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The three-dimensional BF Model with Cosmological Term in the Axial Gauge

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Abstract.
We quantize the three-dimensional BF-model using axial gauge conditions. Exploiting the rich symmetry-structure of the model we show that the Green-functions correspond to tree graphs and can be obtained as the unique solution of the Ward-Identities. Furthermore, we will show that the theory can be uniquely determined by symmetry consideration without the need of an action principle.

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[1] On leave from the Département de Physique

1 Introduction

Topological field models [1] of the Schwarz-type [2] have been the subject of continous investigations over the recent years. These theories are characterized by an invariant action which does not depend on the metric structure of the manifold. Therefore, they are devoid of any local observables. Nevertheless, the metric appears in

the gauge-fixing term which is itself a BRST-variation. The variation of the gauge-fixing term with respect to the metric is a BRST-exact quantity implying the existence of a linear vector-like supersymmetry [4] ν_μ in an elegant manner. Together with

the BRST-symmetry, the symmetry ν_μ forms an algebra of the form

$$\{s, \nu_\mu\} = \partial_\mu \tag{1.1}$$

stating that translations are no physical operations and thus reflecting the topological nature of the theory. Thus one might say that this relation lies at the heart of their topological properties.

The most prominent example of these theories is of course the three-dimensional Chern-Simons theory which has led to the powerful connection between link-invariants and the vacuum expectation value of Wilson lines [3]. The Chern-Simons theory has also been studied extensively from a purely field theoretical point of view. It turns out to be a completely ultraviolet finite theory and that this finiteness is a direct consequence of the topological supersymmetry (1.1). Originally, this supersymmetry has been found using the Landau-gauge [4]. In a serie of papers it has been generalized to other gauge-conditions as well [5, 6, 7]. Of particular interest was the case of the axial gauge where it turned

out that the topological supersymmetry is not only responsible for the finiteness of

the theory, but also allows to compute the Green functions without the use of

an action principle [8]. Let us also mention that it has been shown that the topological supersymmetry exists also in string theories [9] and in two-dimensional chiral W_3-gravity [10] and that it turned out to be an extremely useful tool for solving the descent equations associated with the integrated BRST-cohomology [11].

Another class of Schwarz-type Topological theories [2] are

the BF models. Despite of their simple form they reveal a surprisingly rich

symmetry structure. Indeed, they allow for reducible invariances

[12]. In the particular case of three dimensions, another interesting feature

is that the B-field is a 1-form and therefore allows the addition of a cubic term in B into the action. This term is usually referred to as a cosmological constant term since then the model is related to three-dimensional Einstein-Hilbert gravity with such a term. A detailed investigation of the symmetry stucture and finiteness properties of this model in the Landau-gauge has been given in [13].

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In the case of the Chern-Simons model in the axial gauge, we found that

the supersymmetry has two main consequences. Indeed, it turned out to be

strong enough for fixing all the Green functions of the theory (in this sense, one

can say that it can be substituted to the action principle) and it also imposes

the principal value prescription for the propagators. Therefore, it would be desirable to know wether this remains valid for the BF system in the axial gauge. This is precisely the

question we will address in this paper and we will show that the answer is

positive.

The work is organized as follows. In section 2 we introduce the action and fix the notation and conventions. Section 3 presents the symmetries and all the functional identities. We investigate their

consequences for the equations of motion in section 4.1 and for the calculation of the propagators in section 4.2. At the end, we propose some conclusion.

2 The $3D$ BF model with cosmological term in the axial gauge

The complete action of the $3D$ BF model containing a cosmological term with an axial gauge fixing is given by

$$
S = S_{inv} + S_{gf} \; , \tag{2.1}
$$

with

$$
\begin{aligned}
S_{inv} &= -\tfrac{1}{2} Tr \int d^3x \left[\epsilon^{\mu\nu\rho} (F_{\mu\nu} B_\rho + \frac{2\alpha}{3} B_\mu B_\nu B_\rho) \right] \; , \\
S_{gf} &= Tr \int d^3x \, [b n^\mu A_\mu + d m^\mu B_\mu + \bar{c} n^\mu (D_\mu c + \alpha [B_\mu , \phi]) + \bar{\phi} m^\mu (D_\mu \phi + [B_\mu , c])] \; .
\end{aligned} \tag{2.2}
$$

and $D_\mu \ldots = \partial_\mu \ldots + g[A_\mu, \ldots]$ denoting the gauge covariant derivative. $F_{\mu\nu}$ is the field strength of the gauge field A_μ. Further b, d are the Langrange multipliers imposing the gauge-conditions $n^\mu A_\mu = 0$ and $m^\mu B_\mu = 0$ where n^μ and m^μ are it a priori two independent gauge fixing directions. $\bar{c}$, c and $\bar{\phi}$, ϕ are the

anti-ghost and ghost fields corresponding to the two gauge symmetries of S_{inv}

$$
\begin{aligned}
\delta^1 A_\mu &= -D_\mu \theta \; , \quad \delta^1 B_\mu = - [B_\mu \, , \theta] \; , \\
\delta^2 A_\mu &= -\alpha \, [B_\mu , \lambda] \; , \quad \delta^2 B_\mu = -D_\mu \lambda \; .
\end{aligned} \tag{2.3}
$$

