# The Epsilon Variables for Electroweak Precision Tests: A Reappraisal

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### Abstract

We update the analysis of the precision electroweak tests in terms of the previously introduced epsilon parameters, by taking into account the new experimental information (i.e. the data presented at the Glasgow Conference) and some recent theoretical progress in the computation of radiative corrections in the Standard Model. At the same time we further clarify some important points, as, for example, the dependence of the analysis on the input values of  $\alpha_s(m_Z)$  and  $\alpha(m_Z)$ .

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Recently we have proposed [1] a general strategy for the analysis of precision electroweak tests in view of the search for new physics beyond the Standard Model. Our analysis is based on four parameters,  $\varepsilon_1$ ,  $\varepsilon_2$ ,  $\varepsilon_3$  and  $\varepsilon_b$ . They represent an efficient parameterisation of the small deviations from what is solidly established in a way that is, in particular, unaffected by our relative ignorance of m<sub>t</sub>. In fact, the extremely important new information on m<sub>t</sub> from the CDF events [2] still leaves a considerable uncertainty on the value of m<sub>t</sub>. Indeed the epsilons are defined in such a way that they are exactly zero in the Standard Model in the limit of neglecting all pure weak loop-corrections to a few especially relevant observables (i.e. when only the predictions from the tree level Standard Model plus pure QED and pure QCD corrections are taken into account). This very simple version of improved Born approximation - hereafter simply called Born approximation - is a good first approximation [3], according to the data. The main purpose of this letter is to update the epsilon analysis by taking into account the new experimental information (i.e. the data presented at the Glasgow Conference [4] and displayed in Table 1) and some recent theoretical progress in the computation of radiative corrections in the Standard Model. At the same time we further clarify some important points, as, for example, the dependence of the analysis on the input values of  $\alpha_s(m_7)$  and  $\alpha(m_7)$ .

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In a completely model independent way we have defined [1] four variables, called<sup>\*</sup>  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$  and  $\epsilon_b$ , that are precisely measured and can be compared with the predictions of different theories. The quantities  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$  and  $\epsilon_b$  are defined in ref.1 in one to one correspondence with the set of observables  $m_W/m_{Z_s} \Gamma_l$ ,  $A_{FB}^l$  and  $\Gamma_b$ . The relations between the basic observables and the epsilons can be linearised, leading to the formulae

$$\frac{m_{\rm W}^2}{m_Z^2} = \frac{m_W^2}{m_Z^2} \bigg|_B (1 + 1.43\varepsilon_1 - 1.00\varepsilon_2 - 0.86\varepsilon_3)$$
(1a)

$$\Gamma_{l} = \Gamma_{l} \Big|_{B} \Big( 1 + 1.20\varepsilon_{1} - 0.26\varepsilon_{3} \Big)$$
<sup>(1b)</sup>

$$A_{FB}^{\mu} = A_{FB}^{\mu}|_{R} (1 + 34.72\varepsilon_{1} - 45.15\varepsilon_{3})$$
(1c)

$$\Gamma_b = \Gamma_b|_{\mathcal{B}} \left( 1 + 1.42\varepsilon_1 - 0.54\varepsilon_3 + 2.29\varepsilon_b \right) \tag{1d}$$

The Born approximations, as defined above, of the corresponding quantities on the right hand side of eq. 1 depend on  $\alpha_s(m_Z)$  and also on  $\alpha(m_Z)$ . Defining

$$\delta \alpha_{\rm s} = [\alpha_{\rm s}({\rm m}_Z) - 0.118]/\pi ; \delta \alpha = (\alpha({\rm m}_Z) - 1/128.87)/\alpha$$
(2)

<sup>&</sup>lt;sup>\*</sup> Here we resume the notation  $\varepsilon_i$  for exactly the same quantities as defined in ref.1, where they were denoted  $\varepsilon_{Ni}$  (the index N, for "new", had been inserted to signal some small differences with respect to the original definitions in refs.5,6).

