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Disoriented and Plastic Soft Terms: A Dynamical Solution to the Problem of Supersymmetric Flavor Violations

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Abstract

We postulate that the orientation of the soft supersymmetry breaking terms in flavor space is not fixed by physics at the Planck scale; it is a dynamical variable of the low energy theory which depends on fields that have no potential. These fields can be thought of as either moduli or as the Nambu-Goldstone bosons of the spontaneously broken flavor symmetry which is non-linearly realized by the soft terms. We show that the soft terms align with the quark and lepton masses, just as spins align with an external magnetic field. As a result, the soft terms conserve individual lepton numbers and do not cause large flavor or CP violations. The vacuum adjusts so as to allow large sparticle splittings to naturally coexist with flavor conservation. Consequently, the resulting phenomenology is different from that of minimal supersymmetric theories. We also propose theories in which the shape of the soft terms in flavor space is a dynamical variable of the low energy theory. This dynamically leads to partial degeneracy among sparticles and further supression of flavor violations. We compute the masses and couplings of the nearly massless moduli/goldstones and find that, at distances as large as ~ 30 m, they mediate potentially measurable long range forces and violations of the equivalence principle. The ideas of this paper suggest a connection between the space of moduli and the spontaneously broken flavor group.

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1 Universal versus Disoriented Soft Terms

The soft supersymmetry-breaking terms [1, 2] are important for at least two reasons. First, they are the key ingredient which made the construction of realistic supersymmetric theories possible [1]. Second, they are experimentally measurable quantities since they determine the masses of sparticles. In early works, motivated by the need to avoid large flavor violations, it was postulated that soft terms satisfy universality [1]. Universality states that the squarks and sleptons of the three families are all degenerate in mass at some scale $\sim M_{GUT}$.

Universality has a geometric interpretation which is useful to appreciate. To do this, consider the limit in which all but the gauge couplings of the supersymmetric standard model are set to zero. The resulting theory possesses a $U(3)^5$ global symmetry which is called flavor symmetry. The 3 stands for the number of families and the 5 for the number of $SU(3) \times SU(2) \times U(1)$ members in a family, which will be labelled by $A = Q, \bar{U}, \bar{D}, L, \bar{E}$. The flavor symmetry is simply a manifestation of the fact that gauge forces do not distinguish particles with identical gauge quantum numbers. Universality states that the five 3×3 sparticle squared mass matrices $\tilde{m}^2_A$ are flavor singlets, *i.e.* proportional to the identity. They are spheres in flavor space and they realize the flavor symmetry in the Wigner mode. In this paper we wish to suggest an alternative mechanism to universality for avoiding large flavor violations.

Let Λ be a high-energy scale at which supersymmetry breaking occurs and the soft terms are determined. Λ can be of the order of the Planck mass M_{PL} – as in supergravity – or smaller, equal to the mass of the messengers that communicate supersymmetry breaking to the ordinary particles. Our fundamental hypothesis is that physics at the scale Λ fixes the eigenvalues of the soft terms $\tilde{m}^2_A$ but leaves their direction in flavor $U(3)^5$ space undetermined. In other words, the potential energy V of the sector which determines the soft terms at the scale Λ is flavor $U(3)^5$ invariant. V does not depend on the $U(3)^5$ angles which are flat directions of the potential and which will be called here "moduli". The moduli determine the direction in which the soft terms point in flavor space. They can be thought of as the Goldstone bosons of the flavor group which is spontaneously broken by the soft terms $\tilde{m}^2_A$ themselves. Therefore, the simplest way to state our hypothesis is: the soft terms realize the flavor symmetry in the Goldstone mode. In contrast, universality states that the soft terms realize the flavor symmetry in the Wigner mode.

Our next assumption is that at energies below Λ we have the minimal supersymmetric particle content[1]. We will show that the orientation of the soft terms is determined by the low-energy dynamics – in particular the flavor-breaking fermion masses – in a calculable way.

A simple analogy is to think of the soft terms $\tilde{m}^2_A$ as a spin $\vec{s}$ in space and $U(3)_A$ as ordinary rotational invariance. The magnitude of $\vec{s}$ is determined by some unspecified "high-energy" dynamics to be non-zero. This forces rotational invariance to break spontaneously. $\vec{s}$ can point in any direction until we turn on an external magnetic field $\vec{B}$ which explicitly breaks the rotational invariance and forces $\vec{s}$ to align parallel to $\vec{B}$. Notice that alignment (or anti-alignment) is preferred and the maximal subgroup possible, $SO(2)$, is preserved. This

[1] In Sect. 8 we will also discuss the case of supersymmetric GUTs.

completes the analogy between $\vec{s}$ and the soft terms on one hand and between $\vec{B}$ and the fermion masses on the other. Perfect alignment would mean that the maximal subgroup consisting of the product of all vectorial $U(1)$ quantum numbers is preserved and consequently there is no flavor violation. In the quark sector since the Kobayashi-Maskawa matrix $K \neq 1$ this is not possible, but the dynamics will adjust as to reduce flavor violations.

Finally, the same dynamical mechanism works for the triscalar soft A-terms. If their orientation is not determined by high-energy physics, they too will align parallel to the fermion masses and avoid causing large flavor violations.

2 Alignment

Consider the supersymmetric $SU(3) \times SU(2) \times U(1)$ theory with minimal particle content, whose gauge interactions possess an $U(3)^5$ global flavor symmetry. As in the standard model, the Yukawa couplings break the symmetry. In addition, flavor symmetry is violated here also by the soft supersymmetry-breaking terms which in general lead to phenomenologically unacceptable contributions to flavor-changing neutral current (FCNC) processes. This can be easily understood by inspecting the up and down squark and the charged slepton mass matrices.

$$
\begin{aligned}
\mathcal{M}_u^2 &= \begin{pmatrix} m_u^\dagger m_u + \tilde{m}_Q^2 + D_{u_L} & A_u^\dagger + \frac{\mu}{\tan\beta} m_u^\dagger \\ \\ A_u + \frac{\mu^\star}{\tan\beta} m_u & m_u m_u^\dagger + \tilde{m}_{\bar{U}}^2 + D_{u_R} \end{pmatrix} \\
\mathcal{M}_d^2 &= \begin{pmatrix} K m_d^\dagger m_d K^\dagger + \tilde{m}_Q^2 + D_{d_L} & A_d^\dagger + \mu \tan\beta K m_d^\dagger \\ \\ A_d + \mu^\star \tan\beta m_d K^\dagger & m_d m_d^\dagger + \tilde{m}_{\bar{D}}^2 + D_{d_R} \end{pmatrix} \\
\mathcal{M}_e^2 &= \begin{pmatrix} m_e^\dagger m_e + \tilde{m}_L^2 + D_{e_L} & A_e^\dagger + \mu \tan\beta m_e^\dagger \\ \\ A_e + \mu^\star \tan\beta m_e & m_e m_e^\dagger + \tilde{m}_{\bar{E}}^2 + D_{e_R} \end{pmatrix} .
\end{aligned}
\tag{1}
$$

In Eq. (1), $\tan\beta$ is the ratio of the two Higgs vacuum expectation values and $D = (T_3 - Q \sin^2\theta_W) M_Z^2 \cos^2\beta$ for a sparticle with third component of isospin T_3 and electric charge Q. With a superfield rotation, we choose a basis where the fermion mass matrices m_u, m_d and m_e are diagonal, real and positive and where K is the ordinary Kobayashi-Maskawa matrix and μ, the supersymmetric Higgs mass parameter, is real and positive. If the soft supersymmetry-breaking masses $\tilde{m}^2$ and the trilinear terms A are general matrices in flavor space, squarks and quarks are completely misaligned, allowing for large gluino-mediated contributions to FCNC processes.

