Beyond the Standard Model with Effective Lagrangians[1]

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Abstract

We discuss some applications of the effective quantum field theory to the description of the physics beyond the Standard Model. We consider two different examples. In the first one we derive, at the one-loop level, an effective lagrangian for an extension of the Standard Model with a charged scalar singlet by "integrating out" the heavy scalar. In the second example we illustrate the use of general effective theories at the loop level.

If the physical problem contains several distinct energy scales (masses of the particles etc.) and we are interested in effects at lower energy scale, then the proper language is an effective quantum field theory (EQFT) language [1]. In this case the heavy degrees of freedom can be "integrated out" and the physics at lower energy scale can be described by an effective lagrangian (EL) in the form of the dimensional expansion

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \frac{1}{\Lambda}\mathcal{L}_1 + \frac{1}{\Lambda^2}\mathcal{L}_2 + \cdots \, . \tag{1}$$

Here $\mathcal{L}_0$ contains operators with canonical dimension ≤ 4 (which can be renormalizable). $\mathcal{L}_n$ $(n \geq 1)$ are linear combinations of non-renormalizable operators with dimension $n+4$ which are suppressed by Λ^n, where Λ is an energy scale at which "new physics" starts, and parametrize our ignorance of the dynamics at high energies. Two questions are relevant when the EL is constructed:
- *What is the symmetry of the problem?*
- *What is the (light) particle spectrum?*
For any given accuracy physics at energy scale, E, can be described by a limited number of terms as the contribution of operators of higher dimension is suppressed by the factor $(E/\Lambda)^n$. Obviously, when the energy scale approaches the scale Λ one needs more and more terms in order to describe physics accurately enough. The renormalizability (in the text-book sense) is replaced by the requirement that physics at low scales cannot dramatically depend on the physics at higher scales [2].

[1]Contribution to the Proceedings of the 28th Symposium on the Theory of Elementary Particles, Wendisch-Rietz, August 30 - September 3, 1994, DESY 95-027

The EQFT language can be used in two conceptually different cases. First, when the full theory is known but it contains heavy degrees of freedom which can be "integrated-out" and one can describe low energy physics in a very transparent and economic way with an EL. In this case parameters of EQFT are determined completely by the matching to the full theory. An early example of this case is the lagrangian for low energy light-by-light scattering derived by Euler and Heisebrerg [3] by integrating out the "heavy" electron in QED. The second situation is when the full theory is unknown and an EL is built only from assumptions about symmetries and the particle content. The famous early example of such an approach is the four-fermion Fermi's theory [4] of weak interaction. In this case the parameters of the EQFT can obtained only from experiment.

Although the EQFT approach was used in particle physics for a long time[1] for the description of electroweak interactions the main efforts have been made in the direction of the construction of renormalizable theories by enlarging the symmetry group, particle content etc. But the success of the minimal Standard Model (SM) based on $SU(2)_L \otimes U(1)_Y$ has started to change this point of view. In many recent works the impact of possible "new physics" is analyzed by adding to the SM lagrangian effective non-renormalizable operators built from the standard fields. In this case it is natural to assume the standard $SU(2)_L \otimes U(1)_Y$ symmetry for the new interactions[2]. However, a complication arises due to the fact that the SM symmetry is spontaneously broken to $U(1)_{em}$. Then there are two possibilities:

- The gauge symmetry is realized linearly. It means that the Higgs particle is present in the physical spectrum. This is the simplest *decoupling* situation - effects of non-renormalizable operators disappear when the scale of "new physics" increases (only experiment can tell us something about this scale). The first term in the EL is the usual minimal SM.

- The gauge symmetry is realized non-linearly. There is no elementary scalar in the particle spectrum. The scale of "new physics" cannot be much larger than the Fermi scale as it has to cure the bad behaviour of the model without Higgs. Therefore, the operators of higher dimension become also relevant at the energy scale of modern experiments. Written in unitary gauge, the lagrangian has the most general form consistent with Lorentz invariance and unbroken $U(1)_{em}$ symmetry. The first term in the EL is a non-renormalizable non-linear sigma model [7,8].

