CERN–TH/95–70
hep-th/9503142

A new class of spatially homogeneous 4D string backgrounds

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Abstract

A new class of spatially homogeneous 4D string backgrounds, the $X(d \rightarrow)$ according to a recent classification, is presented and shown to contain only five generic types. In contrast to the case of $X(d \uparrow)$ (which contains as a subclass all possible FRW backgrounds), exact $SO(3)$ isotropy is always broken in the $X(d \rightarrow)$ class. This is due to the H-field, whose dual is necessarily along a principal direction of anisotropy. Nevertheless, FRW symmetry can be attained asymptotically for Bianchi-types I and VII_0 in a rather appealing physical context. Other aspects of the solutions found for types $X = I, II, III, VI_{-1}$, and of the VII_0 case are briefly discussed.

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1 Introduction

One of the major objectives of string theory in recent years has been the development of a well-defined framework which would upgrade the conventional approach to cosmology near the Planck or string scale. This is precisely the region where most of the major cosmological problems arise and thus where increased insight is needed most. The cosmology offered by string theory ought to provide a sufficient understanding of that era, as well as a subsequent 'graceful exit' towards the conventional (general-relativistic) description of the more recent epochs. Such appears to be the general motivation for the study of 4D string backgrounds, whether they descend from a conformal field theory and higher-dimentional compactifications, or simply satisfy the lowest-order string beta function equations [1]-[6]. Next to the Friedmann-Robertson-Walker type of models, the simplest backgrounds to be examined in the above context are apparently those which do not have SO(3) isotropy [7] from the begining but attain that state asymptotically during the later epochs of their evolution. These belong to the wider category of spatially homogeneous (but not necessarily isotropic) 4D string backgrounds [8]-[9], hereafter recaled as HSBs.

All possible HSBs have been recently classified in a total of 576 cases [10], with a major class $X(d)$ therein consisting of those with 'diagonal' metrics. To justify the quotation marks we note that these metrics are always diagonal only in certain *non-holonomic* frames, notably the one supplied by the σ^i forms which respect homogeneity, namely they are invariant under the left action of a *transitive* G_3 group of isometries. The group structure constants C^i_{jk} which typically define the commutation relations of its generators (here the Killing vectors) also define these invariant 1-forms by $d\sigma^i = \frac{1}{2}C^i_{jk}\sigma^j \wedge \sigma^k$. The action of the group is *simply* transitive on its orbits which are precisely the 3D hypersurfaces of spatial homogeneity Σ^3 [5],[9]. This major class $X(d)$ is actually subdivided into three classes, the $X(d \rightarrow)$, $X(d \nearrow)$ and $X(d \uparrow)$. The last one includes the subclass $X(3d \uparrow)$ (here the number 3 shows that there is an extra $SO(3)$ *isotropy* group) which is of special interest because it consists of all possible FRW bacgrounds. The arrows just employed indicate the orientation of the dual H^* of the totally antisymmetric field strength $H_{\lambda\mu\nu}$ with respect to the (pictured as horizontal) hypersurfaces Σ^3. As we will see, the $X(d \uparrow)$ class involves H^* congruences which are *orthogonal* to Σ^3, that being the only orientation which has been examined in the literature on HSBs up to now. In the present paper we study the $X(d \rightarrow)$ class, namely HSBs with 'diagonal' metrics and H^* congruences which lie entirely within Σ^3. We will also see that the presence of such a congruence forbids the attainment of exact SO(3) isotropy so that any FRW-like behavior would in principle be possible in this class only at the limit of a vanishing H. Such is the physically relevant case of an *asymptotically* vanishing H.

Our main results are presented in section 3, preceded by some general definitions and preliminary findings in the following section, and they are further discussed in section 4.