we have

$$\frac{m_W^2}{m_Z^2}\Big|_B = 0.76883[1 - 0.40\delta\alpha]$$
(3a)

$$\Gamma_l|_B = 83.56 [1 - 0.19\delta\alpha] MeV \tag{3b}$$

$$A^{\mu}_{FB}\Big|_{R} = 0.01683(1 - 34\delta\alpha) \tag{3c}$$

$$\Gamma_b|_{\mathcal{B}} = 379.6 [1 + 1.0\delta\alpha_s - 0.42\delta\alpha] MeV$$
(3d)

Note that the dependence on  $\delta\alpha_s$  for  $\Gamma_{b|B}$ , shown in eq.3d, is not simply the one loop result for m<sub>b</sub>=0 but a combined effective shift which takes into account both finite mass effects and the contribution of the known higher order terms.

The important property of the epsilons is that, in the Standard Model, for all observables at the Z pole, the whole dependence on  $m_t$  (and  $m_H$ ) arising from one-loop diagrams only enters through the epsilons. The same is actually true, at the relevant level of precision, for all higher order m<sub>t</sub>-dependent corrections. Recently, within the Standard Model, there has been some additional progress in the control of radiative corrections by new computations of some potentially dominant higher-loop effects: terms of order  $(G_Fm_t^2)^2$  in the Z->bb vertex and in  $\Delta \rho$ , for all values of  $m_H$  [7]; terms of order  $\alpha_s G_Fm_t^2$ in the Z->bb vertex [8] and, for some refinements, in  $\Delta r$  and  $\Delta \rho$  [9]; terms of order  $(\alpha_{s}m_{b}/m_{z})^{2}$  in  $\Gamma(Z \rightarrow hadrons)$  [10]. Very recently the  $o(\alpha_{s}^{2}G_{F}m_{t}^{2})$  corrections to  $\Delta \rho$  have also been computed [11]. We stress that since all of these improvements have to do with vacuum polarisation diagrams or with the Z->bb vertex, the corresponding terms simply affect the theoretical predictions of the epsilons in the Standard Model but do not invalidate the basic property of the epsilons mentioned above. The improved theoretical values of the epsilons in the Standard Model are given in table 2. Actually, the only residual  $m_t$ dependence of the various observables not included in the epsilons is in the terms of order  $\alpha_s^2$  in the pure QCD correction factors to the hadronic widths [12]. But this one is quantitatively irrelevant, especially in view of the errors connected to the uncertainty on the value of  $\alpha_s$ . It is important to remark that the theoretical values of the epsilons in the SM, as defined in eqs. 1 and given in table 2, are not affected, at the percent level or so, by reasonable variations of  $\alpha_s(m_Z)$  and/or  $\alpha(m_Z)$  around their central values. By our definitions, in fact, no terms of order  $\alpha_s^n(m_Z)$  or  $\alpha \log(m_Z/m)$  contribute to the epsilons.

In terms of the epsilons, the following expressions hold, within the SM, for the various precision observables

$$\Gamma_{\rm T} = \Gamma_{\rm T0} [1 + 1.35 \ \epsilon_1 - 0.46 \ \epsilon_3 + 0.35 \ \epsilon_b] \tag{4a}$$

$$\mathbf{R} = \mathbf{R}_0 [1 + 0.28 \ \varepsilon_1 - 0.36 \ \varepsilon_3 + 0.50 \ \varepsilon_b]$$
(4b)

$$\sigma_{\rm h} = \sigma_{\rm h0} [1 - 0.03 \varepsilon_1 + 0.04 \varepsilon_3 - 0.20 \varepsilon_{\rm b}] \tag{4c}$$

$$x = x_0[1 + 17.6 \epsilon_1 - 22.9 \epsilon_3]$$
(4d)

