

CERN-TH/95-85

# Primordial Magnetic Fields from String Cosmology

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## Abstract

Sufficiently large seeds for generating the observed (inter)galactic magnetic fields emerge naturally in string cosmology from the amplification of electromagnetic vacuum fluctuations due to a dynamical dilaton background. The success of the mechanism depends crucially on two features of the so-called pre-big-bang scenario, an early epoch of dilaton-driven inflation at very small coupling, and a sufficiently long intermediate stringy era preceding the standard radiation-dominated evolution.

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HEP-TH-9504083

CERN-TH/95-85

April 1995

It is widely believed that the observed galactic (and intergalactic) magnetic fields, of microgauss strength, are generated and maintained by the action of a cosmic dynamo [1]. Any dynamo model, however, requires a primordial seed field; in spite of many attempts [2]-[5], it is fair to say that no compelling mechanism able to generate the required seed field (coherent over the Mpc scale, and with an energy density to radiation density ratio  $\rho_B/\rho_\gamma \gtrsim 10^{-34}$ ) has yet been suggested.

A priori, an appealing mechanism for the origin of the seed field is the cosmological amplification of the vacuum quantum fluctuations of the electromagnetic field, the same kind of mechanism which is believed to generate primordial metric and energy density perturbations [6]. The minimal coupling of photons to the metric background is, however, conformally invariant (in  $d = 3$  spatial dimensions). As a consequence, a cosmological evolution involving a conformally flat metric (as it is effectively the case in inflation) cannot amplify magnetic fluctuations, unless conformal invariance is broken. Possible attempts to generate large enough seeds thus include considering exotic higher-dimensional scenarios, or coupling non-minimally the electromagnetic field to the background curvature [2] with some “ad hoc” prescription, or breaking conformal invariance at the quantum level through the so-called trace anomaly [4].

In critical superstring theory the electromagnetic field  $F_{\mu\nu}$  is coupled not only to the metric ( $g_{\mu\nu}$ ), but also to the dilaton background ( $\phi$ ). In the low energy limit such interaction is represented by the string effective action [7], which reads, after reduction from ten to four external dimensions,

$$S = - \int d^4x \sqrt{-g} e^{-\phi} (R + \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu}) \quad (1)$$

where  $\phi = \Phi - \ln V_6 \equiv \ln(g^2)$  controls the tree-level four-dimensional gauge coupling ( $\Phi$  being the ten-dimensional dilaton field, and  $V_6$  the volume of the six-dimensional compact internal space).

In the inflationary models based on the above effective action [8, 9] the dilaton background is not at all constant, but undergoes an accelerated evolution from the string perturbative vacuum ( $\phi = -\infty$ ) towards the strong coupling regime, where it is expected to remain frozen at its present value. In this context, the quantum fluctuations of the electromagnetic field

can thus be amplified *directly* through their coupling to the dilaton, according to eq.(1). In the following we will discuss the conditions under which such mechanism is able to produce large enough primordial magnetic fields to seed the galactic dynamo (a scalar-vector coupling similar to that of eq.(1) was previously discussed in [3], but  $\phi$  was there identified with the conventional inflaton undergoing a dynamical evolution much different from the dilaton evolution considered here).

