Emittance issues in the CERN-PS

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1.1 Introduction

Transverse and longitudinal emittances, like intensity and energy, are among the fundamental parameters of a particle beam. In particular for hadron machines, as the CERN-PS, the minimization, the conservation, or more generically the manipulation of the emittances is always a constant care. The utilisation of the PS, as part of the injection chain for the LHC¹, also imposes strong constraints to achieve and preserve beams of high brightness (i.e. intensity/emittance).

This paper is a short review of some encountered problems, conceived solutions and recipes applied to various PS beams.

2.1 Transverse emittance issues

2.2 Measurements of transverse emittance

Transverse emittances of the PS circulating beams are measured essentially with two kind of instruments : wire scanners^{2,3,4} (also called flying wires) and fast moving scrapers (also called measuring targets). Beam size measurements with instruments using collection of ions or electrons generated by ionization processes, were used in the past but space charge effects in high intensity beams produced inaccurate results.

At present four carbon wire scanners are used (two for the horizontal and two for the vertical plane) with moving speeds of 10 or 20 cm/ms. The beam shape, see Fig.1, is obtained by measuring, with photomultipliers, the secondary particles emitted during the wire-beam interaction.



Fig.1: Wire scanner measurements.Upper trace : Hor. plane; lower trace: Vert. plane

In the PS energy range, 1 to 26 GeV, the measurement is perfectly non destructive. The sensitivity is large enough to measure beams with intensities varying between 10^9 to 10^{13} p/pulse. The mechanical life time of the wires, at present, allows several thousand measurements.

After having compared the wire measurements with emittance measurements from other instruments, like scrapers and SEM grids in the injection and extraction transfer lines, the degree of confidence of the measured emittance values is about 10%.

Another important instrument used since a long time in the PS for beam size measurements is the fast moving scrapers. For each plane, a couple of pulsed "blades" intercepts the beam at a given adjustable timing, see Fig. 2a. They move in and out in about 30 ms and stay in position during about 50 ms, see Fig. 2b. The individual blade positions are independently adjusted, with respect to the vacuum chamber center, within a precision of less than 0.1 mm.



Fig.2a: Sketch of the horizontal moving scrapers.



Fig.2b: Cutting 5% of the beam with moving scrapers. Upper trace: beam current. Lower trace: scraper motion. Time scale 100ms/div.

The good reliability and intrinsic simplicity of this instrument has made it a valuable tool for quick checks of beam blow-up and approximate emittance measurements. However if a large amount, say more than ~20%, is cut, local transverse instabilities can be excited which might jeopardize the validity of the measurement, as the threshold for the development of these instabilities is proportional to the derivative of the particle distribution function⁵. For emittance measurements in the injection and extraction transfer lines, classical sets of SEM grids are systematically used, see Fig.3.



Fig.3 Emittance measurements in the extraction transfer line with SEM grids. a) Horizontal plane; b) Vertical plane

Very important for high precision emittance measurements is the number of wires covering the beam 6,7,8,9 . The relative error on the emittance measurement as function of the number of wires overlapping 4σ of the beam is shown in Fig.4. The two curves refer to 1% and 5%, of relative "noise" on the voltage measured on each wire. It can be seen that for a realistic noise value of 5% at least 8 wires are needed to get an emittance error smaller than 10%.



Fig. 4: Relative error on emittance measurement with SEM grids versus number of wires in 4 sigma's of the beam for two values of relative voltage resolution (1% and 5%)⁸.

