

CERN-TH/95-7
NEIP-95-01
IEM-FT-98/95
McGill-95/03
hep-th/9501065

Gaugino Condensates and Chiral-Linear Duality: an Effective Lagrangian Analysis *

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Abstract

We show how to formulate the phenomenon of gaugino condensation in a super-Yang-Mills theory with a field-dependent gauge coupling described with a linear multiplet. We prove the duality equivalence of this approach with the more familiar formulation using a chiral superfield. In so doing, we resolve a longstanding puzzle as to how a linear-multiplet formulation can be consistent with the dynamical breaking of the Peccei-Quinn symmetry which is thought to occur once the gauginos condense. In our approach, the composite gauge degrees of freedom are described by a real vector superfield, V , rather than the chiral superfield that is obtained in the traditional dual formulation. Our dualization, when applied to the case of several condensing gauge groups, provides strong evidence that this duality survives strong-coupling effects in string theory.

CERN-TH/95-7

January 1995

Work supported in part by the Swiss National Foundation, the European Union (contracts SC1-CT92-0789 and CHRX-CT92-0004) CICYT of Spain (contract AEN94-0928), NSERC of Canada and FCAR of Québec.

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1 Introduction

Dynamical supersymmetry breaking has been extensively studied at the non-perturbative level in $N = 1$ super-Yang-Mills theories with and without matter [1]. It is, in particular, known that supersymmetry does not break in the theory without matter. An early analysis of this phenomenon in the pure super-Yang-Mills theory was given by Veneziano and Yankielowicz [2], whose method has been since extended to various renormalizable models with charged matter¹.

The treatment of (non renormalizable) models having a field-dependent gauge coupling is less straightforward. These models have the additional complication that the gauge coupling is induced by expectation values of scalar fields, which are themselves dynamically generated. This is notably the case for superstring theories, where the gauge coupling is determined by the expectation value of the dilaton field, whose potential arises dynamically by gaugino condensation in a hidden confining gauge sector [4, 5, 6, 7]. The understanding of gaugino condensation and supersymmetry breaking in these theories turns on the description of the dilaton sector.

The supersymmetric partners of the dilaton in the gravity sector of superstrings are an antisymmetric tensor $b_{\mu\nu}$, with gauge symmetry $b_{\mu\nu} \longrightarrow b_{\mu\nu} + \partial_{[\mu} b_{\nu]}$ and a Majorana spinor χ . Since an antisymmetric tensor is by duality equivalent to a pseudoscalar σ , there are two dual kinds of descriptions for the dilaton supermultiplet. Using the original variable, $b_{\mu\nu}$, leads to a linear superfield L [8], while the pseudoscalar, σ , arises in a chiral superfield, S , in which $\sigma = \text{Im } s$ is the imaginary part of the lowest scalar component, s . The pseudoscalar that is obtained in this way enjoys a classical ‘Peccei-Quinn’ (PQ) symmetry of the form

$$S \longrightarrow S + i\alpha, \quad \alpha : \text{a real constant}, \quad (1)$$

which is dual to the gauge symmetry acting on the antisymmetric tensor. The transformation from one of these representations to the other is ‘chiral-linear’ duality.

Past examinations of the problem of gaugino condensation with a field-dependent gauge coupling have followed ref. [5] and have used the chiral-superfield representation of the dilaton. The formulation of gaugino condensation using directly the linear multiplet faces an apparent obstacle. The difficulty lies with performing the chiral-linear duality transformation starting with the chiral superfield, S . This duality transformation requires as its starting point the existence of the classical symmetry (1), but this symmetry is apparently broken by anomalies in the strongly-coupled gauge sector. Such considerations have led some workers to entertain the possibility that the dual theories are *inequivalent* once nonperturbative effects are considered.

In this article we readdress this problem in view of the recent progress in the understanding of the gauge sector of the effective supergravity of superstrings due to string loop calculations in (2,2) models [9, 10, 11]. We show how to perform the condensation analysis directly in terms of the linear multiplet, and we demonstrate how chiral-linear duality survives gaugino condensation. We take as our vehicle for demonstration the cases where a gauge sector with a simple gauge group condenses, as

¹See for instance refs. [3] and the review article [1].

well as the more general situation where several commuting factors of the gauge group separately condense.

We start our discussion with the simplest case of a globally supersymmetric model with a simple gauge group and without charged matter. We then extend our results to several gaugino condensates (non-simple gauge groups). In the string context, these could represent the hidden sector of a (2, 2) model. The introduction of additional gauge-singlet matter (moduli) and the generalization to supergravity are reasonably straightforward [12].

It should be emphasized that we do not expect supersymmetry to break in this simple theory. The effective potential for gaugino condensates with a field-dependent gauge coupling described by a chiral multiplet and no further matter exhibits a ‘run-away’ behaviour, towards the zero-coupling limit. The fact that chiral-linear duality survives gaugino condensation implies that the same behaviour will be obtained using the linear multiplet as starting point.

2 Duality

We start with a discussion of duality for the underlying microscopic theory, well above the condensation scale. The two representations of the dilaton supermultiplet are a chiral superfield, S , and a linear superfield, L . S satisfies the chiral constraint $\overline{\mathcal{D}}_{\dot{\alpha}}S = 0$ and has as components a complex scalar, s , a spinor ψ_s and a complex auxiliary field, f_s . The real linear superfield [8], on the other hand, solves the constraints

$$\overline{\mathcal{D}}\mathcal{D}L = \mathcal{D}\mathcal{D}L = 0,$$

and has as particle content a real scalar, C , an antisymmetric tensor, $b_{\mu\nu}$ (which only appears through its curl $v_{\mu} = \frac{1}{\sqrt{2}}\epsilon_{\mu\nu\rho\sigma}\partial^{\nu}b^{\rho\sigma}$), and a Majorana spinor χ . The description of a field-dependent gauge coupling with a linear superfield requires the introduction of the Chern-Simons superfield Ω , which can be defined by its relation with the chiral superfield of the Yang-Mills field strengths:

$$\text{Tr}(W^{\alpha}W_{\alpha}) = \overline{\mathcal{D}}\mathcal{D}\Omega, \quad \text{Tr}(\overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}) = \mathcal{D}\mathcal{D}\Omega. \quad (2)$$

Here, $W^{\alpha} = -\frac{1}{4}\overline{\mathcal{D}}\mathcal{D}(e^{-\mathcal{A}}\mathcal{D}_{\alpha}e^{\mathcal{A}})$ and \mathcal{A} is the matrix-valued gauge vector superfield². These conditions define Ω up to the addition of a real linear superfield Ω_L . Contrary to $\text{Tr}(W^{\alpha}W_{\alpha})$, the Chern-Simons superfield is not gauge invariant. Since its gauge transformation $\delta\Omega$ satisfies $\mathcal{D}\mathcal{D}\delta\Omega = \overline{\mathcal{D}}\mathcal{D}\delta\Omega = 0$, $\delta\Omega$ is a real linear superfield and Ω_L is in some sense a gauge artefact. To construct invariant couplings, one postulates that the gauge transformation of the linear multiplet is

$$\delta L = 2\delta\Omega,$$

so that the combination

$$\hat{L} = L - 2\Omega$$

² The normalization is $\text{Tr}(\mathcal{A}^2) = \sum_a \mathcal{A}^a \mathcal{A}^a$.

is gauge invariant. The natural physical dimension of L and Ω is (energy)².