2 Preliminaries

We want to examine spatially homogeneous 4D metrics which are diagonal in the invariant $\{dt, \sigma^i\}$ basis, namely of the form

$$ds^2 = -dt^2 + a_1^2(t)(\sigma^1)^2 + a_2^2(t)(\sigma^2)^2 + a_3^2(t)(\sigma^3)^2. \tag{1}$$

Such metrics will be considered here as part of a solution for the background fields which satisfies at least the lowest-order string beta-function equations for conformal invariance. The scale factors ('radii') $a_i(t)$ along the principal directions of anisotropy are functions of the time t only. Explicit holonomic expressions for all bases $\{\sigma^i\}$ employed will be supplied in the next section. Refering to the literature for details, we will only add here that, depending on the structure constants C^i_{jk}, two possible G_3-classes are distinguished as $\mathcal{A}$ or $\mathcal{B}$ corresponding to whether the adjoint representation of G_3 is traceless or not [7]. To further fix notation and conventions used, we recall that the background field equations can be derived from the effective action [1],[2]

$$S_{eff} = \int d^4x\sqrt{-g}e^{\phi}(R - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} + \partial_\mu\phi\partial^\mu\phi - \Lambda). \tag{2}$$

In the so-defined 'sigma [conformal] frame' these equations are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{4}H^2_{\mu\nu} - \nabla_\mu\nabla_\nu\phi &= 0, && (3)\\ \nabla^2(e^\phi H_{\mu\nu\lambda}) &= 0, && (4)\\ \nabla^2\phi + (\nabla\phi)^2 - \frac{1}{6}H^2 + \Lambda &= 0, && (5) \end{aligned}$$

where Λ is the cosmological constant emerging as a result of a non-vanishing central charge deficit in the original theory, hereafter set equal to zero. The fundamental fields varied are the gravitational $g_{\mu\nu}$, the dilaton scalar ϕ and, involved in the contractions $H^2_{\mu\nu} = H_{\mu\kappa\lambda}H_\nu{}^{\kappa\lambda}$, $H^2 = H_{\mu\nu\lambda}H^{\mu\nu\lambda}$, the totally antisymmetric field strenght $H_{\mu\nu\lambda}$. This field strength (which may be equivalently viewed here as a closed 3-form H) is defined in terms of the potential $B_{\mu\nu}$ (equivalently the 2-form B) as

$$\begin{aligned} H_{\mu\nu\rho} &= \partial_\mu B_{\nu\rho} + \partial\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu} && (6)\\ (H &= dB) \end{aligned}$$

In addition to specific *coordinate* bases which we will introduce later on as mentioned, we will also employ the general orthonormal frame $\{\omega^\mu\}$ defined so that (1) is equivalently expressed as

$$ds^2 = \eta_{\mu\nu}\omega^\mu\omega^\nu, \tag{7}$$

with $\eta_{\mu\nu}$ the Minkowski signature $(-1,1,1,1)$. We now observe that $H^2_{\mu\nu}$ in (3) is diagonal iff H^* has at most *one* non-vanishing component in the $\{\omega^\mu\}$ frame. Thus, for as long as one remains in the major class of 'diagonal' metrics (1), the most obvious choice involves a H^* which is orthogonal to the hypersurfaces of homogeity, namely

$$H^* = H^*_0 \omega^0. \tag{8}$$

It it this choice which characterizes the $X(d\uparrow)$ class and, as mentioned, the only one which has been investigated in the literature on HSBs so far. It is also the only choice which would respect any existing or eventual $SO(3)$ isotropy in Σ^3. Oviously however, other orientations of H^* are also possible, in particular along the complementary (transverse) directions, as realized in the $X(d\rightarrow)$ class which we examine in this paper. An apparently fundamental feature in the latter case is the fact that the allowed orientations of H^* in Σ^3 are severely restricted. A general restriction is that H^* must be aligned with certain principal directions of anisotropy. To precisely find which one(s), if any, we must examine every possible isometry group (namely Bianchi type) separately. The result of this examination may be summarized as follows. In the simplest case of a fully abelian G_3 which is realized in Bianchi-type I metrics, H^* is allowed to be along *any* one of the three principal directions. This degeneracy is partly lifted in the next case of type-II metrics. For any other Bianchi type there is either exactly one possible orientation for H^* or none at all. As it turns out, for all metrics with isometry groups of G_3-class $\mathcal{A}$ we are obliged to (or without loss of generality may) have