To obtain the real betatron dimension from measurements made in dispersive regions, the contribution for the energy spread has to be subtracted⁹. In principle, for uncorrelated distribution functions in the longitudinal and transverse planes, one has

$$\sigma_{x, y} = \sqrt{\sigma_m^2 - \left(D_{x, y} \frac{\sigma_p}{p}\right)^2}$$
 1)

 $\sigma_{x, y} = rms$ betatron beam size (the transverse emittance is $\varepsilon_{x, y} = \frac{\sigma_{x, y}^2}{\beta_{x, y}}$)

 $\begin{aligned} \beta_{x,y} &= \text{beta function (hor. or vert.)} \\ \sigma_m &= \text{rms measured beam size (hor. or vert.)} \\ D_{x,y} &= \text{dispersion value (hor. or vert.)} \\ \frac{\sigma_p}{p} &= \text{rms momentum spread} \end{aligned}$

where

Sometimes, for particular beam distributions, e.g. in storage or cooling rings whith very large $\frac{o_p}{p}$, another equation is used:

$$\sigma_{x, y} = \sigma_m - D_{x, y} \frac{\sigma_p}{p}$$
 2)

To check which of the formulae 1) or 2) was the most suitable for the PS beam conditions, one experiment was made, where at a given fixed energy the momentum spread of the beam was varied by changing the RF voltage. At the same time the beam size was measured with a wire scanner. The results, shown on Fig.5, confirm the validity of eq. 1), as eq. 2) show the unphysical results of a decreasing transverse emittance.



Fig.5: Transverse emittance versus momentum spread. Top curve, showing a constant emittance, verifies the validity of eq. 1) compared to bottom curve derived from eq. 2), showing an unphysical emittance decreasing.

2.3 Problems with transverse emittance conservation

Beside the obvious problems of injection oscillation minimization and matching optimization, collective effects are the most important source of emittance degradation.

2.3.1 Space charge

For a round beam, the incoherent tune shift $\Delta Qi_{x,y}$, that is the tune depression seen by the particles located in the core of the bunch¹⁰, is approximately given by

$$\Delta Qi_{x,y} \approx -\frac{r_0}{ec} \frac{I_p R}{\varepsilon_{x,y}^* (\beta \gamma)^2}$$
3)

where $r_0 = \text{classical proton radius} = 1.53 \ 10^{-18} \text{ m}$ $e = \text{electron charge} = 1.6 \ 10^{-19} \text{ C}$ $c = \text{speed of light} = 3 \ 10^8 \text{ m/s}$ $I_p = \text{bunch peak current (e.g., for parabolic bunches: } I_p = \frac{3}{2} \frac{N_b e}{\tau_b})$ $\tau_b = \text{total bunch duration}$ $N_b = \text{number of particle in the bunch}$ $\epsilon^*_{x,y} = \text{normalized transverse emittance (} \epsilon^*_{x,y} = \beta \gamma \epsilon_{x,y})$ $\beta, \gamma = \text{usual relativistic parameters}$

When $|\Delta Qi_{x,y}| > 0.2$ and depending on machine nonlinearities, working point and the time during which the beam stays in these conditions, some emittance blow-up can be expected. Machine experiments have been made to quantify the allowable tune spread for the LHC type beam¹¹. The results are shown on Fig.6.



Fig.6a: Transverse averaged emittance blow-up versus vertical incoherent tune shift for two different working points: $Q_x = 6.22$, $Q_y = 6.22$ and an optimized $Q_x = 6.22$, $Q_y = 6.28$



Fig.6b: Transverse emittance blow-up versus time for $\Delta Qi_{x,y} \approx -0.45$

Few cures can be found against such effects. Apart from the obvious solution of increasing the injection energy, another possibility is to reduce the bunch peak current by eventually changing the bunch longitudinal shape (see par. 3.5).

2.3.2 Coupled bunch transverse instabilities

In low energy proton machines, where the bunches are generally relatively long (low frequencies) the responsible impedance for the onset of these coherent instabilities, is the vacuum chamber resistive wall which is peaking around zero frequency . A necessary condition for the instability is 5

A necessary condition for the instability is⁵

$$\operatorname{Re}[Z_{\perp}(\omega)]h(\omega) < 0$$

Where $\operatorname{Re}[Z_{\perp}(\omega)]$ is the real part of the impedance $Z_{\perp}(\omega)$ and $h(\omega)$ is the beam power spectrum envelope, that is a positive value depending on intensity, bunch shape, machine parameters and chromaticity in particular.