A supersymmetric gauge-invariant lagrangian for L takes the form

$$\mathcal{L}_L = 2\mu^2 \int d^2\theta d^2\bar{\theta} \Phi(\hat{L}/\mu^2), \quad (3)$$

where Φ is an arbitrary real function and μ is a scale parameter. If, for instance, \mathcal{L}_L is the low-energy Wilson effective lagrangian for a superstring, in the global supersymmetry limit, then the scale μ can be regarded as the ultra-violet cutoff which defines the Wilson lagrangian.

The action of (3) contains kinetic terms for the components of both superfields L and \mathcal{A} . The gauge kinetic terms are

$$-\frac{1}{2}\Phi_x \text{Tr}(F_{\mu\nu}F^{\mu\nu}), \quad \Phi_x = \left[\frac{d}{dx}\Phi(x) \right]_{x=C\mu^{-2}},$$

which indicates that the gauge coupling constant is

$$\frac{1}{g^2} = 2\Phi_x. \quad (4)$$

It follows that the gauge coupling is a function of the real scalar field C .

We may now dualize this lagrangian to obtain its equivalent in terms of S . To do so, we rewrite (3) in an equivalent form by introducing a real vector superfield V , and replacing (3) by

$$\mathcal{L}_{V+L} = 2\mu^2 \int d^2\theta d^2\bar{\theta} \Phi\left(\frac{V+L}{\mu^2}\right) + \left(\frac{1}{4} \int d^2\theta S\overline{\mathcal{D}\mathcal{D}}(V+2\Omega) + \text{h.c.}\right), \quad (5)$$

where S is a chiral superfield. This new theory is invariant under the gauge transformation

$$V \longrightarrow V + \Delta_L, \quad L \longrightarrow L - \Delta_L, \quad (6)$$

where Δ_L is an arbitrary linear superfield, $\overline{\mathcal{D}\mathcal{D}}\Delta_L = \mathcal{D}\mathcal{D}\Delta_L = 0$. To verify the equivalence with (3), observe that the elimination of S imposes the constraint

$$\overline{\mathcal{D}\mathcal{D}}(V+2\Omega) = \mathcal{D}\mathcal{D}(V+2\Omega) = 0,$$

which in turn indicates that $V+2\Omega = V_L$, a linear multiplet. Gauge invariance (6) allows then the choice $V_L = 0$, which leads again to theory (3). The gauge invariance (6) is a useful tool for the identification of the composite degrees of freedom participating in the effective lagrangian.

To dualize, we instead start by removing V and L in favour of $S + \overline{S}$. Using

$$\frac{1}{4} \int d^2\theta S\overline{\mathcal{D}\mathcal{D}}V + \text{h.c.} = \frac{1}{4} \int d^2\theta S\overline{\mathcal{D}\mathcal{D}}(V+L) + \text{h.c.} = - \int d^2\theta d^2\bar{\theta} (S + \overline{S})(V+L),$$

to rewrite (5), we may in principle perform the integration over $V+L$. Classically, this involves solving the equation of motion for the vector superfield $V+L$ in theory (5), which is

$$2 \left[\frac{d}{dX}\Phi(X) \right]_{X=\frac{V+L}{\mu^2}} = S + \overline{S}. \quad (7)$$

$V + L$ is thereby given implicitly as a function of $S + \bar{S}$: $X(S + \bar{S}) \equiv (V + L)/\mu^2$. The dual theory is then

$$\mathcal{L}_{dual} = \mu^2 \int d^2\theta d^2\bar{\theta} K(S + \bar{S}) + \frac{1}{2} \left(\int d^2\theta S \text{Tr}(W^\alpha W_\alpha) + \text{h.c.} \right), \quad (8)$$

with $K(S + \bar{S})$ the result of integrating over $V + L$. Classically:

$$K(S + \bar{S}) = \left[2\Phi(X) - (S + \bar{S})X \right]_{X=X(S+\bar{S})}. \quad (9)$$

Notice that the lowest component of (7) gives the equality of the two dual expressions for the field-dependent gauge coupling:

$$\frac{1}{g^2} = 2 \text{Re } s = 2\Phi_x. \quad (10)$$

3 Effective Actions

We now turn to a study of gaugino condensation in theory (3). To proceed, we firstly consider the more familiar case of the chiral theory. With chiral multiplets only, a field-dependent gauge coupling can always be introduced with the lagrangian

$$\mathcal{L} = \mathcal{L}_S + \frac{1}{2} \int d^2\theta S \text{Tr}(W^\alpha W_\alpha) + \frac{1}{2} \int d^2\bar{\theta} \bar{S} \text{Tr}(\bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}), \quad (11)$$

The kinetic terms of S are contained in \mathcal{L}_S as well as possible contributions from other chiral multiplets (moduli). Since by assumption the theory does not contain charged matter, \mathcal{L}_S will not depend on \mathcal{A} .

For a non-abelian gauge group, asymptotic freedom of theory (11) leads to a confining regime at a scale where condensates appear. To study the formation of condensates of gaugino bilinears, we follow ref. [13] and compute the generating functional Γ of two-particle-irreducible (2PI) Green's functions³. In the present instance, this is accomplished by coupling an external chiral superfield of currents, J , to $\text{Tr}(W^\alpha W_\alpha)$, which includes gaugino bilinears in its lowest component, as follows:

$$\exp \left\{ i\hat{W}[J, S] \right\} = \int \mathcal{D}\mathcal{A} \exp \left\{ i \int d^4x \int d^2\theta \left(\frac{1}{2} S + J \right) \text{Tr}(W^\alpha W_\alpha) + \text{h.c.} \right\}.$$

We then Legendre transform from the variable J to the variable U , resulting in the 2PI effective action

$$\Gamma[U, S] = \hat{W}[J, S] - \int d^4x \left[\int d^2\theta U J + \text{h.c.} \right],$$

where the chiral superfield U is given by

$$U = \frac{\delta \hat{W}}{\delta J} = \langle \text{Tr}(W^\alpha W_\alpha) \rangle. \quad (12)$$

³This is similar, but not identical [12], to the approach of ref. [2].