Our hypothesis is that the $\tilde{m}_A^2$ are general Hermitian matrices whose eigenvalues are fixed by Planckian physics, but whose orientation is a dynamical variable determined by physics below Λ^2. The soft supersymmetry-breaking masses $\tilde{m}_A^2$ are thus promoted to fields:

$$\tilde{m}_A^2 \to \Sigma_A \equiv U_A^\dagger \bar{\Sigma}_A U_A \qquad\qquad A = Q, \bar{U}, \bar{D}, L, \bar{E}. \tag{2}$$

[2]The possibility that the top-quark Yukawa coupling depends on a flat direction was considered in ref. [3].

$\bar{\Sigma}_A$ are diagonal matrices with real, positive eigenvalues ordered according to increasing magnitude and U_A are 3×3 unitary matrices. Similarly, we promote the trilinear terms to dynamical variables:

$$A_a \rightarrow \Delta_a \equiv V_a^\dagger \bar{\Delta}_a \bar{V}_a \qquad\qquad a = u, d, e, \tag{3}$$

where V_a and $\bar{V}_a$ are unitary 3×3 matrices and $\bar{\Delta}_a$ are diagonal matrices with real and positive eigenvalues ordered in terms of increasing magnitude.

Our fundamental hypothesis can now be restated: $\bar{\Sigma}_A$ and Δ_a are fixed by physics at some very high scale Λ – say $\Lambda \sim M_{PL}$, for concreteness – whereas U_A, V_a and $\bar{V}_a$ are determined only by low energy physics, namely the energetics of the supersymmetric $SU(3) \times SU(2) \times U(1)$ theory.

The classical ground state energy of the system does not depend on U_a, V_a or $\bar{V}_a$ since all the sparticles have vanishing vanishing expectation values (VEVs). The first quantum corrections can be computed by adding the zero point energies $\frac{1}{2}\hbar\omega_k$ of all the oscillators in the system. The result is the one-loop effective potential:

$$V_{1-loop} = \sum_k \frac{1}{2}\hbar\omega_k = \frac{1}{32\pi^2}\Lambda^2 Str\mathcal{M}^2 + \frac{1}{64\pi^2} Str\mathcal{M}^4 \log\frac{\mathcal{M}^2}{\Lambda^2}. \tag{4}$$

Λ is an ultraviolet cut-off beyond which the theory changes and is not well approximated by the supersymmetric $SU(3) \times SU(2) \times U(1)$ with minimal particle content. More precisely, Λ is the scale in which the soft supersymmetry-breaking masses shut off. In theories where the supersymmetry breaking is communicated to the visible sector by messengers lighter than M_{PL}, Λ would be the mass of these messengers [4]. In the case where the messenger is supergravity, $\Lambda \sim M_{PL}$ [5]. Our conclusions do not depend on the value of Λ, as long as it is much larger than the heaviest sparticle mass.

Let us first consider the simple case $A = \mu = 0$. To determine the relative orientation of sparticle and particle masses, we should minimize the effective potential V_{1-loop} of Eq. (4) with respect to U_A. The only terms in V_{1-loop} which depend on U_A are:

$$V_{1-loop}(U) = \frac{1}{64\pi^2}\sum_{a=u,d,e} Tr\mathcal{M}_a^4 \log\frac{\mathcal{M}_a^2}{\Lambda^2}\ , \tag{5}$$

where $\mathcal{M}_u, \mathcal{M}_d, \mathcal{M}_e$ are the 6×6 matrices of Eq. (1). Equation (5) decomposes into a sum of five pieces, one for each A. For instance, for $A = L$, the dependence on U_L is given by

$$V_{1-loop}(U_L) = \frac{1}{64\pi^2} Tr\left(m_e^\dagger m_e + U_L^\dagger \bar{\Sigma}_L U_L + D_{e_L}\right)^2 \log\left(\frac{m_e^\dagger m_e + U_L^\dagger \bar{\Sigma}_L U_L + D_{e_L}}{\Lambda^2}\right), \tag{6}$$

and similar expressions hold for the other sparticle species. Because Λ is a very large number, the logarithm in Eq. (6) is negative and it is therefore evident that $V(U_L)$ is minimized as Σ_L orients itself parallel to $m_e^\dagger m_e$, which means $U_L = 1$. Similarly the other slepton and squark mass matrices choose to align themselves along the corresponding fermion masses.

This can be formally shown as follows. For any given A, let us write

$$U_A = exp(i \sum_\alpha \lambda^\alpha \sigma_A^\alpha) \ , \tag{7}$$

where $\lambda^\alpha/2$ are the generators of the flavor group broken by $\bar{\Sigma}_A$, in short the six generators of $SU(3)/U(1)^2$. Thus σ_A^α can be thought of as the Goldstone bosons of the flavor $U(3)$ group that has been spontaneously broken by the $\bar{\Sigma}_A$ VEV. In reality, the σ^α are pseudo-Goldstone bosons, because quark and lepton masses explicitly break flavor invariance. Dropping for simplicity the index A, the leading σ^α-dependent part of V_{1-loop} extracted from Eq. (5) is

$$V_{1-loop}(\sigma) = \frac{1}{32\pi^2} \sum_{i>j} \sum_\alpha \left| \lambda_{ij}^\alpha \sigma^\alpha \right|^2 \ \frac{(\bar{\Sigma}_i - \bar{\Sigma}_j)(m_i^2 - m_j^2)}{(\bar{\Sigma}_i - \bar{\Sigma}_j + m_i^2 - m_j^2)} (F_i - F_j) + \mathcal{O}(\sigma^3) \ , \tag{8}$$

$$F_i = -(\bar{\Sigma}_i + m_i^2 + D) \left[\frac{1}{2} + \log \ \frac{\bar{\Sigma}_i + m_i^2 + D}{\Lambda^2} \right] . \tag{9}$$

We are working in a basis where both Σ and m^2 are diagonal and the index i refer to their i-th diagonal element.

The absence in Eq. (8) of linear terms in σ^α shows that $U = 1$ is an extremum of the energy. For Λ larger than the heaviest sparticle mass, it actually corresponds to a minimum. This is apparent from Eq. (8), since the diagonal elements of $\bar{\Sigma}$ are ordered in terms of increasing magnitude as are the fermion mass eigenvalues m_i.

The result $U_L = U_{\bar{E}} = 1$ has the important consequence that the e, μ, τ lepton numbers are separately conserved. Since slepton and lepton mass matrices are parallel, they both preserve the same $U(1)^3$ symmetry and individual lepton number violating processes like $\mu \to e\gamma$ do not occur in this theory.

The situation is more involved in the squark sector since Σ_Q couples in V_{1-loop} to both m_u and m_d, which cannot be simultaneously diagonal. However, as it will be clearer in the next section, while $\Sigma_{\bar{D}}$ and $\Sigma_{\bar{U}}$ align respectively with m_d^2 and m_u^2, Σ_Q predominantly aligns with m_u^2, allowing for some gluino-mediated contributions to FCNC processes. It is also interesting to notice that the squark soft masses $\bar{\Sigma}$ becomes larger the higher the generation. This is in contrast to the minimal supersymmetric model where all squarks soft mass parameters are equal at M_{GUT} and, by the effect of the renormalization group equations, become smaller the higher the generation.

