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# A MECHANISM OF LONGITUDINAL SINGLE-BUNCH INSTABILITY IN STORAGE RINGS

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A new mechanism of a longitudinal single-bunch instability in storage rings has been found. This instability results from an interaction between two radial modes which belong to the same azimuthal mode but have different magnitudes of the action. Such a coupling is only possible with potential-well distortion of the bunch. The frequency spread of the incoherent synchrotron motion in a bunch generated by the potential-well distortion plays an essential role in this instability. The system becomes unstable by a coupling of two radial modes when the synchrotron frequencies of two different actions degenerate. In an extreme case of a purely resistive ( $\delta$ -function) wake potential, it is shown that the system is always unstable, *i.e.*, the threshold intensity is zero.

KEY WORDS: Longitudinal single-bunch instability

The longitudinal single-bunch collective motion in a storage ring is usually described by the Vlasov equation<sup>1</sup>

$$-\frac{\partial f}{\partial s} = -p\frac{\partial f}{\partial q} + (q - V(q, s))\frac{\partial f}{\partial p}$$
(1)

for the distribution function f = f(p, q, s) in the longitudinal phase space. The independent variables are  $p \equiv (E - E_0)/E_0\sigma_{\epsilon}$  (relative energy deviation),  $q \equiv z/\sigma_z$  (relative longitudinal position), and  $s \equiv \omega_s t$  (phase of the synchrotron motion). We have introduced  $E_0$  as the nominal beam energy,  $\sigma_{\epsilon}$  the natural relative energy spread,  $\sigma_z$  the natural bunch length, and  $\omega_s$  the unperturbed angular frequency of the synchrotron motion. The positive q corresponds to the head of the bunch. The variables p and q are canonical with the Hamiltonian, which will be shown later (Eq. 5), for the longitudinal single-particle motion. Here the external rf-field is assumed to be linear in the position q. The charge of the bunch induces the relative energy loss -V(q) through the longitudinal wake function (Green's function) W(q) as

$$V(q,s) = k \int_{-\infty}^{+\infty} \rho(q',s) W(q'-q) dq'$$
<sup>(2)</sup>

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where  $\rho(q, s) = \int_{-\infty}^{+\infty} f(p, q, s) dp$  is the longitudinal density of particles, which is normalized  $\int \rho(q, s) dq = 1$ . The wake function introduced here is a normalized one  $W(q) \equiv (e/E_0)ew(q)$ , where w(q) is the usual wake function in the unit of voltage/revolution/charge. The parameter k represents the beam intensity:

$$k = \frac{N}{2\pi\nu_s\sigma_\epsilon},\tag{3}$$

where N is the number of the particles in the bunch, and  $v_s$  is the unperturbed synchrotron tune. We only consider the ultra-relativistic case, which means W(q) = 0 for q < 0.

The equilibrium solution of Eq. 1 is written as

$$f_0(p,q) = g(H(p,q))$$
, (4)

where H(p, q) is the Hamiltonian for the single-particle motion in the bunch:

$$H(p,q) = \frac{p^2}{2} + \frac{q^2}{2} - \int_0^q V_0(q')dq' \,. \tag{5}$$

The equation of motion of a particle with this Hamiltonian is

$$\frac{dp}{ds} = \frac{\partial H}{\partial q} , \qquad \frac{dq}{ds} = -\frac{\partial H}{\partial p} . \tag{6}$$

The wake  $V_0$  in Eq. 5 is determined by the density  $\rho_0(q) = \int f_0(p, q) dp$  using Eq. 2 self-consistently<sup>2</sup>

$$V_0(q) = k \int_{-\infty}^{+\infty} \rho_0(q') W(q'-q) dq'.$$
 (7)

The deformation of the distribution  $f_0$  and the voltage  $V_0$  by the intensity through the wake potential is called "potential-well distortion", which will play an essential role in the longitudinal single-bunch instability. The actual form of the function g in Eq. 4 is not unique, but in the case of an electron-storage ring, the function g must be Gaussian in the p-direction,  $g(H) \propto \exp(-H)$ , to be consistent with the damping and diffusion caused by the synchrotron radiation.