$$(G_3\, class\ \mathcal{A}):\ H^* = H^*_3\omega^3 = A a_3^{-1} e^{-\phi}\omega^3 \tag{9}$$

where A is a constant. For all other cases there exist only two non-trivial possibilities, each one unique for the indicated type, namely

$$\begin{aligned} (type\ III):\ H^* &= H^*_2\omega^2 = A a_2^{-1} e^{-\phi}\omega^2 && (10)\\ (type\ VI_{-1}):\ H^* &= H^*_1\omega^1 = A a_1^{-1} e^{-\phi}\omega^1 && (11)\end{aligned}$$

which we will adopt for the respective G_3-class $\mathcal{B}$ metrics. Turning now to the dilaton field, one realizes that (5) can be significantly simplified and expressed as

$$\phi'' = (H^*)^2 \tag{12}$$

with a *prime* for $d/d\tau$. The coordinate time τ has been defined by

$$dt = a^3 e^\phi d\tau \tag{13}$$

where the 'volume' scale factor

$$a^3 = a_1 a_2 a_3 \tag{14}$$

determines the expansion of any co-moving volume element. The universal time t can be explicitly given in terms of τ once the $a_i(\tau), \phi(\tau)$ functions are known. The set (3) for the metric coefficients in (1) may now be expressed as

$$(\ln a_i^2 e^{\phi})'' + (2V_i - (H^*)^2\delta_{ij})a^6 e^{2\phi} = 0 \tag{15}$$

where $(H^*)^2$ is the length of the dual chosen from (9)-(11). Each particular choice is identified by one of the indices $j = 3, 2, 1$, which also specify (in that order) the particular principal direction associated with it. Each one of the Bianchi-type depended potentials V_i is in general a function of all three a_i. The set of equations (15) is subject to the initial value equation

$$(\ln a_1^2 e^{\phi})'(\ln a_2^2 e^{\phi})' + (\ln a_2^2 e^{\phi})'(\ln a_3^2 e^{\phi})' + (\ln a_3^2 e^{\phi})'(\ln a_1^2 e^{\phi})' + (2\sum V_i - (H^*)^2)a^6 e^{2\phi} = \phi'^2, \tag{16}$$

typically imposing a restriction on the constants of integration. Further restrictions in the form of *constraint equations* emerge in the case of G_3-class $\mathcal{B}$ metrics [7],[9].

3 The $X(d \rightarrow)$ class of 4D HSBs

We want to find all possible $X(d \rightarrow)$ HSBs, where X specifies the Bianchi type, namely the isometry groups G_3 [7],[10] acting on the respective manifold. As we have already implied, solutions exist only in five cases, namely for Bianchi types $I, II, III, VI_{-1}, VII_0$. With the exception of type III, for which a different expression is involved, the dilaton field is given by

$$e^{\phi} = Q^2 e^{2P\tau} coshA(\tau - \tau_0) \tag{17}$$

with τ_0, P, Q constants. It should be noted however that, for $A = 0$, (17) reduces to

$$e^{\phi} = Q^2 e^{2A_0\tau} \tag{18}$$

with A_0 a new constant not necessarily equal to P. We will now present general solutions for types I, II, III, VI_{-1}, while the VII_0 will also be examined but in that case only special solutions (e.g., with higher symmetry) seem attainable in closed form.