3 General Alignment

Consider now the general case where μ and the trilinear terms A are not zero and the sparticle masses are given by the general matrices of Eq. (1). We can write the relevant terms in the effective potential as

$$V = \frac{1}{64\pi^2} Str\mathcal{M}^4 \left[\log\left(\frac{\mathcal{M}^2}{m_s^2} \right) - \tau \right] \ , \tag{10}$$

where m_s^2 is the scale of soft supersymmetry breaking in the low energy theory, and $\tau = \log \Lambda^2/m_s^2$.

We will use the approximation of neglecting the first term in the right-hand side of Eq. (10) with respect to τ. This is justified if $\Lambda \gg m_s$ and if $\Delta \sim \mathcal{O}(mm_s)$. The requirement $\Lambda \gg m_s$ is certainly satisfied in most realistic theories and insures that $\log(m_s^2/\Lambda^2)$ is a number much larger than $\log(M^2/m_s^2)$. The assumption $\Delta \sim \mathcal{O}(Mm_s)$, as opposed to $\Delta \sim \mathcal{O}(m_s^2)$, is also plausible, since as the Yukawa couplings are turned off, the trilinear terms should also disappear from the theory. This assumption is necessary to insure that the U, V and $\bar{V}$-dependent terms in $Str\ M^4 \log(M^2/m_s^2)$ are always proportional to the Yukawa couplings. If this did not hold, they could compete with U, V and $\bar{V}$-dependent terms in $Str\ M^4 \log(m_s^2/\Lambda^2)$ which, although enhanced by the large logarithm, are indeed proportional to the Yukawa couplings.

The large τ approximation is justified provided we work near m_s. Therefore, all of our parameters that enter into V_{1-loop} and into the minimization equations should be interpreted as running parameters evaluated at m_s. With this approximation, the one-loop effective potential for U, V and $\bar{V}$ decomposes into the simple form:

$$64\pi^2 V_{1-loop} = 2\tau \sum_A Tr\ U_A^\dagger \bar{\Sigma}_A U_A m_A^2 - 2\tau \sum_a \left[Tr\ V_a^\dagger \bar{\Delta}_a \bar{V}_a \mu_a + \text{h.c.}\right] \tag{11}$$

$$m_A^2 = \begin{cases} m_u^2 + K m_d^2 K^\dagger & A = Q \\ m_u^2 & A = \bar{U} \\ m_d^2 & A = \bar{D} \\ m_e^2 & A = L, \bar{E} \end{cases} \qquad \mu_a = \begin{cases} \frac{1}{\tan\beta}\ m_u\ \mu & a = u \\ \tan\beta\ K\ m_d\ \mu & a = d \\ \tan\beta\ m_e\ \mu & a = e\ . \end{cases}$$

In the superfield basis we are working in, the minimum of the energy is achieved for

$$\begin{gathered} \langle U_A \rangle = 1 \qquad A = \bar{U}, \bar{D}, L, \bar{E} \qquad \langle U_Q \rangle = S \\ \langle V_i \rangle = \langle \bar{V}_i \rangle = 1 \qquad i = u, e \\ \langle V_d \rangle = 1 \qquad \langle \bar{V}_d \rangle = K^\dagger\ , \end{gathered} \tag{12}$$

where S is such that $S(m_u^2 + K m_d^2 K^\dagger)S^\dagger$ is diagonal. This is easily proven, as in the previous section, first by writing

$$U = \langle U \rangle exp(i \sum_\alpha \lambda^\alpha \sigma^\alpha) \qquad V = \langle V \rangle exp(i \sum_\alpha \lambda^\alpha \delta^\alpha) \qquad \bar{V} = \langle \bar{V} \rangle exp(i \sum_\alpha \lambda^\alpha \bar{\delta}^\alpha)\ , \tag{13}$$

where λ^α are the Gell-Mann matrices and where we have dropped the indices a and A. Next by expanding V_{1-loop} in σ, δ and $\bar{\delta}$ around the vacuum, we write:

$$64\pi^2 V_{1-loop}^{(\sigma)} = 2\tau \sum_\alpha \sigma^{\alpha^2} \sum_{i>j} |\lambda_{ij}^\alpha|^2 (\bar{\Sigma}_i - \bar{\Sigma}_j)(m_i^2 - m_j^2) + \mathcal{O}(\sigma^3) \tag{14}$$

$$\begin{aligned} 64\pi^2 V_{1-loop}^{\delta,\bar{\delta}} = 4\tau \Bigg[& \sum_\alpha \delta_\alpha^{(+)^2} \sum_{i>j} |\lambda_{ij}^\alpha|^2 (\bar{\Delta}_i - \bar{\Delta}_j)(\mu_i - \mu_j) + \\ & \sum_\alpha \delta_\alpha^{(-)^2} \sum_{i>j} |\lambda_{ij}^\alpha|^2 (\bar{\Delta}_i + \bar{\Delta}_j)(\mu_i + \mu_j) + \\ & 2 \sum_i \left| \sum_\beta \delta_\beta^{(-)} \lambda_{ii}^\beta \right|^2 \bar{\Delta}_i m_i \Bigg] + \mathcal{O}(\delta^3) \end{aligned} \tag{15}$$

$$\delta^{(\pm)} \equiv \frac{\delta^{(+)} \pm \delta^{(-)}}{2}, \tag{16}$$

where the sum over α spans the broken $SU(3)/U(1)^2$ generators, while the sum over β spans only the diagonal generators of $U(3)$.

Equations (14) and (15) show that the first derivative of V_{1-loop} vanishes and the second derivative is positive if $\tau > 0$. The alignment is such that Σ_A is parallel to m_A^2 with eigenvalues ordered as in m_A^2. Also Δ_a and μ_a are parallel and the respective eigenvalues have the same phases and are ordered in the same way.

4 Flavor Violating Processes

The first consequence of the results of the previous section is that all three lepton numbers are individually conserved. This follows from the complete alignment of the lepton masses and the slepton soft terms: $U_L = U_E = V_e = \bar{V}_e = 1$. Such a total alignment is obviously not possible in the quark sector, since the up and down quarks themselves do not have parallel mass matrices. The quark flavor violations are best discussed by going, via a superfield rotation, to the quark mass eigenbasis where both up and down masses are diagonal and the squark masses have the form:

$$\mathcal{M}_u^2 = \begin{pmatrix} m_u^2 + S^\dagger \bar{\Sigma}_Q S + D_{u_L} & \bar{\Delta}_u + \frac{\mu}{\tan\beta} m_u \\ \\ \bar{\Delta}_u + \frac{\mu}{\tan\beta} m_u & m_u^2 + \bar{\Sigma}_{\bar{U}} + D_{u_R} \end{pmatrix}$$
$$\mathcal{M}_d^2 = \begin{pmatrix} m_d^2 + K^\dagger S^\dagger \bar{\Sigma}_Q S K + D_{d_L} & \bar{\Delta}_d + \mu \tan\beta m_d \\ \\ \bar{\Delta}_d + \mu \tan\beta m_d & m_d^2 + \bar{\Sigma}_{\bar{D}} + D_{d_R} \end{pmatrix} \tag{17}$$

All flavor violation is contained in S and K. The off-diagonal elements of S are much smaller than those of the Kobayashi-Maskawa matrix:

$$\begin{aligned} S_{23} &\simeq K_{cb} \frac{m_b^2}{m_t^2} \sim 2 \times 10^{-5} \\ S_{13} &\simeq K_{ub} \frac{m_b^2}{m_t^2} \sim 2 \times 10^{-6} \\ S_{12} &\simeq \frac{|K_{us} K_{cs}^\star m_s^2 + K_{ub} K_{cb}^\star m_b^2|}{m_c^2} \sim 5 \times 10^{-3} \ , \end{aligned} \tag{18}$$

and therefore they do not significantly affect FCNC processes, although they may contribute to CP-violating processes. Then, in the approximation $S = 1$, all new flavor violations occur in the D_L sector, as can be seen from the squark mass matrices in Eq. (17).