The equilibrium solution Eq. 4 may exist in most cases for arbitrary intensity k, but this does not guarantee the stability of the solution. The stability of the stationary solution Eq. 4 is examined by a linear perturbation. We expand f around the stationary distribution  $f_0$  as  $f(p, q, s) = f_0(p, q) + f_1(p, q, s)$ , and take the first order terms of  $f_1$  in Eq. 1, then obtain

$$-\frac{\partial f_1}{\partial s} = -p\frac{\partial f_1}{\partial q} + (q - V_0(q))\frac{\partial f_1}{\partial p} - V_1(q, s)\frac{\partial f_0}{\partial p}, \qquad (8)$$

where  $V_1$  is the wake voltage induced by  $f_1$ :

$$V_1(q,s) = k \int_{-\infty}^{+\infty} \rho_1(q',s) W(q'-q) dq' = k \int_{-\infty-\infty}^{+\infty+\infty} f_1(p',q',s) W(q'-q) dp' dq' .$$
(9)

One should not neglect or approximate the term of potential-well distortion  $-V_0(q)\frac{\partial f_1}{\partial p}$  in Eq. 8, because it is of the same order in the intensity k as the term  $-V_1(q, s)\frac{\partial f_0}{\partial p}$ . Moreover once the potential-well term is neglected, the consistency of Eq. 8 will be lost, and it leads to unphysical results. For instance, since the wake potential is an internal force of the bunch, a simple sinusoidal motion of the centroid of the bunch, with the frequency  $\omega_s$ , is never affected by the longitudinal wakefield. Therefore there always exists one trivial solution for Eq. 1, the so-called "rigid-dipole mode", which corresponds to the motion of the centroid of the bunch without deformation. This solution is

$$f(p,q,s) = \Re f_0 \left( p + ia \exp(is), q - a \exp(is) \right), \tag{10}$$

where *a* is an arbitrary amplitude of the motion of the centroid. Thus the first order deviation of Eq. 10 from  $f_0$  for a small *a* 

$$f_1 = \Re \left[ a \, \exp(is) \left( -\frac{\partial f_0}{\partial q} + i \frac{\partial f_0}{\partial p} \right) \right] \tag{11}$$

satisfies the first-order equation Eq. 8. If one modifies the potential-well term in Eq. 8, the centroid motion Eq. 11 becomes no longer its solution. Therefore by changing the potential-well term, one may get an unphysical mode of the motion of the bunch instead of the trivial but physical solution. Couplings of such unphysical modes may give incorrect information on the stability.

The nature of the first-order equation Eq. 8 will become clear by introducing the actionangle variables  $(J, \phi)$  which rewrite the Hamiltonian Eq. 5 as H = H(J).<sup>3</sup> These variables reduce Eq. 8 to

$$-\frac{\partial f_1}{\partial s} = \omega(J)\frac{\partial f_1}{\partial \phi} - V_1(q,s)\frac{\partial f_0}{\partial p}$$
  
=  $\omega(J)\frac{\partial f_1}{\partial \phi} - pV_1(q,s)g'(H(J))$ , (12)

where  $\omega(J) = d\phi/ds = \partial H/\partial J$  is the angular frequency of the single-particle motion in the potential well. In Eq. 12, we have applied Eqs. 4 and 5. The term  $pV_1(q, s)$  is further rewritten as

$$pV_{1}(q,s) = k \iint f_{1}(p',q',s)pW(q'-q)dp'dq'$$

$$= k \iint f_{1}(p',q',s)\omega(J)\frac{\partial}{\partial\phi}F(q'-q)dp'dq',$$
(13)

by making use of the equation of motion  $p = -\omega(J)\partial q/\partial \phi$  with a primitive function F of W, *i.e.*, F'(q) = W(q). Thus Eq. 12 becomes

$$-\frac{\partial f_1}{\partial s} = \omega(J)\frac{\partial}{\partial \phi} \left( f_1 - kg'(H(J)) \iint f_1(p',q',s)F(q'-q)dp'dq' \right), \quad (14)$$

which indicates that the flow of the particles is always along the line J = const. even with the wake field. Now it is natural to expand an eigenfunction of Eq. 14 in terms of the azimuthal modes

$$f_1(J,\phi,s) = \sum_m a_m(J) \exp(im\phi - i\mu s) .$$
(15)

