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# Cosmological Solutions of Higher-Curvature String Effective Theories with Dilatons



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## Abstract

We study the effect of higher-curvature terms in the string low-energy effective actions on the cosmological solutions of the theory, up to corrections quartic in the curvatures, for the bosonic and heterotic strings as well as for the type II superstring. We find that cosmological solutions exist for all string types but they always disappear when the dilaton field is included, a conclusion that can be avoided if string-loop effects are taken into account.

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The fact that string theories, in their low-energy limit, give rise to effective theories of gravity containing higher-curvature corrections to the usual scalar curvature term, has led to renewed interest in such extensions of Einstein gravity [1 – 2]. The leading quadratic correction, proportional to the Gauss-Bonnet combination, in particular, has ben studied in some detail in [3], where it is shown that there are cosmological (de Sitter) solutions in addition to Minkowski space and that they are unstable. For the bosonic string, the next-to-leading corrections are cubic in the curvatures and it has ben shown that cosmological solutions remain unstable at this order [2]. It is, however, possible that higher curvature terms could give rise to stable de Sitter solutions.

In this article, we study the effect of higher order corrections to string low-energy effective actions, up to the quartic curvature level, on the existence and stability of the cosmological solutions to these theories. For the type II superstring there are neither quadratic nor cubic-curvature corrections whereas for the heterotic string only the cubic terms are absent so that the quartic curvatures are actually the leading and next-to-leading corrections to these theories, respectively. We shall study also the consequences of the inclusion of the dilaton in the effective action; for the quadratic correction, it is known that the inclusion of the dilaton has dramatic consequences for the cosmological branches, which become excluded at this level [4]. Finally, we shall analyse the effect of introducing string-loop corrections to the string effective action.

We start with the action in the so-called s-parametrisation:

$$S = \int d^D x \sqrt{-g} \left[ \frac{R}{k} + c_1 \alpha' e^{-2\phi} \mathcal{L}_2 + c_2 \alpha'^2 e^{-4\phi} \mathcal{L}_3 + c_3 \alpha'^3 e^{-6\phi} \mathcal{L}_4 + ... \right] \tag{1}$$

where $k$ is Newton's constant in D dimensions, $\alpha'$ is the string expansion parameter and the dots denote terms involving derivatives of the dilaton which will not be relevant for our discussion. Here and throughout the article we use the signature $(-, +, ..., +)$ and the conventions: $R^{\mu}{}_{\nu\alpha\beta} = \Gamma^{\mu}_{\nu\beta,\alpha} - ...,\ R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$. The $\mathcal{L}_i$ terms involve appropriate powers of the curvature, which have been found by comparison of the amplitudes generated by the action (1) with the string amplitudes (equivalent results have been obtained through the computation of the relevant $\sigma$-model $\beta$-functions), as follows [5]

$$
\mathcal{L}_2 = \Omega_2, \tag{2}
$$

$$
\mathcal{L}_3 = 2\ \Omega_3 + R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\lambda\rho} R^{\lambda\rho}{}_{\mu\nu}, \tag{3}
$$

$$
\begin{aligned}
\mathcal{L}_4 = \zeta(3) & \left[ R_{\mu\nu\rho\sigma} R^{\alpha\nu\rho\beta} \left( R^{\mu\gamma}{}_{\delta\beta} R_{\alpha\gamma}{}^{\delta\sigma} - 2 R^{\mu\gamma}{}_{\delta\alpha} R_{\beta\gamma}{}^{\delta\sigma} \right) \right] \\
& - \xi_H \left[ \frac{1}{8} \left( R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right)^2 + \frac{1}{4} R_{\mu\nu}{}^{\gamma\delta} R_{\gamma\delta}{}^{\rho\sigma} R_{\rho\sigma}{}^{\alpha\beta} R_{\alpha\beta}{}^{\mu\nu} \right. \\
& \qquad \left. - \frac{1}{2} R_{\mu\nu}{}^{\alpha\beta} R_{\alpha\beta}{}^{\rho\sigma} R^{\mu}{}_{\sigma\gamma\delta} R_{\rho}{}^{\nu\gamma\delta} - R_{\mu\nu}{}^{\alpha\beta} R_{\alpha\beta}{}^{\rho\nu} R_{\rho\sigma}{}^{\gamma\delta} R_{\gamma\delta}{}^{\mu\sigma} \right] \\
& - \frac{1}{2} \xi_B \left[ \left( R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right)^2 - 10 R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\sigma} R_{\sigma\gamma\delta\rho} R^{\beta\gamma\delta\rho} \right. \\
& \qquad \left. - R_{\mu\nu\alpha\beta} R^{\mu\nu\rho}{}_{\sigma} R^{\beta\sigma\gamma\delta} R_{\delta\gamma\rho}{}^{\alpha} \right],
\end{aligned} \tag{4}
$$