The use of EQFT at tree level is straightforward. However, during last years, motivated by high precision of the data, people started to bound effective interactions from their contribution in loops. This has to be done with certain caution as such non-renormalizable interactions give, in general, a divergent result. Nevertheless, using the appropriate framework one can obtain finite non-ambiguous results. As we already mentioned, in the EQFT language all operators allowed by the symmetries of the problem are already present in the EL. Therefore, there always exists a counterterm available to absorb any divergence that could appear in loop calculations. The price that has to be payed is that it is not possible to analyze effects of one operator independently of other operators that mix with it under renormalization. Under certain assumptions one can reduce the basis of operators which mix. The more assumptions one makes the stronger will be the bounds one obtains on the couplings of the effective operators. The less assumptions one makes the more reliable will be the bounds obtained. For example, if we want to analyze an operator that contributes to experimental observables at one-loop level we can use a "minimal" set of operators (which, in general, does not form a closed basis) that contains the operator in question plus all the operators that mix "directly" with it at the one-loop level.

In this talk we will illustrate the construction and use of the EQFT by considering two examples: in the first one the EL is derived from a known underlying model; in the second one

[1] One of the most successful application is the so-called Chiral Perturbation Theory [5].

[2] Dimension-six $SU(2)_L \otimes U(1)_Y$ are classified and listed in [6].

the use of general EQFT at the loop level is discussed.

First, we consider the construction of the EL from a renormalizable model which is an extension of the SM with a charged scalar singlet [9]. The full lagrangian for this model is

$$\mathcal{L}_{full} = \mathcal{L}_{SM} + \mathcal{L}_h, \tag{2}$$

where the $\mathcal{L}_{SM}$ refers to the minimal SM part and the $\mathcal{L}_h$ describes the additional charged scalar singlet, h

$$\mathcal{L}_h = (D_\mu h)^\dagger D^\mu h - m^2 |h|^2 - \alpha |h|^4 - \beta |h|^2 \varphi^\dagger \varphi + \left(f_{ab} \bar{\tilde{\ell}}_a \ell_b h^+ + \text{h.c.} \right) \, , \tag{3}$$

where the covariant derivative has the form $D_\mu = \partial_\mu + ig' B_\mu$ (the scalar has hypercharge $Y = -1$); ϕ is a Higgs field, l is a leptonic $SU(2)$ doublet.

This model is one of the simplest extensions of the SM, but in spite of its simplicity it has interesting features common to any extension which contains a large mass scale compared with the Fermi scale (we assume that $m \geq 1TeV$). In addition to the coupling of the scalar to leptons, f, is an antisymmetric complex matrix in flavour space [9] and this leads to flavour-changing interactions in the leptonic sector[3].

If the mass of the scalar, m, is much higher than the energy scale of experiments we can integrate out the scalar. The effective action, $S_{eff} = \int d^4x \mathcal{L}_{eff}(x)$, is defined as

$$e^{iS_{eff}} = e^{iS_{SM}} \int \mathcal{D}h \mathcal{D}h^+ \exp \left\{ i d^4x \mathcal{L}_h(x) \right\} \quad , \tag{4}$$

where $\mathcal{D}h$ represents the functional integration over h. The EQFT represented by the non-local expression (4) is fully equivalent to the original theory as far as Green functions with "light" external particles are considered. As we are interested in the effects of the heavy scalar ($m \geq 1TeV$) on physics around the Fermi scale, we will keep only terms of order $O(1/m^2)$.

Expanding (functionally) the full action around the solution of the classical equation of motion for the scalar field and integrating over h, the one-loop action can be written (in our approximation) as

$$S_{eff} = S_{SM} + S_h[h_0] + i \, \text{Tr} \left\{ \log(O) \right\} \tag{5}$$

with $O = (-D^2 - m^2 - \beta \varphi^\dagger \varphi)$. The last term in eq. (5) takes into account terms which originate from the one-loop diagrams with only heavy scalar in loops. We refer to [9] for the details of calculations of the fluctuation operator and give here the final result which can be split in two parts. The first one includes all dimension-six operators

$$\mathcal{L}^{(1)}_{det} = \frac{1}{m^2} \frac{1}{(4\pi)^2} \left(-\frac{\beta^3}{6} (\varphi^\dagger \varphi)^3 + \frac{\beta^2}{12} \partial_\mu (\varphi^\dagger \varphi) \partial^\mu (\varphi^\dagger \varphi) + \frac{g'^2 \beta}{12} (\varphi^\dagger \varphi) B_{\mu\nu} B^{\mu\nu} - \frac{g'^2}{60} \partial^\mu B_{\mu\nu} \partial_\sigma B^{\sigma\nu} \, . \right) \tag{6}$$