The most stringent constraint comes from the contribution of squark-gluino loops to the real part of the $K^0 - \bar{K}^0$ mixing:

$$\left(\frac{\Delta m_K}{m_K}\right)_{\tilde{g}} = \frac{f_K^2 B_K}{54} \frac{\alpha_s^2}{M_{\tilde{g}}^2} \ Re(X) \tag{19}$$

$$X \equiv \sum_{i,j} K_{is} K^\star_{id} K_{js} K^\star_{jd} \; f\left(\frac{\tilde{m}^2_{Q_i}}{M^2_{\tilde{g}}}, \frac{\tilde{m}^2_{Q_j}}{M^2_{\tilde{g}}} \right) \tag{20}$$

$$f(x,y) \equiv \frac{1}{x-y}\left[\frac{(11x+4)x}{(x-1)^2}\log x - \frac{15}{x-1} - (x \to y) \right], \tag{21}$$

where $f_K = 165$ MeV is the kaon decay constant, B_K parametrizes the hadronic matrix element, and $M_{\tilde{g}}$ is the gluino mass. Assuming $M^2_{\tilde{g}} = \tilde{m}^2_Q$ and keeping the leading contribution in the squark mass splitting, one finds

$$Re(X) = \frac{\sin^2\theta_c}{6}\mathcal{D}^2_{21} \; , \tag{22}$$

where θ_c is the Cabibbo angle and

$$\mathcal{D}_{ij} \equiv \frac{\tilde{m}^2_{Q_i} - \tilde{m}^2_{Q_j}}{\tilde{m}^2_{Q_i}} \; . \tag{23}$$

If we require that the gluino contribution in Eq. (19) does not exceed the experimental value of $\Delta m_K/m_K$, we obtain the constraint:

$$\mathcal{D}_{21} < 0.1 \; \frac{\tilde{m}_Q}{300 \text{ GeV}} \; . \tag{24}$$

The squark-gluino contribution to the imaginary part of $K^0 - \bar{K}^0$ mixing is given by:

$$(|\epsilon|)_{\tilde{g}} = \frac{m_K}{\Delta m_K} \; \frac{f_K^2 B_K}{108\sqrt{2}} \; \frac{\alpha_s^2}{M^2_{\tilde{g}}} \; Im(X) \tag{25}$$

With the same approximation used before, we obtain

$$Im(X) = \frac{1}{3}|K_{us}||K_{ub}||K_{cb}|\sin\delta \; \mathcal{D}_{32}\mathcal{D}_{21} \; , \tag{26}$$

where δ is the CP-violating phase in the Kobayashi–Maskawa matrix. This does not exceed the experimental value for $|\epsilon|$ if

$$\sqrt{\mathcal{D}_{21}\mathcal{D}_{31}} < \frac{\tilde{m}_Q}{300 \text{ GeV}} \; . \tag{27}$$

There is no significant constraint coming from $B^0 - \bar{B}^0$ mixing and, in the limit $S = 1$, there is no new gluino-mediated contribution to $D^0 - \bar{D}^0$ mixing.

The constraints from FCNC on our model are much milder than those on a general supersymmetric $SU(3)\times SU(2)\times U(1)$ theory with minimal particle content and non-universal frozen soft-terms [6]. The reason is that in our theory, just as in the standard model, flavor violations are proportional to the Kobayashi-Maskawa angles; however, they are also suppressed by the large sparticle masses. Therefore, our contributions to rare processes can compete with the standard model contributions only if the latter have light quark suppressions, as in $\Delta m_K/m_K$ where $(\Delta m_K/m_K)_{SM} \sim G_F m_c^2$.

It is noteworthy that we do not obtain any constraints from either $\mu \to e\gamma$ or ϵ. These provide by far the strongest constraints on general supersymmetric models. In our case, $\mu \to e\gamma$ vanishes whereas ϵ is small because it is proportional to the Jarlskog invariant J of the standard model and is further suppressed by sparticle masses. The only significant constraint we have is from Δm_K Eq. (24). It can be accounted for in several ways. One is by invoking heavy gluinos, which cause the squark masses to approach one another in the infrared. Furthermore, in Sect. 7, we will show how the dynamics of the moduli can adjust to render the squarks of the two heavy generations degenerate.

5 Long Range Forces

5.1 Masses of Moduli

Our basic hypothesis so far has been that the matrices U, V and $\bar{V}$ depend on fields that are undetermined by the theory at $\Lambda \sim M_{PL}$ and whose VEVs are determined by the low-energy dynamics that we computed. We did not need to identify what these fields were. In this section we would like to do so. We begin by rewriting:

$$\begin{aligned} V_u &= V'_u U_Q \quad V_d = V'_d U_Q \quad V_e = V'_e U_L \ , \\ \bar{V}_u &= \bar{V}'_u U_{\bar{U}} \quad \bar{V}_d = \bar{V}'_d U_{\bar{D}} \quad \bar{V}_e = \bar{V}'_e U_{\bar{E}} \ . \end{aligned} \tag{28}$$

Let δ' and $\bar{\delta}'$ be the angles corresponding to V' and $\bar{V}$. The matrices U_A are the same as those of Eq. (7) and their angles are σ_A^α. In the limit of vanishing Yukawa couplings, translations in $\sigma_A^\alpha \to \sigma_A^\alpha + \delta\sigma_A^\alpha$ can be compensated by unitary rotations of the quark and lepton superfields. Thus the σ_A correspond to Goldstone bosons of the spontaneously broken $U(3)_A$ flavor group. Our postulate is that they are physical particles; we will now compute their masses and couplings. We will do so in the limit where we ignore the A-terms and set $A = \mu = 0$ for simplicity; this does not affect any of the essential properties of the σ_A. The Yukawa couplings explicitly break the flavor $U(3)_A$ group, and give to each σ_A a mass proportional to the corresponding coefficient in Eq. (14).

To obtain physical masses we need to define canonical fields. Dimensional analysis and the hypothesis that flavor symmetry is broken spontaneously at a scale F suggest that the field Σ' defined by

$$\Sigma = \frac{m_s^2}{F}\Sigma' \tag{29}$$

is canonically normalized. F can be identified with M_{PL} or possibly with some lighter scale connected with flavor breakdown. We want to stress however that our choice of Σ' being the canonical field is arbitrary and different choices can lead to different masses and couplings for the physical particles. The properly normalized kinetic term is:

$$\frac{1}{2}\ Tr\ \partial_\mu \Sigma'_A \partial_\mu \Sigma'_A = \left(\frac{F}{m_s^2}\right)^2 \sum_\alpha \partial_\mu \sigma_A^\alpha \partial^\mu \sigma_A^\alpha \sum_{i>j} |\lambda_{ij}^\alpha|^2 (\bar{\Sigma}_{A_i} - \bar{\Sigma}_{A_j})^2 \ . \tag{30}$$

From this and Eq. (14) we can read the physical squared masses of the σ_A[3]:

$$m^2_{\sigma^\alpha_A} = \frac{\tau}{32\pi^2}\;\frac{m_s^4}{F^2}\;\frac{\sum_{i>j}|\lambda^\alpha_{ij}|^2(m_i^2-m_j^2)}{\sum_{i>j}|\lambda^\alpha_{ij}|^2(\bar\Sigma_{A_i}-\bar\Sigma_{A_j})}\;. \tag{31}$$