Notice that U is not an arbitrary chiral superfield. It follows from relations (2) that a real superfield \tilde{V} exists such that $U = -\frac{1}{2}\overline{\mathcal{D}}\mathcal{D}\tilde{V}$. Clearly \tilde{V} is defined up to the addition of a linear superfield. Integrating over the gauge superfield \mathcal{A} leads to the effective lagrangian for the composite field U and the chiral field-dependent gauge coupling S . One obtains [12]:

$$\begin{aligned}\Gamma[U, S] &= \int d^4x \mathcal{L}_U, \\ \mathcal{L}_U &= \int d^2\theta d^2\bar{\theta} K_U(U, \bar{U}) + \int d^2\theta w_U(U, S) + \int d^2\bar{\theta} \bar{w}_U(\bar{U}, \bar{S}),\end{aligned}\tag{13}$$

with a superpotential given by

$$w_U(U, S) = \frac{1}{2}SU + \frac{A}{12}U \log\left(\frac{U}{M^3}\right) = \frac{A}{12}U \log\left(\frac{Ue^{6S/A}}{M^3}\right), \quad A = \frac{3C(G)}{8\pi^2}.\tag{14}$$

In these expressions, A is the coefficient of the one-loop gauge beta-function, $C(G)$ the quadratic Casimir of the gauge group G and M is a cutoff scale.

The result (14) was first obtained by Veneziano and Yankielowicz [2], without a field-dependent gauge coupling (*i.e.* $S = \text{constant}$), and by Taylor [14], with the chiral superfield S . Although nothing which follows depends on the form taken for the Kähler potential K_U , we assume here for concreteness the expression obtained in ref. [2]: $K_U = h(U\bar{U})^{1/3}$, with h a dimensionless constant.

It is important to recognize [12] that U here is a purely classical field that was obtained by Legendre transforming the externally-applied current, J . It is determined in terms of S by extremizing Γ with respect to variations of U . Once this is done, we can add the result $\mathcal{L}_U(S, U(S))$ to \mathcal{L}_S — the \mathcal{A} -independent part of the original lagrangian (11) — and so obtain an effective lagrangian

$$\mathcal{L}_{chiral} = \mathcal{L}_S + \mathcal{L}_U,\tag{15}$$

which governs the low-energy dynamics of the integration over S .

We now repeat this process using the formulation of the dilaton with a linear multiplet. As point of departure we take expression (5), but this time using the gauge $L = 0$ [or $\Delta_L = L$ in (6)]. With relations (2), lagrangian (5) becomes

$$\mathcal{L}_V = 2\mu^2 \int d^2\theta d^2\bar{\theta} \Phi(V/\mu^2) + \left(\frac{1}{2} \int d^2\theta S \left\{ \text{Tr}(W^\alpha W_\alpha) + \frac{1}{2}\overline{\mathcal{D}}\mathcal{D}V \right\} + \text{h.c.}\right).\tag{16}$$

What is interesting in this last form is that the dependence on the gauge superfield \mathcal{A} is entirely in the term linear in S . And this term is identical to gauge kinetic terms in the usual formulation (11) of the super-Yang-Mills theory, with a field-dependent gauge coupling specified by a chiral superfield. The calculation leading to the effective lagrangian is then identical to the chiral case described above. Borrowing the result, one obtains

$$\begin{aligned}\mathcal{L}_{linear} &= \int d^2\theta d^2\bar{\theta} \left\{ 2\mu^2 \Phi(V/\mu^2) + h(U\bar{U})^{1/3} \right\} \\ &+ \left(\int d^2\theta \left\{ \frac{1}{2}S(U + \frac{1}{2}\overline{\mathcal{D}}\mathcal{D}V) + \frac{A}{12}U \log(U/M^3) \right\} + \text{h.c.} \right).\end{aligned}\tag{17}$$

Recall that the superfield S has been introduced as a Lagrange multiplier in the original theory (16). It plays the same rôle in the effective lagrangian: integrating over S leads to

$$U = -\frac{1}{2}\overline{\mathcal{D}\mathcal{D}}V, \quad (18)$$

and the final form of the effective lagrangian for the linear multiplet theory (3) is

$$\mathcal{L}_{linear} = \int d^2\theta d^2\bar{\theta} \left\{ 2\mu^2\Phi(V/\mu^2) + h(U\bar{U})^{1/3} \right\} + \left\{ \int d^2\theta \frac{A}{12}U \log(U/M^3) + \text{h.c.} \right\}, \quad (19)$$

with U as in eq. (18). We have then obtained an effective theory where the degrees of freedom of the short distance theory, described by a linear multiplet with components $(C, \chi, b_{\mu\nu})$ and a gauge superfield $(\lambda^a, a_\mu^a, D^a)$ (in Wess-Zumino gauge) have been replaced by the components of a complete real vector superfield V (eight bosons and eight fermions).

Having now obtained explicit expressions for the effective (2PI) actions for the low-energy limit of the dual formulations of the underlying microscopic theory, it is instructive to display the equivalence under duality at the level of the effective theories, \mathcal{L}_{linear} of eq. (19) and \mathcal{L}_{chiral} of eqs. (13), (14) and (15), which combine to give:

$$\begin{aligned} \mathcal{L}_{chiral} &= \int d^2\theta d^2\bar{\theta} [\mu^2 K(S + \bar{S}) + h(U\bar{U})^{1/3}] \\ &+ \left(\int d^2\theta \left[\frac{1}{2}SU + \frac{A}{12}U \log(U/M^3) \right] + \text{h.c.} \right). \end{aligned} \quad (20)$$

To obtain \mathcal{L}_{linear} from \mathcal{L}_{chiral} , one uses the fact that the composite superfield U can be obtained from a real vector superfield \tilde{V} by $U = -\frac{1}{2}\overline{\mathcal{D}\mathcal{D}}\tilde{V}$. Then write

$$\begin{aligned} \mu^2 \int d^2\theta d^2\bar{\theta} K + \frac{1}{2} \left(\int d^2\theta SU + \text{h.c.} \right) &= \int d^2\theta d^2\bar{\theta} [\mu^2 K + (S + \bar{S})\tilde{V}] \\ &= \int d^2\theta d^2\bar{\theta} [2\mu^2\Phi(X) + (S + \bar{S})(\tilde{V} - \mu^2 X)]. \end{aligned} \quad (21)$$

The last equality uses the microscopic duality result to express K as a function of X . Integration over S then imposes the constraint $X = \tilde{V}\mu^{-2}$, leading to

$$\mu^2 \int d^2\theta d^2\bar{\theta} K + \frac{1}{2} \left(\int d^2\theta SU + \text{h.c.} \right) = 2\mu^2 \int d^2\theta d^2\bar{\theta} \Phi(\tilde{V}/\mu^2), \quad (22)$$

which completes the proof of the equivalence of theories (19) and (20).

We remark that the shift symmetry (1) has a macroscopic counterpart in the effective lagrangian (20) because of the constraint (18) which is satisfied by the composite chiral superfield U . Imposing that transformation (1) generates an anomaly leads to the condition

$$\frac{i}{2}\alpha \left(\int d^2\theta U - \int d^2\bar{\theta} \bar{U} \right) = 2\alpha \text{Im} f_u = \text{a total derivative } (\partial^\mu \tilde{v}_\mu) \quad (23)$$

(α is a real constant), which in turn implies $U = -\frac{1}{2}\overline{\mathcal{D}\mathcal{D}}\tilde{V}$. The right-hand-side of eq. (23) is the macroscopic version of the anomaly. This indicates that symmetry (1) is broken by non-perturbative effects.