I(d →): The metric (1) may also be expressed in a coordinate basis as

$$ds^2 = -dt^2 + a_1^2(t)(dx^1)^2 + a_2^2(t)(dx^2)^2 + a_3^2(t)(dx^3)^2 \tag{19}$$

and all V_i vanish in (15). From the dual H^* given by (9) one finds the 3-form

$$H = A(a_1 a_2)^2 \; (d\tau \wedge dx^1 \wedge dx^2), \tag{20}$$

for the H field which satisfies (4) and it is obviously exact. It follows that a potential in (6) would be

$$B = A(a_1 a_2)^2 x^2 \; (d\tau \wedge dx^1). \tag{21}$$

It is reminded that the dilaton field is given by (17) while from the set (15) one can determine the a_i. Thus, the rest of the solution is

$$\begin{aligned} a_1^2 e^{\phi} &= Q^2 e^{2(P+M)\tau} \\ a_2^2 e^{\phi} &= Q^2 e^{2(P-M)\tau} \\ a_3^2 &= L^2 e^{2(P+N)\tau} \end{aligned} \tag{22}$$

with M, N constants subject to the restriction

$$16P^2 - 4M^2 + 8PN = A^2 \tag{23}$$

as required by (16) and with L a numerical constant which could be assigned any value. The set (22) represents a Casner-like solution [7] which, together with the results (20),(21),(17) describes the $I(d \rightarrow)$ HSB. This background exhibits asymptotic flat ($k = 0$) FRW behavior with

$$\begin{aligned} M &= 0 \\ N &= -\frac{\sqrt{3}+1}{4} A \\ P &= \frac{\sqrt{3}-1}{4} A, \end{aligned} \tag{24}$$

as discussed in the next section.

II(d $\rightarrow$**)**: The metric (1) may also be expressed as

$$ds^2 = -dt^2 + a_1^2(t)(dx^1 - x^3 dx^2)^2 + a_2^2(t)(dx^2)^2 + a_3^2(t)(dx^3)^2 \tag{25}$$

and it is obviously non-diagonal in this coordinate basis. The V_i in (15) are

$$V_1 = -V_2 = -V_3 = \frac{1}{2}\left(\frac{a_1}{a_2 a_3}\right)^2. \tag{26}$$

From the dual H^* given by (9) one finds the 3-form

$$H = A(a_1 a_2)^2 \; (d\tau \wedge dx^1 \wedge dx^2) \tag{27}$$

for the H field which satisfies (4) and it is obviously exact. A possible potential in (6) would be

$$B = A(a_1a_2)^2x^2\ (d\tau \wedge dx^1). \tag{28}$$

The dilaton field is still given by (17) while from the set (15) one can now find the a_i. Thus, the rest of the solution is

$$\begin{aligned} a_1^2e^{\phi} &= M(\cosh M\tau)^{-1} \\ a_2^2e^{\phi} &= \frac{Q^4}{M}e^{4P\tau}\cosh M\tau \\ a_3^2 &= \frac{L^4}{M}e^{2N\tau}\cosh M\tau \end{aligned} \tag{29}$$

where L, M, N, P, Q are constants, not entirely arbitrary in view of the restriction

$$4P^2 - M^2 + 8PN = A^2 \tag{30}$$

imposed by the initial value equation (16).

III(d $\boldsymbol{\rightarrow}$***)***: The metric (1) may also be expressed in a coordinate basis as

$$ds^2 = -dt^2 + a_1^2(t)(dx^1)^2 + a_2^2(t)(dx^2)^2 + a_3^2(t)(e^{x^1}dx^3)^2 \tag{31}$$

There are non-vanishing potentials V_1, V_3 in (15), namely

$$V_1 = V_3 = -\frac{1}{a_1^2}. \tag{32}$$

This case involves a class-$\mathcal{B}$ isometry group so we also have the constraint equation

$$(\ln\frac{a_3}{a_1})' = 0 \tag{33}$$

which (again without loss of generality) can be integrated to

$$a_1 = a_3. \tag{34}$$

From the dual H^* given by (10) one finds the 3-form

$$H = A(a_1a_3)^2e^{x^1}\ (d\tau \wedge dx^1 \wedge dx^3) \tag{35}$$

for the H field which satisfies (4) and it is obviously exact. It follows that a potential in (6) would be