The second part of the EL, contains operators of dimension *not larger* than four and they have ultraviolet (UV) divergent coefficients[4]. As all $SU(2)_L \otimes U(1)_Y$ operators with dimension ≤ 4 are already present in the SM lagrangian, this part of the EL is absorbed by the redefinition of the SM couplings:

$$\bar{m}^2_\varphi = m^2_\varphi - m^2 \frac{\beta}{(4\pi)^2} (1 + \Delta_\epsilon) \, , \tag{7}$$

[3]Although the generational lepton number is violated, the assignment of the total lepton number 2 to the scalar assures that the total lepton number is conserved; as a consequence neutrinos remain massless at all orders.

[4]We used dimensional regularization. Divergences appear as simple poles in ϵ in the function $\Delta_\epsilon = 1/\epsilon - \gamma + 2log(4\pi\mu/m)$.

$$\overline{\lambda} = \lambda - \frac{\beta^2}{2(4\pi)^2}\Delta_\epsilon \tag{8}$$

and the B_μ field

$$\overline{B}_\mu = \left(1 + \frac{g'^2\Delta_\epsilon}{3(4\pi)^2}\right)^{1/2} B_\mu \ . \tag{9}$$

In order to keep the canonical form for the covariant derivative we have to renormalize the hypercharge coupling, g', as follows,

$$\overline{g'} = \left(1 + \frac{g'^2\Delta_\epsilon}{3(4\pi)^2}\right)^{1/2} g' \ . \tag{10}$$

Note that the above relations are relations between the *bare* couplings of the full lagrangian and the effective one.

Using the equation of motion for the scalar singlet the second term in the effective action, $S_h[h_0]$, can be formally written in the following *non-local* form

$$S_h[h_0] \approx -\int d^4x \overline{\tilde{\ell}}(x) f \ell(x) \frac{1}{(-D^2 - m^2 - \beta\varphi^\dagger(x)\varphi(x))} \overline{\ell}(x) f^\dagger \tilde{\ell}(x) \ + O\left(\frac{1}{m^4}\right) \ . \tag{11}$$

To obtain a *local* approximation to it, one has to make an expansion in $1/m^2$:

$$\frac{1}{(-D^2 - m^2 - \beta\varphi^\dagger(x)\varphi(x))} = -\frac{1}{m^2} + \frac{1}{m^4}(D^2 + \beta\varphi^\dagger(x)\varphi(x)) + \cdots \ . \tag{12}$$

Neglecting all terms but the first, the tree level contribution of the scalar to the EL is:

$$\mathcal{L}^{(0)} = \frac{1}{m^2}(\overline{\tilde{\ell}} f \ell)(\overline{\ell} f^\dagger \tilde{\ell}) = \frac{4}{m^2} f_{ab} f^*_{a'b'} (\overline{\nu^c_{aL}} e_{bL})(\overline{e_{b'L}} \nu^c_{a'L}) \ , \tag{13}$$

where the summation over repeated flavour indices (a, b, a', b') is assumed. It corresponds to the tree-level diagram with scalar exchange between two lepton currents.

However, by using the expansion (12) one does not obtain the complete answer even at order $1/m^2$. Doing this approximation we assumed that $q^2 \ll m^2$ which is not correct when the scalar contributes in loops where the loop momentum runs up to infinity. As a result we missed many operators which correspond to one-loop diagrams in the full theory with heavy-light particles in loops.

In order to find them, we have to consider one-loop diagrams in the full theory with mixed, heavy-light particles in the loops and to subtract the corresponding one-loop contribution in the effective theory using the tree-level lagrangian eq. (13) in loops.

In practice, deriving the matching conditions we can avoid the calculation in the effective theory by splitting the scalar propagator in two parts

$$\frac{1}{k^2 - m^2} = -\frac{1}{m^2} + \frac{1}{m^2}\frac{k^2}{(k^2 - m^2)} \tag{14}$$

and using only the second part in calculations. Doing this we increase the power of the UV-divergence but decrease the power of the infrared (IR) divergence. Thus, all possible small momentum singularities, which have nothing to do with the high-energy behaviour, are transmitted to the low energy EL. Most of the operators obtained by matching have UV-divergent coefficients and serve as counterterms to the divergent loop contributions that appear in the effective theory, some other operators have finite coefficients.