Subsituting Eq. 15 into Eq. 14, then integrating it over  $\phi$  after multiplying  $\exp(-im\phi)$ , we obtain

$$a_{m}(J) = -\frac{mk\omega(J)g'(H(J))}{2\pi (\mu - m\omega(J))} \sum_{m'} \iint a_{m'}(J')F(q' - q)\exp(im'\phi' - im\phi)dJ'd\phi'd\phi ,$$
(16)

where we have changed the integration variables p', q' into  $J', \phi'$ . What Eq. 16 implies is that if the eigenmode is stable, *i.e.*,  $\mu$  is real, the radial function  $a_m(J)$  becomes singular at some value of the action where  $m\omega(J) = \mu$ , unless the numerator of Eq. 16 vanishes at that point. Although there may exist a few stable modes without singularities, for instance the rigid-dipole mode, most of the stable modes are expected to have such singularities. On the other hand, an unstable mode with a complex  $\mu$  cannot be singular, since the denominator of Eq. 16 is always finite for any J. This is the most remarkable nature of the motion with the continuous frequency spectra  $\omega(J)$ , which is created by the potential-well distortion. Therefore it should be very hard to observe or excite a particular stable mode of the bunch below the instability threshold, with exceptions like the rigid-dipole mode, whereas the unstable mode is easy to see once the intensity reaches the threshold.

The standard way to obtain the eigenvalue of Eq. 14 is to expand the radial function  $a_m(J)$  in terms of an orthogonal basis, to rewrite Eq. 14 in a matrix form of the expansion coefficients, then to solve the eigensystem of the matrix. Such a method has been tried numerically using a set of piecewise step-functions as the orthogonal basis.<sup>3</sup> In this method the function  $f_1$  is expanded in terms of the orthogonal basis as

$$f_1 = \sum_{jm} m\omega_j \left( -g'_j \Delta J_j \right)^{1/2} h_j(J) \exp(-i\mu s) (C_{jm} \cos m\phi + S_{jm} \sin m\phi) , \qquad (17)$$

where the function  $h_j(J)$  is a step-like function which takes the value  $1/\Delta J_j$  in the strip around the *j*-th mesh point  $J = J_j$  with the thickness  $\Delta J_j$ , and zero outside. We have also used the normalization factor  $m\omega_j \left(-g'_j \Delta J_j\right)^{1/2}$  with  $g'_j \equiv g'(H(J_j))$  and  $\omega_j \equiv \omega(J_j)$  for convention. Here we choose the origin of  $\phi$  on the *q*-axis so that

$$q \to \sqrt{2J}\cos\phi$$
,  $p \to \sqrt{2J}\sin\phi$ , (18)

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in the limit  $k \to 0$ . Since Eq. 12 contains no derivative by J, we do not have to worry about the discontinuity of  $h_j(J)$ . After substituting Eq. 17 to Eq. 14, we multiply  $\left(-\Delta J_j/g'_j\right)^{1/2} h_j(J) \cos m\phi/\pi m\omega_j$  or  $\left(-\Delta J_j/g'_j\right)^{1/2} h_j(J) \sin m\phi/\pi m\omega_j$  on the both side of them, and integrate them over J and  $\phi$ , then obtain

$$i\mu C_{jm} = m\omega_{j}S_{jm}$$
(19)  

$$i\mu S_{jm} = -m\omega_{j}C_{jm} - \frac{k}{\pi}\sum_{j'm'}m'\omega_{j'}\left(-g'_{j}\Delta J_{j}\right)^{1/2}\left(-g'_{j'}\Delta J_{j'}\right)^{1/2}C_{j'm'}$$
  

$$\times \int_{0}^{2\pi}d\phi \int_{0}^{2\pi}d\phi' F\left(q(J_{j'},\phi') - q(J_{j},\phi)\right)\cos m\phi\cos m'\phi' .$$
(20)