$$R_{bh} = R_{bh0} [1 - 0.06 \epsilon_1 + 0.07 \epsilon_3 + 1.79 \epsilon_b]$$
(4e)

where  $x = \frac{g_V}{g_A}$  as obtained  $A_{FB}^{\mu}$ . The quantities in eqs. 1 and 4 are clearly not independent and the redundant information is reported for convenience. By comparison with the code of Ref. 13 (we also checked the results with the programme of ref.14) we obtain

$$\Gamma_{\rm T0} = 2488.88[1 + 0.73 \,\delta\alpha_{\rm s} - 0.35 \,\delta\alpha] \,\,{\rm MeV} \tag{5a}$$

$$R_0 = 20.8177[1 + 1.05 \,\delta\alpha_s - 0.28 \,\delta\alpha] \tag{5b}$$

$$\sigma_{\rm h0} = 41.4221[1 - 0.41 \,\delta\alpha_{\rm s} + 0.03 \,\delta\alpha] \,\rm nb \tag{5c}$$

$$x_0 = 0.0753142 - 1.32 \,\delta\alpha \tag{5d}$$

$$R_{bh0} = 0.21823$$
 (5e)

Note that the quantities in eqs. 5 should not be confused, at least in principle, with the corresponding Born approximations, due to small "non universal" electroweak corrections. In practice, at the relevant level of approximation, the difference between the two corresponding quantities is in any case significantly smaller than the present experimental error, from a factor of 2 in the case of  $\Gamma_{\rm T}$  up to a factor of 6 in  $R_{bh}$ .

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The properties of the epsilons, as precisely defined from eqs.1, make them suitable for a model independent analysis of the electroweak precision tests. In particular, the fact that, for all observables at the Z pole, the whole relevant dependence on  $m_t$  (and  $m_H$ ) only enters through the epsilons, is true for any extension of the Standard Model with the property that all possible deviations only occur through vacuum polarisation diagrams and/or the Z->bb vertex. In any such model, of course, the actual values of the epsilons will differ in general from the SM ones. As discussed in detail in ref.1, for this kind of models one can compare the theoretical predictions with the experimental determination of the epsilons as obtained from the whole set of LEP/SLC data. If a particular model does not satisfy this requirement, then the comparison is to be made with the epsilons determined from the defining variables only, eqs.1, or with some more limited enlargement of the same set of data, depending on the particular case. For example, if lepton universality is maintained, then the data on  $A_{FB}^{l}$  can be replaced by the combined result on  $g_{V}/g_{A}$  from all lepton asymmetries.

In principle, any four observables could have been picked up as defining variables. In practice we choose those that have a more clear physical significance and are more effective in the determination of the epsilons. In fact, since  $\Gamma_b$  is actually measured by  $R_{bh}$  (which is nearly insensitive to  $\alpha_s$ ), it is preferable to use directly  $R_{bh}$  itself as defining variable, as we shall do hereafter. In practice, since  $R_{bh0}$ , eq.5e is practically indistinguishable from the Born approximation of  $R_{bh}$ , this determines no change in any of the equations given above but simply requires the replacement of eqs. 1d,3d with eqs.4e, 5e among the defining relations of the epsilons. In this way, the equations that have completely general validity are eqs.1a,b,c and 4e together with eqs.3a,b,c and 5e, whereas the remaining observables and the corresponding equations, among which eqs.1d and 3d can be included in the analysis only according to the progression of hypothesis that we shall discuss.

We hope to have made clear by now that our method of analysing the data is more complete and less model dependent than an alternative approach based on the variables S, T and U [15], which, from the start, necessarily assumes dominance of vacuum polarisation diagrams from new physics and truncation of the  $q^2$  expansion of the corresponding amplitudes. Furthermore, the variables S, T, U depend on  $m_t$  and  $m_H$ , being defined as deviations from the complete Standard Model prediction for specified  $m_t$  (and  $m_H$ ). Instead the epsilons are defined with respect to a reference approximation which does not depend on  $m_t$ .