4 Component Expressions

The physical content of these manipulations becomes clearer once the above expressions are expanded in terms of the components of the various superfields, as we do in this section.

The effective lagrangian (17) for the linear multiplet theory describes eight bosonic and eight fermionic degrees of freedom. The vector superfield V replaces the combination $L - 2\Omega$, which appears in the microscopic theory (3). It is to be functionally integrated in the low-energy theory, subject to the constraint, $U = -\frac{1}{2}\overline{\mathcal{D}}\mathcal{D}V$, which fixes four bosonic and four fermionic components in terms of the components of the chiral field $U = \langle \text{Tr}(W^\alpha W_\alpha) \rangle$. Those degrees of freedom in V which are not included in U represent the gauge invariant completion of the degrees of freedom of the original linear superfield L . Indeed, the invariance (6) applied to the lagrangian (5) indicates that L can be absorbed in V . To study the component expansion of the effective theory, it is therefore convenient to use an expansion of V which makes this gauge transformation (6) transparent. If the expansion of the superfield parameter is

$$\Delta_L = c_L + i\theta\varphi_L - i\overline{\theta}\overline{\varphi}_L - \theta\sigma^\mu\overline{\theta}(v_L)_\mu + \frac{1}{2}\theta\theta\overline{\theta}\partial_\mu\varphi_L\sigma^\mu + \frac{1}{2}\overline{\theta}\theta\theta\sigma^\mu\partial_\mu\overline{\varphi}_L + \frac{1}{4}\theta\theta\overline{\theta}\square c_L, \quad (24)$$

with

$$(v_L)_\mu = \frac{1}{\sqrt{2}}\epsilon_{\mu\nu\rho\sigma}\partial^\nu b_L^{\rho\sigma}, \quad [\text{or } \partial^\mu(v_L)_\mu = 0],$$

it is natural to use for V the expansion

$$\begin{aligned} V = & c + i\theta\varphi - i\overline{\theta}\overline{\varphi} - \frac{1}{2}\theta\theta\overline{m} - \frac{1}{2}\overline{\theta}\theta m - \theta\sigma^\mu\overline{\theta}v_\mu \\ & + \frac{1}{2}\theta\theta\overline{\theta}(\overline{\Lambda} + \partial_\mu\varphi\sigma^\mu) + \frac{1}{2}\overline{\theta}\theta\theta(\Lambda + \sigma^\mu\partial_\mu\overline{\varphi}) + \theta\theta\overline{\theta}(d + \frac{1}{4}\square c). \end{aligned} \quad (25)$$

With this choice, the transformation (6) shifts c and φ , while leaving m , Λ and d invariant, Its action on v_μ is

$$v_\mu \longrightarrow v_\mu + \frac{1}{\sqrt{2}}\epsilon_{\mu\nu\rho\sigma}\partial^\nu b_L^{\rho\sigma},$$

which is the gauge transformation of the three-index antisymmetric tensor

$$v_\mu = \frac{1}{\sqrt{2}}\epsilon_{\mu\nu\rho\sigma}h^{\nu\rho\sigma}. \quad (26)$$

The real superfield V with gauge invariance (6) is then the supersymmetric description of the three-index tensor [15], while the invariant chiral superfield $U = -\frac{1}{2}\overline{\mathcal{D}}\mathcal{D}V$ is the supersymmetrization of its gauge-invariant curl. Using the expansion

$$U = u - i\theta\sigma^\mu\overline{\theta}\partial_\mu u - \frac{1}{4}\theta\theta\overline{\theta}\square u + \sqrt{2}\theta\psi_u + \frac{i}{\sqrt{2}}\theta\theta(\partial_\mu\psi_u\sigma^\mu\overline{\theta}) - \theta\theta f_u,$$

its components are given by

$$u = -m; \quad \sqrt{2}\psi_u = \Lambda; \quad f_u = -2d + i\partial^\mu v_\mu = -2d + \frac{i}{\sqrt{2}}\epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma}, \quad (27)$$

where the curl

$$\epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma} \partial^{[\mu} h^{\nu\rho\sigma]} = \sqrt{2} \partial^\mu v_\mu$$

describes a single degree of freedom. The last equation (27) contains an important piece of information: writing a chiral superfield in the form $U = -\frac{1}{2} \overline{\mathcal{D}} \mathcal{D} V$ implies that $\text{Im } f_u = \frac{1}{\sqrt{2}} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho\sigma}$. For all other components, (18) is a simple change of variables, from (m, Λ, d) to $(u, \psi_u, \text{Re } f_u)$. In other words, if a chiral superfield U is such that the imaginary part of its f_u component is a total derivative $\partial^\mu v_\mu$, then there exists a vector superfield V , defined up to the addition of a linear superfield, such that $U = -\frac{1}{2} \overline{\mathcal{D}} \mathcal{D} V$.

Since the fields c , φ and the transverse part v_μ^\perp of the vector v_μ form the components of a linear multiplet [see eq. (24)], they do not appear in (27). The components of V which appear in eq. (27) can be expressed in terms of the underlying composite degrees of freedom, following (12). That is, $U = \langle \text{Tr}(W^\alpha W_\alpha) \rangle$ implies

$$\begin{aligned} m &= \langle \text{Tr}(\lambda\lambda) \rangle, \\ \Lambda &= \langle -2i \text{Tr}(\lambda D) + \text{Tr}(F_{\mu\nu} \sigma^\mu \overline{\sigma}^\nu \lambda) \rangle, \\ d &= \langle \text{Tr}(-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{i}{2} \lambda \sigma^\mu D_\mu \overline{\lambda} - \frac{i}{2} D_\mu \lambda \sigma^\mu \overline{\lambda} + \frac{1}{2} D^2) \rangle, \\ \partial^\mu v_\mu &= \langle \text{Tr}(\frac{1}{4} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} - \partial_\mu [\lambda \sigma^\mu \overline{\lambda}]) \rangle. \end{aligned} \tag{28}$$

We see that the gaugino condensate is described either by the complex scalar m — which is the $\overline{\theta\theta}$ component of V — or the lowest component u of U . Notice also that the quantity $\partial^\mu v_\mu$ (which is constrained to equal the imaginary part of f_u) corresponds as it should to the anomalous divergence of the supersymmetry chiral current [16].