where $\xi_{H(B)} = 1$ for the heterotic (bosonic) string and vanishes for the other string types and $\Omega_2,\ \Omega_3$ are the second and third order Euler densities:

$$
\Omega_2 = R^2_{\mu\nu\alpha\beta} - 4 R^2_{\mu\nu} + R^2, \tag{5}
$$

$$
\begin{aligned}
\Omega_3 = & R^{\mu\nu}{}_{\alpha\beta} R^{\alpha\beta}{}_{\lambda\rho} R^{\lambda\rho}{}_{\mu\nu} - 2 R^{\mu\nu}{}_{\alpha\beta} R_{\nu}{}^{\lambda\beta\rho} R^{\alpha}{}_{\rho\mu\lambda} + \frac{3}{4} R R^2_{\mu\nu\alpha\beta} \\
& + 6 R^{\mu\nu\alpha\beta} R_{\alpha\mu} R_{\beta\nu} + 4 R^{\mu\nu} R_{\nu\alpha} R^{\alpha}{}_{\mu} - 6 R R^2_{\alpha\beta} + \frac{1}{4} R^3.
\end{aligned} \tag{6}
$$

Coefficients $(c_1,\ c_2,\ c_3)$ are different for different string theories

$$
bosonic: \ \left( \frac{1}{4}, \frac{1}{48}, \frac{1}{8} \right), \tag{7}
$$

$$
heterotic: \ \left( \frac{1}{8}, 0, \frac{1}{8} \right), \tag{8}
$$

$$
superstring II: \ \left( 0, 0, \frac{1}{8} \right). \tag{9}
$$

For maximally symmetric spaces

$$
R^{\mu}{}_{\nu\lambda\sigma} = \Lambda \left( \delta^{\mu}_{\lambda} g_{\nu\sigma} - \delta^{\mu}_{\sigma} g_{\nu\lambda} \right), \tag{10}
$$

and the dilaton is constant. Hence, the gravitational and scalar field equations reduce to, respectively

$$f(\Lambda) \equiv \Lambda(c_3\rho\Lambda^3 + c_2\sigma\Lambda^2 + c_1\beta\Lambda + \alpha) = 0, \tag{11}$$

$$g(\Lambda) \equiv \Lambda^2(3c_3\rho\Lambda^2 + 2c_2\sigma\Lambda + c_1\beta) = 0, \tag{12}$$

where

$$\alpha = (D-2)(D-1), \tag{13}$$

$$\beta = \alpha' k e^{-2\phi}(D-4)(D-3)(D-2)(D-1), \tag{14}$$

$$\sigma = (\alpha' k)^2 e^{-4\phi} 2(D-6)(D-1)[2 + (D-5)(D-4)(D-3)(D-2)], \tag{15}$$

$$\rho = (\alpha' k)^3 e^{-6\phi}(D-1)(D-2)\big[-3\zeta(3)(D-3) + \xi_H(D-2)(D-9) - 2\xi_B[(D-2)(D-11)-1]\big], \tag{16}$$

In the absence of the dilaton, only eq. (11) has to be satisfied; clearly, Minkowski space is always a solution and, with our conventions, de Sitter (anti-de Sitter) solutions correspond to the real positive (negative) roots of (11). We find that, for the superstring II in the critical number of dimensions ($D = 10$), there is only one positive real root, $\Lambda_s = \left(\frac{-\alpha}{c_3\rho}\right)^{1/3}$. For the heterotic string, there is only one positive real root in $D = 4$ and three real roots in $D = 10$ (two negative and one positive). For the bosonic string ($D = 26$) there is only one positive real root.

Regarding the stability of these solutions for small graviton excitations about the background, notice that it can be deduced immediately from the observation that the sign of $\delta^2 S$ is determined by the sign of $f'(\Lambda)$ at the roots [3]; in this way, we have checked that all cosmological branches are unstable except for one of the anti-de Sitter branches of the heterotic string in $D = 10$.

When the dilaton is included, eq. (12) has to be taken into account as well; this is a strong constraint and, indeed, it is easy to check that the cosmological branches we found above do not survive and only Minkowsky space remains a solution. We shall see in the following that this conclusion can be evaded if string-loop effects are taken into account. In the so called $\sigma$-parametrization of the effective action, namely

$$S^{(\sigma)} = \int d^D x \sqrt{-g} e^{-2\phi} \left[ \frac{R}{k} + c_1 \alpha' \mathcal{L}_2 + c_2 \alpha'^2 \mathcal{L}_3 + c_3 \alpha'^3 \mathcal{L}_4 + ... \right], \tag{17}$$

these can be included in a rather simple way, i.e. substituting the exponential $e^{-2\phi}$ by the series in the string coupling $g_S \equiv e^{2\phi}$ [6]:

$$B(\phi) \equiv e^{-2\phi} + a_0 + a_1 e^{2\phi} + a_2 e^{4\phi} + ..., \tag{18}$$

where the coefficients $a_0$, $a_1$, ... are, at the present level of understanding of string theory, unknown.

