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Next-to-leading Order Radiative Corrections to the Decay $b \to ccs$

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Abstract:

We calculate the complete $\mathcal{O}(\alpha_s)$ corrections to the quark decay $b \to ccs$ taking full account of the quark masses, but neglecting penguin contributions. For a c to the b quark mass ratio $m_c/m_b = 0.3$ and a strange quark mass of $0.2\,\mathrm{GeV}$, we find that the next-to-leading order (NLO) corrections increase $\Gamma(b \to ccs)$ by $(32 \pm 15)\%$ with respect to the leading order expression, where the uncertainty is mostly due to scale- and scheme-dependences. Combining this result with the known NLO and non-perturbative corrections to other B meson decay channels we obtain an updated value for the semileptonic branching ratio of B mesons, B_{SL}, of $(12.0 \pm 1.4)\%$ using pole quark masses and $(11.2 \pm 1.7)\%$ using running $\overline{\mathrm{MS}}$ masses.

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1. Owing to the newly developed tool of expanding in the inverse heavy quark mass [1], the theoretical description of weak inclusive decays of heavy mesons now rests on more solid ground than ever. Since in such decays the energy release is large compared to the masses of the final state particles, the process takes place essentially at small distances and, in leading order in the heavy quark expansion (HQE), is described by the underlying quark decay. Hadronic corrections only enter at second order in the HQE and should be $\sim 1\,\mathrm{GeV}^2/m_b^2$, which is around 5% for B decays with a b quark mass $m_b \approx 5\,\mathrm{GeV}$. Thus the accuracy of theoretical predictions for hadronic quantities like, say, the semileptonic branching ratio is not so much limited by our necessarily incomplete knowledge of (non-perturbative) hadronic matrix elements, but rather controlled by our knowledge of *perturbative* corrections to the free quark decay.

This issue has recently attracted much attention in connection with the summation of certain terms of the perturbative series, namely the asymptotically leading ones of order $\alpha_s^{n+1}\beta_0^n$ [2]. Although this program can straightforwardly be applied to semileptonic B decays[1] [4], there are severe problems (both technical and conceptual ones) with applying it to nonleptonic channels, so that in this letter we only deal with the first order radiative corrections.

Until recently, full $\mathcal{O}(\alpha_s)$ corrections were only known for the semileptonic decay $b \to ce\nu$ [5, 6] and for $b \to c\tau\nu$ [5]. Although it is known that the exchange of gluons between quarks of unequal masses can yield big effects (cf. the extreme case of an infinitely heavy quark investigated in Ref. [7]), finite c quark mass effects in the $\mathcal{O}(\alpha_s)$ corrections to the nonleptonic decay $b \to cud$ and a rough estimate[2] of c quark mass effects in $b \to ccs$ have only recently been obtained [8]. In Ref. [10] part of the effects of finite charm and strange quark masses in the radiative corrections to $b \to ccs$ were taken into account, based on the calculation done in Ref. [5]. In this letter, we complete the calculation of finite quark mass effects in the $\mathcal{O}(\alpha_s)$ corrections to $b \to ccs$, neglecting penguin corrections. We exploit this result to give an updated prediction for the semileptonic branching ratio B_{SL} of B mesons.

2. In calculating the decay rate $\Gamma(b \to ccs)$, we start from its representation as the imaginary part of the relevant forward-scattering amplitude:

$$\Gamma(b \to ccs) = \frac{1}{m_b}\,\mathrm{Im}\, i\int d^4x\, \langle\, b\,|\, T\, \mathcal{L}_W^{\Delta C=2}(x)\mathcal{L}_W^{\Delta C=2}(0)\,|\, b\,\rangle. \tag{1}$$

$\mathcal{L}_W^{\Delta C=2}$ is the effective Lagrangian that describes the decay process in the limit of an infinite W boson mass and to first order in the weak coupling. It can be written as

$$\mathcal{L}_W^{\Delta C=2} = -\frac{G_F}{\sqrt{2}}\, V_{cs}^*\, V_{cb} \sum_{i=1}^{6} c_i(\mu)\mathcal{O}_i(\mu). \tag{2}$$

In this letter we conform to the notation of [11], where the operators $\mathcal{O}_1$ and $\mathcal{O}_2$ denote current-current operators with a colour-non-singlet and colour-singlet structure, respectively, whereas the remaining operators are due to the admixture of penguin contributions.