The (gauge-invariant) low-energy states c , φ and v_μ^\perp , on the other hand, correspond to the physical degrees of freedom C , χ and $b_{\mu\nu}$ of the linear multiplet present at the microscopic level, supplemented by the gauge-variant part of the Chern-Simons superfield. For instance, computing the Chern-Simons superfield in the Wess-Zumino gauge, the correspondence would be

$$\begin{aligned} c &\longleftrightarrow C, \\ \varphi &\longleftrightarrow \chi + \frac{i}{2} \text{Tr}(\sigma^\mu \overline{\lambda} a_\mu), \\ v_\mu &\longleftrightarrow \frac{1}{\sqrt{2}} \epsilon_{\mu\nu\rho\sigma} (\partial^\nu b^{\rho\sigma} + \sqrt{2} \omega^{\nu\rho\sigma}) - \text{Tr}(\lambda \sigma^\mu \overline{\lambda}), \end{aligned} \tag{29}$$

where the normalization of the bosonic Chern-Simons form is $\epsilon_{\mu\nu\rho\sigma} \text{Tr}(F^{\mu\nu} F^{\rho\sigma}) = 4 \epsilon_{\mu\nu\rho\sigma} \partial^\mu \omega^{\nu\rho\sigma}$. Notice that the expression for the longitudinal part of v_μ is compatible with the last equation (28). In contrast with the components of U , the fields c , φ and v_μ^\perp should be regarded as the quantum fields of the effective theory far below the condensation scale.

The component expansion of the effective lagrangian (19) is now easily obtained, using eqs. (25) and (27). It is as usual the sum of bosonic contributions and terms quadratic or quartic in fermions:

$$\mathcal{L}_{linear} = \mathcal{L}_{bos.} + \mathcal{L}_{quad.} + \mathcal{L}_{quart.} .$$

Using the notation

$$\Phi_x = \left[\frac{d\Phi(x)}{dx} \right]_{x=c\mu^{-2}}, \quad \Phi_{xx} = \left[\frac{d^2\Phi(x)}{dx^2} \right]_{x=c\mu^{-2}}, \quad \dots,$$

the bosonic contributions which determine the vacuum structure of the theory are:

$$\begin{aligned} \mathcal{L}_{bos.} = & \frac{1}{2}\mu^{-2}\Phi_{xx} [m\bar{m} + v^\mu v_\mu - (\partial_\mu c)(\partial^\mu c)] + 2\Phi_x d \\ & + \frac{h}{9}(m\bar{m})^{-2/3} [(\partial_\mu \bar{m})(\partial^\mu m) + 4d^2 + (\partial_\mu v^\mu)^2] \\ & + \frac{A}{6}d \left[2 + \log\left(\frac{m\bar{m}}{M^6}\right) \right] + \frac{A}{12}i(\partial^\mu v_\mu) \log\left(\frac{\bar{m}}{m}\right). \end{aligned} \quad (30)$$

Since a gaugino condensate $\langle \text{Tr}(\lambda\lambda) \rangle$ is described by the expectation value of m , its phase only appears in the coupling $\frac{A}{12}i(\partial^\mu v_\mu) \log(\bar{m}/m)$ which arises in the effective theory as a consequence of the anomaly of R-symmetry [see refs. [2, 16] and the last equation (28)]. The terms quadratic in fermion fields are:

$$\begin{aligned} \mathcal{L}_{quad.} = & -\frac{i}{2}\mu^{-2}\Phi_{xx}(\bar{\varphi}\gamma^\mu\partial_\mu\varphi) + i\frac{h}{36}(m\bar{m})^{-2/3}(\bar{\Lambda}\gamma^\mu\partial_\mu\Lambda) \\ & -\frac{1}{4}\mu^{-4}\Phi_{xxx} [m\bar{\varphi}_R\varphi_L + \bar{m}\bar{\varphi}_L\varphi_R - v^\mu(\bar{\varphi}\gamma_\mu\gamma_5\varphi)] \\ & -\frac{i}{2}\mu^{-2}\Phi_{xx} [\bar{\varphi}_R\Lambda_L - \bar{\varphi}_L\Lambda_R] + i\frac{h}{108}(m\bar{m})^{-2/3}(\bar{\Lambda}\gamma^\mu\gamma_5\Lambda)\partial_\mu(\log\frac{\bar{m}}{m}) \\ & -\frac{h}{27}(m\bar{m})^{-2/3}d [m^{-1}\bar{\Lambda}_R\Lambda_L + \bar{m}^{-1}\bar{\Lambda}_L\Lambda_R] \\ & -i\frac{h}{54}(m\bar{m})^{-2/3}(\partial^\mu v_\mu) [m^{-1}\bar{\Lambda}_R\Lambda_L - \bar{m}^{-1}\bar{\Lambda}_L\Lambda_R] \\ & + \frac{A}{48}m^{-1}\bar{\Lambda}_R\Lambda_L + \frac{A}{48}\bar{m}^{-1}\bar{\Lambda}_L\Lambda_R. \end{aligned} \quad (31)$$

Finally, the quartic fermionic terms are simply

$$\mathcal{L}_{quart.} = \frac{1}{8}\mu^{-6}\Phi_{xxxx}(\bar{\varphi}_R\varphi_L)(\bar{\varphi}_L\varphi_R) - \frac{19h}{324}(m\bar{m})^{-5/3}(\bar{\Lambda}_R\Lambda_L)(\bar{\Lambda}_L\Lambda_R). \quad (32)$$

We may now use this lagrangian to determine the theory's vacuum structure. The first step is to eliminate the constrained field, d , using its equation of motion. Since d is auxiliary its equation may be solved algebraically, with solution

$$d = -\frac{9}{8h}(m\bar{m})^{2/3} \left\{ 2\Phi_x + \frac{A}{6} \left[2 + \log\left(\frac{m\bar{m}}{M^6}\right) \right] \right\} + \frac{1}{24} \left\{ m^{-1}\bar{\Lambda}_R\Lambda_L + \bar{m}^{-1}\bar{\Lambda}_L\Lambda_R \right\}. \quad (33)$$

Using this to eliminate d in the lagrangian gives the following scalar potential for the fields c and m^4

$$\begin{aligned} V_{linear} = & \frac{4h}{9}(m\bar{m})^{-2/3}d^2 - \frac{1}{2}\mu^{-2}\Phi_{xx}m\bar{m} \\ = & \frac{9}{16h}(m\bar{m})^{2/3} \left\{ 2\Phi_x + \frac{A}{6} \left[2 + \log\left(\frac{m\bar{m}}{M^6}\right) \right] \right\}^2 - \frac{1}{2}\mu^{-2}\Phi_{xx}m\bar{m}. \end{aligned} \quad (34)$$

⁴The potential depends on c via $x = c\mu^{-2}$ and $\Phi = \Phi(x)$.