The s-parametrisation of action (1), is obtained from the $\sigma$-parametrization, eq. (17), through the conformal transformation $g_{\mu\nu} \to e^{-2\phi} g_{\mu\nu}$, or, if string-loop effects are included:

$$g_{\mu\nu} \to B(\phi) g_{\mu\nu}, \tag{19}$$

which, in turn, translates into the substitutions

$$e^{-2n\phi} \to B^n(\phi), \qquad n = 1, 2, 3, ... \tag{20}$$

in (1). As a consequence, eqs. (11) and (12) now become

$$\bar{f}(\bar{\Lambda}) \equiv \bar{\Lambda}(\bar{\rho} c_3 \bar{\Lambda}^3 + \bar{\sigma} c_2 \bar{\Lambda}^2 + \bar{\beta} c_1 \bar{\Lambda} + \alpha) = 0, \tag{21}$$

$$\bar{g}(\bar{\Lambda}) \equiv \frac{B'(\phi)}{B(\phi)} \bar{\Lambda}^2 (3\bar{\rho} c_3 \bar{\Lambda}^2 + 2\bar{\sigma} c_2 \bar{\Lambda} + \bar{\beta} c_1) = 0, \tag{22}$$

where $\bar{\rho} = B^3(\phi) e^{6\phi} \rho$, $\bar{\sigma} = B^2(\phi) e^{4\phi} \sigma$ and $\bar{\beta} = B(\phi) e^{2\phi} \beta$. Notice now the appearence of the factor $B'(\phi)$ in front of the dilaton equation of motion; this allows us to require that this factor vanishes in order to satisfy eq. (22), which fixes the value of the dilaton field, $\phi = \phi_0$, so that there is only one algebraic equation for $\bar{\Lambda}$. The analysis then proceeds as for the solutions to eq. (11), except that now results depend on the value of $B(\phi_0)$. For instance, for the superstring II, where $\bar{\Lambda}_s = B^{-1}(\phi_0) \Lambda_s$ and $f'(\bar{\Lambda}_s) = -3\alpha$, so that the sign of the solution indeed depends on $B(\phi_0)$ but the stability analysis does not change as compared to the case with no dilaton. The same result holds for the heterotic string in $D = 4$.

We stress that our conclusions regarding the existence and stability of cosmological solutions to higher-curvature string effective theories, upon inclusion of next-to-leading corrections, support and reinforce the ones that were already pointed out at the leading-order level, namely that there exist de Sitter branches that are, however, erased in the presence of the dilaton; given that this is basically due to the fact that the dilaton equation of motion represents an additional strong constraint on possible maximally symmetric solutions, a feature that remains when higher order corrections are included, it is therefore natural to conjecture that this may be a rather general result. Our analysis shows, however, that this conclusion is not unescapable if string-loop corrections are taken into account.

Thus, our results show that string theory has no intrinsic cosmological problem, at least in $D = 4$. Furthermore, it is rather easy to show that the inclusion of an explicit cosmological constant may change considerably our conclusions as stable cosmological branches can be then found. This implies, for instance, that in order to achieve a sufficiently long lasting period of inflation a potential for the dilaton with at least one single local minimum is required. This indicates the prominance of the issue of supersymmetry breaking and the ensued cosmological constant problem in any realistic stringy cosmological scenario.

There is, nonethess, a basic ambiguity in our analysis, associated with the fact that the form we have chosen for the higher-curvature corrections, eqs. (2)–(4), is just one among various possible parametrizations that are related by metric redefinitions of the type $g_{\mu\nu} \to g_{\mu\nu} + \alpha'(a_1 R_{\mu\nu} + a_2 g_{\mu\nu} R) + \alpha'^2(R_{\mu\alpha\beta\gamma} R_\nu{}^{\alpha\beta\gamma} + ...) + ...$, which do not change the S-matrix. In fact, it is possible to show that there are both ambiguous and fixed terms according to whether they are changed by field redefinitions or not. At order $\alpha'$, only the $R^2_{\mu\nu\rho\sigma}$ term is fixed by the string amplitudes and the Gauss-Bonnet form is singled out only if we introduce the further requirement that the effective theory be unitary at this level [7]. However, it has been shown that at cubic and higher orders, it is not possible to write the effective action only in terms of Euler densities as required by unitarity. The ambiguous terms are then often chosen so as to keep the effective action as simple as possible at each order; however, even this choice is not unique since such terms cannot, in general, be all set to zero simultaneously given that there are relations among their variations. We have studied other possible forms for the effective action at the cubic curvature level, where a complete field redefinition analysis has been performed in Refs. [8, 9], and we find that our conclusions do not basically change, i.e. although there indeed exist different cosmological solutions for different actions, they are still always erased in the presence of the dilaton and saved by the inclusion of string-loop effects.

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