By combining the value of  $m_W/m_Z$  [1] with the LEP results on the charged lepton partial width and the forward-backward asymmetry, all given in table 1, one obtains from eqs. 1a,b,c and 3a,b,c:

$$\begin{aligned} \varepsilon_1 &= \Delta \rho = (4.7 \pm 2.2) \ 10^{-3} \\ \varepsilon_2 &= (-3.2 \pm 5.0) \ 10^{-3} + 0.23 \ \delta \alpha \\ \varepsilon_3 &= (3.4 \pm 3.0) \ 10^{-3} \ -0.77 \ \delta \alpha \end{aligned} \tag{6}$$

Finally, by adding the value of R<sub>bh</sub> listed in table 1 and using eqs. 4e, 5e one finds :

$$\varepsilon_{\rm b} = (2.5 \pm 4.6) \ 10^{-3} \tag{7}$$

The central values of the epsilons, as determined experimentally, depend on the chosen value  $\alpha(m_Z)$ , since the Born approximation of the defining variables does. As before, we have taken  $\alpha(m_Z)=1/128.87$  [16] but, in eqs.6,7, we have given the variation induced on the epsilons by corresponding shifts  $\alpha(m_Z)$ . At present there is a lively debate in the

literature on the best value of  $\alpha(m_Z)$  that can be extracted from the data on e<sup>+</sup>e<sup>-</sup>->hadrons and on the corresponding uncertainty [17]. By using eqs. 6,7 the reader can easily adapt the results to his/her preferred values.

In fig.1 the experimental  $1\sigma$  ellipse in the  $\varepsilon_1$ - $\varepsilon_3$  plane is shown and compared, as a particularly relevant example, with the Standard Model predictions for different  $m_t$  and  $m_H$  values. We recall that  $\varepsilon_1$  and  $\varepsilon_3$  are completely determined by  $\Gamma_1$  and  $A_{FB}^l$ . In fig.2 the experimental value of  $\varepsilon_2$  is compared with the Standard Model prediction as a function of  $m_t$ . There is consistency at all practical values of  $m_t$ . Note that  $\varepsilon_2$  also depends on  $m_W/m_Z$  and better measurements of this quantity are needed in order to make this test more stringent. Finally, in fig.3 we compare the experimental value of  $\varepsilon_b$  with the Standard Model prediction. Here we see that  $\varepsilon_b$  would prefer relatively small values of  $m_t$ . This result is a simple and direct consequence of the fact that the measured value of  $R_{bh}$  is a bit high ( for  $m_t \sim 170$  GeV,  $\Gamma_b$  is about  $2\sigma$  larger than the Standard Model prediction).

To proceed further, and include other measured observables in the analysis we need to make some dynamical assumptions. The minimum amount of model dependence is introduced by including other purely leptonic quantities at the Z pole such as  $A_{pol}^{\tau}$ ,  $A_e$  (measured [4] from the angular dependence of the  $\tau$  polarisation) and  $A_{LR}$  (measured by SLD [18]). At this stage, one is simply relying on lepton universality. With essentially the same assumptions one can also include the data on the b-quark forward backward asymmetry  $A_{FB}^{b}$ . In fact it turns out that  $A_{FB}^{b}$  is almost unaffected by the Z->b $\bar{b}$  vertex correction.

