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Bose–Einstein Effects and W Mass Determinations

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Abstract

In $e^+e^- \to W^+W^- \to q_1\overline{q}_2\, q_3\overline{q}_4$ events at LEP 2, the two W decay vertices are much closer to each other than typical hadronization distances. Therefore the Bose–Einstein effects, associated with the production of identical bosons (mainly pions), may provide a 'cross-talk' between the W^+ and the W^- decay products. If so, the observable W masses are likely to be affected. We develop algorithms for the inclusion of Bose–Einstein effects in multi-hadronic events. In this way we can study potential uncertainties in the W mass determination. In some scenarios the effects are significant, so that this source of uncertainty cannot be neglected.

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CERN–TH/95–17

1 Introduction

Today we have a theory for multiparticle production, QCD, but no associated computational scheme to predict the properties of exclusive hadronic final states. Instead, phenomenological models are used to describe the hadronization process [1]. Where the correct theory is an amplitude-based quantum-mechanics description, the models are formulated in a probabilistic language. The surprise is therefore not that models ultimately break down, but that they have been so successful in predicting a host of event properties.

Bose–Einstein (BE) effects occur because the production amplitude should be symmetrized for identical bosons ($\pi^\pm$, π^0, $\mathrm{K}^\pm$, …) [2]. These effects are therefore absent in probabilistic descriptions. They are, however, observed in the data in a host of different processes at high energies [3, 4, 5]. The crucial question is whether one can include BE effects with a minimal shake-up of our current (probabilistic) understanding of hadronization — if not, we are at a dead end.

One conventional approach to BE effects is the geometrical picture [6]. If, for instance, particle production vertices have a Gaussian distribution in space and time, the enhancement in the two-particle correlation (relative to an imagined reference world without BE effects) takes the form

$$f_2(Q) = 1 + \lambda \exp(-Q^2 R^2) \ . \tag{1}$$

Here λ is the incoherence parameter, in the range $0 \leq \lambda \leq 1$, R is the source radius, and Q is the relative difference in four-momenta, $Q^2 = Q_{12}^2 = (p_1 - p_2)^2 = m_{12}^2 - 4m^2$.

The geometrical approach is very convenient to interpret data from different reactions. However, it is limited in applicability, in that it has only been used to study specific particle correlations, not generic event properties. In this paper we consider ways to overcome this limitation.

An alternative approach is the one proposed by Andersson and Hofmann [7], and further developed by Bowler and Artru [8]. In this AHBA model, the space–time history of string fragmentation uniquely predicts the relative amplitudes for different particle configurations, and therefore also the magnitude of BE effects. An enhancement $f_2(Q)$ is obtained, which may be written in a way akin to the geometrical one, but without the incoherence interpretation of λ. The effects of resonance decays can be included [9]. While ideologically appealing, and more predictive than the geometrical approach, it is not without its problems. For instance, the source radii of several fermis observed in heavy-ion collisions [3] are too large to be associated with this mechanism. One possibility is to supplement the AHBA model with a final-state-interaction source of BE effects.

The study of BE effects is interesting in its own right, but it should also be noted that consequences may spill over into other fields of research, seemingly unrelated. In this letter we will take the issue of W-mass determinations at LEP 2 as one specific example of topical interest.

We have in mind the process $\mathrm{e^+e^-} \to \mathrm{W^+W^-} \to \mathrm{q_1\overline{q}_2\, q_3\overline{q}_4}$, i.e. where both W's decay hadronically. The typical separation, in space and time, between the W^+ and W^- decay vertices is smaller than 0.1 fm at LEP 2 energies [10], much smaller than typical hadronic sizes and source radii of $R \sim 0.5$ fm. The W^+ and W^- source regions are therefore on top of each other, so that BE effects in the hadronization stage can couple identical particles from the W^+ and the W^-. For a pair of π^0's, for instance, it would no longer be allowed to think of one of them as produced by the W^+ and the other by the W^-. Even when we define a pragmatic subdivision of particles into two groups, the redistribution of particle momenta implied by eq. (1) could mean that the hadrons that come from the W^+ (W^-) decay do not add up to the same invariant mass as the original W^+ (W^-) had.