To find the operators that appear as a consequence of the matching procedure we first consider one-loop diagrams with the minimal number of the external particles keeping track of the external momenta (to order p^2/m^2). Then we write effective operators which correspond to such amplitudes. After this we reconstruct the gauge invariance by promoting simple derivatives to covariant derivatives. Sometimes, however, there is an ambiguity to do this and we need to calculate diagrams with more external (gauge) particles. The full matching procedure can be found in [9].

As an example, we construct operators which correspond to the lepton self-energy with the h-scalar in the loop. In the effective theory this contribution is zero because it is a massless tadpole-like diagram. However, in the full theory we have a non-trivial result:

$$T_{self-energy} = \frac{(f^\dagger f)_{ab}}{(4\pi)^2}\left(2\left(\Delta_\epsilon + \frac{1}{2}\right) + \frac{2}{3}\frac{p^2}{m^2}\right)\overline{u}(p)\, \not{p}\frac{1}{2}(1-\gamma_5)u(p)\ . \tag{15}$$

For the first term in (15) we have the following operator

$$2(\Delta_\epsilon + \frac{1}{2})i(\overline{\ell}F\, \not{D}\ell)\ , \tag{16}$$

with $F_{ab} \equiv (f^\dagger f)_{ab}/(4\pi)^2$. Evidently this operator can be absorbed in the standard kinetic term of the lepton doublet[5].

The second term in eq. (15) is proportional to $\not{p}p^2$, which requires an effective operator of the form $i(\overline{\ell}_a\ \not{\partial}\partial^2\ell_b)$. However, the promotion of this term to covariant derivatives is ambiguous: should we use $\not{D}D^2$, $\not{D}^3$ or $D_\mu\ \not{D}D^\mu$? The only way to resolve this ambiguity is to perform a full calculation with one external gauge boson [9].

There are many other effective operators obtained by matching which correspond to diagrams with external leptons and Higgs particles and different four-fermion operators corresponding to box diagrams with a heavy line.

Before presenting the final form of the EL for the model we discuss the renormalization of the EL. We use the $\overline{MS}$-scheme for the renormalization of both the full and effective theories and, in this case, we obtain matching equations for the renormalized couplings in both theories [10,**?**]. For example, we have the standard relations for the gauge coupling in the $\overline{MS}$-scheme[6]

$$\begin{aligned} g'\mu^\epsilon &= g'(\mu) + \frac{1}{2\hat{\epsilon}}\, b_{g'}g'^3(\mu) + \cdots\ , \\ \overline{g}'\mu^\epsilon &= \overline{g}'(\mu) + \frac{1}{2\hat{\epsilon}}\, \overline{b}_{g'}\overline{g}'^3(\mu) + \cdots\ , \end{aligned}$$

where $b_{g'}$ and $\overline{b}_{g'}$ are the lowest-order coefficients of the β-functions for the coupling constants in the full and the effective theories, respectively. Substituting these equations in the relation between bare couplings in the full and effective theories, eq. (10), and equating finite terms we obtain the desired matching condition for the *renormalized* couplings:

$$\overline{g}'(\mu) = g'(\mu) - \frac{g'(\mu)^3}{3(4\pi)^2}\log(\mu/m) + \cdots\ . \tag{17}$$

Note that this equation can be obtained by just dropping the $1/\hat{\epsilon}$ contained in Δ_ϵ in eq. (10). This is not surprising since the divergent term in eq. (10) gives just the charged scalar contribution

[5]As a consequence we have to redefine the standard Yukawa couplings and the coupling of the tree-level four-fermion operator, f_{ab}.

[6]We denote the $\overline{MS}$ renormalized quantities with the same symbol as the bare quantities, but adding an additional dependence on the renormalization scale μ. All effective theory quantities will be distinguished by a bar; $D = 4 - 2\epsilon$ and $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \gamma + \log(4\pi)$.

to the beta function of g' in the full theory. For the coefficient of the quadratic term in the Higgs potential we have

$$\bar{m}_\varphi^2(\mu) = m_\varphi^2(\mu) - m^2(\mu)\frac{\beta(\mu)}{(4\pi)^2}(1 + 2\log(\mu/m)) \; , \tag{18}$$

and similar equations can be written for other couplings (and fields).