We have again assumed smoothness for  $\omega$ , q, and F in the strip  $h_j(J)$  to evaluate their integrals with the values at  $J_j$  and  $J_{j'}$ . Combining Eqs. 19 and 20, we get a linear equation for  $C_{jm}$ :

$$\mu^2 C_{jm} = \sum_{j'm'} M_{jmj'm'} C_{j'm'} , \qquad (21)$$

with

$$M_{jmj'm'} = m^2 \omega_j^2 \delta_{jj'} \delta_{mm'} + \frac{k}{\pi} mm' \omega_j \omega_{j'} \left( -g'_j \Delta J_j \right)^{1/2} \left( -g'_{j'} \Delta J_{j'} \right)^{1/2}$$

$$\times \int_0^{2\pi} \int_0^{2\pi} \cos m\phi \cos m' \phi' F \left( q(J_{j'}, \phi') - q(J_j, \phi) \right) d\phi d\phi' ,$$
(22)

where  $\delta_{jj'}$  is Kronecker's delta. The system becomes unstable when the matrix *M* has a negative or a complex eigenvalue.

Once the wake potential W(q) of a machine is given, we can examine the stability of the equilibrium of Eq. 4 by solving the matrix of Eq. 22 for a given number of mesh points in the *J*-direction and azimuthal modes. This method has been applied to several cases.<sup>4,5</sup> Since the singular nature of the stable modes described above, most eigenfunctions corresponding to stable modes shrink around particular actions  $m\omega(J) \sim \mu$  as pointed out in Ref. 6.

To proceed further in an analytical way, we apply this formulation to special forms of the wake potential: pure-capacitive:  $W(q) = C\theta(q)$ , pure-resistive:  $W(q) = R\delta(q)$ , and pure-inductive:  $W(q) = L\delta'(q)$  wakes. The function  $\theta(q)$  is the step function defined as  $\theta(q) = 1$  when q > 0 and zero otherwise. First in the cases of the pure-capacitive and the pure-inductive wakes, the matrix  $M_{jmj'm'}$  becomes completely symmetric under the exchange of (j, m) and (j', m') indices. In Eq. 22, it is easy to see that the matrix Mbecomes symmetric when F is an even function. In the case of the pure-capacitive wake,

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we can choose F(q) = C(|q|+q)/2. Its first term is even in q and the second term vanishes in the integral of Eq. 22, thus M is symmetric. Also in the pure-inductive case, the matrix becomes symmetric by simply choosing  $F(q) = L\delta(q)$ . Therefore all the eigenvalues of M are real, which results in a stable system for the pure-capacitive and the pure-inductive wakes.

On the other hand, the pure-resistive wake brings a quite different situation. In this case we can use  $F(q) = R(\theta(q) - 1/2)$  which makes the second term of  $M_{jmj'm'}$  in Eq. 22 antisymmetric by exchanging (j, m) and (j', m'). Thus the matrix M becomes antisymmetric except the diagonal elements. The main characteristics of the pure-resistive case can be understood by looking at a 2 by 2 sub-matrix of the big matrix M. Let us pick up 2 by 2 elements of M which belong to the same azimuthal mode m and have different actions  $j_1$  and  $j_2$ . Such a sub-matrix has the form

$$M_{j_1 j_2} = m^2 \begin{pmatrix} \omega_{j_1}^2 & b_m(k) \\ -b_m(k) & \omega_{j_2}^2 \end{pmatrix}$$
(23)

where  $b_m(k)$  is a quantity given by the integral in Eq. 22. The matrix  $M_{j_1j_2}$  is unstable when

$$(\omega_{j_1}^2 - \omega_{j_2}^2)^2 - 4b_m(k)^2 < 0.$$
<sup>(24)</sup>

If the frequencies at two actions are equal or close to each other, *i.e.*,  $\omega_{j_1} \approx \omega_{j_2}$ , the submatrix  $M_{j_1j_2}$  is unstable for any azimuthal mode number *m*. Then the entire matrix *M* can be unstable, if the contributions of other components more or less cancel each other. The condition of Eq. 24 becomes rigorous in the case of the "double-waterbag model", which assumes the equilibrium distribution to be a double-step function.<sup>7</sup> Note that the coupling between two modes with the same azimuthal mode number *m* is only possible under the potential-well distortion, since the coupling terms in Eq. 22 for the same *m* vanishes when there is no distorsion, *i.e.*,  $q = \sqrt{2J} \cos \phi$ .