We choose the gauge group to be simple, all fields belong to the adjoint representation and are written as Lie algebra matrices $\varphi(x) = \varphi^a(x) t_a$, with

$$[t_a, t_b] = f^c{}_{ab} t_c \ , \qquad Tr(t_a t_b) = \delta_{ab}. \tag{2.4}$$

Finally α is some numerical constant. We summarize the canonical dimensions and the ghost numbers of the various fields in Table 1.

	A	B	b	d	c	$\bar{c}$	ϕ	$\bar{\phi}$
Dimension	1	1	2	2	0	2	0	2
Ghost number	0	0	0	0	1	-1	1	-1

Table 1: Dimensions and ghost numbers.

3 Symmetries of the action and Ward identities

The action (2.1) is invariant under the BRST transformation s :

$$\begin{aligned}
&sA_\mu = -D_\mu c - \alpha[B_\mu, \phi], && sB_\mu = -D_\mu \phi - [B_\mu, c], \\
&sc = c^2 + \alpha\phi^2, && s\phi = \{\phi, c\}, \\
&s\bar{c} = b, && s\bar{\phi} = d, \\
&sb = 0, && sd = 0.
\end{aligned} \tag{3.1}$$

Since we are dealing with a topological field theory of Schwarz type, the only metric dependence arises from the gauge fixing part of the action. Therefore, the energy momentum tensor is BRST exact:

$$T_{\alpha\beta} = s\Lambda_{\alpha\beta} \tag{3.2}$$

with

$$\Lambda_{\alpha\beta} = \ Tr(\eta_{\alpha\beta}\bar{c}n^\rho A_\rho - \bar{c}n_\alpha A_\beta - \bar{c}n_\beta A_\alpha + \eta_{\alpha\beta}\bar{\phi}n^\rho B_\rho - \bar{\phi}n_\alpha B_\beta - \bar{\phi}n_\beta B_\alpha). \tag{3.3}$$

Using the equations of motion, one gets for the divergence

of (3.3) the following expression

$$\begin{aligned}
\partial^\alpha\Lambda_{\alpha\beta} = \ & Tr(\partial_\beta\bar{c}\frac{\delta S}{\delta b} - A_\beta\frac{\delta S}{\delta c} - n^\alpha\bar{c}\varepsilon_{\rho\beta\alpha}\frac{\delta S}{\delta B_\rho} + \partial_\beta\bar{\phi}\frac{\delta S}{\delta b} - B_\beta\frac{\delta S}{\delta\phi} - \\
& -m^\alpha\bar{\phi}\varepsilon_{\rho\beta\alpha}\frac{\delta S}{\delta A_\rho} + n^\alpha\bar{c}\varepsilon_{\rho\beta\alpha}m^\rho d - n^\alpha\bar{\phi}\varepsilon_{\rho\beta\alpha}m^\rho b) + \text{tot. der.}
\end{aligned} \tag{3.4}$$

Integrating (3.4) allows to derive the usual form for the topological supersymmetry only for the case where[1] $n^\mu = m^\mu$ which we

will assume for the rest of the paper. Thus we have the following form for

[1] Actually one
could also insist in keeping different gauge vectors since the breaking term is BRST exact. This breaking could be controlled by coupling it to an
additional source and adding it to the action [7]

the vector supersymmetry transformations ν_α :

$$
\begin{aligned}
&\nu_\alpha A_\mu = -\varepsilon_{\alpha\beta\mu} n^\beta \bar{\phi}, \qquad && \nu_\alpha B_\mu = -\varepsilon_{\alpha\beta\mu} n^\beta \bar{c}, \\
&\nu_\alpha c = A_\alpha, && \nu_\alpha \phi = B_\alpha, \\
&\nu_\alpha \bar{c} = 0, && \nu_\alpha \bar{\phi} = 0, \\
&\nu_\alpha b = -\partial_\alpha \bar{c}, && \nu_\alpha d = -\partial_\alpha \bar{\phi}.
\end{aligned}
\tag{3.5}
$$

The transformations s and the supersymmetry transformations ν_μ form an algebra which closes on-shell:

$$
s^2 = \{\nu_\mu, \nu_\nu\} = 0, \quad \{s, \nu_\mu\} = \partial_\mu + \text{Eq. of motion.} \tag{3.6}
$$

