CERN-TH.7467/94
McGill/94–51

Black Hole Solutions in String Theory

Ramzi R. Khuri†
CERN, Theory Division
CH-1211 Geneva 23, Switzerland
and
McGill University, Physics Department
Montreal, PQ, H3A 2T8, Canada

ABSTRACT

We present two-parameter solutions of the low-energy four-dimensional heterotic string which in the extremal limit reduce to supersymmetric monopole, string and domain wall solutions. The effective scalar coupling to the Maxwell field, $e^{-\alpha\phi}F_{\mu\nu}F^{\mu\nu}$, gives rise to a new string black hole with $\alpha=\sqrt{3}$, in contrast to the pure dilaton black hole solution which has $\alpha=1$. Implications of string/fivebrane duality in $D=10$ to four-dimensional dualities are discussed.

In recent work[1], supersymmetric soliton solutions of the four-dimensional heterotic string were presented, describing monopoles, strings and domain walls. These solutions admit the $D=10$ interpretation of a fivebrane wrapped around 5, 4 or 3 of the 6 compactified dimensions and are arguably exact to all orders in α'. In this talk, we extend all three solutions to two-parameter solutions of the low-energy equations of the four-dimensional heterotic string[2]. The two-parameter solution extending the supersymmetric monopole corresponds to a magnetically charged black hole, while the solution extending the supersymmetric domain wall corresponds to a black membrane. By contrast, the two-parameter string solution does not possess a finite horizon and corresponds to a naked singularity.

All three solutions involve both the dilaton and the modulus fields, and are thus to be contrasted with pure dilaton solutions[3]. In particular, the effective scalar coupling to the Maxwell field, $e^{-\alpha\phi}F_{\mu\nu}F^{\mu\nu}$, gives rise to a new string black hole with $\alpha=\sqrt{3}$, in contrast to the pure dilaton solution of the heterotic string which has $\alpha=1$ [3]. It thus resembles the black hole arising from Kaluza-Klein theories which also has $\alpha=\sqrt{3}$, and which reduces to the Pollard-Gross-Perry-Sorkin[4] magnetic monopole in the extremal limit. The fact that the heterotic string admits $\alpha=\sqrt{3}$ black holes also has implications for string/fivebrane duality[5]. Both electric/magnetic duality and string/string duality in $D=4$ may be seen as a consequence of string/fivebrane duality in $D=10$.

We begin with the two-parameter black hole. Inspired by the wrapping of a

Based on talks given at MRST 94, McGill University, May 1994 and at Mini-workshop on Extended Objects, Newton Institute, March 1994 and condensed from work already published[1,2].
† Supported by a World Laboratory Fellowship.

fivebrane around five of the six compactified dimensions $(x_5, x_6, x_7, x_8, x_9)$, it was shown[1] that the tree-level effective action for the $D = 10$ heterotic string may be reduced to the following four-dimensional form

$$S_1 = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}e^{-2\Phi-\sigma_1}\left(R + 4(\partial\Phi)^2 + 4\partial\sigma_1\cdot\partial\Phi - \frac{1}{4}e^{2\sigma_1}F_{\mu\nu}F^{\mu\nu}\right), \tag{1}$$

where $\mu, \nu = 0, 1, 2, 3$. Here $g_{\mu\nu}$ is the string sigma-model metric and Φ is the dilaton. In the case of toroidal compactification, with $N = 4$ supersymmetry in $D = 4$, σ_1 is a modulus field, $g_{44} = e^{-2\sigma_1}$, and $F_{\mu\nu} = H_{\mu\nu 4}$ where $H = dB$ and B is the string antisymmetric tensor. However, actions of this type also appear in a large class of $N = 1$ supergravity theories[6]. The solution is given by[2]

$$\begin{aligned}
e^{-2\Phi} &= e^{2\sigma_1} = \left(1 - \frac{r_-}{r}\right),\\
ds^2 &= -\left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{-1}dt^2 + \left(1 - \frac{r_+}{r}\right)^{-1}dr^2 + r^2\left(1 - \frac{r_-}{r}\right)d\Omega_2^2,\\
F_{\theta\varphi} &= \sqrt{r_+r_-}\sin\theta,
\end{aligned} \tag{2}$$

where here, and throughout this paper, we set the dilaton vev Φ_0 equal to zero. This represents a magnetically charged black hole with event horizon at $r = r_+$ and inner horizon at $r = r_-$. The magnetic charge and mass of the black hole are given by

$$\begin{aligned}
g_1 &= \frac{4\pi}{\sqrt{2}\kappa}(r_+r_-)^{\frac{1}{2}},\\
\mathcal{M}_1 &= \frac{2\pi}{\kappa^2}(2r_+ - r_-).
\end{aligned} \tag{3}$$