Let us first define a few important parameters of the inflationary scenario (also called "pre-big-bang" scenario) discussed in [9]. The phase of growing curvature and dilaton coupling ( $\dot{H} > 0$ ,  $\dot{\phi} > 0$ ), driven by the kinetic energy of the dilaton field, is correctly described in terms of the lowest order string effective action only up to the conformal time  $\eta = \eta_s$  at which the curvature reaches the string scale  $H_s = \lambda_s^{-1}$  ( $\lambda_s \equiv \sqrt{\alpha'}$  is the fundamental length of string theory). A first important parameter of this cosmological model is thus the value  $\phi_s$  attained by the dilaton at  $\eta = \eta_s$ . Provided such value is sufficiently negative (i.e. provided the coupling  $g = e^{\phi/2}$  is sufficiently small to be still in the perturbative region at  $\eta = \eta_s$ ), it is also arbitrary, since there is no perturbative potential to break invariance under shifts of  $\phi$ . For  $\eta > \eta_s$  high-derivatives terms (higher orders in  $\alpha'$ ) become important in the string effective action, and the background enters a genuinely "stringy" phase of unknown duration. It was shown in [10] that it is impossible to have a graceful exit to standard cosmology without such an intermediate stringy phase. An assumption of string cosmology is that the stringy phase eventually ends at some conformal time  $\eta_1$  in the strong coupling regime. At this time the dilaton, feeling a non-trivial potential, freezes to its present constant value  $\phi = \phi_1$  and the standard radiation-dominated era starts. The total duration  $\eta_1/\eta_s$ , or the total red-shift  $z_s$  encountered during the stringy epoch (i.e. between  $\eta_s$  and  $\eta_1$ ), will be the second crucial parameter besides  $\phi_s$  entering our discussion. For the purpose of this paper, two parameters are enough to specify completely our model of background, if we accept that during the string phase the curvature stays controlled by the string scale, that is  $H \simeq gM_p = \lambda_s^{-1}$  ( $M_p$  is the Planck mass) for  $\eta_s < \eta < \eta_1$ .

We will work all the time in the String (also called Brans-Dicke) frame, in which test strings move along geodesic surfaces. In this frame the string scale  $\lambda_s$  is constant, while

the Planck scale  $\lambda_p = e^{\frac{\phi}{2}}\lambda_s$  grows from zero (at the initial vacuum) to its present value, reached at the end of the string phase. We have explicitly checked that all our results also follow in the more commonly used (but less natural in a string context) Einstein frame, in which the gravi-dilaton action is diagonalized in standard canonical form. Moreover, our parameterization in terms of  $\phi_s$  and  $\eta_1/\eta_s$  is completely frame-independent, since the dilaton field and the conformal time coordinate are the same in the String and Einstein frame [9].

We shall now consider, in the above background, the amplification of the quantum fluctuations of the electromagnetic field, assuming that, at the very beginning, it was in its vacuum state. In a four-dimensional, conformally flat background, the Fourier modes  $A_k^\mu$  of the (correctly normalized) variable corresponding to the standard electromagnetic field, and obeying canonical commutation relations, satisfy the equation

$$A_k'' + [k^2 - V(\eta)]A_k = 0 \quad , \quad V(\eta) = g(g^{-1})'' \quad , \quad g(\eta) \equiv e^{\frac{\phi}{2}} \quad (2)$$

This equation is valid for each polarization component, and is obtained from the action (1) with the gauge condition  $\partial_\nu[e^{-\phi}\partial^\mu(e^{\frac{\phi}{2}}A^\nu)] = 0$  (a prime denotes differentiation with respect to  $\eta$ ). Note the analogy with the equation for the tensor part of metric perturbations [6]. The latter have the same form as (2) with the inverse of the string coupling,  $g^{-1}$ , replaced simply by the Einstein-frame scale factor  $a_E = ae^{-\phi/2}$ , expressed in conformal time. Equation (2) clearly shows the absence of parametric amplification whenever  $\phi$  is constant.

The effective potential  $V(\eta)$  grows from zero like  $\eta^{-2}$ , for  $\eta \rightarrow 0_-$ , in the phase of dilaton-driven inflation (where [8, 9]  $\phi \sim -\sqrt{3}\ln|\eta|$ ), is expected to reach some maximum value during the string phase, and then goes rapidly to zero at the beginning of the radiation-dominated era (where  $\phi = \text{const}$ ). The approximate solution of eq. (2), for a mode  $k$  "hitting" the effective potential barrier at  $\eta = \eta_{ex}$ , and with initial conditions corresponding to vacuum fluctuations, is given by:

$$\begin{aligned} A_k &= \frac{e^{-ik\eta}}{\sqrt{k}} \quad , \quad \eta < \eta_{ex} \\ A_k &= g^{-1}(\eta)[C_k + D_k \int^\eta d\eta' g^2(\eta')] \quad , \quad \eta_{ex} < \eta < \eta_{re} \\ A_k &= \frac{1}{\sqrt{k}}[c_+(k)e^{-ik\eta} + c_-(k)e^{ik\eta}] \quad , \quad \eta > \eta_{re} \end{aligned} \quad (3)$$