Since the quantities $h(m\bar{m})^{-2/3}$ and $-\Phi_{xx}$ appear in the kinetic terms for m and c respectively [see (30)], these quantities must be positive. The scalar potential is therefore the sum of two non-negative terms, and so is minimized when they both vanish. We are therefore led to the conditions $d = m = 0$, leaving only the expectation value of the lowest component, c , undetermined (so far). The condition $d = 0$ implies

$$m\bar{m} = M^6 e^{-2} \langle e^{-32\pi^2 \phi_x / C(G)} \rangle. \quad (35)$$

Since the gauge coupling constant in the microscopic theory is $g^{-2} = 2\langle \Phi_x \rangle$, which is a function of the free quantity $\langle c \rangle / \mu^{-2}$, we see that (35) has the usual form

$$|m| = M^3 e^{-1} e^{-8\pi^2 g^{-2} / C(G)}.$$

Equation (35) exhibits the familiar ‘runaway’ behaviour of the theory with a field-dependent gauge coupling and no matter. That is, the minimization condition $m = 0$ now implies that $\langle c \rangle$ prefers to take values for which the gauge coupling vanishes, for which condensates do not form and supersymmetry does not break.

It is natural to identify the scale μ which appears in the microscopic lagangian (3) with the ultra-violet cutoff, M , of the macroscopic theory. In this case, the Wilson gauge coupling, $2\Phi_x$, is the Wilson gauge coupling defined at scale M , and condensates should form at the renormalization-group invariant scale $M_{cond.} \sim M \exp[-1/Ag^2(M)]$.

To further sharpen the previous discussion concerning the duality between the two descriptions of the gaugino condensation, we now compare the above discussion with the equivalent analysis using the chiral superfield S . For the chiral theory defined by eq. (20), the scalar potential (after eliminating the auxiliary fields f_u and f_s) is

$$\begin{aligned} V_{chiral} &= \frac{h}{9} (u\bar{u})^{-2/3} f_u \bar{f}_u + \mu^2 K_{s\bar{s}} f_s \bar{f}_s \\ &= \frac{9}{4h} (u\bar{u})^{2/3} \left| s + \frac{A}{6} \left[1 + \log\left(\frac{u}{M^3}\right) \right] \right|^2 + \frac{1}{4} \mu^{-2} K_{s\bar{s}}^{-1}(u\bar{u}), \end{aligned} \quad (36)$$

where s and u are the lowest complex scalar components of the chiral superfields S and U . Since $K_{s\bar{s}} = \frac{\partial^2}{\partial s \partial \bar{s}} K(s + \bar{s})$ is the kinetic metric for s , this potential is once more the sum of two non-negative terms, and so is minimized by $f_u = f_s = 0$.

These conditions can be compared to those obtained using the linear multiplet by using the duality relations (7) and (9), which imply

$$K_{s\bar{s}} = -\frac{1}{2} [\Phi_{xx}]_{x=x(s+\bar{s})}^{-1}.$$

Since $u\bar{u} = m\bar{m}$, the second term in V_{chiral} is clearly identical to the second term in V_{linear} . In contrast with the similar term in eq. (34), the first contribution in V_{chiral} is proportional to the square of the absolute value of a complex quantity. Cancelling the first term in V_{chiral} requires

$$u = M^3 e^{-1} \langle e^{-16\pi^2 s / C(G)} \rangle, \quad (37)$$

an equation relating the *complex* quantities u and s . Its absolute value is identical to (35) since $|u| = |m|$ and $\text{Re } s = \Phi_x$ according to eq. (7). But the argument of eq. (37),

$$\langle s - \bar{s} \rangle - \frac{A}{6} \log(\bar{m}/m) = 0, \quad (38)$$

which relates the phase of the gaugino condensate m to $\langle \text{Im } s \rangle$, does not exist in the case of the linear multiplet.

To make the comparison explicit, separate $\text{Re } f_u$ and $\text{Im } f_u$ in V_{chiral} , and write

$$\begin{aligned} V_{chiral} = & \frac{9}{16h} (m\bar{m})^{2/3} \left\{ s + \bar{s} + \frac{A}{6} [2 + \log(\frac{m\bar{m}}{M^6})] \right\}^2 - \frac{1}{2} \mu^{-2} \Phi_{xx} m\bar{m} \\ & - \frac{9}{16h} (m\bar{m})^{2/3} \left\{ s - \bar{s} - \frac{A}{6} \log(\frac{\bar{m}}{m}) \right\}^2. \end{aligned} \quad (39)$$

Since eq. (7) allows us to rewrite the potential (34) as

$$V_{linear} = \frac{9}{16h} (m\bar{m})^{2/3} \left\{ s + \bar{s} + \frac{A}{6} [2 + \log(\frac{m\bar{m}}{M^6})] \right\}^2 - \frac{1}{2} \mu^{-2} \Phi_{xx} m\bar{m}, \quad (40)$$

one obtains

$$V_{chiral} = V_{linear} + \frac{h}{9} (m\bar{m})^{-2/3} (\text{Im } f_u)^2. \quad (41)$$

The two potentials (39) and (40) differ by the last term in eq. (39), which leads to the supplementary vacuum equation (38). Since we have explicitly proven the equivalence of the two dual theories, see (22), the information contained in both theories must however be the same.

To understand the meaning of the vacuum equation (38), which is equivalent to $\text{Im } f_u = 0$, it is useful to recall two facts about the chiral effective lagrangian (20). Firstly, the superpotential (14) is invariant under an R-symmetry rotation of the composite superfield U , which rotates the gaugino condensate by a phase β , combined with the shift $S \rightarrow S - \frac{A}{6} i\beta$, which only acts on $\text{Im } s$. This invariance is apparent in eq. (38). Secondly, apart from the last term in potential (39), \mathcal{L}_{chiral} only depends on quantities which are invariant under a phase rotation of m or a shift of $\text{Im } s$, like $(m\bar{m})$, $\partial_\mu \log(\bar{m}/m)$, $s + \bar{s}$ or $\partial_\mu(\text{Im } s)$. The effective theory \mathcal{L}_{chiral} is then completely insensitive to the choice of the phase of $\langle m \rangle$: it is the rôle of $\text{Im } s$ to cancel any anomalous dependence on the phase of the gaugino condensate. The minimum equation (38) ensures, then, that there exists a vacuum for all choices of the phase of the gaugino condensate, and that the physics of the effective theory does not depend on this phase. In the dual effective lagrangian \mathcal{L}_{linear} , this information is hidden in the use of the real vector superfield V for describing the condensate. As a consequence, all terms in \mathcal{L}_{linear} are explicitly invariant under a phase rotation of m , and the potential (34) only depends on $m\bar{m}$.

It appears then that the PQ symmetry (1) is not broken by the potential for $\text{Im } s$. This is no surprise, since this symmetry is actually used to construct the U dependence of the original 2PI action (13). The same is not true in more general cases, such as for

several condensates which we consider next, where our duality construction nevertheless applies equally well, even though the symmetry (1) can be broken⁵.

