J. Dai, J. F. Gunion and R. Vega, Phys. Lett. B315 (1993) 355.

[76] B. Kileng, Z. Phys. C63 (1994) 87.

[77] G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189;
P. Breitenlohner and D. Maison, Commun. Math. Phys. 52 (1977) 11.

[78] S. A. Larin, Phys. Lett. B303 (1993) 113; Preprint NIKHEF–H 92/18.

[79] S. L. Adler and W. A. Bardeen, Phys. Rev. 182 (1969) 1517;
R. Jackiw, Lectures on Current Algebra and its Applications, Princeton University Press 1972.

[80] A. Sommerfeld, Atombau und Spektrallinien, Vieweg, Braunschweig 1939.

Figure Captions

Fig. 1: Generic diagrams of the gluon fusion mechanism $gg \to H$ for the production of Higgs bosons: lowest order amplitude (a), and QCD radiative corrections (b).

Fig. 2: Generic diagrams for the amplitude of the Higgs boson decay into two photons $H \to \gamma\gamma$: (a) lowest order W–boson amplitude, (b) lowest order quark amplitude, and (c) QCD radiative corrections to the quark amplitude.

Fig. 3: Generic Feynman diagrams for the amplitude of the Higgs boson decay into gluons $H \to gg$: (a) lowest order amplitude, and (b) QCD radiative corrections.

Fig. 4: The real and imaginary parts of the lowest order amplitudes A_f (a) and A_W (b) of the $H\gamma\gamma$ vertex as a function of $\tau_{f,W} = m_H^2/4m_{f,W}^2$.

Fig. 5: Real and imaginary parts of the radiative correction factor to the quark amplitudes for the $H\gamma\gamma$ coupling; the renormalization point for the quark mass is taken to be $\mu_Q = m_H/2$ in (a) and $\mu_Q = m_Q$ in (b).

Fig. 6: (a) The QCD corrected partial decay width of the Higgs boson to two photons as a function of the Higgs mass, and (b) the size of the QCD radiative correction factor (in %).

Fig. 7: (a) Comparison of the size of the infrared regularized virtual QCD corrections to the quark amplitude for the pole mass $m_Q(m_Q)$ and the running quark mass $m_Q(m_H/2)$; for large quark–loop masses the coefficient C approaches the value $C = \pi^2/2+11/4$; (b) The deviation ΔE of the radiative QCD correction to the decay $H \to gg$ from its value in the heavy quark–loop limit; the renormalization scale is taken to be $\mu = m_H$.

Fig. 8: (a) The QCD corrected partial decay width of the Higgs boson into two gluons (in MeV) as a function of the Higgs mass, and (b) the size of the QCD radiative correction factor; the renormalization scale is taken to be $\mu = m_H$.

Fig. 9: (a) Total decay width (in GeV) of the Standard Model Higgs boson as a function of its mass, and (b) the branching ratios (in %) of the dominant decay modes ($m_t = 174$ GeV). All known QCD and leading electroweak radiative corrections are included.

Fig. 10: (a) Feynman diagram for the effective couplings of the Higgs boson to gluons in the heavy–quark–loop limit, and (b) generic Feynman diagrams of the effective QCD corrections to the decay $H \to gg$ in the heavy–quark–loop limit.

Fig. 11: K factors of the QCD corrected cross section $\sigma(pp \to H + X)$ at the LHC with c.m. energy $\sqrt{s} = 14$ TeV. K_{virt} and K_{AB} $(A, B = q, g)$ are the regularized virtual correction and the real correction factors, respectively; K_{tot} is the ratio of the QCD

corrected total cross section to the lowest order cross section. The renormalization and factorization scales are taken to be $\mu = M = m_H$ and the GRV parameterizations for the parton densities have been used.

Fig. 12: (a) The spread of the Higgs boson production cross section at the LHC with c.m. energy of $\sqrt{s} = 14$ TeV for two parameterizations of the parton densities. (b) The total Higgs production cross section at the LHC for two different c.m. energy values: $\sqrt{s} = 14$ TeV and $\sqrt{s} = 10$ TeV.

Fig. 13: The renormalization and factorization scale dependence of the Higgs production cross section at lowest and next–to–leading order; the Higgs mass is chosen to be (a) $m_H = 150$ GeV, and (b) $m_H = 500$ GeV.

Fig. 14: (a) The upper limit of the lightest scalar Higgs boson mass in the $\mathcal{MSSM}$ as a function of the top quark mass for two values of tg$\beta = 1.5$ and 30; the top quark and the common squark masses are taken to be $m_t = 174$ GeV and $M_S = 1$ TeV, respectively. The dashed line corresponds to the case where $A_t = A_b = \mu = 0$ (only the leading radiative correction is included), while the full lines correspond to the case where $A_t = A_b = 1$ TeV and $\mu = -200, 0, +200$ GeV (from top to bottom). The masses of the $\mathcal{MSSM}$ Higgs bosons h^0, H^0 and $H^\pm$, as a function of the pseudoscalar Higgs mass for the two previous values of tgβ and for $A_t = A_b = \mu = 0, M_S = 1$ TeV, are displayed in (b), (c) and (d), respectively.