[1]Cf. [3] for an explicit calculation of the $\alpha_s^2\beta_0$ term in the decays $b \to ue\nu$ and $b \to ce\nu$.
[2]See also [9].

The c_i are perturbatively calculable short-distance coefficients that describe the physics between the scale of the W boson and the characteristic hadronic scale of the process, which is of the order of the b quark mass. If we neglect the effects of strong interactions, then only $\mathcal{O}_2$ has a non-vanishing Wilson-coefficient. In next-to-leading order (NLO), both the coefficients c_i and the operators $\mathcal{O}_i$ depend on the scheme used to deal with γ_5. To that accuracy, the decay rate $\Gamma(b \to ccs)$ can be written as

$$\Gamma(b \to ccs) = \frac{G_F^2 m_b^5}{64\pi^3} |V_{cb}|^2 |V_{cs}|^2 \, \mathrm{PH}(x_c, x_c, x_s) \sum_{i=1}^{6} \sum_{j=1}^{i} f_{ij} c_i(\mu) c_j(\mu) d_{ij} \tag{3}$$

$$\equiv \frac{G_F^2 m_b^5}{64\pi^3} |V_{cb}|^2 |V_{cs}|^2 \, \mathrm{PH}(x_c, x_c, x_s) \kappa(x_c, x_s, \mu) \, K(x_c, x_s, \mu), \tag{4}$$

where $d_{ij} \equiv 1 + r_{ij} \left[\alpha_s(m_W) - \alpha_s(\mu)\right]/\pi + k_{ij} \, \alpha_s(\mu)/\pi$. The function κ is defined in such a way as to contain the LO effects, whereas the product κK covers the complete NLO terms. In Eq. (3) the c_i denote the *leading order* Wilson-coefficients, so that both the r_{ij} and the k_{ij} are scheme-independent. PH is the tree-level phase-space factor given by

$$\mathrm{PH}(x_1, x_2, x_3) = 12 \int\limits_{(x_2+x_3)^2}^{(1-x_1)^2} \frac{ds}{s} \, (s - x_2^2 - x_3^2)(1 + x_1^2 - s) w(s, x_2^2, x_3^2) w(s, x_1^2, 1) \tag{5}$$

with

$$w(a, b, c) = (a^2 + b^2 + c^2 - 2ab - 2ac - 2bc)^{1/2}. \tag{6}$$

The arguments of the phase-space factor are ratios of the quark masses, $x_c = m_c/m_b$ and $x_s = m_s/m_b$. The weight functions f_{ij} are tabulated in Table 1; they depend on[3]

$$f = \frac{1}{\mathrm{PH}(x_c, x_c, x_s)} \int\limits_{(x_c+x_s)^2}^{(1-x_c)^2} ds \, \frac{6x_c^2}{s^2} \, w(s, x_c^2, x_s^2) w(1, s, x_c^2)(s + x_s^2 - x_c^2)(1 + s - x_c^2). \tag{7}$$

f describes the interference of operators with Dirac structure $(V-A) \otimes (V+A)$ with those of the form $(V-A) \otimes (V-A)$ and vanishes for zero final state quark masses. The coefficients r_{ij} can be obtained from Ref. [11]. In particular, we find

$$\sum_{i=1}^{2} \sum_{j=1}^{i} f_{ij} c_i(\mu) c_j(\mu) r_{ij} =$$

$$\frac{10863 - 1278 n_f + 80 n_f^2}{162 \beta_0^2} \left[\frac{\alpha_s(m_W)}{\alpha_s(\mu)}\right]^{4/\beta_0} - \frac{15021 - 1530 n_f + 80 n_f^2}{162 \beta_0^2} \left[\frac{\alpha_s(m_W)}{\alpha_s(\mu)}\right]^{-8/\beta_0}, \tag{8}$$

[3]Note that we have corrected a sign error in Ref. [10] in all the terms containing f. We are grateful to G. Buchalla for pointing out this mistake.