As a result, we can combine the values of  $x = g_V/g_A$  from the whole set of asymmetries measured at LEP (obtaining the value given in table 1) and we can include, in the fit of the epsilons, eqs. 4d, 5d, valid in a more general theory fulfilling the stated assumptions. At this stage, with the SLD result also taken into account, the best values of  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$  are modified according to

$$\begin{aligned} \varepsilon_1 &= \Delta \rho = (5.1 \pm 2.2) \ 10^{-3} \\ \varepsilon_2 &= (-4.1 \pm 4.8) \ 10^{-3} \\ \varepsilon_3 &= (5.1 \pm 2.0) \ 10^{-3} \\ \varepsilon_b &= (2.4 \pm 4.6) \ 10^{-3} \end{aligned} \tag{8}$$

with a similar dependence on  $\alpha(mZ)$  as in eqs.6,7. In fig.4 we report the two ellipses in the  $\epsilon_1$ - $\epsilon_3$  plane that correspond to the data with and without A<sub>LR</sub> from SLD.

All observables measured on the Z peak at LEP can be included in the analysis provided that we assume that all deviations from the Standard Model are only contained in vacuum polarisation diagrams (without demanding a truncation of the  $q^2$  dependence of the corresponding functions) and/or the Z->bb vertex. Note that this is true for whatever partition of the new effect between  $g_{bV}$  and  $g_{bA}$ , because only one combination of them is measured in  $\Gamma_b$ , while, as already mentioned,  $A_{FB}^b$  is nearly independent of the Z->b b vertex.

For a global fit of all high energy data we consider  $m_W/m_Z$ ,  $\Gamma_T$ ,  $R_h$ ,  $\sigma_h$ ,  $R_{bh}$  and  $x=g_V/g_A$  given in table 1. The relations between these quantities and the epsilons, valid in any model of the assumed type, are given in eqs.1a,3a,4,5. For LEP data, we have taken the correlation matrix for  $\Gamma_T$ ,  $R_h$  and  $\sigma_h$  given by the LEP experiments [4], while we have considered the additional information on  $R_{bh}$  and x as independent. We obtain (SLD is also included):

$$\begin{aligned} \varepsilon_1 &= \Delta \rho = (4.2 \pm 1.8) \ 10^{-3} \ -0.27 \ \delta \alpha_s \\ \varepsilon_2 &= (-4.9 \pm 4.8) \ 10^{-3} \ -0.24 \ \delta \alpha_s + 0.23 \ \delta \alpha \\ \varepsilon_3 &= (4.5 \pm 1.8) \ 10^{-3} \ -0.17 \ \delta \alpha_s \ -0.77 \ \delta \alpha \\ \varepsilon_b &= (-0.2 \pm 4.1) \ 10^{-3} \ -1.23 \ \delta \alpha_s \end{aligned}$$
(9)

At this stage, the epsilons have acquired also a dependence on  $\alpha_s(m_Z)$ . We have taken  $\alpha_s(m_Z) = 0.118$  [19] and we have given the variation induced on the epsilons by a shift of  $\alpha_s(m_Z)$ , as defined in eq.2. The comparison of theory (the SM) and experiment in the planes  $\varepsilon_1$ - $\varepsilon_3$ ,  $\varepsilon_b$ - $\varepsilon_3$  and  $\varepsilon_b$ - $\varepsilon_1$  is shown in fig.5,6 and 7, respectively. We see that the inclusion of all LEP quantities does not change the epsilons very much. The effect of a  $\pm 0.007$  uncertainty on  $\alpha_s(m_Z)$  is included in the quoted error for  $\varepsilon_b$ . Note that  $\varepsilon_b$  moves in the direction of the Standard Model prediction. This is because  $\Gamma_T$ ,  $\sigma_h$  and  $R_h$  (or equivalently the ratios of  $\Gamma_Z$ ,  $\Gamma_h$  and  $\Gamma_l$ ), which also depend on  $\varepsilon_b$ , are normal.