In other words the instabilities appear, in frequency domain, at lines with frequencies f_n for n < 0 and $|n| > Q_{x,y}$ with

$$f_n = \left(n + Q_{x,y}\right) f_0 + m f_s \tag{4}$$

where

 $\begin{array}{l} f_n = \mbox{frequency of the instability} \\ n = \mbox{coupled-bunch mode} = ...-2,-1,0,1,2,... \\ Q_{x,y} = \mbox{betatron tune} \\ f_0 = \mbox{revolution frequency} \\ m = \mbox{oscillation mode} = 0,1,2,... \\ f_s = \mbox{synchrotron frequency} \end{array}$

In the PS, instabilities with n = -7 and -8 and m = 0 and 1 are typical for beam intensities larger than ~ 0.5 10¹³ p/pulse in 20 bunches ($\tau_{\rm b} \sim 50$ ns).

Classical cures for these instabilities are transverse feedback's and / or octupoles¹².

2.3.3 Single bunch head-tail instabilities

Usually driven by the broad band vacuum chamber impedance, single bunch head tail instabilities are cured by an accurate chromaticity control along the accelerating cycle. A negative chromaticity is required below transition energy and a positive one if above. Nevertheless, for very long bunches, higher modes of oscillation as m = 5, 6 and 7, excited by the resistive wall impedance, have been observed in the PS beam for LHC with bunches of $\tau_b \sim 200$ ns and 2 10¹² p/bunch¹³.

2.4 The PS beam for LHC

An example of cautious transverse emittance conservation is the results of an experiment recently made in the PS Booster (PSB) and PS^{14,15}. In this experiment a single bunch beam was accelerated in the PSB from 50MeV to 1 or 1.4 GeV(kinetic energy) then injected into the PS. After having spent 1.2s on a flat bottom at injection energy, the beam was then accelerated, in about 1s, to 26 GeV and finally fast extracted on an external dump. The characteristics of such a beam were the following: $N_b = 2 \ 10^{12} \text{ p/bunch}$, $\epsilon_x^* \approx \epsilon_y^* \approx 2.5 \,\mu\text{m}$, $\epsilon_l \approx 1.2 \text{ eVs}$

A careful optimization of working point, chromaticity, transverse feedback, longitudinal parameters etc. allowed to transfer the beam with a negligible transverse emittance blow-up. See Fig.7.



Fig 7: Results of transverse emittance measurements, at 1 and 1.4 GeV inj. energy, during the "LHC test" experiment. Vertical axis : Averaged normalized emittance value: $\left(\epsilon_x^* + \epsilon_y^*\right)/2$

Horizontal axis: measurement points during the acceleration, that is:

1: in the PSB before extraction, with "Beamscope"

- 2: in the PS at injection, with SEM grids
- 3: """ " just after injection, with wire scanner
 4: "" " at the end of the 1.2s injection flat bottom, with wire scanner

5: """ at 2.7GeV, with wire scanner (NB: large dp/p)
6: """ before extraction at 26 GeV, with wire scanner
7: extracted beam in transfer line, with SEM grids

3.1 Longitudinal emittance issues

3.2 Longitudinal emittance measurements

Longitudinal emittance measurements in circular machines are in principle relatively easy. The classical method consists in measuring the bunch current line density, or bunch shape. This gives the bunch length or equivalently the maximum phase excursion inside the RF bucket. The integration of the phase space trajectory of the extreme particle, obtained by tracking or by analytical approximation, gives finally the emittance. In the computation one can even take into account collective effects like potential well bunch lengthening or shortening depending on the impedance value and if above or below transition energy. However usually these effects account for less than 10%. From the hardware point of view, the detector commonly adopted is a wideband (e.g. 0.1-1000 MHz) resistive wall monitor connected to a fast transient digitizer ¹⁶. Today digital oscilloscopes, with analog bandwidth larger than 1000 Mhz and sampling frequencies larger than 5 GHz, are excellent for measuring of 1-100 ns long bunches. An example of such measurements, done in the PS machine, is in Fig.8.