where $\beta_0 = 11 - 2n_f/3$ is the lowest order coefficient of the QCD β-function; in our case we have $n_f = 5$ quark flavours. k_{11} and k_{22} were already given in Ref. [10]; k_{12} can be obtained from the diagrams shown in Fig. 1 as[4]

$$k_{12} = k_{22} + \frac{2}{3}\,(H_e + B) \text{ with } H_e\,\mathrm{PH}(x_c, x_c, x_s) = \frac{768\pi^5}{g_s^2 m_b^6}\,\mathrm{Im}(\mathrm{VI}+\mathrm{VIII}+\mathrm{X}+\mathrm{X}^\dagger+\mathrm{XI}+\mathrm{XI}^\dagger). \quad (9)$$

Here B is a scheme-dependent constant that removes the scheme-dependence of H_e; in naïve dimensional regularization (see below) one finds $B = 11$ [11]. Note that H_e is independent of the definition of the quark mass, which only affects k_{11} and k_{22} through the self-energy diagrams.

3. In the following, we present our results in the limit of vanishing strange quark mass (as will be discussed below, their dependence on this parameter is small); the full results are available from the authors as a Mathematica file. We have checked that all formulæ coincide with the corresponding ones in Ref. [8] when the appropriate limit is taken.[5]

Without going into too many details, we present first a short outline of the method of calculation of H_e which is described at length in Ref. [8]. In calculating the imaginary parts of the diagrams of Fig. 1, we use $\overline{\mathrm{MS}}$ subtraction and control the ultraviolet divergences through dimensional regularization with an anticommuting γ_5, often referred to as naïve dimensional regularization (NDR). NDR is applicable if one uses Fierz-transformations to relate diagrams with closed fermion loops, which are ambiguous in NDR, to diagrams which are well-defined in NDR. As shown in Ref. [12], Fierz-transformations are only valid diagram by diagram with a correct choice of the so-called evanescent operators. We have verified that in the limits $m_c, m_s \to 0$ our procedure yields the same results as obtained in other schemes [13]. Technically, we calculate the imaginary parts of the forward-scattering amplitudes by applying Cutkosky rules. We regularize intermediate infra-red singularities by introducing small quark and gluon masses, denoted by ρ and λ respectively, which allows phase-space integration to be done in four dimensions.

For the sake of compactness in displaying the formulæ, the square masses of the heavy quarks, c, b, are denoted by c, b. In the same spirit, we define $\Delta = (\sqrt{b} - \sqrt{c})^2$; $w = w(b, c, t)$ (or $w = w(p_1^2, p_2^2, l^2)$ in Eqs. (12), (13) below), where w was defined in Eq. (6), and similarly $v = w(c, c, t)/t$. Finally, we omit the arguments of the functions. Our results are written in terms of the functions A, B, C, $\tilde{B}$, $\tilde{K}$, which were defined in Ref. [8], Eqs. (A.1)–(A.4), with $M^2 \equiv p_1^2$ and $\mu^2 \equiv p_2^2$. Hence, throughout this letter we consider the external square momenta p_1^2, p_2^2 and $(p_1 + p_2)^2$ to be the natural arguments of those functions. The integrals can be computed following standard techniques and be expressed in terms of logarithmic and dilogarithmic functions. The final analytic expressions for A, B, C, $\tilde{B}$, $\tilde{K}$ are rather involved; we will give them elsewhere along with details of the calculation. The functions $\mathcal{K}_j$, $j = 0, 1, \ldots, 7$, denote certain phase-space integrals defined as

$$(\mathcal{K}_0, \ldots, \mathcal{K}_7) = \int \mathrm{LIPS}(p_1, p_2, k) \left(1, \frac{1}{2p_1k + \lambda^2}, p_1k, \frac{p_2k}{2p_1k + \lambda^2}, \frac{1}{\lambda^2 - 2kl}, \frac{p_2k}{\lambda^2 - 2kl},\right.$$

[4] We use the same notations as in Ref. [8].

[5] Note a misprint in Eq. (C.9) in Ref. [8]: the factor $(2m_c^2 + s)$ should read $(m_c^2 + 2s)$.