Because of the fact that  $\Gamma_T$ ,  $\sigma_h$ ,  $R_h$  depend on  $\alpha_s$  much more than  $R_{bh}$ , the fitted value of  $\varepsilon_b$  in eq.9 depends on the assumed value of  $\alpha_s(m_Z)$ , that we have taken as in table 1. If we repeat the fit of high energy data with  $\alpha_s(m_Z)$  free,  $\varepsilon_b$  moves up to  $\varepsilon_b \cdot 10^3 = 2.6 \pm 4.8$  (to fix  $R_{bh}$  which is nearly independent of  $\alpha_s(m_Z)$ ) while  $\alpha_s(m_Z)$  goes down to  $\alpha_s(m_Z) = 0.111 \pm 0.009$ . Finally,  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$  are quite insensitive to  $\alpha_s(m_Z)$  and closely keep their values in eq.9.

To include in our analysis lower energy observables as well, a stronger hypothesis needs to be made: vacuum polarization diagrams are allowed to vary from the Standard Model only in their constant and first derivative terms in a q<sup>2</sup>-expansion. In such a case, one can, for example, add to the analysis the ratio  $R_V$  of neutral to charged current processes in deep inelastic neutrino scattering on nuclei [20], the "weak charge"  $Q_W$  measured in atomic parity violation experiments on Cs [21] and the measurement of  $g_V/g_A$  from  $v_{\mu}e$  scattering [22]. The expressions of these quantities in terms of the epsilons are given in ref.1. In this way one obtains the global fit (also including SLD):

$$\begin{aligned} \varepsilon_1 &= \Delta \rho = (3.6 \pm 1.7) \ 10^{-3} \\ \varepsilon_2 &= (-5.3 \pm 4.7) \ 10^{-3} \\ \varepsilon_3 &= (4.0 \pm 1.7) \ 10^{-3} \\ \varepsilon_b &= (0.2 \pm 4.0) \ 10^{-3} \end{aligned} \tag{10}$$

with the same dependence on  $\alpha_s(m_Z)$  and  $\alpha(m_Z)$  as in eqs.9. With the progress of LEP the low energy data, while important as a check that no deviations from the expected q<sup>2</sup> dependence arise, play a lesser role in the global fit. The  $\varepsilon_1$ - $\varepsilon_3$  plot for all data is shown in fig.8. We observe no drastic change in the epsilons and we take this fact as evidence that no exotic q<sup>2</sup> dependence is visible. The inclusion of more parameters to describe the possible departure from the q<sup>2</sup> behaviour predicted by the Standard Model was discussed in refs.

23,24. Their conclusion coincides with ours that no sign of special  $q^2$  dependent non standard effects is observed. Any attempt of significantly constraining the additional parameters is frustrated by the limited precision of the low energy data.

Note that the present ambiguity on the value of  $\alpha(m_Z) = (128.87 \pm 0.12)^{-1}$  [16] corresponds to an uncertainty on  $\varepsilon_3$  (the other epsilons are not much affected) given by  $\Delta \varepsilon_3 \cdot 10^3 = \pm 0.7$  Thus the theoretical error is still confortably less than the experimental error but the two will become close at the end of the LEP1 phase. The values of  $\varepsilon_2$  and  $\varepsilon_b$  in eq.10 were compared with the Standard Model predictions in figs. 2 and 3.

\* \* \*

Finally we would like to add some comments.

As is clearly indicated in figs.5-12 there is by now a solid evidence for departures from the "improved Born approximation", defined as including the predictions from the tree level Standard Model plus pure QED and pure QCD corrections only, where all the epsilons vanish. Such evidence comes from  $\varepsilon_1$  and  $\varepsilon_3$ , both measured with an absolute error below 2 10<sup>-3</sup> and shown to be different from zero at more than the  $2\sigma$  level for each of them. In this way one has obtained a strong evidence for pure weak radiative corrections, thus fulfilling one of the explicit goals of the precision electroweak tests. LEP and SLC are now measuring the different components of the radiative corrections.