where  $\eta_{ex}$  and  $\eta_{re}$  are the times of exit and reentry of the comoving scale associated with  $k$ , defined by the conditions  $k^2 = |V(\eta_{ex})| = |V(\eta_{re})|$  ( $C, D, c_{\pm}$  are integrations constants). We are following here the usual convention for which a mode in the underbarrier region is referred to, somewhat improperly, as being "outside the horizon". Hence the names "exit" and "reentry". Moreover, we are considering a background in which the potential  $V(\eta)$  keeps growing in the string phase until the final time  $\eta_1$ , so that a mode crossing the horizon during dilaton-driven inflation remains outside the horizon during the whole string phase, i.e.  $\eta_{re} \geq \eta_1$ .

The Bogoliubov coefficients  $c_{\pm}(k)$ , determining the parametric amplification of a mode  $k < |V(\eta_1)|$ , are easily determined by matching these various solutions. One finds:

$$2ik e^{-ik(\eta_{ex} \mp \eta_{re})} c_{\pm} = \mp \frac{g_{ex}}{g_{re}} \left( -\frac{g_{re}'}{g_{re}} \mp ik \right) \pm \frac{g_{re}}{g_{ex}} \left( -\frac{g_{ex}'}{g_{ex}} + ik \right) \pm \frac{1}{g_{ex}g_{re}} \left( -\frac{g_{ex}'}{g_{ex}} + ik \right) \left( -\frac{g_{re}'}{g_{re}} \mp ik \right) \int_{\eta_{ex}}^{\eta_{re}} g^2 d\eta \quad (4)$$

If we remember that reentry occurs during the radiation epoch in which the dilaton freezes to a constant value ( $g'_{re} \simeq 0$ ), it is easy to estimate the complicated looking expression (4) and to obtain, for the leading contribution, the amazingly simple and intuitive result:

$$|c_{-}| \simeq \frac{g_{re}}{g_{ex}} \equiv e^{-\frac{1}{2}(\phi_{ex} - \phi_{re})} \quad (5)$$

expressing the fact that the amplification of the electromagnetic field depends just (to leading order) on the ratio of the gauge couplings at reentry and at exit.

The coefficient (5) defines the energy density distribution ( $\rho_B(\omega)$ ) over the amplified fluctuation spectrum,  $d\rho_B/d \ln \omega \simeq \omega^4 |c_{-}(\omega)|^2$ , where  $\omega = k/a$  is the red-shifted, present value of the amplified proper frequency. We are interested in the ratio

$$r(\omega) = \frac{\omega}{\rho_{\gamma}} \frac{d\rho_B}{d\omega} \simeq \frac{\omega^4}{\rho_{\gamma}} |c_{-}(\omega)|^2 \simeq \frac{\omega^4}{\rho_{\gamma}} \left( \frac{g_{re}}{g_{ex}} \right)^2 \quad (6)$$

measuring the fraction of electromagnetic energy stored in the mode  $\omega$  (in particular, for the intergalactic scale,  $\omega_G \simeq (1Mpc)^{-1} \simeq 10^{-14}$  Hertz), relative to the background radiation energy  $\rho_{\gamma}$ .

The ratio  $r(\omega)$  stays constant during the phase of matter-dominated as well as radiation-dominated evolution, in which the universe behaves like a good electromagnetic conductor [2]. In terms of  $r(\omega)$  the condition for a large enough magnetic field to seed the galactic dynamo is [2]

$$r(\omega_G) \gtrsim 10^{-34} \quad (7)$$

Using the known value of  $\rho_\gamma$  and  $e^{\phi_{re}}$  we thus find, from eqs.(6, 7):

$$g_{ex}(\omega_G) < 10^{-33} \quad (8)$$

i.e. a very tiny coupling at the time of exit of the (inter)galactic scale  $\omega_G$  (we may note, incidentally, that it looks very unlikely that such a large ratio of  $g_{re}/g_{ex}$  can be obtained through the trace anomaly mechanism of Ref. [4]).