It does not have to be this way. In the AHBA approach each string can be considered as a system of its own, with separate BE effects. There is some experimental support for such a decoupling of BE effects between the W^+ and W^- strings, from the UA1 observation that the λ parameter is smaller in events with larger multiplicity [11]. Such a trend arises naturally if large-multiplicity events have several string pieces (for instance in a multiple-interactions scenario [12]), so that a larger fraction of all pairs involve particles from two different strings.

One could also imagine many intermediate scenarios. For instance, a large fraction of all final-state pions are secondary decay products of short-lived resonances, and LEP data indicate that also these pions contribute to the BE effect [5, 13]. It could then be that primary pions follow from the AHBA approach and do not cross-talk between the W^+ and the W^-, while secondary decays (and other final-state interactions) introduce a geometrical component that does involve a cross-talk.

In the rest of this paper we will therefore adopt a simple geometrical approach, involving cross-talk between *all* pions and kaons (except those produced in the decay of long-lived resonances), as most likely to provide a conservative upper limit of possible effects on W-mass determinations.

BE effects are not the only theoretical uncertainty in W-mass determinations. Another possibility is that the colour flow is mixed up in the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2\, q_3\overline{q}_4$, so that the fragmenting colour singlet systems could be $q_1\overline{q}_4$ and $q_3\overline{q}_2$ rather than the 'original' $q_1\overline{q}_2$ and $q_3\overline{q}_4$ systems [10, 14]. Such a phenomenon is related to an 'early' part of the fragmentation process, where colour degrees of freedom are still important, whereas the BE effects appear at a 'later' stage of colour-singlet hadrons. In experimental observables the two aspects would appear intermingled.

2 A Bose–Einstein Algorithm

Almost all theoretical studies in the past have been concentrated on the shape of correlation functions, such as $f_2(Q)$. We do not have a corresponding formalism for how events differ in a global sense between a world without and one with BE effects. One of the few algorithms that have been proposed [15] is included in the JETSET code [16], and is the starting point for the studies in this paper. (We are only aware of one other algorithm [17].) The philosophy behind this algorithm, when applied to e.g. an $e^+e^- \to \gamma^*/Z^0 \to q\overline{q}$ event, where it is very successful phenomenologically [13], is the following.

Conventionally, the BE phenomenon is associated with an additional event weight. For a given pair at separation Q this weight is $f_2(Q)$. For a complete event the total weight may be approximated by the product of the $f_2(Q)$ values for all pairs of identical particles. (In fact, this overestimates allowed BE effects; for n identical particles of the same momentum, i.e. $Q = 0$, and for $\lambda = 1$, the correct maximum weight is $n!$ while the above approximation gives $2^{n(n-1)/2}$. For typical particle configurations the difference need not be as drastic.)

Such a BE event weight *cannot* be given the interpretation of an additional factor to be included in the total cross section of a given final state: if so, it would e.g. imply that the total hadronic decay width of the Z^0 would be (much!) larger than that given by perturbative QCD. Instead, perturbation theory is expected to give the composition of partonic states (including the primary flavours, the gluon emission, etc.), with hadronization thereafter proceeding with unit probability (factorization [18]). So the BE weight can only correspond to a redistribution among possible hadronic final states allowed for a fixed partonic final state.

In principle, the BE weight could change the multiplicity distribution. Configurations with more particles would then gain a higher weight on two counts, firstly because there are more pairs that can contribute to the total weight and secondly because particles are packed closer to one another. The increase in average multiplicity is typically very large if reweighting is applied. To compensate, one could imagine that the 'no BE' input distribution has a significantly lower average multiplicity and is shifted up to the observed one by the BE weights. However, it seems very unlikely that such an approach is workable: even if a retuning were possible for a given parton configuration at a given energy, this tuning would additionally have to reproduce the correct energy dependence, the correct dependence on parton topology (two-jet vs. three-jet events), the correct distribution of charged vs. neutral energy (so as to avoid e.g. events consisting entirely of π^0's), and all the other multiplicity features so well predicted by conventional approaches.