Of great significance is also the fact that both  $\varepsilon_1$  and  $\varepsilon_3$  are reproduced in the Standard Model with an appropriate choice of  $m_t$  and  $m_H$ . This can be interpreted as an indirect but nevertheless significant evidence for the description of the electroweak symmetry breaking sector of the theory in terms of fundamental Higgs(es), as in the Standard Model or its supersymmetric extension. This is true in spite of the fact that the dependence of  $\varepsilon_1$  and  $\varepsilon_3$  on the Higgs mass is rather weak. One should consider in fact that, in most examples of Higgs-less theories that can be found in the literature [25,26],  $\varepsilon_1$  and  $\varepsilon_3$ , when they can be computed [27], show relatively large deviations from the predictions of the Standard Model. In this respect a further reduction of the errors on  $\varepsilon_1$  and  $\varepsilon_3$ , together with an improved direct determination of  $m_t$  at the Tevatron, are extremely important. Similarly, it would also be interesting to have a clear evidence for a deviation from zero of the remaining parameters,  $\varepsilon_2$  and  $\varepsilon_b$ . These important goals of the electroweak precision tests are indeed possible in a near future.

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m <sub>Z</sub> (GeV)	$91.1888 \pm 0.0044$
$\Gamma_{\rm T}({\rm MeV})$	$2497.4 \pm 3.8$
$R = \Gamma_h / \Gamma_l$	$20.795 \pm 0.040$
$\sigma_h = 12 \pi \Gamma_e \Gamma_h / m_Z^2 \Gamma_T^2$ (nb)	$41.49 \pm 0.12$
$\Gamma_{l}(MeV)$	$83.96 \pm 0.18$
$\Gamma_{\rm h}({\rm MeV})$	$1745.9 \pm 4.0$
$\Gamma_{b}(MeV)$	$382.7 \pm 3.1$
$R_{bh} = \Gamma_b / \Gamma_h$	$0.2192 \pm 0.0018$
A <sup>l</sup> <sub>FB</sub>	$0.0170 \pm 0.0016$
$A_{pol}^{\tau}$	$0.143\pm0.010$
A <sup>e</sup>	$0.135 \pm 0.01091$
A <sup>b</sup> <sub>FB</sub>	$0.0967 \pm 0.0038$
A <sup>c</sup> <sub>FB</sub>	$0.0760 \pm 0.0091$
$g_v/g_a$ (all asymmetries -LEP)	$0.0716 \pm 0.0020$
A <sub>LR</sub> (SLD)	$0.1637 \pm 0.0075$
g <sub>v</sub> /g <sub>a</sub> (all asymmetries-LEP+SLD)	$0.0738 \pm 0.0018$
m <sub>W</sub> /m <sub>Z</sub> (UA2+CDF+D0) [28]	$0.8798 \pm 0.0020$
$\alpha_{\rm s}({\rm m_{Z}})$ [19]	$0.118 \pm 0.007$

## Table 1

Summary of the data [4] used in the present paper

		ε <sub>1</sub>			$\epsilon_2$			83		Еb	
m <sub>t</sub> (GeV)	m <sub>H</sub> = 65 GeV	300	1000	65	300	1000	65	300	1000	All m <sub>H</sub>	
120	1.51	0.888	-0.23	-5.72	-5.40	-5.25	5.04	6.4	7.07	-2.29	
130	2.19	1.54	0.413	-6.10	-5.74	-5.56	4.96	6.3	6.96	-2.98	
140	2.93	2.25	1.10	-6.46	-6.07	-5.86	4.88	6.21	6.85	-3.71	
150	3.72	3.00	1.84	-6.80	-6.38	-6.15	4.81	6.12	6.75	-4.48	
160	4.56	3.81	2.63	-7.13	-6.70	-6.45	4.74	6.03	6.65	-5.30	
170	5.47	4.68	3.47	-7.48	-7.03	-6.76	4.68	5.95	6.57	-6.15	
180	6.43	5.60	4.36	-7.84	-7.36	-7.07	4.63	5.88	6.49	-7.05	
190	7.44	6.57	5.29	-8.23	-7.71	-7.39	4.58	5.81	6.41	-7.99	
200	8.53	7.6	6.27	-8.64	-8.08	-7.72	4.54	5.76	6.35	-8.98	
210	9.67	8.69	7.30	9.08	-8.47	-8.08	4.51	5.72	6.29	-10.0	
220	10.9	9.83	8.37	9.55	-8.9	-8.45	4.49	5.69	6.23	-11.1	
230	12.2	11.0	9.49	-10.0	-9.36	-8.85	4.49	5.67	6.18	-12.2	