$$B = A(a_1a_3)^2e^{x^1}\ (d\tau \wedge dx^3). \tag{36}$$

One can now proceed with (15),(12) to easily determine that

$$a_2 = Qe^{P\tau}, \tag{37}$$

where Q, P are constants, while the rest of these equations reduce to

$$(\ln a_1^2 e^{\phi})'' - 2Q^2 e^{2P\tau}\, a_1^2 e^{2\phi} = 0, \tag{38}$$

coupled to

$$\phi'' - A^2 a_1^4 = 0 \tag{39}$$

A solution to this system is

$$\begin{aligned} a_1^2 &= a_3^2 = \frac{\sqrt{2}M}{A}(\sinh 2M\tau)^{-1} \\ e^{2\phi} &= \frac{3MA}{\sqrt{2}Q^2} e^{-2P\tau}(\sinh 2M\tau)^{-1} \end{aligned} \tag{40}$$

where $7M^2 = P^2$ as required by the initial value equation (16). The above solution was not obtained by quadratures, so we have no proof that it is the most general to the (38),(39) system.

$\boldsymbol{VI_{-1}(d \longrightarrow)}$: The metric (1) may also be expressed in a coordinate basis as

$$ds^2 = -dt^2 + a_1^2(t)(dx^1)^2 + a_2^2(t)(e^{-x^1}dx^2)^2 + a_3^2(t)(e^{x^1}dx^3)^2. \tag{41}$$

The only non-vanishing potential in (15) is

$$V_1 = -\frac{2}{a_1^2}. \tag{42}$$

It should be noted, however, that in this case we also have the constraint equation

$$(\ln \frac{a_3}{a_2})' = 0 \tag{43}$$

which can be integrated (essentially without loss of generality) to

$$a_2 = a_3 \tag{44}$$

From the dual H^* given by (11) one finds the 3-form

$$H = -Aa_2^4 e^{x^1}\ (d\tau \wedge dx^1 \wedge dx^3), \tag{45}$$

for the H field which satisfies (4) and it is obviously exact, so that a possible potential in (6) would be

$$B = Aa_2^4 x^2 e^{x^1}\ (d\tau \wedge dx^3). \tag{46}$$

The dilaton field is again given by (17) while the essentially remaining a_1 function can be obtained by direct integration in the set (15) with (42). We thus find the rest of the solution which may be expressed as

$$a_1^2 = L^2 \exp(M\tau + \frac{Q^2}{4P^2} e^{4P\tau})$$
$$a_2^2 e^{\phi} = a_2^2 e^{\phi} = Q^2 e^{2P\tau}, \tag{47}$$

where L, M, Q, P are constants. These are not entirely arbitrary in view of the restriction

$$8P^2 + 8PM = A^2 \tag{48}$$

imposed by the initial value equation (16).

$\boldsymbol{VII_0(d \rightarrow)}$: The metric (1) may also be expressed in a coordinate basis as

$$ds^2 = -dt^2 + a_1^2(t)(dx^1)^2 + a_2^2(t)(dx^2)^2 + a_3^2(t)(dx^3)^2 \tag{49}$$

and the V_i in (15) are

$$V_1 = -V_2 = \frac{a_1^4 - a_2^4}{2a^6}, \quad V_3 = -\frac{(a_1^2 - a_2^2)^2}{2a^6}. \tag{50}$$

From the dual H^* given by (9) one finds the 3-form

$$H = A(a_1 a_2)^2 \ (d\tau \wedge dx^2 \wedge dx^3), \tag{51}$$

for the H field which satisfies (4) and it is obviously exact, so that a possible potential in (6) would be

$$B = A(a_1 a_2)^2 x^2 \ (d\tau \wedge dx^3) \tag{52}$$

It is reminded that the dilaton field is given by (17) and, as in the two other G_3-class $\mathcal{A}$ cases, we also have

$$a_1 a_2 e^{\phi} = Q^2 e^{2P\tau}. \tag{53}$$

However, it seems unlikely that the general $VII_0(d \rightarrow)$ HSB can be found in closed form from the remaining integration of (15) with (50). We observe, however, that under the $SO(2)$ partial isotropy realized if $a_1 = a_2$, these equations reduce to the $SO(2)$-symmetric type-I set, which we have already at our disposal from (22). In other words, the backgrounds $VII_0(2d \rightarrow)$ and $I(2d \rightarrow)$ are identical, and so are their respective subcases which involve asymptotic flat FRW behaviour. The same behavior is expected from the general $VII_0(d \rightarrow)$ bacground, under a choice of constants analogous to (24). The above results will be further discussed in the next section.