There is another possible critique of the event-weight approach. The BE phenomenon is local: it affects particles produced nearby in coordinate and momentum space. Therefore any reweighting of events should be considered on a local basis, rather than be associated with a global event weight. What one has in mind is a redistribution between events that globally look almost the same, but locally differ as regards the size of the momentum difference Q between nearby identical particles. Instead of defining event weights, it is therefore more convenient to generate conventional events with unit weight in a world without BE effects, and then to perturb individual momenta slightly in such a way that two-particle (and, if possible, multiparticle) correlations are redistributed to give the desired shape. In the end, the BE effect is then included almost as a classical force acting on the final state, shifting momenta of the outgoing identical particles.

An appropriate momentum shift may be obtained as follows. Consider a pair of relative separation Q in a world without BE effects, which is to be shifted to some Q' due to the inclusion of BE effects. If the inclusive distribution in Q values is assumed to be given just by phase space, $\mathrm{d}^3p/E \propto Q^2\mathrm{d}Q/\sqrt{Q^2+4m^2}$, then the Q' is found as the solution to the equation

$$\int_0^Q \frac{q^2\mathrm{d}q}{\sqrt{q^2+4m^2}} = \int_0^{Q'} f_2(q)\,\frac{q^2\mathrm{d}q}{\sqrt{q^2+4m^2}}\;. \tag{2}$$

For an arbitrary $f_2(Q) \geq 1$, the $Q \to Q'$ shift means that pairs are pulled closer in such a way that the inclusive distribution would be enhanced precisely by the factor $f_2(Q)$. The assumption of uniform phase-space density is acceptably well obeyed for momentum separations in the range 0.1–1 GeV, where the bulk of the BE effects occur. Local charge and momentum conservation issues (e.g., the fact that you cannot produce two adjacent π^+ in string fragmentation, but have to have a π^- in between) leads to fewer pairs than expected at small Q, and therefore to the possibility of some exaggeration of BE effects in this region. Similarly, global momentum conservation and the one-dimensional nature of string fragmentation (for distances larger than the string width) leads to a steeper-than-phase-space fall-off at large Q. Equation (2) could have been modified to take into account the actual density of pairs in Q [5], but this could be a disadvantage. Specifically, conservation of the total multiplicity implies that the pair density has to be decreased somewhere to compensate for the introduction of a BE enhancement at small Q. The whole compensation is put at the upper edge of the Q range if the correct density is used in a modified eq. (2), which means that BE effects are non-negligible at all Q values, a rather awkward situation. The use of a phase space that opens up faster than the actual density ensures that the shift $\delta Q = Q' - Q$ rapidly becomes small outside the BE enhancement region. The compensation of total multiplicity is smeared over a broad range of intermediate Q values, where the multiplicity is high and the relative change therefore modest.

The shift in a Q value can be translated into a change of particle momenta, but the recipe for this is not unique: since the invariant mass of a pair is changed, it is not possible to conserve both momentum and energy simultaneously, and so some compromises are necessary. The simplest alternative is to conserve three-momentum $\mathbf{p}$ in the rest frame of the Z^0. Then, for a pair of particles i and j, the change is $\mathbf{p}'_i = \mathbf{p}_i + \delta\mathbf{p}_i^j$, $\mathbf{p}'_j = \mathbf{p}_j + \delta\mathbf{p}_j^i$, with $\delta\mathbf{p}_i^j + \delta\mathbf{p}_j^i = 0$. The simplest choice is $\delta\mathbf{p}_i^j = c(\mathbf{p}_j - \mathbf{p}_i)$, with c determined to give the desired $Q \to Q'$ shift; i.e. the $\mathbf{p}_i$ and $\mathbf{p}_j$ are pulled closer to each other along the line connecting them in the rest frame of the Z^0. Presumably a better procedure is to use the line connecting them in the rest frame of the pair, but for our studies here this choice does not seem to make much difference.