## Table 2

Values of the epsilons in the Standard Model as functions of  $m_t$  and  $m_H$  as obtained from recent versions of ZFITTER [13] and TOPAZO [14]. These values are obtained for for  $\alpha_s(m_Z)=0.118$ ,  $\alpha(m_Z)=1/128.87$  but are essentially independent of these input parameters.

## **Figure Captions.**

1. The  $1\sigma$  ellipse in the plane  $\varepsilon_1$ - $\varepsilon_3$  obtained from the data on the defining variables  $\Gamma_l$  and  $A_{FB}^l$  compared with the Standard Model predictions for the indicated values of  $m_t$  and  $m_H$ .

2. The  $1\sigma$  data on  $\varepsilon_2$  obtained from the data on the defining variables  $\Gamma_1$ ,  $A_{FB}^1$  and  $m_W/m_Z$  compared with the Standard Model predictions as functions of  $m_t$  for the indicated values of  $m_H$ . The arrows indicate the experimental  $1\sigma$  band from the fit in eq.10 to all electroweak data.

3. The 1 $\sigma$  data on  $\epsilon_b$  obtained from the data on the defining variables  $\Gamma_l$ ,  $A_{FB}^l$ ,  $m_W/m_Z$  and  $R_{bh}$  compared with the Standard Model predictions as functions of  $m_t$ . The arrows indicate the experimental 1 $\sigma$  band from the fit in eq.10 to all electroweak data.

4. The  $1\sigma$  ellipses in the plane  $\varepsilon_1$ - $\varepsilon_3$  obtained from the data on  $\Gamma_1$  and  $g_V/g_A$  derived from all the asymmetries (see table 1), both with SLD included or not, compared with the Standard Model predictions for the indicated values of  $m_t$  and  $m_H$ .

5. The  $1\sigma$  ellipses in the plane  $\varepsilon_1$ - $\varepsilon_3$  obtained from the data on  $m_W/m_Z$ ,  $\Gamma_T$ ,  $\sigma_h$ ,  $R_h$ ,  $R_{bh}$  and  $g_V/g_A$  derived from all the asymmetries (see table 1), both with SLD included or not, compared with the Standard Model predictions for the indicated values of  $m_t$  and  $m_H$ .

6. The  $1\sigma$  ellipses in the plane  $\varepsilon_b$ - $\varepsilon_3$  obtained from the data on  $m_W/m_Z$ ,  $\Gamma_T$ ,  $\sigma_h$ ,  $R_h$ ,  $R_{bh}$  and  $g_V/g_A$  derived from all the asymmetries (see table 1), both with SLD included or not, compared with the Standard Model predictions for the indicated values of  $m_t$  and  $m_H$ .

7. The  $1\sigma$  ellipses in the plane  $\varepsilon_1$ - $\varepsilon_b$  obtained from the data on  $m_W/m_Z$ ,  $\Gamma_T$ ,  $\sigma_h$ ,  $R_h$ ,  $R_{bh}$  and  $g_V/g_A$  derived from all the asymmetries (see table 1), both with SLD included or not, compared with the Standard Model predictions for the indicated values of  $m_t$  and  $m_H$ .

8. The  $1\sigma$  ellipse in the plane  $\varepsilon_1$ - $\varepsilon_3$  obtained from all the data also including the low energy data compared with the Standard Model predictions for the indicated values of  $m_t$  and  $m_H$ .