In order to avoid large logarithms, the matching conditions should be evaluated at some scale around the charged scalar mass[7]. Then, using the SM renormalization group, run all the couplings down in order to obtain their values at lower scales.

Eq. (18) is very interesting and a similar equation can be found in most theories with (at least) two different mass scales. This equation clearly exhibits the so-called *naturalness problem* of the SM. $\bar{m}_\varphi(\mu)$ is the mass parameter that appears in the Higgs potential part of the effective Lagrangian, and it has to be of the order of the electroweak scale. However, if $m(\mu)$ is very large, one should also take $m_\varphi(\mu)$ large in order to have $\bar{m}_\varphi(\mu)$ small enough. But even if we do so at some scale μ, it will be very difficult to keep $\bar{m}_\varphi(\mu)$ small at any other scale. This represents a serious fine-tuning problem, which appears when the standard model is embedded in another model containing mass scales much larger than the Fermi scale. It is important to note that by using $\overline{MS}$-scheme the problem appears only in the matching conditions.

The final EL in terms of physical fields has the form (flavour indices are suppressed)

$$\begin{aligned}
\mathcal{L}^{(1)} \;=\; & -\frac{g^2}{2m_W^2}\delta_Z(c_W^2 J_A^\mu - J_Z^\mu)(c_W^2 J_{A\mu} - J_{Z\mu}) + \frac{g}{c_W}\delta_Z Z_\mu(c_W^2 J_A^\mu - J_Z^\mu) \\
& +\frac{2}{3}\frac{g}{m^2 c_W}\left(-(1-2s_W^2)\left(\Delta_\epsilon + \frac{4}{3}\right) + s_W^2\frac{1}{3}\right)\left(M_Z^2 Z^\mu + \frac{g}{\hat{c}_W}J_Z^\mu\right)(\overline{\nu_L}F\gamma_\mu\nu_L) \\
& +\frac{2}{3}\frac{g}{m^2 c_W}\left(\left(\Delta_\epsilon + \frac{4}{3}\right) + s_W^2\frac{1}{3}\right)\left(M_Z^2 Z^\mu + \frac{g}{c_W}J_Z^\mu\right)(\overline{e_L}F\gamma_\mu e_L) \\
& -\frac{2}{3}\left(\Delta_\epsilon + \frac{4}{3}\right)\frac{g}{m^2}\left((\sqrt{2}M_W^2 W_\mu^+ + J_\mu^\dagger)(\overline{\nu_L}F\gamma^\mu e_L) + \text{h.c.}\right) \\
& -\frac{2}{9}\frac{e^2}{m^2}J_A^\mu(\overline{e}F\gamma_\mu e) \; - \frac{2}{3}\frac{e^2}{m^2}\left(\Delta_\epsilon + \frac{5}{3}\right)J_A^\mu(\overline{\nu_L}F\gamma_\mu\nu_L) \qquad (19) \\
& -\frac{1}{6}\frac{e}{m^2}A^{\mu\nu}\left((\overline{e_L}FM_e\sigma_{\mu\nu}e_R) + \text{h.c.}\right) \\
& +\frac{1}{6}\frac{g}{m^2 c_W}(1+s_W^2)Z^{\mu\nu}\left((\overline{e_L}FM_e\sigma_{\mu\nu}e_R) + \text{h.c.}\right) \\
& -\frac{1}{3\sqrt{2}}\frac{g}{m^2}\left(W_{\mu\nu}^+(\overline{\nu_L}FM_e\sigma_{\mu\nu}e_R) + \text{h.c.}\right) \\
& -\frac{(4\pi)^2}{m^2}\left((\overline{e_L}F\gamma^\mu e_L)(\overline{e_L}F\gamma_\mu e_L) + (\overline{\nu_L}F\gamma^\mu\nu_L)(\overline{\nu_L}F\gamma_\mu\nu_L)\right. \\
& \qquad \left. + \, 2(\overline{e_L}F\gamma^\mu e_L)(\overline{\nu_L}F\gamma_\mu\nu_L)\right) \; . \qquad (20)
\end{aligned}$$

Here M_e is the charged lepton mass matrix and $A^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ and $Z^{\mu\nu} = \partial^\mu Z^\nu - \partial^\nu Z^\mu$ are the field strengths of the photon and the Z boson, respectively; $J_A^\mu,\ J_Z^\mu,\ J_\mu^\dagger$ are the standard electromagnetic, neutral and charge currents.