A given particle is likely to belong to several pairs. If the momentum shifts above are carried out in some specific order, the end result will depend on this order. Instead all pairwise shifts are evaluated on the basis of the original momentum configuration, and only afterwards is each momentum $\mathbf{p}_i$ shifted to a $\mathbf{p}'_i = \mathbf{p}_i + \sum_{j\neq i} \delta\mathbf{p}_i^j$. That is, the net shift is the composant of all potential shifts due to the complete configuration of identical particles. This procedure is strictly valid only for large source radii, when the BE-enhanced region in Q is small, so that the momentum shift of each particle receives contributions only from very few nearby identical particles. For normal-sized radii, $R \sim 0.5$ fm, the method introduces complex effects among triplets and higher multiplets of nearby identical particles, which may be reflected both in the emergence of non-trivial higher-order correlations (which could be either a bonus or a drawback [19]) and in some changes between the input $f_2(Q)$ and the final output.

The above procedure preserves the total momentum, while the shift of particle pairs towards each other reduces the total energy. For a $\mathrm{Z}^0 \to \mathrm{q}\overline{\mathrm{q}}$ event this shift is typically a few hundred MeV, and so is small in relation to the Z^0 mass. In practice, the mismatch has been removed by a rescaling of all three-momenta by a common factor (very close to unity). As a consequence, also the Q values are changed by about the same small amount, whether the pairs are at low or at high momenta. That is, the local changes due to the energy conservation constraint have been minimized by spreading the corrections globally.

This approach could be wrong. We have argued that BE effects should be local; so should not also energy be compensated locally? This would then imply that non-identical particles would have to be moved apart from each other in the neighbourhood of a pair of identical particles. While not unreasonable as an idea, it is very difficult to develop a precise and consistent algorithm along these lines. Somewhat less ambitious is to give up momentum conservation for each individual pair, i.e. allow $\delta\mathbf{p}_i^j + \delta\mathbf{p}_j^i \neq 0$, and with some clever choice try to minimize the final energy and momentum imbalance that is to be compensated globally. As a first try in this direction, we have compared with an alternative 'energy conservation' algorithm, where the momentum shifts $\delta\mathbf{p}_i^j$ are determined by boosting the pair to its rest frame, reducing momenta there, and then boosting back along the same direction but with a magnitude determined so as to restore the original energy of the pair (rather than the momentum). Such a procedure is unstable for a pair with low momentum, since there is then no well-defined direction of motion of the pair; a more stable modified expression is thus used, which does not quite preserve energy. The net momentum imbalance of the event is compensated by an overall boost before the common momentum rescaling for full energy conservation (as before).

This completes the description of the algorithm for $\mathrm{Z}^0 \to \mathrm{q}\overline{\mathrm{q}}$. As we have noted before, new uncertainties enter when this approach is generalized to $e^+e^- \to W^+W^- \to q_1\overline{q}_2\, q_3\overline{q}_4$. In one extreme, the two W systems completely decouple, and so the W mass is unaffected. To simulate this scenario, the above algorithm can be used once for the W^+ particles and once for the W^- ones. At the other extreme, the two W's appear as one single common source of BE effects. The BE algorithm above can then be used, in the rest frame of the W^+W^- pair, without ever knowing which particle comes from which W. This means that neither momentum nor energy is conserved for the two groups of particles, but only for the event as a whole. In going from the 'no BE' to the 'with BE' world, the W masses are then changed when defined in terms of the final-state particles. For simplicity we here keep the original assignment of each particle as coming either from the W^+ or from the W^-, although strictly speaking this is no longer legitimate.