$$\frac{1}{(\lambda^2 - 2kl)(2p_2k + \lambda^2)}, \frac{1}{(2p_1k + \lambda^2)(2p_2k + \lambda^2)}\Bigg), \quad (10)$$

where $l = p_1 + p_2 + k$. $\mathcal{K}_0, \ldots, \mathcal{K}_5$, can be obtained from the results given in Ref. [14], whereas the calculation of $\mathcal{K}_6$ and $\mathcal{K}_7$ requires some effort. The prime symbol ($'$), as in $\mathcal{C}'$ or $\mathcal{K}_2'$ below, will always denote the replacement $p_1 \leftrightarrow p_2$. After a tedious calculation, one finds

$$(\mathcal{K}_6, \mathcal{K}_7) = \frac{\pi^2}{4l^2} \left(-\mathcal{C}, [\mathcal{C} + \mathcal{C}']\right), \quad (11)$$

where $\mathcal{C}$ is given by

$$\begin{aligned}
\mathcal{C} = & -2\ln\frac{m_1K_+ + m_2}{m_1} \ln\frac{K_+ l^2}{w(K_+ - 1)} - 2L_2\left(\frac{K_+ - 1}{K_+ + m_2/m_1}\right) \\
& - 2\ln\frac{m_1K_- + m_2}{m_1} \ln\frac{K_+ - K_-}{1 - K_-} + \frac{5}{2}L_2\left(\frac{K_- - K_+}{K_- + m_2/m_1}\right) - 2L_2\left(\frac{K_- - 1}{K_- + m_2/m_1}\right) \\
& + 2\ln\frac{m_2}{m_1}\ln K_+ - 2L_2\left(\frac{-m_1K_+}{m_2}\right) + 2L_2\left(\frac{-m_1}{m_2}\right) + \ln\frac{l^2}{m_1^2}\ln\frac{l^2}{l^2 - (m_1 + m_2)^2} \\
& + L_2\left(\frac{l^2 - (m_1 + m_2)^2}{l^2}\right) + \ln\frac{K_- + m_2/m_1}{K_+ + m_2/m_1}\ln\frac{\lambda}{\sqrt{l^2}} - \frac{1}{2}L_2\left(\frac{K_+ - K_-}{K_+ + m_2/m_1}\right). \quad (12)
\end{aligned}$$

In the above formula, we have introduced the obvious notation $\sqrt{p_j^2} = m_j$. The functions $K_\pm$ are given by

$$K_\pm = \frac{l^2 - p_1^2 - p_2^2 \pm w}{2m_1m_2}. \quad (13)$$

Now that we have calculated the $\mathcal{K}_i$ phase-space integrals, we may give the imaginary parts of the relevant diagrams, where we denote the sum of all j-particle cuts for a given diagram by the superscript $^{(j)}$. To start with, we find for diagram VI

$$\mathrm{Im}\,\mathrm{VI}^{(j)} = \frac{1}{8\pi b}\int_{4c}^{b} \mathrm{d}t\ (b-t)^2 \left[b\rho_1^{(j-1)} - 2\rho_2^{(j-1)}\right], \quad (14)$$

where the spectral densities $\rho_1^{(j)}$ and $\rho_2^{(j)}$ are given by

$$\begin{aligned}
\rho_1^{(2)} &= \mathrm{Re}\,\frac{g^2v}{24\pi^4 t}\left\{(4c - t)[t(A + B) + 2(t + c)\tilde{B}] - 2(t + 2c)\left(C + \frac{1}{2}\right) + (t^2 - 4c^2)\tilde{K}\right\}, \\
\rho_2^{(2)} &= \mathrm{Re}\,\frac{g^2v}{48\pi^4}\left\{2(t - c)[t(A + B) + 2C + 1 - (t - 2c)\tilde{K}] + (4t^2 - 15ct + 2c^2)\tilde{B}\right\}, \\
\rho_1^{(3)} &= \frac{g^2}{6\pi^6 t^2}\left\{t(t^2 - 4c^2)\mathcal{K}_7 - 2t(t + c)\mathcal{K}_1 - 2t\mathcal{K}_0 + 8c\mathcal{K}_3 + 8\mathcal{K}_2\right\}, \\
\rho_2^{(3)} &= \frac{g^2}{12\pi^6 t}\left\{-2t(t - c)(t - 2c)\mathcal{K}_7 + 2t(2t - c)\mathcal{K}_1 + t\mathcal{K}_0 - 4c\mathcal{K}_3 - 4\mathcal{K}_2\right\}. \quad (15)
\end{aligned}$$