In addition there exist two discrete symmetries [2] which leave the action (2.1) invariant:

$$
c \longleftrightarrow \alpha \bar{c} \quad , \quad \phi \longleftrightarrow \bar{\phi} \tag{3.7}
$$

and

$$
c \longleftrightarrow \bar{\phi} \quad , \quad \bar{c} \longleftrightarrow \phi \tag{3.8}
$$

At the level of the generating functional of the connected Green

functions Z_C, all these symmetries leads to a set of WI. The one which correspond to

the vector supersymmetry takes the form

$$
\begin{aligned}
\mathcal{V}_\alpha Z_C = \quad & Tr \int d^3x \left(J_b \partial_\alpha \frac{\delta Z_C}{\delta J_{\bar{c}}} + \varepsilon_{\alpha\mu\nu} n^\mu J_B^\nu \frac{\delta Z_C}{\delta J_{\bar{c}}} + J_c \frac{\delta Z_C}{\delta J_A^\alpha} + \right. \\
& \left. + J_d \partial_\alpha \frac{\delta Z_C}{\delta J_{\bar{\phi}}} + \varepsilon_{\alpha\mu\nu} n^\mu J_A^\nu \frac{\delta Z_C}{\delta J_{\bar{\phi}}} + J_\phi \frac{\delta Z_C}{\delta J_B^\alpha} \right) = 0.
\end{aligned}
\tag{3.9}
$$

In this formalism, the axial gauge is imposed by the two gauge

conditions :

$$
\begin{aligned}
J_b + n^\mu \frac{\delta Z_C}{\delta J_A^\mu} = 0, \\
J_d + n^\mu \frac{\delta Z_C}{\delta J_B^\mu} = 0.
\end{aligned}
\tag{3.10}
$$

As in any linear gauge there exist antighost equations which in the case of the axial gauge are local [5]. In our case we have

two of them and they take the following form:

$$
\begin{aligned}
J_{\bar{c}} - n^\mu \partial_\mu \frac{\delta Z_C}{\delta J_c} + \left[J_b, \frac{\delta Z_C}{\delta J_c} \right] + \alpha \left[J_d, \frac{\delta Z_C}{\delta J_\phi} \right] = 0, \\
J_{\bar{\phi}} - n^\mu \partial_\mu \frac{\delta Z_C}{\delta J_\phi} + \left[J_b, \frac{\delta Z_C}{\delta J_\phi} \right] + \left[J_d, \frac{\delta Z_C}{\delta J_c} \right] = 0.
\end{aligned}
\tag{3.11}
$$

[2] In fact, (3.7) and (3.8) imply the existence of additional anti-BRST-like symmetries and anti-vector-like supersymmetries as in [4, 5].

Finally, it is well known that in the axial gauge, due to the

decoupling of the ghosts, the Slavnov identity which express the invariance of the theory under the BRST-transformation (3.1) takes the form of a

local WI. Therefore, one get the two following local gauge WI's

$$\begin{aligned} &\partial_\mu J^\mu_A - \left[J^\mu_A, \frac{\delta Z_C}{\delta J^\mu_A}\right] - \left[J^\mu_B, \frac{\delta Z_C}{\delta J^\mu_B}\right] - \left[J_b, \frac{\delta Z_C}{\delta J_b}\right] - \left[J_d, \frac{\delta Z_C}{\delta J_d}\right] - \\ &- \left\{J_c, \frac{\delta Z_C}{\delta J_c}\right\} - \left\{J_{\bar c}, \frac{\delta Z_C}{\delta J_{\bar c}}\right\} - \left\{J_\phi, \frac{\delta Z_C}{\delta J_\phi}\right\} - \left\{J_{\bar\phi}, \frac{\delta Z_C}{\delta J_{\bar\phi}}\right\} + (n\cdot\partial)\frac{\delta Z_C}{\delta J_b} = 0 \end{aligned} \tag{3.12}$$

and

$$\begin{aligned} &\partial_\mu J^\mu_B - \alpha\left[J^\mu_A, \frac{\delta Z_C}{\delta J^\mu_B}\right] - \left[J^\mu_B, \frac{\delta Z_C}{\delta J^\mu_A}\right] - \left[J_b, \frac{\delta Z_C}{\delta J_d}\right] - \alpha\left[J_d, \frac{\delta Z_C}{\delta J_b}\right] - \\ &-\alpha\left\{J_c, \frac{\delta Z_C}{\delta J_\phi}\right\} - \left\{J_{\bar c}, \frac{\delta Z_C}{\delta J_{\bar\phi}}\right\} - \left\{J_\phi, \frac{\delta Z_C}{\delta J_c}\right\} - \alpha\left\{J_{\bar\phi}, \frac{\delta Z_C}{\delta J_{\bar c}}\right\} + (n\cdot\partial)\frac{\delta Z_C}{\delta J_d} = 0. \end{aligned} \tag{3.13}$$

