



# OBSERVATION OF HIGH-ORDER HEAD–TAIL INSTABILITIES AT THE CERN-PS

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Transverse head–tail instabilities, induced by the broad-band vacuum chamber impedance, are observed in proton bunches at modes generally lower than  $m=3$ . In a PS experiment, with a beam of special characteristics (LHC), higher order modes have been observed up to  $m=7$ . The resistive wall has been identified as the guilty impedance. This paper describes observations, theoretical explanation, and possible cures.

KEY WORDS: Impedance, instabilities, synchrotrons: superconducting

## 1 INTRODUCTION

The use of the PS machine as injector for the LHC<sup>1</sup> requires the injection of  $2 \times 4$  bunches from the PS Booster (PSB) at 1 GeV or 1.4 GeV. To study the beam behaviour, in particular space-charge effects, during the 1.2 s long flat bottom, a test was performed in December 1993 where only one bunch, with the ‘LHC characteristics’, was injected and even accelerated to 26 GeV. The ‘LHC characteristics’ of such a bunch are shown in Table 1 below. Note that this bunch is four times longer than the present standard operational beam.

TABLE 1: Beam and machine parameters during the December 1993 LHC test compared with the present typical operational beam.

	LHC test	Typical operation
Inj. kin. energy: $T$ [GeV]	1 or 1.4	1
Bunch intensity: $N_b$ [p/b]	$2 \times 10^{12}$	$1 \times 10^{12}$
Norm. tr. em.: $\epsilon_x^* \sim \epsilon_y^*$ [ $\mu\text{m}$ ]	2.5	8
Total bunch length: $\tau_b$ [ns]	200	50
Number of bunches: $k_b$	1	20
RF harm. number: $h$	8	20
Tunes: $Q_x \sim Q_y$	6.25	6.25
Chromaticity: $\xi_x \sim \xi_y$	-1	-1

We define

$$\varepsilon_{x,y}^* = \beta\gamma \frac{\sigma_{x,y}^2}{\beta_{x,y}}$$

and

$$\xi_{x,y} = \frac{dQ_{x,y}/Q_{x,y}}{dp/p}$$

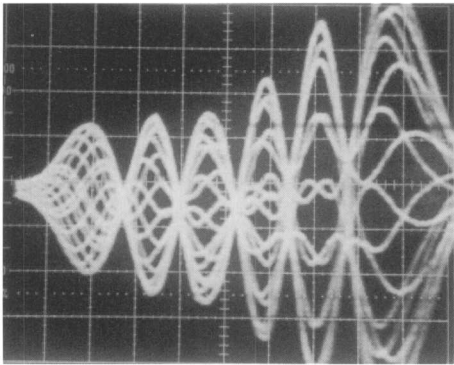
where

$\beta$  and  $\gamma$  are the usual relativistic factors;

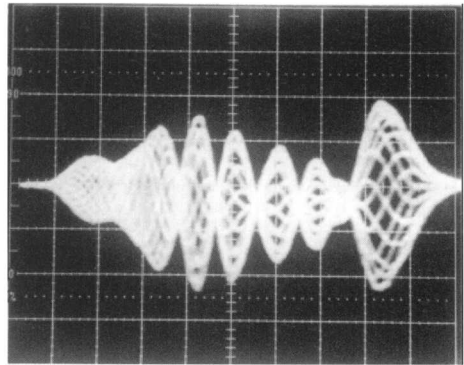
$\sigma_{x,y}$  are the r.m.s. horizontal and vertical beam dimensions;

$\beta_{x,y}$  are the amplitudes of the beta functions.

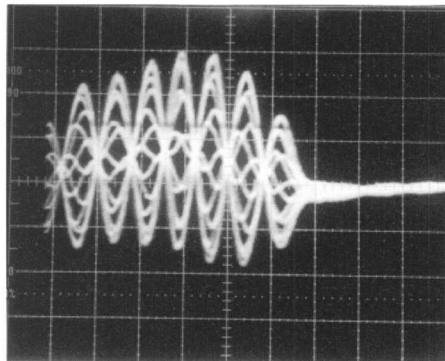
During operation at 1 GeV it was observed that the single-bunch beam was sometimes transversally unstable along the 1.2 s injection flat bottom. This transverse instability, yielding occasionally beam losses of 20–30%, was observed to happen only in the horizontal plane and with a rise time of  $\sim 100$ –200 ms. Figures 1(a)–1(c) show a  $\Delta R$  signal from a beam-position monitor during several consecutive turns and on different machine cycles.