The external momenta in A, B, C, $\tilde{B}$, $\tilde{K}$ satisfy $p_1^2 = p_2^2 = c$, $(p_1+p_2)^2 = t$, whereas in $\rho_i^{(3)}$, $t \equiv (p_1+p_2+k)^2$. For the remaining diagrams, we obtain:

$$\begin{aligned}
\mathrm{Im}\,\mathrm{VIII}^{(3)} &= \mathrm{Re}\,\frac{-g^2}{32\pi^5 b}\int_c^\Delta \frac{\mathrm{d}t}{t}(c-t)^2(t-c-b)w \\
&\quad\times\left\{\,2tA+2tB-2(c-t)\tilde{B}+8\left(C+\frac{1}{16}\right)+(c-t)\tilde{K}\right\}, \qquad (16)\\
\mathrm{Im}\,\mathrm{VIII}^{(4)} &= \frac{g^2}{8\pi^7 b}\int_c^\Delta \mathrm{d}t\,(t-c-b)w\left\{\mathcal{K}_0-(t-c)\mathcal{K}_1-t\mathcal{K}_1'+(t-c)^2\mathcal{K}_7\right\}, \qquad (17)\\
\mathrm{Im}\,[\mathrm{X}+\mathrm{X}^\dagger]^{(3)} &= \frac{g^2}{32\pi^5 b}\int_c^\Delta \frac{\mathrm{d}t}{t}(t-c)^2 w\left\{(b+c-t)\left[2t(A+B)+8C+\frac{1}{2}\right.\right.\\
&\quad\left.\left.+(b+c-t)\tilde{K}\right]-2(t^2-2tc-2tb+c^2+b^2)\tilde{B}\,\right\}, \qquad (18)\\
\mathrm{Im}\,[\mathrm{X}+\mathrm{X}^\dagger]^{(4)} &= \frac{g^2}{8\pi^7}\int_c^\Delta \frac{\mathrm{d}t}{t}(t-c)^2\left\{(t-c-b)^2\mathcal{K}_6+\mathcal{K}_0-(t-b)\mathcal{K}_1-(t-c)\mathcal{K}_4\right\}, \qquad (19)\\
\mathrm{Im}\,[\mathrm{XI}+\mathrm{XI}^\dagger]^{(3)} &= \frac{g^2}{192\pi^5 b}\int_{4c}^b \frac{\mathrm{d}t}{t}\,(b-t)^2 v\left\{(t+2c)b\left[t(A+4B)+(t+2b)\tilde{B}\right.\right.\\
&\quad\left.-\;2C-1-(b-t)\tilde{K}\right]\\
&\quad\left.-\;2t(t-c)\left[(t+b)(A+B)-(b-t)(2\tilde{B}-\tilde{K})+2C+1\right]\right\}, \qquad (20)\\
\mathrm{Im}\,[\mathrm{XI}+\mathrm{XI}^\dagger]^{(4)} &= \frac{g^2}{48\pi^7}\int_{4c}^b \frac{\mathrm{d}t}{t}\,v\left\{(t+2c)\left[t\mathcal{K}_0+b(t-b)\mathcal{K}_1+2\mathcal{K}_2'+b^2\mathcal{K}_4-2b\mathcal{K}_5\right.\right.\\
&\quad\left.\left.-\;b(t-b)^2\mathcal{K}_6\right]+2t(t-c)\left[(t-b)\mathcal{K}_1+t\mathcal{K}_4-(t-b)^2\mathcal{K}_6\right]\right\}. \qquad (21)
\end{aligned}$$

In Eq. (16), the arguments of A, B, C, $\tilde{B}$, $\tilde{K}$ satisfy $p_1^2 = c$, $p_2^2 = \rho^2 \to 0$. As mentioned above, ρ (and also λ) regularizes the infra-red singularities arising in intermediate steps of the calculation. They cancel upon addition of the 3- and 4-particle cut contributions to each diagram. In Eq. (18), we set $p_1^2 = c$, $p_2^2 = b$. Finally, in Eq. (20), we have $p_1^2 = \rho^2 \to 0$, $p_2^2 = b$, and in all three equations $(p_1+p_2)^2 = t$. In Eq. (17), the arguments of the functions $\mathcal{K}_j$ are $p_1^2 = \rho^2 \to 0$, $p_2^2 = c$ and $l^2 = t$. In Eq. (19), $p_1^2 = c$, $p_2^2 = t$ and $l^2 = b$. Finally, in Eq. (21), $p_1^2 = \rho^2 \to 0$, $p_2^2 = t$ and $l^2 = b$.