Fig.8: Various bunches measured in the PS machine, making use of a wideband (0.1-2000 MHz) resistive wall monitor connected to a TEKTRONIX[®] 7912AD transient digitizer (analog bandwidth 0-1200 Mhz).

3.3 Longitudinal emittance manipulations

3.3.1 Controlled longitudinal blow-up

Frequently it is not absolutely necessary to preserve the longitudinal emittance but it is always very important to deliver a beam which is stable and reproducible from bunch to bunch and from pulse to pulse. Blowing up longitudinal emittance increases Landau damping and strongly enhances beam stability. However such beam manipulations, also called "controlled longitudinal blow-up", must satisfy some requirements: the amount of blow-up has to be adjustable, reproducible and keep the original distribution unchanged, in particular without generating long tails.

The classical recipe adopted since a long time in the $PS^{17,18}$, is the following: at a given constant energy, during a time Δt , on top of the normal RF system (h_0, V_0), a second RF system is applied with a much higher harmonic number (h_1, V_1). This second RF voltage is phase modulated with a sinewave of adjustable frequency f_m and amplitude θ_m .

The amount of blow-up is generally adjusted by changing the ratio V_1/V_0 or Δt , while the distribution is optimized by varying f_m . Typical numerical values in the PS at 1GeV injection energy are the following:

$$\begin{array}{l} h_0 = 20 \ ; \ V_0 = 50 \ kV \\ h_1 = 478 \ ; \ V_1 = 6 \ kV \ ; \ \Delta t = 30 \ ms \\ f_m \sim \ 6 \ kHz \ (\sim 3f_s \ where \ f_s \ is \ the \ synchrotron \ frequency) \ ; \ \theta_m = \pi \end{array}$$

the resulting blow-up is $\varepsilon_{\text{final}} / \varepsilon_{\text{initial}} \approx 3$. See Fig.9.



Fig.9: Bunch shape before and after blow-up

The results on beam stability are striking, see Fig.10.



Fig.10: Detected wide band resistive wall monitor signal (amplitude proportional to the bunch peak current) during the first ~200 ms of acceleration.. The noisy signal corresponds to a bunch without blow-up, that is a higher peak current (i.e. lower emittance) but with some bunch shape instabilities (noise). The clean signal corresponds to a blown up beam done in the first 30 ms after injection. The emittance is larger (lower voltage) but the beam is stable: the final result is better.

3.3.2 Bunch splitting

To facilitate transition crossing and to improve debunching at high energy a bunch splitting RF gymnastic has been tested in the PS beam for LHC¹⁹. The method consists in slowly reducing the main RF voltage while increasing the voltage on a second RF system at double frequency. If the operation is done adiabatically (with respect to synchrotron period), the bunches split in two equal part and the longitudinal emittance of each of the final bunches is just half the emittance of the initial bunch. See Fig.11.



Fig.11: Mountain range display of the PS resistive wall monitor signal showing bunch splitting at 1GeV. From top to bottom: one trace every 800 turns (~2 ms). Hor. scale: 50 ns/div.

3.3.3 e+- emittance control

The PS machine belongs to the chain of the LEP injectors and accelerates also e+- from 500 MeV to 3.5 GeV.

The main beam parameters are:

$$\begin{split} & E = 0.5 \Longrightarrow 3.5 \text{ GeV} \\ & K_b = 8 \ (= \text{ number of bunches}) \\ & N_b = 2 \ 10^{11} \ \text{p/b} \\ & V_{\text{RF1}} = 200 \ \text{kV}, \ \text{h}_1 = 8 \\ & V_{\text{RF2}} = 500 \ \text{kV}, \ \text{h}_2 = 240 \end{split}$$