It is instructive to see how the usual conclusions concerning supersymmetry breaking in this scenario emerge in the linear-multiplet formulation. As is well known, unlike the case for constant gauge couplings [2], a nonvanishing value of the gaugino condensate necessarily breaks supersymmetry. We see this in the linear multiplet approach because the gaugino condensate is represented by the field m , which is not the first component of a superfield. As a result any nonvanishing value for m must necessarily break supersymmetry. The same is not so in the chiral representation, where the condensate is the lowest component of a supermultiplet. In the chiral case, supersymmetry breaking by gaugino condensation instead emerges because a nonzero condensate generates a scalar potential for s which would not be minimized at zero energy. (This is reminiscent of the discussion in [10, 11] where a similar conclusion was reached in a toy model.) Notice, however, that a nonvanishing condensate is not obtained in either theory because of the runaway behaviour of both potentials, which drives the gaugino condensate to zero.

5 Several condensates

The extension of our results to a semi-simple gauge group⁶ and with one linear multiplet is simple and interesting. The microscopic lagrangian is again (3), but with

$$\hat{L} = L - 2 \sum_a c_a \Omega_a,$$

the index a labelling the simple group factors. The positive constants c_a specify the normalization of the unified (Wilson) couplings, $g_a^{-2} = 2c_a \Phi_x$. The equivalent lagrangian \mathcal{L}_V , as in eq. (16), is

$$\mathcal{L}_V = 2\mu^2 \int d^2\theta d^2\bar{\theta} \Phi(V/\mu^2) + \left(\frac{1}{2} \int d^2\theta S \left\{ \sum_a c_a \text{Tr}_a(W_a^\alpha W_{a\alpha}) + \frac{1}{2} \overline{\mathcal{D}\mathcal{D}V} \right\} + \text{h.c.} \right).$$

And the effective lagrangian (17) becomes

$$\begin{aligned} \mathcal{L}_{eff.} = & \int d^2\theta d^2\bar{\theta} \left\{ 2\mu^2 \Phi(V/\mu^2) + \sum_a h_a (U_a \bar{U}_a)^{1/3} \right\} \\ & + \left(\int d^2\theta \left\{ \frac{1}{2} S (\sum_a c_a U_a + \frac{1}{2} \overline{\mathcal{D}\mathcal{D}V}) + \frac{1}{12} \sum_a A_a U_a \log(U_a/M^3) \right\} + \text{h.c.} \right). \end{aligned} \tag{42}$$

⁵A recent attempt to describe gaugino condensation with a linear multiplet [17] only applied to the simplest case of one gaugino condensate. This corresponds to an effective theory for S with a simple exponential superpotential. The possibility of an overall exponential superpotential compatible with symmetry (1) was already pointed out in refs. [18, 19, 20, 11]. We are also aware that P. Binétruy, M.K. Gaillard and T. Taylor are making progress along a similar direction.

⁶ Abelian factors are not asymptotically-free.

The S -dependent term generates as before the anomaly of the PQ symmetry. Integration over S imposes the constraint

$$\sum_a c_a U_a = -\frac{1}{2} \overline{\mathcal{D}\mathcal{D}} V, \quad (43)$$

which generalizes eq. (18) and corresponds to $-\frac{1}{2} \overline{\mathcal{D}\mathcal{D}} \hat{L} = \sum_a c_a \text{Tr}_a (W_a^\alpha W_{a\alpha})$, which is valid in the microscopic theory.

Before proceeding to analyze the vacuum structure of this model, recall first the results for the chiral version of the theory. The chiral version is obtained by dualizing the above results, giving

$$\begin{aligned} \mathcal{L}_{chiral} = & \int d^2\theta d^2\bar{\theta} \left[\mu^2 K(S + \bar{S}) + \sum_a h_a (U_a \bar{U}_a)^{1/3} + (S + \bar{S})V \right] \\ & + \left(\frac{1}{12} \sum_a A_a \int d^2\theta U_a \log(U_a/M^3) + \text{h.c.} \right), \end{aligned} \quad (44)$$

where the vector superfield V is related to the chiral U_a by the constraint (43). The anomaly of the PQ symmetry (1) has been transformed into the invariant term $(S + \bar{S})V$ by a partial integration.

The scalar potential for the chiral version of the theory, (44), taking (43) into account, is

$$\begin{aligned} V = & \mu^2 K_{s\bar{s}} f_s \bar{f}_s + \frac{1}{9} \sum_a h_a (u_a \bar{u}_a)^{-2/3} f_a \bar{f}_a \\ = & \sum_a \frac{9}{4h_a} (u_a \bar{u}_a)^{2/3} \left| c_a s + \frac{A_a}{6} \left[1 + \log\left(\frac{u_a}{M^3}\right) \right] \right|^2 + \frac{1}{4} \mu^{-2} K_{s\bar{s}}^{-1} \left(\sum_a c_a u_a \right) \left(\sum_b c_b \bar{u}_b \right), \end{aligned} \quad (45)$$

where u_a and f_a are the lowest and highest components of U_a . This potential is the sum of non-negative terms, and so is minimized when each vanishes. The condition for the vanishing of each f_a is

$$u_a = M^3 e^{-1} e^{-6c_a s/A_a} = M^3 e^{-1} e^{-16\pi^2 s/C(G_a)}, \quad (46)$$

where we recall that the gauge coupling is $g_a^{-2} = 2 \text{Re } s$, as in eq. (10). This establishes the size of each condensate as a function of the scalar field s . The other condition for minimizing the potential is the vanishing of the auxiliary field f_s , which gives:

$$\sum_a c_a u_a = 0, \quad (47)$$

Notice that this sum can be zero without requiring each of the condensates, u_a , to separately vanish, because $\text{Im } s$ — which controls the phases of the u_a 's through (46) — can adjust to permit their cancellation. But given our experience with the linear multiplet, for which no analogue of $\text{Im } s$ exists in the scalar potential, it would appear to be problematic to similarly ensure the cancellation of the u_a 's. As we shall see shortly, however, a careful analysis reveals similar conditions for the linear multiplet.

In order to perform the analysis for the linear multiplet, we have to use two of the components of constraint (43):

$$\sum_a c_a u_a = -m, \quad \text{and} \quad \sum_a c_a f_a = -2d + i\partial^\mu v_\mu, \quad (48)$$

Unlike the single condensate case, we cannot use these relations to eliminate all the auxiliary fields f_a as functions of d and $\partial^\mu v_\mu$. Instead, we can use them to eliminate m and d in terms of u_a and $R_a \equiv \text{Re } f_a$ and reintroduce the Lagrange multiplier $\text{Im } s$ (the axion) to impose the restriction on $I_a \equiv \text{Im } f_a$. With this information we can write the bosonic part of the lagrangian (42) as:

$$\begin{aligned} \mathcal{L}_{bos.} = & \frac{1}{2}\mu^{-2}\Phi_{xx} \left[|\sum_a c_a u_a|^2 + v^\mu v_\mu - (\partial_\mu c)(\partial^\mu c) \right] + \sum_a K_a (\partial_\mu u_a)(\partial^\mu \bar{u}_a) - \Phi_x \sum_a c_a R_a \\ & + \sum_a K_a (R_a^2 + I_a^2) - 2 \sum_a (R_a \Gamma_a + I_a \Lambda_a) + \text{Im } s (\sum_a c_a I_a - \partial_\mu v^\mu) \end{aligned} \quad (49)$$