When we present numerical results on W-mass shifts in the next section, it will be in the context of preserved assignments of particles. In an experimental mass determination this need not give the full story. Imagine, for instance, an intermediate scenario where identical particles from the W^+ and the W^- are pulled closer together, but where these shifts are compensated within each W system, so that W masses are unaffected. An ideal detector would then perfectly reconstruct the $W^\pm$ masses if the final state could be correctly separated into two groups of particles. A smearing would come from misassignments of particles. With BE effects included, this misassignment rate is increased, since there would be more nearby pairs that would correctly have to be split up with one for the W^+ and the other for the W^-, rather than e.g. both from the W^-. This smearing need not be symmetric, but could well introduce a systematic bias.

3 Results

To study the possible Bose–Einstein effects on the W-mass determination we have used Pythia and Jetset to generate $e^+e^- \to W^+W^- \to$ hadrons events at 170 GeV. The algorithm described above was applied to all pairs of identical pions with the parameters in eq. (1) set to $\lambda = 1$ and $R = 0.5$ fm, following the 'tuning' in ref. [20]. Keeping the original assignment of particles to each of the W's, we find a shift in the resulting W mass of -0.026 GeV.

The sign of the shift is somewhat surprising. The particles from the W^+ and the W^- predominantly overlap in the low-momentum region, with the motion of the two W's away from each other giving a relative displacement of the W^+ and W^- particle distributions. The BE effects are expected to pull the soft particles closer to their common origin, i.e. reduce the momenta of the W's and hence increase their masses:

$$E_{\mathrm{cm}} = \sqrt{m_+^2 + \mathbf{p}_{\mathrm{W}}^2} + \sqrt{m_-^2 + \mathbf{p}_{\mathrm{W}}^2}\ , \tag{3}$$

where $\pm\mathbf{p}_{\mathrm{W}}$ are the $W^\pm$ momenta. However, the algorithm also contains an overall momentum rescaling to conserve total energy, and this could give an opposite effect on the W-mass shift.

To examine how the momentum rescaling affects the W-mass shift, we generated events according to four different strategies as follows:

(a) 'Full Bose–Einstein': the algorithm is applied to all pairs of identical pions from both W^+ and W^-.

(b) 'Completely separated W's': the algorithm is applied twice for each event, once on the decay products from the W^+, and once for the W^-.

Method	$\langle \delta p \rangle$	$\langle \delta E \rangle$	$\langle \delta m_{\mathrm{W}} \rangle$
Conserve momentum	0.000	-0.92	-0.059
Conserve energy	0.48	0.018	-0.087

Table 1: *The momentum, energy and mass shift in GeV for different methods of shifting the momentum of pairs of identical particles in strategy (c) at 170 GeV.*

c.m. energy	170 GeV	180 GeV	190 GeV	200 GeV
$\langle \delta E \rangle$ (GeV)	-0.92	-0.88	-0.86	-0.82
$\langle \delta m_{\mathrm{W}} \rangle$ (GeV)	-0.059	-0.101	-0.141	-0.175

Table 2: *Energy dependence of the energy and mass shift for the momentum-conserving method in strategy (c).*

(c) 'Not quite separated W's': the algorithm is applied only once per event, but only pairs of identical pions from the same W are considered. This differs from (b) only in that the final rescaling, to conserve total energy, is applied to the whole event instead of to the W^+ and W^- separately. This will give an artificial mass shift for the W which we will use for reference.

(d) '$\mathrm{W}^+ - \mathrm{W}^-$ effects only': the algorithm is applied only to identical pions stemming from different W's. This is used to cross-check how much of the mass shift is due to the Bose–Einstein correlations as such, and how much is due to the global momentum rescaling.

We also used different methods for the shifting of momentum in the algorithm to try to minimize the global momentum rescaling needed to conserve the total energy and momentum. Here we present the result from two methods: one conserves the total momentum, and the other tries to conserve the total energy as described above. To examine how these different methods work, we generated events according to strategy (c) and measured the average of the total momentum imbalance δp before the global boost, which is needed in the second method to ensure momentum conservation. We also measured the average of the total energy imbalance δE after the global boost, but before the global momentum rescaling. These values are then compared to the average shift δm_{W} in the W mass.