(a)



(b)



(c)

FIGURE 1:  $\Delta R$  signal from a beam-position monitor on several consecutive turns. Time scale: 20 ns/div. (a)  $m = 5$  (most common); (b)  $m = 5$  & 6; (c)  $m = 7$ .

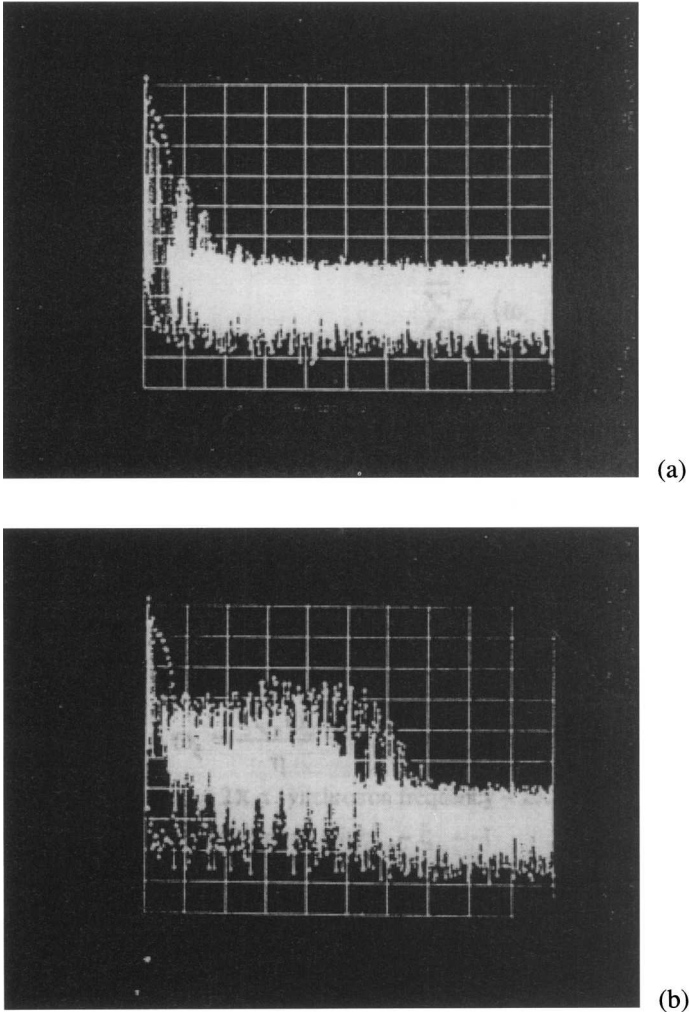


FIGURE 2: Frequency spectrum of the signal on Fig. 1. Total frequency range 0–100 MHz. Vertical scale: 10 dBm/div. (a) stable beam; (b) unstable beam.

Figures 2(a) and 2(b) show the same signal in frequency domain (frequency spectrum) without and with the presence of the instability.

The instability was not observed during the test with the injection energy at 1.4 GeV (with similar bunch parameters). The transverse feedback was of little effect in trying to cure it.

By coupling the  $x$  and  $y$  planes (that is by adjusting the same tune in both planes, i.e.  $Q_x \sim Q_y \sim 6.24$ ) it was sometimes possible to stabilize the beam.

## 2 ANALYSIS

From Figure 1, the instability can be identified as a head–tail type with mode  $m = 5$  (sometimes  $m = 6$  or  $7$ ).<sup>2–4</sup>

The instability occurs if, by the beam spectrum–impedance spectrum interaction, the imaginary part of the coherent frequency shift is positive:

$$\text{Im} \left( \Delta\omega_m^\perp \right) > 0,$$

where

$$\Delta\omega_m^\perp = -i \frac{1}{(m+1)} \frac{ecN_b}{4\pi Q_x \tau_b E/e} \frac{\sum_{p=-\infty}^{+\infty} Z_\perp(\omega_p^\perp) h_m(\omega_p^\perp - \omega_\xi)}{\sum_{p=-\infty}^{+\infty} h_m(\omega_p^\perp - \omega_\xi)}$$

and

$$\omega_p^\perp = (p + Q_x)\omega_0 + m\omega_s,$$

with

$$p = \dots - 2, -1, 0, 1, 2, \dots$$

$$m = 0, 1, 2, \dots$$

$$\omega_0 = 2\pi \times \text{revolution frequency} \sim 2\pi \times 0.417 \text{ MHz}$$

$e$  = electron charge

$c$  = speed of light

$E$  = total energy = 1.93 GeV

$$\omega_\xi = \frac{\xi Q_x \omega_0}{\eta} = \text{chromatic frequency}$$

$$\omega_s = 2\pi \times \text{synchrotron frequency} \sim 2\pi \times 0.9 \text{ kHz}$$