4 Consequences of the Symmetries

4.1 Equations of motion

Let us now investigate in some detail the meaning of the

relations of the last section, starting with the projection of the WI for the supersymmetry(3.9), along the gauge fixed direction. Without loss of generality we can choose the gauge vector n^μ to be $(0,0,1)$. We

will denote this gauge fixed direction by u and the transverse coordinates by $x^{tr} = (x^i), i = 1,2$. Then,

$$\mathcal{V}_u Z_C = Tr\int d^3x \left(J_b\partial_u\frac{\delta Z_C}{\delta J_{\bar c}} + J_c\frac{\delta Z_C}{\delta J^u_A} + J_d\partial_u\frac{\delta Z_C}{\delta J_{\bar\phi}} + J_\phi\frac{\delta Z_C}{\delta J^u_B}\right) = 0. \tag{4.1}$$

Taking into accout the gauge conditions (3.10), the latter can

be written as

$$Tr\int d^3x\,(J^b\mathcal{X} + J^d\mathcal{Y}) = 0.$$

where $\mathcal{X}$ and $\mathcal{Y}$ are the most general forms compatible

with (4.1)

$$\begin{aligned} \mathcal{X} &= \partial_u\frac{\delta Z_C}{\delta J_{\bar c}} + \lambda_1\left[J^d, \frac{\delta Z_C}{\delta J_{\bar c}}\right] + \lambda_2\left[J^d, \frac{\delta Z_C}{\delta J_{\bar\phi}}\right] + \lambda_3\left[J^b, \frac{\delta Z_C}{\delta J_{\bar c}}\right] + \lambda_4\left[J^b, \frac{\delta Z_C}{\delta J_{\bar\phi}}\right] - J_c = 0 \\ \mathcal{Y} &= \partial_u\frac{\delta Z_C}{\delta J_{\bar\phi}} + \xi_1\left[J^d, \frac{\delta Z_C}{\delta J_{\bar c}}\right] + \xi_2\left[J^d, \frac{\delta Z_C}{\delta J_{\bar\phi}}\right] + \xi_3\left[J^b, \frac{\delta Z_C}{\delta J_{\bar c}}\right] + \xi_4\left[J^b, \frac{\delta Z_C}{\delta J_{\bar\phi}}\right] - J_\phi = 0 \end{aligned}$$

At this level, one can use the consistency condition between the two equations we just found and the ghost equation (4.2) in order to fix the arbitrary parameters. Then, the result is

$$
\begin{aligned}
&J_c - n^\mu \partial_\mu \frac{\delta Z_C}{\delta J_{\bar c}} + \left[J_b, \frac{\delta Z_C}{\delta J_{\bar c}}\right] + \left[J_d, \frac{\delta Z_C}{\delta J_{\bar\phi}}\right] = 0, \\
&J_\phi - n^\mu \partial_\mu \frac{\delta Z_C}{\delta J_{\bar\phi}} + \left[J_b, \frac{\delta Z_C}{\delta J_{\bar\phi}}\right] + \alpha \left[J_d, \frac{\delta Z_C}{\delta J_{\bar c}}\right] = 0.
\end{aligned}
\tag{4.2}
$$

which are nothing else than the ghost equations. Thus, the equations of motion for the ghost sector are a consequence of the gauge-fixed component of the supersymmetry WI, the gauge condition and the Slavnov identity.

For the gauge sector, let us consider the transverse component of (3.9)

$$
\begin{aligned}
\mathcal{V}_i Z_C = Tr\int d^3x \Big(& J_b \partial_i \frac{\delta Z_C}{\delta J_{\bar c}} + \varepsilon_{ji} J_B^j \frac{\delta Z_C}{\delta J_{\bar c}} + J_c \frac{\delta Z_C}{\delta J_A^i} + \\
& + J_d \partial_i \frac{\delta Z_C}{\delta J_{\bar\phi}} + \varepsilon_{ji} J_A^j \frac{\delta Z_C}{\delta J_{\bar\phi}} + J_\phi \frac{\delta Z_C}{\delta J_B^i} \Big) = 0.
\end{aligned}
\tag{4.3}
$$

together with the antighost equations (3.11) written as functional operators acting on Z_C

$$
\begin{aligned}
&\left\{ \partial_u \frac{\delta}{\delta J_c} - \left[J_b, \frac{\delta}{\delta J_c}\right] - \alpha \left[J_d, \frac{\delta}{\delta J_\phi}\right] \right\} Z_C = J_{\bar c}, \\
&\left\{ \partial_u \frac{\delta}{\delta J_\phi} - \left[J_b, \frac{\delta}{\delta J_\phi}\right] - \left[J_d, \frac{\delta}{\delta J_c}\right] \right\} Z_C = J_{\bar\phi}.
\end{aligned}
\tag{4.4}
$$