The result is presented in table 1. Note that the W mass is shifted to lower values, although the two W's are treated almost separately. This may be understood as follows. The BE momentum shifts make jets narrower and therefore decrease the invariant mass of each jet. Since jet directions are well preserved in both methods, the total energy can only be conserved if jet momenta are increased. This implies an increase of the W momenta and therefore a net decrease of their masses. The critical step for this effect differs between the two methods: the common rescaling of momenta at the end for the first, the BE shifts at the beginning for the second. In table 2 we show how the shift in W mass for the momentum-conserving method depends on the c.m. energy of the event. It is seen that the larger the W velocity, the larger the fraction of the momentum rescaling that is consumed by the increase of this boost.

The shift in the W mass in strategy (c) is of course completely unphysical and is an artefact of the global boost and momentum rescaling. We will have to correct for this when we try to estimate the shift due to the actual Bose–Einstein correlation *between* the

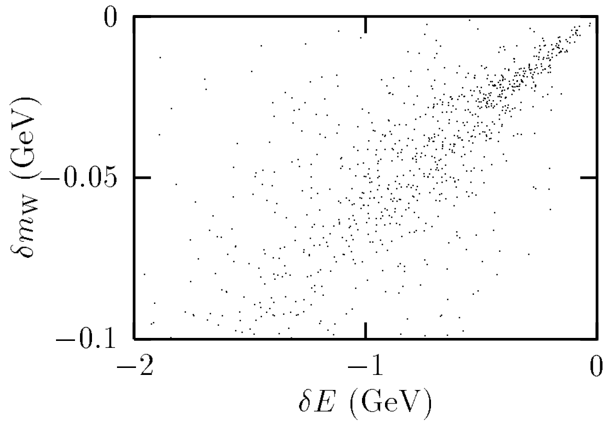

Figure 1: *Scatter plot of mass versus energy shift in 170 GeV events generated according to strategy (c).*

c.m. energy	170 GeV	180 GeV	190 GeV	200 GeV
$\langle \Delta m_{\rm W}^{(a)} \rangle$ (GeV)	0.095	0.164	0.219	0.290
$\langle \Delta m_{\rm W}^{(d)} \rangle$ (GeV)	0.095	0.163	0.228	0.282

Table 3: *Energy dependence of the corrected shift in the* W *mass.*

W's. Since the momentum-conserving method gives the lowest artificial W-mass shift, we will only consider that in the following.

It turns out that the energy and mass shifts in strategy (c) are very well correlated, even on an event-by-event basis. The scatter plot in fig. 1 shows the shift in the average W mass in each event versus the corresponding shift in energy, for events with 170 GeV c.m. energy. The relationship is clearly linear, so we can correct for the spurious momentum conservation effects by generating events both with strategy (a) and with strategy (c) and get the corrected shift in the W mass as

$$\Delta m_{\rm W} = \delta m_{\rm W}^{(a)} - \delta m_{\rm W}^{(c)} \frac{\delta E^{(a)}}{\delta E^{(c)}} \ . \tag{4}$$

In table 3 we present the result for the corrected mass shift according to eq. (4) for different c.m. energies. Also the corrected mass shifts obtained by using strategy (d) instead of (a) in eq. (4) are shown as a cross check, and it is clear that the two results are consistent.

The resulting W-mass shift is positive, in line with our above argument that low-momentum particles from the $\rm W^+$ and the $\rm W^-$ get to be pulled closer to each other, thus reducing the W momenta. This effect should vanish in the limit that the two W's are at rest with respect to each other, since then there is no preferred direction for any systematic shifts. Also small random shifts have a vanishing effect in this limit since, keeping the energy of a W fixed, the relation $E^2 = m^2 + p^2$ implies that $m\delta m + p\delta p \simeq 0$, or $\delta m \simeq -p\delta p/m$. As the W's move faster away from each other, the increased W mass shift may then be seen as a combination of a dynamic effect — a systematic shift δp of slow particles in the direction of smaller W momenta — and a kinematic one — the multiplication of any effect by a factor p/m.