$\xi$  = chromaticity;  $\xi_x \sim \xi_y \sim -1$

$$\eta = \text{frequency slip factor}, \eta = \gamma_t^{-2} - \gamma^{-2} = -0.209$$

all this resulting in:

$$\omega_\xi \sim 2\pi \times 12.5 \text{ MHz} .$$

Moreover, for a parabolic bunch, the oscillation spectrum has the form

$$h_m(\omega) = (m+1)^2 \frac{1 + (-1)^m \cos(\omega\tau_b)}{[(\omega\tau_b/\pi)^2 - (m+1)^2]^2} .$$

A plot of  $h_m(\omega_p^\perp - \omega_\xi)$  for our 200 ns bunch is shown on Figure 3.

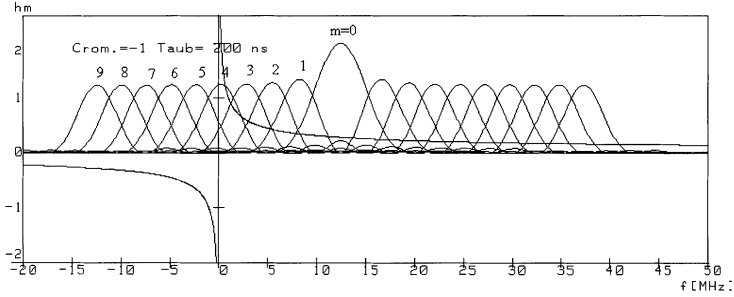


FIGURE 3: Relative amplitude of the oscillation modes versus frequency for a 200 ns parabolic bunch in the PS with  $\xi_x = -1$ .

Considering  $Z_{\perp}$  as the sum of the resistive wall impedance  $Z_{RW\perp}$  and the transverse broad band impedance  $Z_{BB\perp}$ , where:

$$Z_{RW\perp} = (\text{sgn } \omega - i) \frac{R}{b^3} \sqrt{\frac{2\rho}{\epsilon_0 |\omega|}}$$

with

$R$  = machine radius = 100 m

$b$  = vacuum chamber radius = 3.5 cm

$\rho$  = vacuum chamber resistivity =  $9 \times 10^{-7} \Omega \text{ m}$

$\epsilon_0$  = permittivity of free space =  $8.85 \times 10^{-12} \text{ Fm}^{-1}$

and

$$Z_{BB\perp} = \frac{\omega_r}{\omega} \frac{R_T}{\left[1 + iQ \left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)\right]}$$

with

$R_T$  = shunt resistance  $\sim 3 \text{ M}\Omega/\text{m}$

$\omega_r = 2\pi \times$  vacuum chamber cut-off freq.  $\sim 2\pi \times 1.4 \text{ GHz}$

$Q$  = quality factor  $\sim 1$

the condition for the onset of the instability is:

$$\text{Im} \left( \Delta \omega_m^{\perp} \right) \propto -\text{Re} \left[ Z_{\perp} \left( \omega_m^{\perp} \right) \right] h_m \left( \omega_p^{\perp} - \omega_{\xi} \right) > 0$$

and this relation is satisfied in the region where  $\text{Re}[Z_{\perp}(\omega)] < 0$ , that is for modes  $m \geq 5$  (see Figure 3).

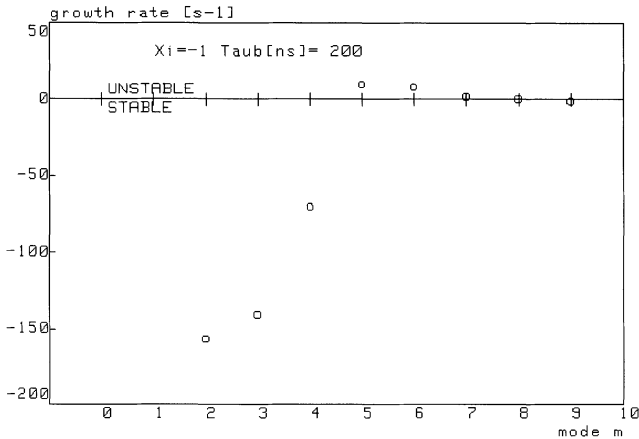


FIGURE 4: Instability growth rate versus mode number.

Computations of growth rate ( $= 1/\text{rise time}$ ) versus mode number are shown on Figure 4. The measured values (rise time  $\sim 100\text{--}200$  ns) confirm the validity of this model.