The numerical results of our calculation are presented in Table 2, namely k_{12} as a function of the charm quark mass for zero strange quark mass and $m_s = 0.2\,\mathrm{GeV}$, respectively. For comparison and completeness we likewise give the coefficients k_{11} and k_{22} referring to the on-shell definition of quark masses. The table shows also the leading order correction κ and the ratio $\Gamma_{NLO}/\Gamma_{LO} = K$. In the latter quantity, we have estimated the unknown NLO penguin contributions very conservatively by assuming $0 < d_{ij} < 2$, which corresponds to $|k_{ij}| < 15$ for $\alpha_s = 0.2$. The other input parameters are given in the table caption. The effect of a finite value of the strange quark mass is tiny for the rate (though appreciable for the NLO corrections after dividing out the phase-space factor), and less than 5% for $x_c \leq 0.4$, which is less than the estimated uncertainty from the unknown NLO penguin contributions. Using pole masses and a renormalization scale $\mu = m_b$, we thus observe that $\Gamma(b \to ccs)$ increases by $(32 \pm 7)\%$ through NLO corrections for a reasonable choice of quark masses $x_c = 0.3$. If we allow the renormalization scale to vary in the range[6] $m_b/2 < \mu < 2m_b$ and take the uncertainty in x_c to be ± 0.05, we obtain $\Gamma_{NLO}/\Gamma_{LO} = 1.32 \pm 0.15$.

So far we have used the on-shell definition of the quark mass. However, far from being compulsory, this definition most likely introduces artificially large higher order perturbative corrections (cf. [4]). It is therefore most instructive to eliminate the pole mass in favour of an off-shell renormalized mass, such as, e.g. the $\overline{\rm MS}$ mass. As discussed in Ref. [15], this amounts to the replacement

$$m_b^5\,\mathrm{PH}(x_c,x_c,0) \longrightarrow \bar{m}_b^5\,\mathrm{PH}(\bar{x}_c,\bar{x}_c,0)\left\{1+\frac{\alpha_s}{\pi}\left(\frac{20}{3}-5\ln\frac{\bar{m}_b^2}{\mu^2}-2\bar{x}_c\ln\bar{x}_c\,\frac{d\ln\mathrm{PH}(\bar{x}_c,\bar{x}_c,0)}{d\bar{x}_c}\right)\right\} \qquad (22)$$

in the decay rate, where $\bar{x}$ denotes a running quantity evaluated at the scale μ. We then obtain[7] $\Gamma_{NLO}/\Gamma_{LO} = 1.2 \pm 0.4$, which indicates that the uncertainty due to unknown higher order corrections is appreciable. We shall come back to this point in the next section.

4. With the results for $\Gamma(b \to ccs)$ in hand, we are ready to give an updated value for the semileptonic branching ratio B_{SL} of B mesons defined as

$$B_{SL} \equiv \frac{\Gamma(B \to Xe\nu)}{\sum_{\ell=e,\mu,\tau}\Gamma(B\to X\ell\nu_\ell)+\Gamma(B\to X_c)+\Gamma(B\to X_{c\bar{c}})+\Gamma(\text{rare decays})}. \qquad (23)$$

Performing an expansion in the inverse b quark mass, it is possible to show [1] that the inclusive decay rate of a B meson into a final state X coincides with that of the underlying b quark decay up to corrections of order $1/m_b^n$ $(n \geq 2)$:

$$\Gamma(B \to X) = \Gamma(b \to x)\left(1+\mathcal{O}(1/m_b^2)\right). \qquad (24)$$

The power-suppressed correction terms to the total inclusive widths of both semi– and nonleptonic decays were calculated in Refs. [1, 16]. They depend on two hadronic matrix elements, λ_1 and λ_2. While the latter is related to the squared mass difference of the B and the B* meson,

$$\lambda_2 \approx \frac{1}{4}(m_{B^*}^2 - m_B^2) = 0.12\,\mathrm{GeV}^2, \qquad (25)$$

[6]We conform here to the conservative choice of "characteristic scales" that is preferred if one follows standard renormalization group improvement arguments, where large logarithms of type $\ln m_b^2/\mu^2$ are to be avoided. The results obtained from summing the asymptotically leading part of the perturbative series seem, however, to indicate a lower scale, at least in semileptonic decays, cf. [2, 3, 4]. It remains to be seen if those scales necessarily have to coincide or not.