This lagrangian shows all phenomenological consequences of the model in a very transparent way. The most interesting ones are different processes with generational *lepton number violations*. For example, assuming (without loss of generality) a diagonal form for the matrix

[7] Although, in principle, they are valid for an arbitrary value of the renormalization scale μ

M_e, from the fifth line of (20) we have the amplitude of the decay

$$T(e_b \to e_a\,\gamma) = -i\frac{e}{3}F_{ab}\overline{u}(p_a)\sigma_{\mu\nu}q^\nu(m_b R + m_a L)u(p_b)\epsilon^\mu(q)\,; \tag{21}$$

L and R are, respectively, the left-handed and right-handed chirality operators. The amplitude eq. (21) leads to the process $\mu \to e\gamma$ without neutrino masses. Other terms lead to decays $\mu^- \to e^-e^-e^+$ and similar processes.

Another interesting process is the flavour changing Z-decay, $Z \to e_a^+\,e_b^-$. To consider this decay we have to take into account not only the contribution at tree level (third line in (20)) but also the contribution given by the tree-level four-fermion lagrangian at one loop. By construction the sum is UV finite and depends only on the few parameters of the full model. For example, from the upper bounds on the branching ratios for the decays $Z \to e\mu,\ e\tau,\ \mu\tau$ measured at LEP [12] one gets $m \geq 1TeV$ (for coupling $f \approx 1$).

However, when the full theory is unknown, the situation is more complicated. Assume we are in a two-operator mixing situation (like in the above case). Then for their renormalized couplings at some scale μ we have

$$\begin{aligned} c_1(\mu) &= c_1(\mu_0)\left(1+\gamma_{11}\log\frac{\mu}{\mu_0}\right) + c_2(\mu_0)\gamma_{12}\log\frac{\mu}{\mu_0} \\ c_2(\mu) &= c_1(\mu_0)\gamma_{21}\log\frac{\mu}{\mu_0} + c_2(\mu_0)\left(1+\gamma_{22}\log\frac{\mu}{\mu_0}\right)\ . \end{aligned} \tag{22}$$

which are the solutions of the general renormalization group equation

$$\mu\frac{dc_i(\mu)}{d\mu} = \gamma_{ij}c_j(\mu)\ , \tag{23}$$

valid only in the case that $\gamma_{ij}\log(\mu/m) \ll 1$. Let us suppose that at experiment we measure the coupling $c_2(\mu)$ at some energy scale μ. If we want to extract bounds on $c_1(\mu_0)$ we need to know the initial condition, $c_2(\mu_0)$, as an effective theory predicts only the anomalous dimensions γ_{ij}. Thus, in this case we need to add $c_2(\mu_0)$ in the analysis and we either have to consider more experimental data or make additional assumptions. As we will see in the next example such an analysis is more complicated but nevertheless one can get useful bounds on effective couplings.

One of the most elusive among the non-standard four-fermion interactions is that which involves only neutrinos. Best bounds on the effective coupling of the V-A form (we assume lepton universality for simplicity)

$$\mathcal{L}^{\nu-\nu} = c_1\,G_F\sum_{i,j=e,\mu,\tau}(\overline{\nu_i}\gamma_\alpha(1-\gamma_5)\nu_i)(\overline{\nu_j}\gamma_\alpha(1-\gamma_5)\nu_j)\ , \tag{24}$$

were obtained [13] from its tree-level contribution to the invisible width of the Z-boson via the decay $Z \to \nu\overline{\nu}\nu\overline{\nu}$:

$$|c_1| \leq 390\ . \tag{25}$$

When the right-handed neutrinos are involved in the interaction much stronger bounds were obtained recently [14] from the primordial nucleosynthesis.

Thus, in the case of $V-A$ structure the interaction may be rather strong. One can ask on the possible bounds one could obtain on this interaction via its one-loop contribution to the

$Z \to \nu\bar{\nu}$. Using the fact that the invisible Z-width is measured at LEP with an accuracy better than one-percent [12] one can get a simple estimate:

$$\frac{\Delta\Gamma_{\bar{\nu}\nu}}{\Gamma_{\bar{\nu}\nu}} \approx \frac{c_1 G_F M_Z^2}{(4\pi)^2} \qquad \to \qquad c_1 \leq (1\text{–}10) \cdot . \tag{26}$$

This estimate suggests that one can obtain good bounds by considering the four-neutrino operator at the loop level.