Changing coordinates via $y = r - r_-$ and taking the extremal limit $r_+ = r_-$ yields:

$$\begin{aligned}
e^{2\Phi} &= e^{-2\sigma_1} = \left(1 + \frac{r_-}{y}\right),\\
ds^2 &= -dt^2 + e^{2\Phi}\left(dy^2 + y^2 d\Omega_2^2\right),\\
F_{\theta\varphi} &= r_-\sin\theta,
\end{aligned} \tag{4}$$

which is just the tree-level supersymmetric monopole solution without a Yang-Mills field which saturates the Bogomol'nyi bound $\sqrt{2}\kappa\mathcal{M}_1 \geq g_1$. Note that the monopole arises in the gravitational sector of the string, as can be seen from an earlier solution found in purely bosonic string theory[7]. Supersymmetric extensions of this solution both with and without gauge fields were found in [8].

Next we derive a two-parameter string solution which, however, does not possess a finite event horizon and consequently cannot be interpreted as a black string. This is inspired by the wrapping of the fivebrane around four of the compactified dimensions (x_6, x_7, x_8, x_9). The action is given by

$$S_2 = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}e^{-2\Phi-2\sigma_2}\left(R + 4(\partial\Phi)^2 + 8\partial\sigma_2\cdot\partial\Phi + 2(\partial\sigma_2)^2 - \frac{1}{2}e^{4\sigma_2}F_\mu F^\mu\right), \tag{5}$$

In the case of the torus, σ_2 is the modulus field $g_{44} = g_{55} = e^{-2\sigma_2}$ and $F_\mu = H_{\mu 45}$. A two-parameter family of solutions is now given by[2]

$$\begin{aligned} e^{2\Phi} &= e^{-2\sigma_2} = (1 + k/2 - \lambda \ln y), \\ ds^2 &= -(1+k)dt^2 + (1+k)^{-1}(1+k/2-\lambda \ln y)dy^2 + y^2(1+k/2-\lambda\ln y)d\theta^2 + dx_3^2, \\ F_\theta &= \lambda\sqrt{1+k}, \end{aligned} \tag{6}$$

where for $k = 0$ we recover the supersymmetric string soliton solution[1] which is dual to the elementary string solution of Dabholkar *et al*[9]. The solution shown in Eq.(6) in fact represents a naked singularity, since the event horizon is pushed out to $r_+ = \infty$, which agrees with the Horowitz-Strominger "no-4D-black-string" theorem[10].

Finally, we consider the two-parameter black membrane solution. In this case, we wrap the fivebrane around three of the compactified dimensions (x_7, x_8, x_9). However, the four-dimensional action necessary to yield membrane solutions is not obtained by a simple dimensional reduction of the ten-dimensional action because of the non-vanishing of $F = H_{456}$. Instead, the effective action is obtained by treating F^2 as a cosmological constant and is given by

$$S_3 = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}e^{-2\Phi-3\sigma_3}\left(R + 4(\partial\Phi)^2 + 12\partial\sigma_3\cdot\partial\Phi + 6(\partial\sigma_3)^2 - e^{6\sigma_3}\frac{1}{2}F^2\right), \tag{7}$$

In the case of the torus, σ_3 is the modulus field $g_{44} = g_{55} = g_{66} = e^{-2\sigma_3}$. The two-parameter black membrane solution is then[2]

$$\begin{aligned} e^{-2\Phi} &= e^{2\sigma_3} = \left(1 - \frac{r}{r_-}\right), \\ ds^2 &= -\left(1-\frac{r}{r_+}\right)\left(1-\frac{r}{r_-}\right)^{-1}dt^2 + \left(1-\frac{r}{r_+}\right)^{-1}\left(1-\frac{r}{r_-}\right)^{-4}dr^2 + dx_2^2 + dx_3^2, \\ F &= -(r_+r_-)^{-1/2}. \end{aligned} \tag{8}$$

This solution represents a black membrane with event horizon at $r = r_+$ and inner horizon at $r = r_-$. Changing coordinates via $y^{-1} = r^{-1} - r_-^{-1}$ and taking the extremal limit yields

$$\begin{aligned} e^{2\Phi} &= e^{-2\sigma_3} = \left(1 + \frac{y}{r_-}\right), \\ ds^2 &= -dt^2 + dx_2^2 + dx_3^2 + e^{2\Phi}dy^2, \\ F &= -\frac{1}{r_-}. \end{aligned} \tag{9}$$

which is just the supersymmetric domain wall solution[1].