Their consistency gives rise to the following identities

$$
\begin{aligned}
&\left\{ \partial_u \frac{\delta}{\delta J_A^i} - \left[J_d, \frac{\delta}{\delta J_A^i}\right] - \alpha \left[J_b, \frac{\delta}{\delta J_B^i}\right] \right\} Z_C = \varepsilon_{ji} J_B^j - \partial_i J_d, \\
&\left\{ \partial_u \frac{\delta}{\delta J_B^i} - \left[J_d, \frac{\delta}{\delta J_B^i}\right] - \left[J_b, \frac{\delta}{\delta J_A^i}\right] \right\} Z_C = \varepsilon_{ji} J_A^j - \partial_i J_b.
\end{aligned}
\tag{4.5}
$$

which correspond to the equations of motion for the gauge fields. This concludes the analysis of the consequences of the supersymmetry for the equations of motion.

4.2 Calculation of the Propagators

Gauge conditions

We begin by looking at the gauge conditions (3.10) which imply the vanishing of the connected Green functions involving the components A_3 or B_3

$$\begin{aligned} &\langle A_3^a(x)\textstyle\prod_i \varphi_i(x_i)\rangle = 0 \\ &\langle B_3^a(x)\textstyle\prod_i \varphi_i(x_i)\rangle = 0 \end{aligned} \qquad \forall\varphi_i \tag{4.6}$$

with two exceptions

$$\begin{aligned} \langle A_3^a(x)b^b(y)\rangle &= -\delta^{ab}\delta^{(3)}(x-y), \\ \langle B_3^a(x)d^b(y)\rangle &= -\delta^{ab}\delta^{(3)}(x-y). \end{aligned} \tag{4.7}$$

Antighost equations

The antighost equations (4.2) give the following differential equations for the connected Green functions involving one pair of ghost fields:

$$\partial_{x^3}\langle \bar{c}^a(x)c^b(y)\rangle = \delta^{ab}\delta^{(3)}(x-y), \tag{4.8}$$

$$\partial_{x^3}\langle \bar{\phi}^a(x)\phi^b(y)\rangle = \delta^{ab}\delta^{(3)}(x-y) \tag{4.9}$$

and

$$\begin{aligned} &\partial_{x^3}\langle d^{c_1}(z_1)..d^{c_r}(z_r)b^{b_1}(y_1)..b^{b_r}(y_r)c^b(y)\bar{c}^a(x)\rangle = \\ &\sum_{j=1}^{s} f^{ab_jc}\ \langle d^{c_1}(z_1)..d^{c_r}(z_r)b^{b_1}(y_1)..\widehat{b^{b_j}}(y_j)..b^{b_r}(y_r)c^b(y)\bar{c}^c(y_j)\rangle\ \delta(x-y_j)\ + \\ &\sum_{i=1}^{r} f^{ac_ic}\ \langle d^{c_1}(z_1)..\widehat{d^{c_i}}(z_i)..d^{c_r}(z_r)b^{b_1}(y_1)..b^{b_r}(y_r)c^b(y)\bar{\phi}^c(z_i)\rangle\ \delta(x-z_i) \end{aligned} \tag{4.10}$$

and

$$\begin{aligned} &\partial_{x^3}\langle d^{c_1}(z_1)..d^{c_r}(z_r)b^{b_1}(y_1)..b^{b_r}(y_r)c^b(y)\bar{\phi}^a(x)\rangle = \\ &\sum_{j=1}^{s} f^{ab_jc}\ \langle d^{c_1}(z_1)..d^{c_r}(z_r)b^{b_1}(y_1)..\widehat{b^{b_j}}(y_j)..b^{b_r}(y_r)c^b(y)\bar{\phi}^c(y_j)\rangle\ \delta(x-y_j)\ + \\ &\alpha\sum_{i=1}^{r} f^{ac_ic}\ \langle d^{c_1}(z_1)..\widehat{d^{c_i}}(z_i)..d^{c_r}(z_r)b^{b_1}(y_1)..b^{b_r}(y_r)c^b(y)\bar{c}^c(z_i)\rangle\ \delta(x-z_i). \end{aligned} \tag{4.11}$$