From Eq. (31), we see that $\sigma^4_A,\sigma^5_A,\sigma^6_A,\sigma^7_A$ get masses proportional to the third generation fermion mass of species A, while σ^1_A,σ^2_A get masses proportional only to the second generation fermion mass of species A. A convenient expression is:

$$m_{\sigma^\alpha_A} \simeq 6\times 10^{-8}\; eV\; \left(\frac{m_s}{300\; GeV}\right)\left(\frac{m^A_{f\alpha}}{1\; GeV}\right)\sqrt{\frac{m_s^2}{\Delta m_s^2}}\left(\frac{M^R_{PL}}{F}\right)\;. \tag{32}$$

Here, $M^R_{PL}=(8\pi)^{-1/2}M_{PL}$ are the reduced Planck mass, and $\Delta m_s^2/m_s^2$ is the relevant sparticle mass splittings. $m^A_{f_\alpha}$ denotes the third (second) generation fermion mass of species A if $\alpha=4,5,6,7$ ($\alpha=1,2$). The Compton wavelengths of the σ particles are

$$\lambda_{\sigma^\alpha_A} \simeq 3\; m\; \left(\frac{300\; GeV}{m_s}\right)\left(\frac{1\; GeV}{m^A_{f\alpha}}\right)\sqrt{\frac{\Delta m_s^2}{m_s^2}}\left(\frac{F}{M^R_{PL}}\right)\;. \tag{33}$$

Thus the σ can mediate forces between two objects separated by a macroscopic distance and lead to deviations from the equivalence principle.

In the absence of Yukawa couplings the σ are exactly massless Goldstone bosons; thus they are coupled derivatively and mediate short-range forces with potentials $\propto 1/F^2r^3$. Once the Yukawas are turned on, they can mediate $1/r^2$ forces that mimic gravity at distances shorter than their Compton wavelengths. Their CP properties are essential in determining the nature of the forces[4]. The σ^α that correspond to imaginary λ^α, namely σ_2,σ_5 and σ_7, are CP-even scalars and they can mediate $1/r^2$ forces. The rest are CP-odd pseudoscalars; they couple to spin and do not mediate long-range forces in ordinary non-magnetized matter. Note that the "scalar" generators λ_2,λ_5 and λ_7 are all off-diagonal. This means that the couplings of σ_2,σ_5 and σ_7 to interaction eigenstates are off-diagonal. Diagonal couplings to ordinary matter will arise because of the mismatch between mass and interaction eigenstates. Diagonal $1/r^2$ forces mediated by the exchange of $\sigma_{2,5,7}$ will have to involve some off-diagonal entry of the Kobayashi-Maskawa matrix K. Since for leptons the mixing angles vanish, there are no diagonal long-range forces coupled to lepton number.

We will work in the basis defined after Eq. (1), which is particularly convenient because it approximately corresponds to the mass eigenbasis for all squarks[5]. The coupling of the properly normalized σ to squarks ϕ is given by

$$\mathcal{L}_{\sigma\phi\phi} = \frac{i}{\sqrt{2}}\frac{m_s^2}{F}\sum_{\alpha,i,j}\frac{\lambda^\alpha_{ij}\;(\bar\Sigma_i-\bar\Sigma_j)}{\sqrt{\sum_{r>s}|\lambda^\alpha_{rs}|^2(\bar\Sigma_r-\bar\Sigma_s)^2}}\;\phi_i^\star\phi_j\sigma^\alpha\;. \tag{34}$$

[3]The non-linear Goldstone parametrization used here makes sense only if the explicit symmetry breaking $(m_i^2-m_j^2)$ is not larger than the spontaneous breaking $(\bar\Sigma_i-\bar\Sigma_j)$. This is why Eq. (31) apparently blows up as $\bar\Sigma_i-\bar\Sigma_j\to 0$.

[4]In the following, we neglect small effects coming from CP violation.

[5]This is true unless the splittings of the squarks soft masses are smaller than the corresponding quark mass splittings, a case too close to universality to be considered here.

From this we see explicitly that imaginary generators lead to scalar couplings and real generators to pseudoscalar couplings. Let us focus on σ_Q for definitness. Its interaction with squarks, Eq. (34), can be converted into a diagonal coupling to ordinary matter exploiting the Kobayashi-Maskawa angles which rotate the down quarks from the basis we are working in to their mass eigenbasis. This can be done via one-loop diagrams mediated either by gluinos (for the coupling to down quarks) or by charginos (for both up and down quarks). It is reasonable to expect that strong interactions make the gluino exchange dominant over the chargino, although this may depend on the various parameters. The gluino-exchange produces a scalar effective coupling between σ_Q^α and a pair of down quarks d_k, with identical flavor index k, given by:

$$\mathcal{L}_{\sigma\bar{d}d} = i\frac{\sqrt{2}\alpha_s}{9\pi}\frac{m_s^2}{M_{\tilde{g}}^2}\frac{m_{d_k}}{F}\ \bar{d}_k d_k \sigma_Q^\alpha\ \frac{\sum_{i>j}(\bar{\Sigma}_i-\bar{\Sigma}_j)\ Im(\lambda^\alpha_{ij}K_{jk}K^\star_{ik})\ f\left(\frac{\bar{\Sigma}_i}{M_{\tilde{g}}^2},\frac{\bar{\Sigma}_j}{M_{\tilde{g}}^2}\right)}{\sqrt{\sum_{r>s}|\lambda^\alpha_{rs}|^2(\bar{\Sigma}_r-\bar{\Sigma}_s)^2}}\ , \tag{35}$$

where

$$f(x,y)=\frac{3}{2(x-y)}\left[\frac{1}{x-1}+\frac{x(x-2)}{(x-1)^2}\log x-(x\to y)\right] \tag{36}$$

is normalized so that $f(1,1)=1$. It is apparent from Eq. (35) that if CP is conserved, or in other words if K is real, only imaginary λ^α can generate scalar couplings. Equation (35) is proportional to the down quark mass, because only m_d allows flavor transitions. Both light and heavy quarks contribute to the σ_Q coupling to nuclei. Again, depending on the different squark mass splittings and on the σ field under consideration, either contribution can be the most important. Heavy quarks contribute to the σ coupling to nucleons via the gluon anomaly, with the result [7]:

$$\mathcal{L}_{\sigma^\alpha\bar{N}N}=\frac{m_N}{M_{PL}}\mathcal{G}_{\sigma^\alpha}\sigma_Q^\alpha\bar{\psi}_N\psi_N\ , \tag{37}$$

where m_N and ψ_N are respectively the mass and wavefunction of the nucleon N, and $\mathcal{G}_{\sigma^\alpha}$ measures the strength of the σ coupling relative to gravity:

$$\mathcal{G}_{\sigma^\alpha}=i\frac{2\sqrt{2}\alpha_s}{243\pi}\frac{m_s^2}{M_{\tilde{g}}^2}\frac{M_{PL}}{F}\frac{\sum_{i>j}(\bar{\Sigma}_i-\bar{\Sigma}_j)\ Im(\lambda^\alpha_{ij}K_{jk}K^\star_{ik})\ f\left(\frac{\bar{\Sigma}_i}{M_{\tilde{g}}^2},\frac{\bar{\Sigma}_j}{M_{\tilde{g}}^2}\right)}{\sqrt{\sum_{r>s}|\lambda^\alpha_{rs}|^2(\bar{\Sigma}_r-\bar{\Sigma}_s)^2}}\ . \tag{38}$$

Since we are dealing with heavy quarks, the index α can be equal to 5 or 7. The largest of the two couplings can be estimated to be:

$$\mathcal{G}_{\sigma_7}\simeq\frac{2\sqrt{2}}{243\pi}\alpha_s K_{cb}\ \frac{M_{PL}}{F}\simeq 10^{-4}\ \frac{M^R_{PL}}{F}\ . \tag{39}$$

Similarly, we can estimate the direct coupling of σ_Q with the light down quark from Eq. (35). We find that this coupling for $\alpha=2$ is $\sim 10^{-5}\ M_{PL}/F$. Since the coupling is to down quarks only, the σ_2-neutron coupling is twice as large as the σ_2-proton coupling. This leads to violations of the equivalence principle whose magnitude depends on F. If $F\sim M_{GUT}\sim 2\times 10^{16}$ GeV, the σ_2 coupling is 5×10^{-3} times smaller than gravity and the Compton wavelength of σ_2 is of the order of 30 cm.