Consider the generic toroidal compactification of the heterotic string, where the four-dimensional theory is given by $N = 4$ supergravity coupled to 22 $N = 4$ vector multiplets. Then the Maxwell field $F_{\mu\nu}$ in $D = 4$ (or its dual $\tilde{F}_{\mu\nu}$) and the scalar field ϕ come from the $D = 10$ 3-form (or 7-form) and dilaton plus modulus field of the heterotic string (or heterotic fivebrane). Thus, the $D = 4$ electric/magnetic duality can be interpreted as a reduction of $D = 10$ string/fivebrane duality.

Another reduction of string/fivebrane duality is $D = 4$ string/string duality[1]. The compactified heterotic string displays a target space duality $O(6, 22, Z)$. It is also conjectured to display the strong/weak coupling $SL(2, Z)$ S-duality relating the dilaton and the axion, which is certainly there in the field theory limit. The "duality of dualities" suggestion[11,1] is that, under string/fivebrane duality, the roles of S and T dualities are interchanged. The picture that emerges is one in which the massive states of the string correspond to extreme black holes.

We have shown only that these two-parameter configurations are solutions of the field theory limit of the heterotic string. Although the extreme one-parameter solutions are expected to be exact to all orders in α', the same reasoning does not carry over to the new two-parameter solutions. It would be also interesting to see whether the generalization of the one-parameter solutions to the two-parameter solutions can be carried out when we include the Yang-Mills coupling. This would necessarily involve giving up the self-duality condition on the Yang-Mills field strength, however, since the self-duality condition is tied to the extreme, $\sqrt{2}\kappa\mathcal{M}_{p+1} = g_{p+1}$, supersymmetric solutions. Finally, there is the question of whether these solutions are peculiar to the toroidal compactification or whether they survive in more realistic orbifold or Calabi-Yau models. Although the actions S_1, S_2 and S_3 were originally derived in the context of the torus[1], they also appear in a large class of $N = 1$ supergravity theories.

References

1. M. J. Duff and R. R. Khuri, *Nucl. Phys.* **B411** (1994) 473.
2. M. J. Duff, R. R. Khuri, R. Minasian and J. Rahmfeld, *Nucl. Phys.* **B418** (1994) 195.
3. G. W. Gibbons, *Nucl. Phys.* **B207** (1982) 337; G. W. Gibbons and K. Maeda, *Nucl. Phys.* **B298** (1988) 741; D. Garfinkle, G. T. Horowitz and A. Strominger, *Phys Rev.* **D43** (1991) 3140; R. Kallosh, A. Linde, T. Ortin, A. Peet and A. Van Proeyen, *Phys Rev.* **D46** (1992) 5278.
4. D. Pollard, *J. Phys.* **A16** (1983) 565; D. J. Gross and M. J. Perry, *Nucl. Phys.* **B226** (1983) 29; R. D. Sorkin, *Phys. Rev. Lett.* **51** (1983) 87.
5. M. J. Duff, *Class. Quantum Grav.* **5** (1988) 189; A. Strominger, *Nucl. Phys.* **B343** (1990) 167; M. J. Duff and J. X. Lu, *Nucl. Phys.* **B354** (1991) 129.
6. E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, *Nucl. Phys.* **B212** (1983) 413.
7. R. R. Khuri, *Phys. Lett.* **B259** (1991) 261.
8. R. R. Khuri, *Phys. Lett.* **B294** (1992) 325; *Nucl. Phys.* **B387** (1992) 315.
9. A. Dabholkar, G. Gibbons, J. A. Harvey and F. R. Ruiz, *Nucl. Phys.* **B340** (1990) 33.
10. G. T. Horowitz and A. Strominger, *Nucl. Phys.* **B360** (1991) 2930.
11. J. H. Schwarz and A. Sen, *Phys. Lett.* **B312** (1993) 105; P. Binétruy, *Phys. Lett.* **B315** (1993) 80; A. Sen *Int. J. Mod. Phys.* **A9** (1994) 3707.