[7]Note that $x_c = 0.30 \pm 0.05$ translates to $\bar{x}_c(\mu = m_b) = 0.28 \pm 0.05$.

the former, λ_1, is difficult to measure; in this letter we use $\lambda_1 = -(0.6 \pm 0.1)\,\mathrm{GeV}^2$ as determined from QCD sum rules [17].

The one-loop corrections to the partial decay widths in Eq. (23) can be found tabulated in Ref. [10] except for $\Gamma(b \to ccs)$, which was a rough estimate. In Table 2 we give the corrections to this partial width. As in Ref. [10], we neglect the rare decays in the present analysis because of their smallness. Using the same input parameters as in the last section, we find

$$B_{SL} = 12.0 \pm 0.9^{+0.9}_{-1.3}, \qquad \bar{B}_{SL} = 11.2 \pm 1.0^{+1.0}_{-2.2}. \tag{26}$$

Here the first error comes from the uncertainty in the input parameters x_c, λ_1 and $\Gamma(b \to ccs)$ (in which the effect of the penguin operators has been estimated), whereas the second one indicates the variation of the result with the renormalization scale μ. Both results are in agreement with the most recent experimental data and the particle data group world average $B_{SL} = (10.43 \pm 0.24)\%$ [18]. Nevertheless, we observe a nonnegligible scheme-dependence of the two results. Although at the considered order in α_s it is difficult to judge which scheme is "best", we remark that at least for the semileptonic width one can sum up a certain class of terms, which are of order $\beta_0^n \alpha_s^{n+1}$. One observes that both, the explicit coefficients multiplying $\beta_0^n \alpha_s^{n+1}$ with n not too big (say $n < 5$), and the resummed all-order expression are smaller in the $\overline{\mathrm{MS}}$ than in the on-shell scheme ([4], see also [2]). Interpreting this result with due caution, since the evidence that these terms are dominant already in low orders comes from empirical observation (of quantities with known complete α_s^2 corrections) rather than from a theoretical principle, we still feel that it favours the $\overline{\mathrm{MS}}$ scheme. Any further discussion would require the knowledge of complete α_s^2 or even higher order terms, whose calculation is a formidable task.

Finally, we would like to discuss shortly the average charm quark content of B decays, which is defined by

$$\langle\, n_c \,\rangle = 1 + \frac{\Gamma(B \to X_{c\bar{c}})}{\Gamma_{tot}}. \tag{27}$$

We obtain (using again the same input parameters as in the last section):

$$\langle\, n_c \,\rangle = 1.27 \pm 0.07, \qquad \langle\, \bar{n}_c \,\rangle = 1.35 \pm 0.19, \tag{28}$$

which has to be compared with the experimental result $\langle\, n_c \,\rangle^{exp} = 1.04 \pm 0.07$ [18]. The experimental and the theoretical numbers differ by 3 standard deviations. Unfortunately, we do not see any natural theoretical explanation for that fact, unless x_c were much smaller than we assumed, which is however in conflict with the results obtained from the phenomenology of charmed particles.