When the W's get boosted away further from each other, their points of decay will also become more separated and the strength of the Bose–Einstein effect should decrease. This mechanism is not included in the algorithm used here, but as the aim of this paper is

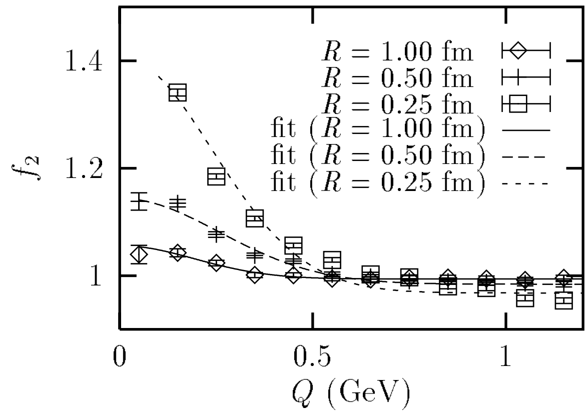

Figure 2: *The ratio between the two-particle correlation functions for strategies (a) and (b) as a function of Q for different values of R in eq. (1).*

R_{input} (fm)	0.25	0.5	1.0
$\langle \Delta m_{\mathrm{W}} \rangle$ (GeV)	0.325	0.095	0.021
R_{fit} (fm)	0.66	0.54	0.74
λ_{fit}	0.56	0.16	0.07

Table 4: *The corrected shift in the* W *mass for different values of the input R together with the fitted values for R and λ.*

to study the effect at LEP 2, where the two W decay vertices are very close, the algorithm should at least be able to give us an estimate of an upper limit of the effect.

It may be possible to directly measure the Bose–Einstein effects between the W^+ and W^- decay products at LEP2. One could e.g. try to measure the enhancement in the two-particle correlation in $\mathrm{W}^+\mathrm{W}^- \to$ hadrons events relative to fictitious events made up by the hadrons from two semileptonic $\mathrm{W}^+\mathrm{W}^-$ events. In that case the result should look as in fig. 2, where the ratio between the two-particle correlation functions for strategy (a) and (b) is shown as a function of Q for different values of R in eq. (1).

In fig. 2 are also shown lines corresponding to a fit of the function in eq. (1) to the points and in table 4 we show the fitted values of R and λ together with the corresponding corrected shifts in the W mass. Notice that the fitted values of R and λ do not agree with the input ones. We expect λ to differ because the algorithm acts only on those pions produced directly and in the decays of short-lived resonances. But also the reconstructed R value differs, due to the breakdown of the procedure of adding up the momentum shifts when the BE-enhanced region in Q becomes large, as described above. Particles are then pulled closer together than they should, which gives a decreased $1/R$ and an increased λ. These two aspects tend to compensate each other for the W-mass shift, however, so that $\langle \Delta m_{\mathrm{W}} \rangle$ shows the inverse-square-like dependence on the input R that could be expected from phase-space arguments.

4 Conclusions

Bose–Einstein effects are visible in particle physics processes, but are still very poorly known. Specifically, what has been studied is BE effects on n-particle *correlations* (n =

$2, 3, \ldots$), not on n-particle *events* ($n = 20, 30, \ldots$). That is, we have no standard formalism that allows us to know how particle four-momenta differ between a world without and one with BE effects. It should therefore come as no big surprise that we also do not know whether W-mass determinations will be affected by BE. This paper contains a first study to see what the consequences on the observable W mass would be in a 'worst-case' scenario, where the W^+ and W^- hadronic systems 'cross-talk' maximally.