In the case of the pure-resistive wake, the behavior of  $\omega(J)$  is shown in Figure 1(a). The function actually gives the same frequency for two different actions. This situation suggests that the stationary solution with the pure-resistive wake is always unstable, and we confirmed it by the numerical calculation for the large matrix M. Figure 2 shows the growth rate of several unstable modes for the pure-resistive wake obtained by the large-matrix method with a Gaussian distribution (in this paper we used 60 mesh points in the range  $0 \le J \le 8$ , and azimuthal modes of  $m \le 5$ ). According to the analysis above, this instability can be understood as a coupling of coherent modes with the same azimuthal mode number m.

It is remarkable that all the intensity dependences of the matrix M start from the order of  $k^2$  in the case of the pure-resistive wake. It is due to the fact that the coupling between two radial modes is only caused by the potential-well distortion which raises the order from k to  $k^2$ . As the result the growth rate of the pure-resistive instability starts at the order of  $k^2$ , which agrees with Figure 2. Then the growth of the instability can be very weak at low intensity. In particular in an electron ring, the actual threshold of the instability is determined by the balance between the growth rate and the radiation damping rate of the mode. Although this instability appears in any azimuthal mode number m, their growth and damping rates



FIGURE 1: The normalized synchrotron frequency  $\omega(J)$  of the single-particle motion in the bunch as the function of the action J. (a): the pure-resistive wake  $W(q) = R\delta(q)$ . (b): the pure-inductive wake  $W(q) = L\delta'(q)$ .



FIGURE 2: Growth rates of unstable modes with the pure-resistive wake  $W(q) = R\delta(q)$  obtained from the matrix in Eq. 22. The parameter *m* specifies the nearest integer of the frequency of each mode. It is seen that the growth rate is roughly proportional to  $k^2$ .

are different. We see in Figure 2 that the growth rate is highest for m = 2 especially in the region  $kR \gtrsim 5$ . The threshold becomes lowest for the m = 2 mode with an estimation of the radiation damping rates for these unstable modes.

Next we discuss on the combined wake of the pure-inductive and the pure-resistive cases, *i.e.*,  $W(q) = R\delta(q) + L\delta'(q)$ . In the analysis for the pure-resistive wake, we have made the hypothesis that the degeneration of the synchrotron frequencies for two different actions makes the instability. If the hypothesis is true, the additional inductive part can suppress the instability by boosting the frequency spread in the bunch. Figure 1(b) shows  $\omega(J)$  for the pure-inductive wake. The frequency  $\omega(J)$  for the pure-inductive wake starts below 1 at J = 0, and simply raises toward 1 as the action J increases. Thus when we add the inductive term to the resistive term, the minimal point of  $\omega(J)$  shifts left (toward smaller J). As we increase the inductive part more, the minimal point eventually vanishes, so that the frequency is always different for two distinct values of the action. The condition for a monotonic growth of  $\omega(J)$  is equivalent to

$$\left. \frac{d\omega(J)}{dJ} \right|_{J=0} \ge 0 , \qquad (25)$$

in the combined case of the pure-resistive and the pure-inductive wakes. Now we have reached a hypothetical condition which gives the stability criterion of the pure-resistive+inductive wake. To verify that Eq. 25 gives the threshold of the instability, we have to express it in terms of the intensity and the magnitudes of the wakes. So far we use a Gaussian bunch, but the method is applicable to any distribution with minor changes. First we rewrite the Hamiltonian around its fixed point  $q_0$  for the given intensity and wakes, using the new coordinate  $\overline{q} \equiv q - q_0$ :