The solutions **of** the equations (4.8) and

(4.9) are

$$\langle \bar{c}^a(x)c^b(y)\rangle = \delta^{ab}[\theta(x^3-y^3)+\beta_1]\delta^{(2)}(x^{tr}-y^{tr}), \tag{4.12}$$

$$\langle \bar{\phi}^a(x)\phi^b(y)\rangle = \delta^{ab}[\theta(x^3-y^3)+\beta_2]\delta^{(2)}(x^{tr}-y^{tr}) \tag{4.13}$$

The form of the terms proportional to β_1, β_2 is dictated by transverse two dimensional Poincaré

invariance and scale invariance. Indeed, the latter forbids solutions of the type $1/(x^{tr}-y^{tr})^2$ because this term is not a well defined distribution. To give it a meaning would need the introduction of UV subtraction point, i.e., of a dimensionful parameter which would break scale invariance. The integration constant can be fixed with the help of the discrete symmetry of the action (3.7) to be $\beta_1 = \beta_2 = -\frac{1}{2}$.
Integration of the equations (4.10) and (4.11) yields the following recursion relations for the Green functions with one pair of ghosts:

$$
\begin{aligned}
&\langle d^{c_1}(z_1)..d^{c_r}(z_r)b^{b_1}(y_1)..b^{b_s}(y_s)c^b(y)\bar{c}^a(x)\rangle = \\
&\quad = \sum_{j=1}^{s} f^{ab_jc}\ [\theta(x^3-y_j^3)+\beta(r,s)]\delta^{(2)}(x^{tr}-y_j^{tr})\times \\
&\quad \times\langle d^{c_1}(z_1)..d^{c_r}(z_r)b^{b_1}(y_1)..\widehat{b^{b_j}}(y_j)..b^{b_s}(y_s)c^b(y)\bar{c}^c(y_j)\rangle\ + \\
&\quad + \sum_{i=1}^{r} f^{ac_ic}\ [\theta(x^3-z_i^3)+\beta(r,s)]\delta^{(2)}(x^{tr}-z_i^{tr})\times \\
&\quad \times\langle d^{c_1}(z_1)..\widehat{d^{c_i}}(z_i)..d^{c_r}(z_r)b^{b_1}(y_1)..b^{b_s}(y_s)c^b(y)\bar{\phi}^c(z_i)\rangle
\end{aligned}
\tag{4.14}
$$

and

$$
\begin{aligned}
&\langle d^{c_1}(z_1)..d^{c_r}(z_r)b^{b_1}(y_1)..b^{b_s}(y_s)c^b(y)\bar{\phi}^a(x)\rangle = \\
&\quad = \sum_{j=1}^{s} f^{ab_jc}\ [\theta(x^3-y_j^3)+\beta(r,s)]\delta^{(2)}(x^{tr}-y_j^{tr})\times \\
&\quad \times\langle d^{c_1}(z_1)..d^{c_r}(z_r)b^{b_1}(y_1)..\widehat{b^{b_j}}(y_j)..b^{b_s}(y_s)c^b(y)\bar{\phi}^c(y_j)\rangle\ + \\
&\quad + \alpha\sum_{i=1}^{r} f^{ac_ic}\ [\theta(x^3-z_i^3)+\beta(r,s)]\delta^{(2)}(x^{tr}-z_i^{tr})\times \\
&\quad \times\langle d^{c_1}(z_1)..\widehat{d^{c_i}}(z_i)..d^{c_r}(z_r)b^{b_1}(y_1)..b^{b_s}(y_s)c^b(y)\bar{c}^c(z_i)\rangle\ .
\end{aligned}
\tag{4.15}
$$

Using the discrete symmetry one could produce two additonal recursion relations which are not written explicitly since they are not needed for our calculations. The integration constants are all fixed to $\beta(r,s) = -\frac{1}{2}$ by the discrete symmetry (3.7) and Bose symmetry of the Lagrange multipliers b and d.
Now we will discuss the recursion relations for some special values of r and s.

The case $r = 0$

For this discussion we will use a more symbolic notion i.e. we will drop indices and variables because we only want to find the vanishing Green functions whereas the non vanishing Green functions can always be obtained from the explicit recursion relations (4.14) and (4.15). From (4.14) we get $\langle(b)^s c\bar{c}\rangle = \sum\langle(b)^{s-1}c\bar{c}\rangle$ which ends up after s steps with the $\langle c\bar{c}\rangle$ propagator defined in (4.8). On the contrary $\langle(b)^s\phi\bar{c}\rangle = 0$ since the recursion relation stops with the vanishing Green function $\langle\phi\bar{c}\rangle$. Using again the discrete symmetry we obtain $\langle(b)^s\bar{\phi}c\rangle = 0$ and $\langle(b)^s\phi\bar{\phi}\rangle = \sum\langle(b)^{s-1}\phi\bar{\phi}\rangle$.