In order see whether or not the previous condition can be fulfilled we go back to our two-parameter cosmological model. We recall that, in the string phase, the curvature stays controlled by the string scale, so that  $H \simeq \lambda_s^{-1} = \text{constant}$  in the String frame, and  $a_1/a_s = \eta_s/\eta_1 = z_s$ . The discussion is greatly helped by looking at **Fig.1** where we plot, on a double-logarithmic scale against the scale factor  $a$ , the evolution of the coupling strength (i.e. of  $e^{\phi/2}$ ) and that of the ‘‘horizon’’ size (defined here by  $a|V|^{-1/2}$ , whose behavior coincides with that of the Hubble radius  $H^{-1}$  during the dilaton driven epoch). The horizon curve has an inverted trapezoidal shape, corresponding to the fact that  $V = 0$  during the radiation era, that  $\dot{\phi}$  and  $H$  are approximately constant during the string era, and that, during the dilaton-driven era [8, 9],

$$a = (-t)^\alpha, \quad \alpha = -\frac{1}{\sqrt{3}}\sqrt{1-\Sigma}, \quad a|V|^{-1/2} \simeq a(t) \int_t^0 dt' a^{-1}(t') \simeq a^{\frac{1}{\alpha}} \quad (9)$$

Here  $\Sigma \equiv \sum_i \beta_i^2$  represents the possible effect of internal dimensions whose radii  $b_i$  shrink like  $(-t)^{\beta_i}$  for  $t \rightarrow 0_-$  (for the sake of definiteness we show in the figure the case  $\Sigma = 0$ ). The shape of the coupling curve corresponds to the fact that the dilaton is constant during the radiation era, that  $\dot{\phi}$  is approximately constant during the string era, and that it evolves like

$$g(\eta) = a^\lambda, \quad \lambda = \frac{1}{2}\left(3 + \frac{\sqrt{3}}{\sqrt{1-\Sigma}}\right) \quad (10)$$

during the dilaton-driven era [8, 9] ( $\Sigma = 0$  is the case shown in the picture).

We can now easily see when a sufficient amplification is achieved. The galactic scale of length  $\omega_G^{-1}$  was about  $10^{25}$  in string (or Planck) units at the beginning of the radiation era. By definition, at earlier times it evolves as a straight line with a positive slope on our plot and thus inevitably hits the horizon curve sometimes during the string or the dilaton-driven era. At that time, the value of  $g$  should have been smaller than  $10^{-33}$ . One can easily convince her/himself that this is all but impossible provided: i)  $z_s = a_1/a_s$  is a sufficiently large, and ii) the dilaton evolution during the string era is sufficiently fast. For the first condition a red-shift  $z_s$  of  $10^{10}$  is necessary, while for the average ratio  $\dot{\phi}/H$  during the string era a value below but not too far from the one just before  $\eta_s$  is sufficient. The combination of i) and ii) also implies that the coupling at the onset of the string era has to be smaller than  $10^{-20}$  or so, which thus supports the scenario advocated in [10] for the gracious exit problem.

We can express our results more quantitatively by showing the allowed region in the  $g_s - z_s$  plane in order to have sufficiently large seeds. Considering the possibility of galactic scale exit during the string or the dilaton-driven phase, we find from eq.(6) that  $r(\omega_G)$  can be expressed, in the two cases, respectively as:

$$r(\omega_G) \simeq \left(\frac{\omega_G}{\omega_1}\right)^4 e^{-\phi_{ex}(\omega_G)} \simeq \left(\frac{\omega_G}{\omega_1}\right)^{4+\frac{\phi_s}{\ln z_s}} \quad , \quad \omega_s < \omega_G < \omega_1 \quad (11)$$

and

$$r(\omega_G) \simeq \left(\frac{\omega_G}{\omega_1}\right)^{4-2\gamma} z_s^{-2\gamma} e^{-\phi_s} \quad , \quad \omega_G < \omega_s \quad (12)$$

where  $\gamma = \lambda\alpha/(\alpha - 1)$ ,  $\omega_1 = H_1 a_1/a \simeq 10^{11}$  Hertz is the maximal amplified frequency, and  $\omega_s = \omega_1/z_s$ . In the previous formulae (11), (12) we used the fact that, according to our model of background, the transition scale  $H_1$  has to be of the order of the Planck mass  $M_p$ , so that  $\rho_\gamma(t) \simeq H_1^4 [a_1/a(t)]^4 = \omega_1^4$ .