### 3 QUESTIONS AND POSSIBLE EXPLANATIONS

*Why did transverse feedback have no action?*

The present PS transverse feedback has a ‘low’ frequency bandwidth (0–1.5 MHz) just enough to cure the transverse coupled bunch instabilities observed up to now only on modes  $n = -7, -8$  and  $-9$  (occasionally). For these high-order head–tail modes such a bandwidth is probably too low.

*Why was the instability present only in the horizontal plane and not in the vertical?*

A possible explanation could be that as the horizontal and vertical coherent tune shifts are about  $+0.001$  and  $-0.025$ , respectively, then in the vertical plane the conditions for Landau damping (coherent and incoherent tune shifts with the same sign) are better satisfied than in the horizontal plane.

*Why did the instability disappear (sometimes) by coupling the x and y plane?*

This is not very clear, also because it was not always true and we did not have much time during the experiment to correlate it with other parameter variations. A possible explanation could be that owing to strong coupling the (natural) chromaticity was strongly changed locally (rough measurements suggest  $\xi_x \sim 0$ ) and this could put the beam in a more stable condition (see following paragraph). Another explanation could be a ‘sharing’ of the vertical Landau damping.

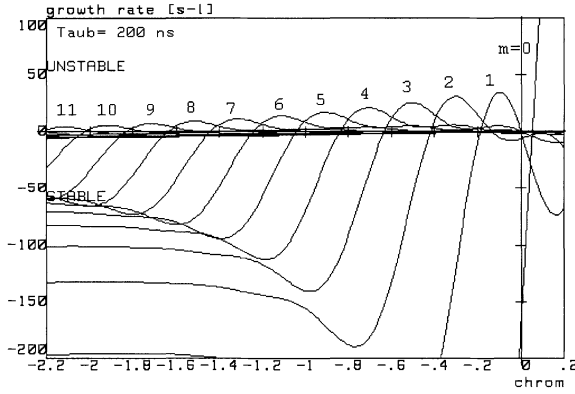


FIGURE 5: Instability growth rates for various modes ( $m=0-11$ ) versus chromaticity.

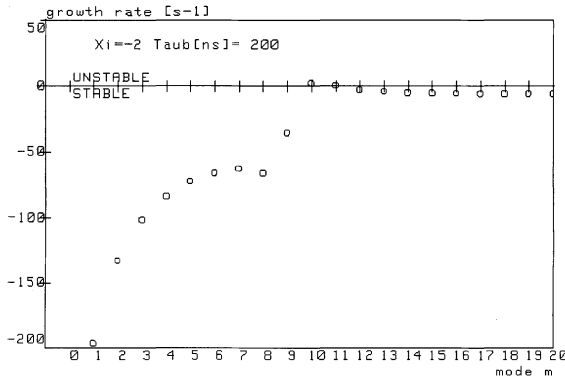


FIGURE 6: Instability growth rate versus mode numbers with  $\xi_x = -2$ .

#### 4 POSSIBLE CURES

The instability could, in principle, be cured by changing the chromaticity to a small positive value (e.g.  $\xi_x \sim +0.02$ ) or to a large negative value (e.g.  $\xi_x \sim -2$ ). In fact computing the growth rate as a function of the chromaticity one can notice a small gap ( $\sim 0 < \xi_x < \sim 0.05$ ) where all modes are stable, see Figure 5. However, this working point can be considered fairly dangerous as small chromaticity changes can produce a fast growth of modes  $m = 0$  or 1. On the contrary, adjusting  $\xi_x \sim -2$ , only much higher order modes (e.g.  $m \sim 10, 11, \dots$ ), with small growth rates (see Figure 6), are excited and these could eventually be Landau-damped more easily.

## 5 CONCLUSIONS

If, as foreseen, the injection energy is increased to 1.4 GeV, the LHC beam should be stable. While for operations at 1 GeV, some cures, like those indicated above, should be investigated in detail and eventually tried.

## REFERENCES

1. R. Capi, R. Garoby, S. Hancock, M. Martini, N. Rasmussen, T. Risselada, J.P. Riunaud, K. Schindl, H. Schonauer and E.J.N. Wilson, *The CERN PS Complex as Part of the LHC Injector Chain*, IEEE Particle Accelerator Conference, San Francisco, 1991, p. 171.
2. B. Zotter and F. Sacherer, *Transverse Instabilities of Relativistic Particle Beams in Accelerators and Storage Rings*, CERN 77-13, p. 175.
3. M. Gygi-Hanney, A. Hofmann, K. Hübner and B. Zotter, Program BBI, CERN/LEP TH/83-2.
4. M. Zisman, S. Chattopadhyay and J. Bisognano, ZAP User's Manual, LBL-2127.