Fig. 15: The coupling parameters of the $\mathcal{MSSM}$ neutral Higgs bosons as functions of the pseudoscalar A^0 Higgs mass for two values of tg$\beta = 1.5$ and 30 and for $A_t = A_b = \mu = 0, M_S = 1$ TeV. The couplings are normalized to the $\mathcal{SM}$ couplings as defined in Table 1.

Fig. 16: The branching ratios of the $\mathcal{MSSM}$ Higgs bosons h^0 (a), H^0 (b), A^0 (c) and $H^\pm$ (d) as functions of their masses for two values of tg$\beta = 1.5$ and 30; the values $A_t = A_b = \mu = 0$ and $M_S = 1$ TeV have been chosen. [The arrows in (a) denote the branching ratios in the $\mathcal{SM}$ limit of large A^0 masses.]

Fig. 17: The total decay widths of the $\mathcal{MSSM}$ Higgs bosons h^0, H^0, A^0 and $H^\pm$ as functions of their masses for two values of tg$\beta = 1.5$ (a) and tg$\beta = 30$ (b); the values $A_t = A_b = \mu = 0$ and $M_S = 1$ TeV have been chosen.

Fig. 18: Real and imaginary parts of the QCD radiative correction factor to the quark amplitudes of the two–photon couplings for the $\mathcal{MSSM}$ neutral Higgs bosons: (a) h^0 and H^0 and (b) A^0; the renormalization scale for the quark mass is taken to be $\mu_Q = m_\Phi/2$.

Fig. 19: The QCD corrected partial decay widths into two photons of the $\mathcal{MSSM}$ Higgs bosons h^0, H^0, A^0 for (a) tg$\beta = 1.5$ and (b) tg$\beta = 30$, and the the size of the QCD radiative corrections to the processes $h^0/H^0 \to \gamma\gamma$ and $A^0 \to \gamma\gamma$ (in %) as

functions of the Higgs boson masses for two values of tg$\beta = 1.5$ (c) and 30 (d). The renormalization scale for the quark mass is taken to be $\mu_Q = m_\Phi/2$.

Fig. 20: The deviation $\Delta E_{\mathcal{H}}$ (a) and ΔE_A (b) of the coefficients E_Φ of the radiative QCD correction factors to the process $\Phi \to gg$ from their values in the heavy quark limit, for two values of tg$\beta = 1.5$ and 30; the renormalization scale is taken to be $\mu = m_\Phi$.

Fig. 21: The QCD corrected gluonic partial decay widths of the $\mathcal{M}SSM$ neutral Higgs bosons h^0, H^0 (a) and A^0 (b), for two values of tg$\beta = 1.5$ and 30; the size of the QCD radiative correction factor for $h^0/H^0 \to gg$ (c) and $A^0 \to gg$ (d). The renormalization scale is taken to be $\mu = m_\Phi$.

Fig. 22: K factors of the QCD corrected cross sections $\sigma(pp \to h^0/H^0 + X)$ (a) and $\sigma(pp \to A^0 + X)$ (b) for two values of tg$\beta = 1.5$ and 30; K_{virt} and K_{AB} $(A, B = q, g)$ are the regularized virtual correction and real correction factors, respectively, and K_{tot} is the ratio of the QCD corrected total cross section to the lowest order cross section. The renormalization and factorization scales are taken to be $\mu = M = m_\Phi$ and the GRV parameterization for the parton densities have been used.

Fig. 23: The dependence of the total K factors for the processes $\sigma(pp \to \Phi + X)$ on the value of tgβ for a characteristic set of Higgs boson masses.

Fig. 24: The spread of the $\mathcal{M}SSM$ Higgs production cross sections $\sigma(pp \to h^0/H^0 + X)$ (a) and $\sigma(pp \to A^0 + X)$ (b) for two parameterizations of the parton densities.

Fig. 25: The total production cross sections of the scalar $\mathcal{C}P$–even Higgs bosons h^0, H^0 (a) and the pseudoscalar Higgs boson A^0 (b) at the LHC for two different c.m. energy values: $\sqrt{s} = 14$ TeV and $\sqrt{s} = 10$ TeV.

Fig. 26: The renormalization/factorization scale dependence of the $\mathcal{M}SSM$ Higgs boson production cross sections at lowest and next–to–leading order, for a characteristic set of Higgs boson masses and tgβ values.