$$H(p,\overline{q}) = \frac{p^2}{2} + \frac{(\overline{q}+q_0)^2}{2} - \int_0^{\overline{q}} V_0(\overline{q}'+q_0)dq'$$

$$= \frac{p^2}{2} + \frac{(\overline{q}+q_0)^2}{2} - kR \int_0^{\overline{q}} \rho_0(\overline{q}'+q_0)d\overline{q}' + kL \left(\rho_0(\overline{q}+q_0) - \rho_0(q_0)\right) ,$$
(26)

where we have used Eq. 7 and the combined wake. From the definition of the fixed point  $q_0$ ,

$$0 = \left. \frac{\partial H}{\partial \bar{q}} \right|_{\bar{q}=0} = q_0 - k R \rho_0(q_0) , \qquad \rho_0'(q_0) = 0 , \qquad (27)$$

where Eq. 4 has been applied. We also assume a Gaussian distribution

$$\rho_0(\overline{q}) = A \int_{-\infty}^{+\infty} \exp\left(-H(p,\overline{q})\right) dp , \qquad (28)$$

where A is the normalization factor. To obtain the derivative Eq. 25, we need terms up to the fourth order in  $\overline{q}$  in  $H(p, \overline{q})$ . Such an expansion of  $H(p, \overline{q})$  can be obtained repeatedly by combining Eqs. 26, 27, and 28. The result is

$$H(p,\overline{q}) = \frac{p^2}{2} + \frac{a}{2}\overline{q}^2 + \frac{a^{5/2}q_0}{6}\overline{q}^3 + \frac{a^4q_0(q_0 + 3L/R)}{24}\overline{q}^4 + O(\overline{q}^5), \qquad (29)$$

where  $a \equiv (R/(R + Lq_0))^{1/2}$ . From Eq. 29 it is not difficult to obtain the derivative

$$\left. \frac{d\omega(J)}{dJ} \right|_{J=0} = \frac{a^3 q_0}{24R} \left( 9L - 2Rq_0 \right) \ . \tag{30}$$

Thus the stability condition Eq. 25 is written in a simple form

$$kL \ge \frac{2}{9}kRq_0 \,. \tag{31}$$

Note that the equilibrium position  $q_0$  is a function of kR and kL. Its lowest-order term is given by Eq. 27 as

$$q_0 = \frac{kR}{\sqrt{2\pi}} + \mathcal{O}(k^2) . \tag{32}$$

To examine the validity of the stability condition Eq. 31, we show the growth rate of the combined wake, obtained by the large-matrix method, as a function of both kR and kL in Figure 3. Here we draw the m = 2 mode which gives the highest growth rate among all unstable modes in most cases. We also superimposed the curve given by Eq. 31 on the plot. The fixed point  $q_0$  is obtained by solving Eq. 27 numerically.



FIGURE 3: Contour plot of the growth rate  $Im(\mu)$  of m=2 mode of the combined wake  $W(q)=R\delta(q)+L\delta'(q)$ , obtained from the matrix in Eq. 22. The pitch of the contour is  $\Delta$  Im ( $\mu$ )=0.006. The dashed curve is the stability condition Eq. 31.

This figure shows that Eq. 31, which is derived from Eq. 25, gives a fairly good criterion of stability. This result justifies our basic hypothesis that the degeneration of the synchrotron frequency for two different actions is the source of the weak longitudinal single-bunch instability. Figure 3 and Eq. 31 also suggest that reducing the wakefield by smoothing the beam duct does not always improve the threshold of the instability, unless the resistive part is significantly reduced. Although the bunch-lengthening below threshold is improved, the threshold itself is lowered by reducing only the inductive part of the wake. Even with a more general form of the wake potential than the pure-resistive+inductive model, reducing the frequency spread inside a bunch can be dangerous from the point of view of the stability condition. Indeed an additional frequency spread with a higher harmonic rf accelerating voltage can possibility remove the instability.

The recent results seen at the SLC damping rings with new smooth chambers may be caused by the coupling between radial modes with a wake field close to the pure-resistive one as described in this paper.<sup>5,8</sup>

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