In our model the BE effects do not change the multiplicity or particle content of events. If this assumption is wrong, it will show up in a comparison between doubly hadronic and mixed hadronic–leptonic decays of W^+W^- events. By the standard assumption of factorization, BE effects do not change the perturbative partonic processes either, i.e. the global event shapes. (In so far as an observable obtains both perturbative and non-perturbative contributions, a BE modification of the non-perturbative aspects could have some small implications for the experimental determination of an α_s value, however.)

To be more precise, we assume that BE effects correspond to a *local* reweighting of allowed particle momentum configurations, whereby those that correspond to two nearby identical bosons get to be enhanced. To achieve this reweighting, a momentum shift procedure is used, so that the Q value between any pair of identical bosons is reduced in a simple, deterministic way (in the absence of the influence of all the other identical bosons). In this paper we have throughout used a Gaussian enhancement as input, but results would not have been significantly different for any other similar shape. Unfortunately, results are more sensitive to the main limitation of the algorithm: it is not possible to conserve both energy and momentum in the pairwise shifts, so some *ad hoc* rescaling procedure is needed afterwards. One of the main technical issues addressed in this letter has therefore been the way of separating the 'true' BE-induced W-mass shifts from the 'spurious' ones.

The final result is a surprisingly large shift in the W mass, by about 100 MeV at 170 GeV c.m. energy. If this is taken as a measure of a net uncertainty on the W mass that can be determined from double hadronic W^+W^- events, then the mixed leptonic–hadronic W^+W^- decay modes could offer the best chance for a precision determination. (In passing we note that, although we have not addressed it here, also a direct determination of the W width could be affected.)

Such an attitude is probably too pessimistic, however. Based on measured two-particle correlations at LEP 1, we can predict what to expect at LEP 2 if the W^+ and W^- do not cross-talk. It should therefore be possible to tell from the data whether there is any further enhancement (and how large), which would then be associated with a cross-talk and could lead to a W-mass shift. The above numbers give maximal effects; presumably the true answer is some fraction thereof. A reduction could come in an AHBA-type scenario, where primary particle production occurs independently for the W^+ and the W^-. It could also come because of the simplified geometrical picture we have used here: for a single string, two particles with comparable momenta are also likely to be produced nearby in the string, while the effective space-time separation may be larger in the W^+W^- case. (We have in mind topologies where a W^+ jet and a W^- one are produced at an angle but, because of transverse momentum fluctuations, still produce particles that move in the same direction.) Results are sensitive to the assumed source radius R — phase space roughly predicts a scaling like $\langle \Delta m_W \rangle \propto 1/R^2$ — so a radius larger than our assumed 0.5 fm would lead to reduced effects. Finally, Coulomb repulsion has been neglected in our studies, but presumably this is not of any practical importance [21].

It is interesting to note that we obtain a mass shift that increases with the c.m. energy. Ultimately the separation between the W^+ and W^- decay vertices has to lead to a decoupling of effects, but over the LEP 2 energy range the calculated trend should be taken seriously, and could provide some interesting tests if LEP 2 is run at different energies. Both the sign of the W-mass shift and the energy dependence could be understood as follows: The particles from the W^+ and the W^- predominantly overlap in the central region of small momenta. The average motion of such slow particles is given by the velocities of the respective W; the mutual BE-induced momentum shifts therefore slow down these particles and thereby increase the observable W masses. If the W's already are at rest, there is no systematic direction of shifts, and so effects are smallest close to threshold. (However, note that the W's never are completely at rest with respect to each other, even at the nominal threshold [10].)

In conclusion, further studies are needed. Alternative models should be developed and the current model studied in more detail to understand what is going on. The studies at LEP 1 should lead to an understanding of the shape (or shapes, if e.g. charged and neutral pions behave differently) of the BE enhancement and a convergence of related parameter values. The observability of BE effects at LEP 2 (even with the limited statistics that will be available) should be seriously explored. This way, we have a hope both of doing a high-precision measurement of the W mass and of understanding the Bose–Einstein phenomenon better.

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