Where we have set $K_a \equiv \frac{h_a}{g} (u_a \bar{u}_a)^{-2/3}$ and we had decomposed the derivatives of the superpotential $W \equiv \sum_a A_a U_a \log(U_a/M^3)$ in their real and imaginary parts: $W_a \equiv \Gamma_a + i\Lambda_a$. We can now solve for the auxiliary fields $R_a, I_a, \text{Im } s$ leading to a scalar potential⁷:

$$\begin{aligned} V_{linear} = & -\frac{1}{2}\mu^{-2}\Phi_{xx} |\sum_a c_a u_a|^2 + \sum_a K_a^{-1} \left(\Gamma_a + \frac{1}{2}\Phi_x c_a \right)^2 \\ & + Q^{-2} \sum_a K_a^{-1} \left[\sum_b K_b^{-1} c_b (c_b \Lambda_a - c_a \Lambda_b) \right]^2, \end{aligned} \quad (50)$$

with $Q \equiv (\sum_a c_a^2 K_a^{-1})$, and to the following derivative couplings for the v^μ field.

$$\mathcal{L}_{der.} = Q^{-1} \left[(\partial_\mu v^\mu)^2 - 2 \sum_a K_a^{-1} c_a \Lambda_a (\partial_\mu v^\mu) \right] \quad (51)$$

Since the potential is the sum of three positive terms, its minimum corresponds to the points where each of the three terms vanishes. This solutions is of course supersymmetric. Vanishing of the second term fixes the magnitude of u_a , vanishing of the third term implies $\Lambda_a = \xi c_a$ with ξ arbitrary, this implies:

$$u_a = M^3 e^{-1} e^{-\frac{c_a}{2A_a}(\Phi_x - 2i\xi)} \quad (52)$$

Finally vanishing of the first term in (50) fixes Φ_x and ξ , reproducing the situation of the chiral case in (47).

Therefore we have seen that the structure of the scalar potential is identical in the two dual theories, in the sense that they both give the same vacuum with nonvanishing gaugino condensates and unbroken supersymmetry as long as $\sum_a c_a u_a = 0$. For the linear multiplet case this conclusion follows quite naturally, since it is only this particular susy-breaking combination that corresponds to the $\theta\theta$ component of a superfield. This precisely corresponds to the situation in the dual chiral version for which the potential

⁷Had we solved for the field v^μ instead of $\text{Im } s$ we would have recovered the potential for $\text{Im } s$ which is the axion field in the chiral version.

can develop a minimum at $\sum_a c_a u_a = 0$ without breaking supersymmetry [21]. This is true even though in the linear version there is no axion field developing a potential and therefore a mass, as it happens in the chiral case.

Also one can easily see from (45) that, as anticipated, the PQ symmetry (1) is *not* conserved in the presence of several condensates. In fact the first term in (45) is invariant under (1) if we make the shifts $u_a \rightarrow u_a \exp\{-i(16\pi^2\alpha/C(G_a))\}$. However since the shifts are G_a -dependent, the second term in (45) is not invariant under the shift (1). This explains why the potential develops a mass term for the axion field. It is remarkable, then, that the theory in this case nevertheless has a dual version.

It is instructive to ask how this propagating massive degree of freedom looks in the dual theory. This is particularly so considering that the usual mechanism for giving mass to an antisymmetric tensor gauge field — the Higgs mechanism whereby it and an ordinary spin-one gauge field combine into a massive spin-one particle — is not available in the present case. Inspection of the previous formulae shows that this degree of freedom is described by the vector field v^μ , in the linear-multiplet version, and that it describes a massive spinless particle in a somewhat unusual way.

These conclusions follow from the lagrangian for v^μ , which is

$$\mathcal{L}_{v^\mu} = \frac{1}{2}\mu^{-2}\Phi_{xx}v^\mu v_\mu + Q^{-1} \left[(\partial_\mu v^\mu)^2 - 2 \sum_a K_a^{-1} c_a \Lambda_a (\partial_\mu v^\mu) \right], \quad (53)$$

with the coefficients of each term now taken to be the value of the corresponding function in the vacuum. Clearly, this lagrangian describes a propagating massive scalar degree of freedom given by the longitudinal part of v^μ (the transverse spin one components do not propagate) corresponding to the axion. The only component of this field whose time derivatives appear in \mathcal{L}_{v^μ} is v^0 , so the other three components can be considered to be auxiliary fields. The propagator for v^μ which follows from \mathcal{L}_{v^μ} can be computed, and is (taking, for simplicity, $\frac{1}{2}\mu^{-2}\Phi_{xx} = -1$), $\delta_{\mu\nu} - \frac{k_\mu k_\nu}{Q+k^2}$. A similar propagator for the massive axion has also recently been used by R. Kallosh *et al* [22] in their discussion of the axion mass ($m^2 = Q$). We have, therefore, a lagrangian description of a massive axion in terms of a vector field or, equivalently, a massive 3-index antisymmetric tensor field.

Supersymmetrically, the vector superfield V which (off-shell) had eight bosonic degrees of freedom, has only two bosonic degrees of freedom on-shell, corresponding to the massive scalar fields c and the longitudinal component of v^μ . The three transverse components of v^μ as well as m, \bar{m} and d do not propagate. We are led to a formulation for which, after gaugino condensation, the original $b_{\mu\nu}$ field of the linear multiplet is projected out of the spectrum in favour of a massive scalar field inside v^μ . This solves the puzzle of the axion mass, and gives a novel mechanism for $b_{\mu\nu}$ to acquire a mass.

6 Conclusions

We have shown that, contrary to previous belief, it is possible to formulate gaugino condensation directly in the linear multiplet formalism. This permits the analysis to be performed using the multiplet in which string theory presents the dilaton. It is our

hope that the existence of this alternative point of view may lead to new insights into the physics of dynamical supersymmetry breaking in string-like theories.

A possible advantage of using the linear multiplet in string theory is that it is the function $\Phi(L)$ which is directly obtained from perturbative string calculations. Working directly with $\Phi(L)$ obviates the necessity for eliminating V as a function of $S + \bar{S}$, as is required to determine the lagrangian in its chiral form. As may be seen from equation (7), this can usually *not* be done exactly.

We show the result which we obtain to be dual to the standard chiral approach, even though the PQ symmetry on which this duality is based in the microscopic theory is broken, in general, in the low energy lagrangian. We find in so doing a novel mechanism for giving a mass to an antisymmetric tensor gauge field. As a result, we give the first good evidence that chiral-linear duality is an exact symmetry of string theories, which survives strong coupling effects.

Acknowledgements

We wish to thank L. Alvarez-Gaumé for useful discussions. One of us (C.B.) also wishes to thank V. Kaplunovsky for useful conversations.

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