The resulting limits obtained by imposing eq.(7) are plotted in **Fig.2**, where they provide the right-side border of the allowed region (the shaded area). According to the previous spectrum, however, the amplitude of the electromagnetic perturbations is not constant outside the horizon, but tends to grow, asymptotically, with a power-like behavior (similar effects are known to occur also for metric perturbations in string cosmology backgrounds [11]). A

further bound on the allowed region thus emerges in the context of this model, as we must require for consistency that our perturbative treatment of the vacuum fluctuations remains valid at all the times, or, in other words, that the energy density of the produced photons is smaller than critical also in the radiation era, i.e.

$$r(\omega) < 1 \tag{13}$$

at all  $\omega$ . For electromagnetic perturbations crossing the horizon during the dilaton-driven period, in particular, this condition is most stringent at  $\omega = \omega_s$ .

Using eqs. (11), (12) and (13) we find that the two different branches of the photon spectrum give the same bounds,  $g_s < 1$  and  $\log_{10} g_s > -2 \log_{10} z_s$ , the latter determining the left border of the allowed region. Actually, a slightly tighter bound ( $r \lesssim 0.5$ ) could also be imposed in order to avoid perturbations of the standard nucleosynthesis scenario [12], but this does not affect in significant way the result presented in **Fig.2**, which should be regarded, in any case, as an order of magnitude estimate of the allowed region.

We want also to mention that in the described scenario is not only possible to seed the galactic dynamo (according to eq.(7)), but also to produce directly fields of the same strength as that of the galactic magnetic field, by requiring [2]  $r(\omega_G) \simeq 10^{-8}$ . In our case this is achieved if  $z_s > 10^{21}$  and if the coupling is sufficiently tiny at the end of the dilaton driven era,  $\log_{10} g_s < -43$ . The possible impact of such a small value on the problem of freezing out the classical oscillations of the dilaton background, in the presence of a "realistic" supersymmetry breaking, non-perturbative potential, is left as an interesting subject for future research.

We want to recall, finally, that our results were obtained in the framework of the tree-level, string effective lagrangian. We know that we could have corrections coming either from higher loops (expansion in  $e^\phi$ ) or from higher curvature terms ( $\alpha'$  corrections). Since we work in a range of parameters where the dilaton is deeply in his perturbative regime we expect our results to be stable against loop corrections, at least for scales leaving the horizon during the dilaton driven phase. As to the  $\alpha'$  corrections, they are instead invoked in the basic assumption that the dilaton-driven era ends when the curvature reaches the string scale  $\lambda_s^{-2}$ , and leads to a quasi-de Sitter epoch. It should be clear, nevertheless, that



the basic ideas of our mechanism and its implementation do not depend too strongly on the detailed features of the string epoch.

## **Acknowledgements**

It is a pleasure to thank R. Brustein and V. Mukhanov for a fruitful collaboration on the spectral properties of metric perturbations in string cosmology, which inspired part of this work.

## **Note added**

While this paper was being written, we received a paper by D. Lemoine and M. Lemoine, "Primordial magnetic fields in string cosmology", whose content overlap with ours for what concerns the effects of dilaton-driven inflation on the amplification of electromagnetic perturbations. Their model of background does not include, however, a sufficiently long, intermediate stringy era whose presence is instead crucial to produce the large amplification discussed here.

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## Figure captions

### Fig. 1

Plot showing the evolution of the horizon scale  $H^{-1}$  (thick lines) and of the coupling scale  $g = e^{\phi/2}$  (dashed lines) while the galactic scale  $\omega_G^{-1}$  (thin line) was beyond the horizon. Two cases with a different  $z_s$  are compared, showing that, for a sufficiently fast variation of the dilaton during the string era, a larger  $z_s$  helps achieving the bound  $g_{ex}(\omega_G) < 10^{-33}$ .

### Fig. 2

The shaded area represents the allowed region determined by the conditions (7) and (13), and defines the values of  $z_s$ ,  $g_s$  compatible with an amplification of the electromagnetic vacuum fluctuations large enough to seed the galactic magnetic field.