The machine combined function lattice generates a negative horizontal partition number: $J_x=-1$ (while $J_y=1$ and $J_{\epsilon}=4$) leading to an unstable (expanding) beam in the horizontal plane. The use of a Robinson wiggler²⁰ permits to vary J_x and J_{ϵ} within the following values:

$$-1 < J_x < 3$$
$$0 < J_\varepsilon < 4$$
$$J_y = 1$$

The wiggler not only stabilizes the beam but also allows to set the beam energy spread, before extraction, according to the equation²¹

$$\frac{\sigma_{\rm E}}{\rm E} = \frac{\rm E}{\rm E_0} \sqrt{\frac{\rm C_q}{\rho J_{\rm E}}}$$
5)

where

 $\begin{array}{l} \displaystyle \frac{\sigma_E}{E} & \text{is the bunch rms energy spread} \\ E \text{ is the beam energy} \\ \displaystyle E_0 & \text{is the electron rest energy (= 0.511 MeV)} \\ \displaystyle C_q = \text{quantum constant (= 3.84 10^{-13} m)} \\ \displaystyle \rho = \text{radius of curvature of the machine} \\ \end{array}$

To achieve good beam stability in the receiving machine (SPS) usually the energy spread is set as $\frac{\sigma_E}{E}$ ~1. 10⁻³ by adjusting J_{ϵ} ~ 0.2 and the rms bunch length as σ_t ~ 0.8 ns by regulating V_{RF2}.

3.4 Longitudinal emittance conservation

Similarly to the transverse plane, apart obvious problems of RF matching, collective effects are the phenomena that most affect longitudinal emittance conservation. We mention here the most common.

3.4.1 Coupled bunch instabilities

The main ferrite RF cavities have a bandwidth large enough to show a rather high impedance still at harmonics close to the main one (e.g. 19 and 21). They can excite, for example, longitudinal coupled bunch instabilities on mode 19. Recently a fast feedback²² has successfully been installed on each one of the ten cavities. The feedback, working similarly to a notch filter, reduces the impedance at these nearby harmonics by a factor ~5.

3.4.2 Microwave instabilities

They appears when, in longitudinal phase space, dense particle concentrations with small energy spread are formed ²³(for example during a debunching or some other special beam gymnastics, etc.). They normally provoke an uncontrolled longitudinal emittance blow-up. To prevent this phenomena, a controlled longitudinal blow-up, as described in par. 3.3.1, is a viable cure.

3.4.3 Quadrupole oscillations

Quadrupole oscillations are efficiently reduced by a simple feedback acting on the amplitude (or on the phase during the acceleration) of the RF voltage.

Moreover improved beam stability by increasing Landau damping is done by carefully adjusting the RF voltage all along the accelerating cycle to obtain a bucket acceptance just larger than the beam emittance.

3.5 Reduction of the bunch peak current

A way to reduce the incoherent space charge tune shift is to reduce the bunch peak current. This can be done by reducing the RF voltage or blowing up the longitudinal emittance or by making use of a 2nd harmonic cavity^{24,25}. Recently a new method has been successfully tested in the PS¹⁹where the particle distribution is modified by depopulating the core of the bunch without increasing significantly the bunch longitudinal emittance. The method requires the use of a second higher harmonic cavity, similarly to the controlled longitudinal blow-up. In this case the main RF system, $h_0 = 20$ and $V_0 = 44$ kV, in the PS, is phase modulated, during a time $t = ~7/f_s$, by a sinewave with a frequency $f_m ~ f_s ~ 1.6$ kHz and an amplitude $\Delta f_m ~ 0.5\pi$. The consequence of this modulation is to depopulate the bunch core. Shifting the frequency of the higher harmonic cavity, $h_1 = 479$ and $V_1 = 6.5$ kV, by $\Delta f ~ 10$ kHz helps to "smooth" the final distribution. See Fig.12.



Fig. 12a: Longitudinal phase space for bunch flattening



Fig. 12b: Reduction of the bunch peak current. Total bunch length ~ 200ns.

The flat-topped bunch shape is conserved during the acceleration and even after a transition crossing. Another advantage of this method, compared for example to a 2nd harmonic cavity method, is that in transferring a beam to a another accelerator, the longitudinal matching in the receiving machine can be more easily achieved.

Acknowledgments

I have incompletely reported on the work made, during many years, by many people. Most of their names appear in the list of References. The good present PS performance are the result of their dedication.

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