The experimental upper limits on $\mathcal{G}_\sigma^2$ are about 10^{-4} at distances of 3 m and about 10^{-5} at distances of 30 m [8]. This means that the moduli forces could be detected in future experiments if F is not much larger than the GUT scale, which is still a possibility.

The numerical estimates for the coupling strength and Compton wavelength of the σ have large uncertainties associated with the overall scale F as well as the supersymmetry-breaking mass m_s^2 and sparticle splittings Δm_s^2. Nevertheless, we hope that these estimates will motivate renewed efforts for searches of new long-range forces and violations of the equivalence principle at distances from ~ 100 m to few cm. It would be fascinating if one of the first indications for supersymmetry comes from the discovery of such forces. It is also interesting to notice that, because of the diverse moduli mass spectrum and the variety of their couplings, we could have a complicated pattern of different deviations from gravity at different length scales.

Finally, if $A \neq 0$ and V', $\bar{V}'$ are dynamical then δ' and $\bar{\delta}'$ are new moduli. They get their mass predominantly from the quadratic soft terms $\phi^\dagger \tilde{m}^2 \phi$ and not the Yukawa couplings. Their Compton wavelengths are:

$$\lambda_{\delta',\bar{\delta}} \simeq 8\pi\hbar c \frac{F}{m_s^2} \simeq 0.1\ m\ \left(\frac{300\ GeV}{m_s}\right)^2 \frac{F}{M_{PL}^R}\ . \tag{40}$$

The couplings of the δ are similar to those of the σ, so we will not discuss them in detail.

6 Moduli as Goldstone Bosons of the Spontaneously Broken Flavor Group

The alert reader has already noticed that the unitary matrices U_A are non-linear representations of the flavor group $U(3)_A$. In other words, the soft masses spontaneously break $U(3)_A$ and σ_A are the Goldstone bosons. The subsequent definitions

$$\tilde{m}_A^2 = U_a^\dagger \bar{\Sigma}_A U_A \qquad \text{and} \qquad \Sigma_A = \frac{m_s^2}{F}\Sigma'_A \tag{41}$$

suggest that we think of Σ'_A as a field which transforms as a $(1{+}8)_A$ and acquires a VEV Σ' at the scale $F \sim M_{PL}$ which spontaneously breaks $U(3)_A$. The high-energy ($\sim F$) dynamics $V(\Sigma'_A)$ that caused Σ'_A to get its VEV was postulated to be $U(3)_A$ invariant. Thus the orientation of the Σ'_A VEV in $U(3)_A$ space is not determined by high-energy dynamics. Σ'_A can point in any $U(3)_A$ direction, until we turn on the quark and lepton masses which align Σ'_A as computed in Sect. 2.

For the triscalar A-terms there are three possibilities. Because they break both supersymmetry and chirality, they resemble the soft masses $\tilde{m}_A^2$ in one sense and the Yukawa couplings in another.

One possibility is that V and $\bar{V}$ are fully dynamical just like the U. In particular, we can define fields $\Delta'_a = \frac{F}{m_s^2}\Delta_a$ $(a = u, d, e)$ which get VEV $\sim \mathcal{O}(F)$ and break the flavor symmetry spontaneously. Δ'_u transforms as a $(3,\bar{3})$ representation of $U(3)_Q \times U(3)_{\bar{U}}$; similarly for Δ'_d and Δ'_e. Again, some high-energy dynamics fix the magnitude of $\Delta' \neq 0$ but does not specify its direction. This means in particular that the high-energy potential V decomposes into a sum $V = V(\Sigma') + V(\Delta')$ which does not lock the orientations of Σ' and Δ' together. As before, once we turn on the quark and lepton masses, Δ' aligns with them as computed in Sect. 3.

The second possibility is that the A-terms act like Yukawas. They are "frozen" – by some unknown high-energy dynamics – and have specific values at low energies. This situation is identical to that of the minimal supersymmetric model. One simply has to hope that the high-energy dynamics aligns the A-term with the Yukawas. The A-terms now themselves break the flavor $U(3)^5$ symmetry and thus contribute to the mass of the σ_A. If they are proportional to fermion masses, their contributions are small and they do not affect the alignment of the Σ'.

Finally, there is a third possibility in which the A-terms are neither totally free to rotate nor frozen in flavor space, but they are "thawed". They are coupled to the Σ' direction in the way indicated in Eq. (28), where V' and $\bar{V}'$ are now frozen but the U_A are not. In this case, again, one has to hope that the frozen combination $V'^{\dagger}\bar{\Delta}\bar{V}'$ lines up with the Yukawas. In this "thawed" scenario the A-terms do not break the $U(3)^5$ symmetry realized by the Σ'.

We end with a cosmological caveat. The potential energy that we computed for the moduli/Goldstones, see Eq. (11), explicitly demonstrates that the energy difference between the minimum and a non-aligned configuration is $\sim m_W^4$. The minimum can be reached, by the emission of Goldstone particles, even at arbitrarily small energy or temperature and presumably is reached, given enough time. The amount of time depends on how rapidly the moduli lose energy as well as on cosmo-historical questions. Because the moduli couple very weakly with strength $F^{-1} \sim M_{PL}^{-1}$, they do not efficiently lose energy. As a result, they do not reach their minima in simple cosmologies [9], unless they happen to accidentally start out near their vacuum. Recently, there have been a revival of suggestions [10] on how to solve the problem and to allow the moduli to cosmologically relax to their ground state. Such a mechanism is clearly necessary to ensure flavor alignment. Even more, it is necessary to ensure that the Universe is not overclosed by coherent oscillations of the moduli.

7 Plastic Soft Terms

In previous sections we have conjectured that physics above Λ leaves the orientation of the soft terms undetermined, but fixes their eigenvalues. In this section we wish to relax the latter hypothesis. We envisage that the supersymmetry-breaking dynamics at Λ provide the low-energy theory with a constraint which fixes the overall scale m_s but does not necessarily freeze all three eigenvalues. Some functions of the eigenvalues can correspond to flat directions which remain undetermined until we turn on the low-energy dynamics. Of course, our postulate that the supersymmetry-breaking mechanism respects the flavor symmetry requires that the constraints that fix m_s have to be flavor singlets.

Let us consider first the case of vanishing left-right mixings in the squark and slepton mass matrices and focus on the fields Σ. Suppose that the dynamics at the scale Λ fixes the two lowest-dimension flavor-singlet operators:

$$Tr\Sigma = T \ , \quad Tr\Sigma^2 = T_2 \ , \tag{42}$$

where T^2 and T_2 are numbers of order m_s^4 and the first constraint ensures the absence of field-dependent quadratic divergences.