The case $s = 0$

In this case we have have to use (4.14) and (4.15) iteratively e.g. $\langle (d)^r c\bar{c}\rangle = \sum\langle (d)^{r-1} c\bar{\phi}\rangle = \langle (d)^{r-2} c\bar{c}\rangle = \ldots$. The final result depends on wether the recursion relation ends up with $\langle c\bar{c}\rangle$ which gives a non vanishing result or with $\langle c\bar{\phi}\rangle$ which gives zero. Here we only want to compile the zero results:
$\langle (d)^r c\bar{c}\rangle = 0$ and $\langle (d)^r \phi\bar{\phi}\rangle = 0$ if r is an odd integer.
$\langle (d)^r \phi\bar{c}\rangle = 0$ and $\langle (d)^r c\bar{\phi}\rangle = 0$ if r is an even integer.

The Green functions with additional ghosts, gauge fields or Lagrange multipliers vanish in general as a consequence of the antighost equations (4.2):

$$\langle X c\bar{c}\rangle = \langle X c\bar{\phi}\rangle = \langle X \bar{c}\phi\rangle = \langle X \bar{c}\bar{\phi}\rangle = 0 \ , \ \text{unless} \ X = (b)^m (d)^n \ . \tag{4.16}$$

Transverse supersymmetry

From equation (4.3) we get further relations along the same lines. For reasons of simplification we use the sloppy notation from the discussion above whenever possible.
The results for the two-point functions are:

$$\langle A_i^a(x) B_j^b(y)\rangle = \varepsilon_{ij}\delta^{ab}[\theta(x^3 - y^3) - \frac{1}{2}]\delta^{(2)}(x^{tr} - y^{tr}) \ , \tag{4.17}$$

$$\langle b^a(x) A_i^b(y)\rangle = -\delta^{ab}[\theta(x^3 - y^3) - \frac{1}{2}]\partial_{x^i}\delta^{(2)}(x^{tr} - y^{tr}) \ , \tag{4.18}$$

$$\langle d^a(x) B_i^b(y)\rangle = -\delta^{ab}[\theta(x^3 - y^3) - \frac{1}{2}]\partial_{x^i}\delta^{(2)}(x^{tr} - y^{tr}) \tag{4.19}$$

and $\langle AA\rangle = \langle BB\rangle = \langle bb\rangle = \langle dd\rangle = \langle dA\rangle = \langle bB\rangle = \langle c\bar{\phi}\rangle = \langle \bar{c}\phi\rangle = 0$. Furthermore we observe that all two-point functions with one ghost and one bosonic field vanish.
For the higher Green functions we obtain the recursion relations

$$\begin{aligned}
&\langle b^{d_1}(w_1)..b^{d_r}(w_r) d^{c_1}(z_1)..d^{c_s}(z_s) A_{l_1}^{b_1}(y_1)..A_{l_t}^{b_t}(y_t) B_{n_1}^{a_1}(x_1)..B_{n_u}^{a_u}(x_u) \left\{ \begin{array}{c} A_i^b(y) \\ B_i^b(y) \end{array} \right\}\rangle = \\
&= \sum_{k=1}^{r}\partial_{w_k^i}\langle b^{d_1}(w_1)..\widehat{b^{d_k}}(w_k)..b^{d_r}(w_r)(d)^s(A)^t(B)^u \left\{ \begin{array}{c} c^b(y) \\ \phi^b(y) \end{array} \right\} \bar{c}^{d_k}(w_k)\rangle + \\
&+\sum_{k=1}^{s}\partial_{z_k^i}\langle (b)^r d^{c_1}(z_1)..\widehat{d^{c_k}}(z_k)..d^{c_s}(z_s)(A)^t(B)^u \left\{ \begin{array}{c} c^b(y) \\ \phi^b(y) \end{array} \right\} \bar{\phi}^{c_k}(z_k)\rangle + \\
&+\sum_{k=1}^{t}\varepsilon_{il_k}\langle (b)^r(d)^s A_{l_1}^{b_1}(y_1)..\widehat{A_{l_k}^{b_k}}(y_k)..A_{l_t}^{b_t}(y_t)(B)^u \left\{ \begin{array}{c} c^b(y) \\ \phi^b(y) \end{array} \right\} \bar{\phi}^{b_k}(y_k)\rangle + \\
&+\sum_{k=1}^{u}\varepsilon_{in_k}\langle (b)^r(d)^s(A)^t B_{n_1}^{a_1}(x_1)..\widehat{B_{n_k}^{a_k}}(x_k)..B_{n_u}^{a_u}(x_u) \left\{ \begin{array}{c} c^b(y) \\ \phi^b(y) \end{array} \right\} \bar{c}^{b_k}(y_k)\rangle.
\end{aligned} \tag{4.20}$$

In the following we want to specify these recursion relations for special values of r, s, t and u to demonstrate that all Green functions can be obtained from our recursion relations. All Green functions with one pair of ghosts have been obtained in the previous subsection. Now we want to calculate the remaining Green functions with only bosonic fields and at least one A or B field.