The above estimate is rather naïve because inserting the non-renormalizable vertex in the loop diagram we get a divergent result. It should be renormalized by adding a derivative coupling of the Z-boson to neutrinos [15]

$$\mathcal{L}^{\nu-Z} = -\frac{g}{2c_W}\mu^\epsilon G_F \sum_{i=e,\mu,\tau,..} (c_2 + \Delta c_2)\,(\bar{\nu_i}\gamma^\alpha L\nu_i)\partial^\beta Z_{\beta\alpha}\ , \tag{27}$$

where c_2 is the $\overline{\text{MS}}$ renormalized coupling and the corresponding counterterm is

$$\Delta c_2 = -c_1\gamma_{12}\frac{1}{\hat{\epsilon}}\ .$$

Moreover, since by using only four-neutrino interactions we do not assume $SU(2)$ symmetry, we have to also add a non-standard direct (non-derivative) coupling of the Z-boson to neutrinos, c_3 (there is no symmetry which forbids it). Then the full renormalized vertex $Z\bar{\nu}\nu$ is given by

$$\hat{T} = -\frac{g}{2c_W}G_F[q^2\left(c_2(\mu) + c_1(\mu)\left(\gamma_{12}\left(\log(\mu^2/|q^2|) + i\pi\theta(q^2)\right) + \kappa_{12}\right)\right) + c_3(\mu)]\ , \tag{28}$$

with

$$\gamma_{12} = \frac{1}{3\pi^2}\ , \quad \kappa_{12} = \gamma_{12}\frac{17}{12}\ . \tag{29}$$

The running couplings in our approximation (we neglect all contributions with gauge bosons running in the loops) are given by

$$c_1(\mu) \approx c_1(\mu_0)\ , \tag{30}$$

$$c_2(\mu) = c_2(\mu_0) + c_1(\mu_0)\gamma_{12}\log\left(\frac{\mu_0^2}{\mu^2}\right)\ , \tag{31}$$

where μ_0 is some reference scale. The effective four-neutrino operator at the one-loop level contributes to the running of the coupling of the operator (27) and we have to consider mixing between at least these two operators[8]. The coupling $c_3(\mu)$ does not mix with the other couplings because it corresponds to an operator of different dimension, then $c_3(\mu) \approx c_3(\mu_0)$.

Obviously, we need several experimental data in oder to put bounds on these couplings in a model independent way. As the q^2 dependence of the coefficients in front of the various couplings is different, we can separate different couplings by considering their contribution to the observables at distinct energy scales: apart from the invisible Z-width measured at LEP we consider data from high energy deep inelastic scattering (DIS).

The details of our analysis can be found in [15] here we list only final results. The three-parameter (model independent) fit to the full body of data gives the following bounds at 68% C.L.

$$c_3(\mu_0) = 0.004 \pm 0.009 \qquad c_2(\mu_0) = 4.7 \pm 7 \qquad c_1(\mu_0) = -100 \pm 140\ . \tag{32}$$

[8] Obviously, there are many other four-fermion operators like $(\bar{l}l)(\bar{\nu}\nu)$, etc., which also mix with the Z-neutrino coupling (27). But as we are neglecting loops with gauge bosons, they do not mix directly at the one-loop level with the four-neutrino operator and, as they can be strongly bounded from other processes, we will disregard them.

The extreme values of $c_1(\mu_0)$, of order ~ 240, are possible only because of large cancellations between the contributions of the three non-standard couplings. If one decides that such cancellations are unnatural, then one obtains a much better bound for the contact four-neutrino interaction. The complete analysis gives in this case

$$|c_1(\mu_0)| \leq \ 2 \, . \tag{33}$$

which is 200 times better than bounds on the four-neutrino coupling from the tree level analysis eq. (25).

In this talk we illustrate by two examples the construction and use of the effective field theory approach to the description of physics beyond the minimal Standard Model. In the first example we sketch the construction of the one-loop effective lagrangian for the extension of the Standard Model with a heavy charged scalar singlet [9]. We discuss the matching of the effective theory to the underlying full theory. In the second example [15] we illustrate the use of the general effective lagrangian at the loop level by bounding elusive four-fermion neutrino operator from its contribution in loops to the invisible width of the Z-boson and to the neutral to charged currents ratio measured in the deep inelastic scattering.

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