4 Conclusions

The $X(d \rightarrow)$ class of 4D HSBs has been investigated, with explicit solutions found for all but the last one of five possible generic cases, realized at $X = I, II, VI_{-1}, III, VII_0$. For the last type only specialized (that is, with more symmetry) solutions could be found in closed form, such as the $VII_0(2d \rightarrow)$ which essentially coincides with $I(2d \rightarrow)$. Although there are no FRW backgrounds in $X(d \rightarrow)$ (unless on goes over to trivial limits), there exists a subclass therein with *asymptotic* flat FRW behavior. As we will see shortly, this subclass can be viewed as a counterpart of the FRW models. It can be obtained from $I(d \rightarrow)$ and $VII_0(d \rightarrow)$ with proper choice of constants, such as the one in (24) made for the type-I case. This choice may appear as a *fine-tuning*, but such a characterization is in a sense misleading. To appreciate this rather important point, we may now compare with the case of the FRW models. As recently shown, the far richer $X(d \uparrow)$ class contains as a subclass all possible FRW models [9]. Every one of these models is obtained by a specific choice of constants (entirely analogous to the one made in (24)) which is equivalent to introducing three extra Killing isometries, namely the $SO(3)$ isotropy of the FRW regime. It is then clear that the choice (24) involves the same or perhaps even *less* of a fine tuning because it introduces three *asymptotic* Killing vectors [7]. The impossibility to establish full (rather than just asymptotic) $SO(3)$ isotropy in $X(d \rightarrow)$ is of course due to the presence of the H fiefld, visualized by its H^* congruence within the hypersufaces of homogeneity Σ^3. On physical grounds such behaviour may be more interesting than the FRW one, in view of the *dynamical* attainment of isotropy predicted for the later epochs in the cosmic evolution.

Other gross physical aspects of the solutions presented here firstly include a confirmation of the expected presence of an initial singularity. One can also establish the absence of inflation in the entire $X(d \rightarrow)$ class. It should be noted, however, that the latter result has been established only in terms of the scale factor a introduced in (14), because inflation has actually *not* been studied in the context of anisotropic cosmology (incidentally, this default is also a paradox if one recalls that, for example, the horizon problem is intimately related to the issue of spatial anisotropy) [7]. It should also be noted that any string background may appropriately be viewed as the counterpart of a general relativistic *vacuum*. In that sense, many $X(d \rightarrow)$ configurations which by our present findings are charactrized as non-existent (namely those with $X = IV, V, VI_h, VII_h, VIII, IX$), could be realized in the presence of appropriate sources added to the effective action (2).

It is conceivable that some of the backgrounds presented here may discend from a CFT and, further, one would like to examine the behavior of these backgrounds under various duality transformations (cf., eg., [6] and refs cited therein). Here we will briefly comment on the type of new backgrounds which can be obtained under *abelian* target-space duality. In the $X(d \uparrow)$ class, duality transformations largerly reproduced backgrounds with metrics in the same class [9]. In the present case the situation is complementary, in the sense that the $X(d \rightarrow)$ HSBs in general poduce duals with metrics outside that class. At the same time, the original symmetry can be severely reduced or virtually lost [6]. We also observe that due to the presence of non-vanishing B_{0i} components of the potentials in (21),(28) etc., the new metrics will involve g_{0i} components so that even if a Σ^3 has survived as a hypersurface of homogeneity, it will cease to also be a hypersurfaces of simultaneity. One consequence of that would be the impossibility to define a cosmic time t in the respective manifold. These aspects should be examined in detail, especially in the context of ref. [5].

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