The case $t = u = 0$

We obtain from (4.20):

$$\langle (b)^r (d)^s A \rangle = \sum \partial \langle (b)^{r-1} (d)^s c \bar{c} \rangle + \sum \partial \langle (b)^r (d)^{s-1} c \bar{\phi} \rangle$$

so the calculations breaks down to summing over already known Green functions. The same holds for $\langle (b)^r (d)^s B \rangle$.

The case $r = s = 0$

We obtain

$$\langle (A)^t (B)^u A \rangle = \varepsilon \langle (A)^t (B)^{u-1} c \bar{c} \rangle + \varepsilon \langle (A)^{t-1} (B)^u c \bar{\phi} \rangle$$

Using (4.16) we find $\langle (A)^a (B)^b \rangle = 0$ unless $a = 1$ and $b = 1$ which yields the two-point function $\langle AB \rangle$.

The case $s = u = 0$

In this case the relation (4.20) takes the form:

$$\langle (b)^r (A)^t \begin{pmatrix} A \\ B \end{pmatrix} \rangle = \sum \partial \langle (b)^{r-1} (A)^t \begin{pmatrix} c \\ \phi \end{pmatrix} \bar{c} \rangle + \sum \varepsilon \langle (b)^r (A)^{t-1} \begin{pmatrix} c \\ \phi \end{pmatrix} \bar{\phi} \rangle$$

For $t = 0$ we get the results:
$\langle (b)^r A \rangle = \sum \partial \langle (b)^{r-1} c \bar{c} \rangle$ and $\langle (b)^r B \rangle = 0$
For $t = 1$:
$\langle (b)^r AB \rangle = \sum \varepsilon \langle (b)^r \phi \bar{\phi} \rangle$ and $\langle (b)^r AA \rangle = 0$.

The case $r = u = 0$

In this case the relation (4.20) takes the form:

$$\langle (d)^s (A)^t \begin{pmatrix} A \\ B \end{pmatrix} \rangle = \sum \partial \langle (d)^{s-1} (A)^t \begin{pmatrix} c \\ \phi \end{pmatrix} \bar{\phi} \rangle + \sum \varepsilon \langle (d)^s (A)^{t-1} \begin{pmatrix} c \\ \phi \end{pmatrix} \bar{\phi} \rangle$$

For $t = 0$ we get the results:
$\langle (d)^s A \rangle = 0$ for s odd and $\langle (d)^s B \rangle = 0$ for s even.
For $t = 1$:
$\langle (d)^s AA \rangle = 0$ for s even and $\langle (d)^s AB \rangle = 0$ for s odd.

The case $r = t = 0$

In this case the relation (4.20) takes the form:

$$\langle (d)^s (B)^u \begin{pmatrix} A \\ B \end{pmatrix} \rangle = \sum \partial \langle (d)^{s-1} (B)^u \begin{pmatrix} c \\ \phi \end{pmatrix} \bar{\phi} \rangle + \sum \varepsilon \langle (d)^s (B)^{u-1} \begin{pmatrix} c \\ \phi \end{pmatrix} \bar{c} \rangle$$

For $t = 0$ we get the results:
$\langle (d)^s A \rangle = 0$ for s odd and $\langle (d)^s B \rangle = 0$ for s even.
For $t = 1$:
$\langle (d)^s BA \rangle = 0$ for s odd and $\langle (d)^s BB \rangle = 0$ for s even.

Gauge invariance

The two gauge symmetries (3.12) and (3.13) do not give a lot of new information besides consistency checks and the fact that all correlators consisting only of the Lagrange multipliers vanish:

$$\langle b^{a_1}(x_1)..b^{a_m}(x_m) d^{b1}(y_1)..d^{b_n}(y_n) \rangle = 0 \qquad \forall\ m,\ n$$

This is the unique solution obeying dimensional and scaling arguments.

5 Concluding Remarks

The main result of our study is that the Green functions of the model are the unique solutions of the Ward-identities defining the theory. Furthermore it turned out that the topological vector supersymmetry imposed a rather unexpected restriction on the

a priori independent gauge vectors. It is also worth noticing that the Green functions correspond to tree graphs only. Note also that in principle there are loop graphs with external b and d fields only, however, as in Chern-Simons theory [14], the gauge-field and the ghost field contributions to these graphs cancel exactly due to the topological supersymmetry. Having investigated here the three-dimensional BF model it is now natural to apply the axial gauge also to higher dimensional BF models. It would be highly interesting to know wether the methods developped

here and in [8] are also applicable to these cases.

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