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Figure

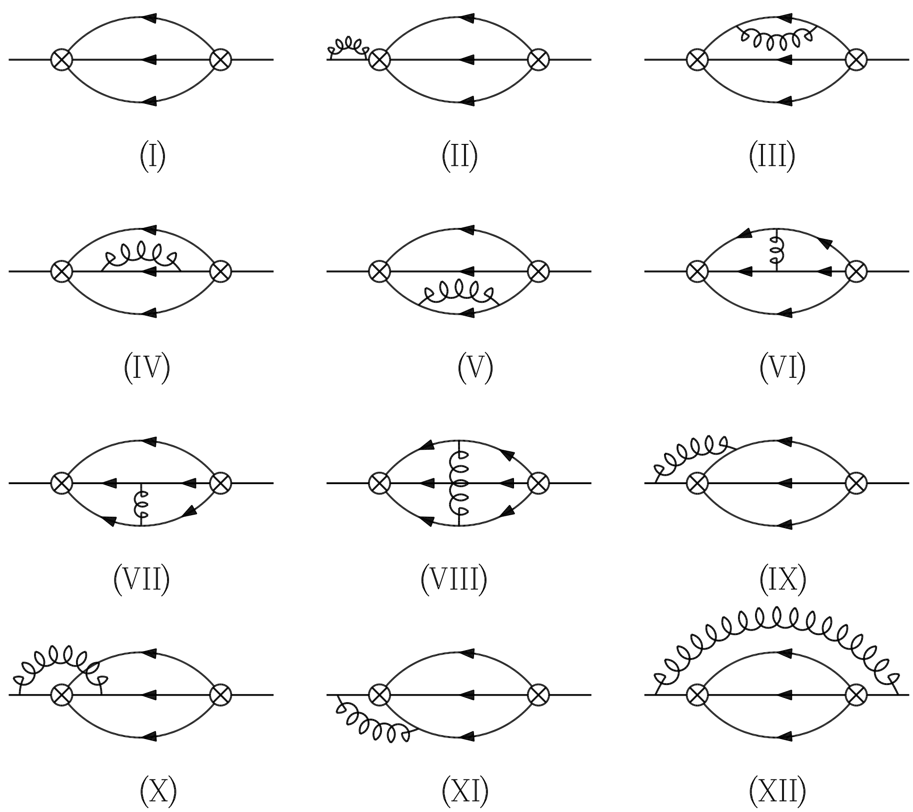

Figure 1: The diagrams contributing to the forward–scattering amplitude Eq. (1) up to order α_s without penguins. The crossed circles denote insertions of any of the operators $\mathcal{O}_i$. Of the three internal quark lines, the upper one denotes the c quark, the lower one the s quark, and the middle one the c antiquark.

Tables

f_{ij}	1	2	3	4	5	6
1	1					
2	$\frac{2}{3}$	1				
3	2	$\frac{2}{3}$	1			
4	$\frac{2}{3}$	2	$\frac{2}{3}$	1		
5	$2f$	$\frac{2}{3}f$	$2f$	$\frac{2}{3}f$	1	
6	$\frac{2}{3}f$	$2f$	$\frac{2}{3}f$	$2f$	$\frac{2}{3}$	1

Table 1: Coefficients f_{ij} defined in Eq. (3).

x_c	$\kappa(x_c, 0\ , m_b)$	k_{11}	$k_{12}(\mu = m_b)$	k_{22}	$K(x_c, 0\ , m_b)$	$\kappa\, K$ PH
0	1.054	−1.34	−7.75	−1.41	1.01 ± 0.05	1.065 ± 0.059
0.1	1.052	−0.14	−6.31	−0.53	1.07 ± 0.06	0.959 ± 0.052
0.2	1.047	2.40	−3.50	0.99	1.17 ± 0.06	0.634 ± 0.035
0.3	1.040	6.44	0.82	2.99	1.29 ± 0.07	0.263 ± 0.015
0.4	1.032	14.76	9.50	5.83	1.45 ± 0.08	0.039 ± 0.002

x_c	$\kappa(x_c, x_s, m_b)$	k_{11}	$k_{12}(\mu = m_b)$	k_{22}	$K(x_c, x_s, m_b)$	$\kappa\, K$ PH
0	1.054	−1.33	−7.63	−1.26	1.02 ± 0.05	1.062 ± 0.058
0.1	1.052	−0.05	−6.20	−0.35	1.08 ± 0.06	0.956 ± 0.052
0.2	1.047	2.53	−3.36	1.23	1.18 ± 0.06	0.631 ± 0.034
0.3	1.040	6.69	1.08	3.41	1.32 ± 0.07	0.259 ± 0.014
0.4	1.032	15.68	10.43	7.09	1.54 ± 0.08	0.037 ± 0.002

Table 2: The LO and NLO corrections to the nonleptonic decay $b \to ccs$ as a function of $x_c = m_c/m_b$. The penultimate column gives the increase of the decay rate $\Gamma(b \to ccs)$ in NLO if we include finite c and s quark effects in the radiative corrections. In the upper table we have put $m_s = 0$. The errors in K represent a conservative estimate of the unknown parts of the NLO penguin contributions. The input parameters are $\mu = m_b = 4.8\,\mathrm{GeV}$, $x_s = 0.04$ and $\Lambda^{(4)}_{\overline{\mathrm{MS}}} = 312\,\mathrm{MeV}$, which corresponds to $\alpha_s(m_Z) = 0.117$. A comparison of the last column in both tables shows that the effect of the strange quark mass is negligible and actually below 5% for all values of x_c.