These are two constraints on three eigenvalues, thus one combination of eigenvalues remains a flat direction whose VEV will be determined by low-energy physics in a calculable way. It is easy to identify the flat direction. The above constraints are not just $SU(3)$ invariant, but are $SO(8)$ invariant, and they force the spontaneous breakdown $SO(8) \to SO(7)$, giving rise to seven Goldstone bosons. Six of them are a consequence of the breaking $SU(3) \to U(1)^2$ and can be identified with the fields σ. The seventh is the new flat direction θ which allows the eigenvalues of Σ to slide along a valley which preserves the above constraints. Notice that θ acquires mass already from the soft term, which preserves $SU(3)$ but violates $SO(8)$, as opposed to the σ fields which get mass only from Yukawa interactions.

The field Σ satisfying Eq. (42) can be expressed as

$$\Sigma = TU^\dagger \left[\frac{1}{3} - x(\cos\theta\lambda_8 + \sin\theta\lambda_3) \right] U \ , \tag{43}$$

where $\lambda_{3,8}$ are the two diagonal Gell-Mann matrices, U denotes an $SU(3)/U(1)^2$ rotation, and

$$x \equiv \sqrt{\frac{3T_2 - T^2}{6T^2}} \ , \quad 0 \leq x \leq \frac{1}{\sqrt{3}} \ . \tag{44}$$

Our assumption is that the six parameters contained in U and the angle θ are dynamical variables, related to flat directions of the moduli fields. The soft term Σ is not only "disoriented" in flavor space, but is also "plastic", since the pattern of eigenvalues can be deformed.

The effective potential for Σ can be approximated as

$$64\pi^2 V_{1-loop} = Tr(\Sigma + m^2)^2 \log\frac{\Sigma + m^2}{\Lambda^2} \simeq Tr\Sigma^2 \log\Sigma - 2\tau Tr\Sigma m^2 \ , \tag{45}$$

where we have kept only the contribution from the fermion mass term m^2 leading in $\tau \equiv \log\Lambda^2/m_s^2$. The minimization with respect to the $SU(3)/U(1)^2$ angles is analogous to the case of the disoriented soft terms and aligns Σ parallel to m^2 with eigenvalues ordered as in m^2. The θ-dependent part of V_{1-loop} is more easily identified in Eq. (45) by means of a series expansion in x:

$$\begin{aligned} 64\pi^2 V_{1-loop} &= \frac{2}{\sqrt{3}} x^3 T^2 \cos\theta(4\cos^2\theta - 3) + \mathcal{O}(x^5) \\ &- \frac{4}{\sqrt{3}} \tau x T \left[\cos\theta(m_3^2 - m_1^2) + \frac{1}{2}(\sqrt{3}\sin\theta - \cos\theta)(m_2^2 - m_1^2) \right] . \end{aligned} \tag{46}$$

This expansion is reasonable, since x must be somewhat smaller than $\frac{1}{\sqrt{3}}$ or the lightest sfermion becomes too light. In the limit $m_3^2 = m_2^2 = m_1^2$, there are three equivalent vacua:

$$\langle\theta\rangle = \pi \qquad \langle\theta\rangle = \pm\frac{\pi}{3} \; , \tag{47}$$

which correspond to a spontaneous breakdown of flavor as $SU(3) \to SU(2)\times U(1)$. Notice that the two *heaviest* sfermions transform as a doublet under the remaining $SU(2)$.

After we turn on the Yukawa of the heaviest fermion, the vacuum for θ becomes:

$$\langle\cos\theta\rangle = \begin{cases} 1 & \text{if } \xi > 1 \\ \frac{1}{2}\sqrt{1+3\xi} & \text{if } \xi < 1 \end{cases} \tag{48}$$

$$\xi = \frac{2\tau m_3^2}{9x^2 T} \; . \tag{49}$$

If the third generation fermion is heavy enough ($\xi > 1$), the vacuum has an $SU(2)$ invariance, where now the two *lightest* squarks transform as a doublet. For moderate fermion masses ($\xi < 1$), flavor is spontaneously broken as $SU(3) \to U(1)^2$. The critical value of m_3 determining which flavor subgroup is left invariant is given by, including all orders in x:

$$m_3^2 = \frac{T}{\tau}\left[\left(\frac{2\sqrt{3}x+1}{3}\right)\log\left(\frac{1+2\sqrt{3}x}{1-\sqrt{3}x}\right) - \sqrt{3}x\right] = \frac{9}{2}\frac{T}{\tau}x^2 + \mathcal{O}(x^4) \; , \tag{50}$$

which corresponds to $\xi = 1$ at leading order in x. For reasonable values of m_s, we expect that the top fulfills the requirement $\xi > 1$. This is welcome because the remaining $SU(2)$ implies that the masses of the two light squark doublets Σ_{Q_1} and Σ_{Q_2} are approximately equal, providing the desired suppression of the real part of $K^0 - \bar{K}^0$ mixing. Neither the bottom quark nor the tau lepton are heavy enough to expect a large value of ξ. This means that, unless x is very small, $\tilde{\tau}$ and $\tilde{\mu}$ are nearly degenerate in mass, while $\tilde{e}$ is considerably lighter; similarly $m_{\tilde{b}_R} \simeq m_{\tilde{s}_R} > m_{d_R}$. This is clearly in contrast with the minimal supersymmetric model prediction of near degeneracy between the first and second generation of each species of squarks and sleptons.

Since the Yukawa couplings of the second generation are much smaller than the corresponding ones for the third, their effect is just a small perturbation on the θ vacuum of Eq. (48). However, this is an important perturbation as it fixes the sign of $\sin\theta$, which is undetermined in the limit $m_2 = m_1 = 0$ at all orders in x, and breaks the residual $SU(2)$ of the case $\xi > 1$:

$$\langle\sin\theta\rangle = \begin{cases} \frac{\sqrt{3}}{2}\,\frac{m_2^2-m_1^2}{m_3^2-m_1^2}\,\frac{\xi}{\xi-1} & \text{if } \xi > 1 \\ \frac{1}{2}\left(3\sqrt{3}\frac{m_2^2-m_1^2}{m_3^2-m_1^2}\right)^{1/3} & \text{if } \xi = 1 \\ \frac{\sqrt{3}}{2}\sqrt{1-\xi} + \frac{\sqrt{3}}{8}\frac{\xi}{1-\xi}(\sqrt{1+3\xi}+\sqrt{1-\xi})\frac{m_2^2-m_1^2}{m_3^2-m_1^2} & \text{if } \xi < 1 \end{cases} \tag{51}$$

The mass splitting between the first two squark generation is given by

$$\frac{\Delta m^2}{m^2} \equiv \frac{\Sigma_2 - \Sigma_1}{\Sigma_2} \simeq 6x\langle\sin\theta\rangle \; . \tag{52}$$

The contribution to Δm_K is safely suppressed. If the top-quark mass is such that $\xi > 1$, the splitting between the soft masses of the first two generations of left-handed down squarks satisfies $\mathcal{D}_{21} \sim m_c^2/m_t^2$, which has to be compared with Eq. (24).

We now want to extend our considerations to the case of Δ. The lowest-dimensional flavor-singlet constraint which involves Δ is:

$$\mathrm{Tr}\Delta\Delta^\dagger = D. \tag{53}$$

In a manner consistent with the hypothesis $\Delta \sim \mathcal{O}(mm_s)$ stated in Sect. 3, we define

$$D = d \ \mathrm{Tr}\ m^2 \ , \tag{54}$$

with $d \sim \mathcal{O}(m_s^2)$. With approximations analogous to those used to derive Eq. (45), we can write the effective potential for Δ as

$$64\pi^2 V_{1-loop} \simeq \mathrm{Tr}\ \mathcal{M}^4 \ln \mathcal{M}^2 - 2\tau\mu\ \mathrm{Tr}(\Delta + \Delta^\dagger)m \tag{55}$$
$$\mathcal{M}^2 = \begin{pmatrix} \Sigma_L & \Delta^\dagger \\ \Delta & \Sigma_R \end{pmatrix} \ ,$$

where for simplicity we take $\tan\beta = 1$. As shown in Sect. 3, the minimization of the flavor rotation angles brings $\Sigma_{L,R}$ and Δ to the diagonal form. Working in the limit $\Delta \ll \Sigma_{L,R}$, justifed by Eq. (54), the first term in the right-hand side of Eq. (55) becomes

$$\begin{aligned} \mathrm{Tr}\mathcal{M}^4 \ln \mathcal{M}^2 &= 2\mathrm{Tr}\left(\frac{\Sigma_L \log \Sigma_L - \Sigma_R \log \Sigma_R}{\Sigma_L - \Sigma_R}\right)\Delta^2 + \mathcal{O}(\Delta^4) \\ &= 2\mathrm{Tr}\ \log \Sigma \Delta^2 + \mathcal{O}(\Delta^4) \qquad \text{if } \Sigma_L \simeq \Sigma_R = \Sigma \ . \end{aligned} \tag{56}$$

The minimization of V_{1-loop} with respect to Δ under the constraint of Eq. (53), gives, at leading order in τ:

$$\Delta_i \simeq \sqrt{d} m_i \ , \tag{57}$$

where the index $i = 1, 2, 3$ spans the diagonal elements of the corresponding matrix. We recover therefore the minimal supersymmetric model result that the trilinear terms are proportional to the corresponding Yukawa couplings.

8 Minimal Unification

Until now we have been working under the hypothesis that below the scale Λ, where the supersymmetry breakdown occurs, we have the minimal supersymmetric $SU(3) \times SU(2) \times U(1)$ particle content. We now consider the possibility that the theory below Λ is some minimal supersymmetric GUT.

In minimal supersymmetric GUTs the gauge symmetry is increased to $SU(5)$ or $SO(10)$ and the number of chiral multiplets decreases. This means that the flavor group is no longer

$U(3)^5$, but it is smaller: $U(3)_{\bar{5}} \times U(3)_{10}$ in the case of $SU(5)$, and just $U(3)_{16}$ for $SO(10)$. If we also assume that the soft terms are as minimal as possible, namely singlets under the GUT group, then we have a very constrained system with a small flavor group and a small number of parameters in the soft terms. Are there enough moduli/Goldstones available to align sufficiently and avoid problems with flavor violations ?

For simplicity, let us discuss the minimal $SO(10)$ model in which the Yukawa coupling superpotential between the ordinary fermions in the 16 representation and the Higgs fields $H_{u,d}$ is

$$W_Y = 16\lambda_u 16 H_u + 16 K \lambda_d K^T 16 H_d \ . \tag{58}$$

For simplicity, we will ignore the A trilinear terms and write the soft supersymmetry-breaking Lagrangian as:

$$\mathcal{L}_{Soft} = \frac{m_s^2}{F} 16^\dagger U^\dagger \bar{\Sigma} U 16 \ . \tag{59}$$

The crucial difference between this minimal-GUT case, with gauge-singlet Σ, and the previous $SU(3) \times SU(2) \times U(1)$ analysis is apparent from Eq. (59). Now there is just one U available, instead of 5, to do all the alignments necessary to reduce flavor violations. To see how U chooses to orient we can work at the scale m_s, where the large τ approximation is valid. It is clear that U will align Σ parallel to m_u, since m_u gives the largest contribution to the energy. Neglecting the small off-diagonal effects caused by the running from M_{GUT} to m_s, this implies that all sparticle mass matrices will be parallel to m_u whereas the down-quark and charged-lepton mass matrices will be misaligned from m_u by angles of the order of the Kobayashi-Maskawa angles.

Thus, unless sleptons are highly degenerate in mass, $\mu_{L,R} \to e_{R,L}\gamma$ transitions are proportional to a mixing angle $K_{e\mu} = K_{us} \simeq \sin\theta_c$ and occur at an unacceptable rate. In $SU(5)$ only the right-handed sleptons are misaligned from the lepton mass matrix, and the amplitude for $\mu_L \to e_R + \gamma$ is again proportional to the Cabibbo angle $\sin\theta_c \simeq \sqrt{d/s}$. Of course, minimal $SO(10)$ and $SU(5)$ theories have a problem: they predict $m_d = m_e$ and this is the reason why they give $\mu \to e\gamma$ proportional to $\sqrt{d/s}$. However even if we extend the theory *à la* Georgi–Jarlskog, the $\mu \to e\gamma$ amplitude is still problematic, being proportional to $\sqrt{e/\mu}$.

The reason for this failure is that in minimal supersymmetric GUTs with minimal GUT-invariant soft terms, the few available soft terms just align with m_u, leaving some mismatch between down quarks and squarks and more importantly between leptons and sleptons. This causes difficulties with individual lepton violating processes, which were not originally present in supersymmetric GUTs with universality at M_{GUT}.

The problem could be cured in more complicated GUTs with a larger flavor structure, necessary perhaps to explain the fermion mass pattern, which would allow for more freedom in the low-energy alignment of the soft-breaking masses.

A strong degeneracy between the first two generations of sleptons and down squarks suppresses the most dangerous processes and could therefore represent an alternative solution. In the previous section we have shown that this occurs in the plastic soft-term scenario if the corresponding Yukawa coupling of the third generation is strong enough. In the $SU(3)\times SU(2)\times U(1)$ theory, this is not the case for sleptons. However, in the $SO(10)$ example, the slepton mass alignment feels the strong top-quark Yukawa coupling and the degeneracy between the first two generations is *predicted*. The dynamics of the plastic soft terms cures the disease in the dynamics of the disoriented soft terms: in GUTs the large up-type quark Yukawa couplings force the sleptons to misalign, but insure that the first two generations are almost degenerate in mass. This is completely analogous to the suppression of K^0–$\bar{K}^0$ which we have discussed in the previous section.

9 Conclusions

We proposed "disorientation" as an alternative to universality for suppressing flavor violation in supersymmetric theories. Universal soft terms realize the flavor symmetry in the Wigner mode. Disoriented soft terms realize it in the Nambu-Goldstone mode; this allows large sparticle splittings and has the appeal that the absence of flavor violations is a consequence of a dynamical calculation.

The Goldstone particles can be thought of as either the consequence of a spontaneously broken flavor symmetry or perhaps could be identified with some of the flat directions (moduli) that frequently occur in supersymmetric or superstring theories. In the latter case there would be an important connection between the space of the moduli and the flavor group.

Why did our mechanism work? Promoting some of the parameters of the low-energy theory to fields allowed us to exploit nature's preference for states of maximal possible symmetry. This is the reason why: the spin aligns with an external magnetic field, preserving SO(2); sleptons align with leptons, preserving individual lepton number conservation $U(1)^3$; squarks align –as much as possible– with the quarks, preserving an approximate $U(1)^3$; the 7th goldstone boson of the plastic scenario chooses to relax at its special value where the symmetry is enhanced to $SU(2)\times U(1)$ and pairs of sparticles are degenerate. Nature's frequent preference for states of higher symmetry fully accounts for our mechanism for the suppression of flavor violation. More importantly, it leads us to novel phenomena: long range forces, new supersymmetric phenomenology and the peaceful coexistence of split sparticles and flavor conservation.

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