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Neutrino Masses*

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Abstract.

Even if neutrino masses are unknown, we know neutrinos are much lighter than the other fermions we know, and we do not have a good explanation for it. In the Standard Model of elementary particles neutrinos are exactly massless, although this is not insured by any basic principle. Non-zero neutrino masses arise in many extensions of the Standard Model. Massive neutrinos and their associated properties, such as the Dirac or Majorana character of neutrinos, their mixings, lifetimes and magnetic or electric moments, may have very important consequences in astrophysics, cosmology and particle physics. Here we explore these consequences and the constraints they already impose on neutrino properties, as well as the large body of experimental and observational efforts currently devoted to elucidate the mystery of neutrino masses. Several hints for non-zero masses in solar and atmospheric neutrinos, that will be confirmed or rejected in the near future, make this field of research particularly exciting at present.

Contents

1. Introduction

Pauli proposed in 1930 the existence of neutrinos as a "desperate way out" to explain the continuous spectrum of electrons emitted in β-decay. This idea was considered at the time almost as revolutionary as the alternative explanation, the violation of the principle of energy conservation. In this way the neutrino became the first particle proposed as the solution to an elementary particle problem, an idea that proved to be useful many times later and now commonly used (and even over used). Pauli postulated the existence of a new neutral light fermion. Even if we now know that neutrinos exists and come in three varieties or "flavours" (as charged leptons do) and we have no doubts about their neutrality, we are still debating their masses.

In fact, fermions come in families or generations, each one a repetition of the others, except for their mass. The lighter charged member of each family is heavier than the heaviest of the previous one. This is an inter-familial mass hierarchy that has no explanation so far, since in the Standard Model (SM) of elementary particles all masses are free parameters (see section 3.1). Why the generations are three is another mystery. In any case, we know neutrinos come only in the three known varieties, unless the additional ones are inert, i.e. do not have weak interactions unlike the known neutrinos, or they are heavier than $M_Z/2 \simeq 45$ GeV. This results from the measurement at the CERN e^+e^- collider LEP of the width of the weak boson Z^o into "invisible" particles (neutrinos or other exotic particles that do not interact within the LEP detectors), $Z^o \rightarrow \nu\nu$.

At present we only know upper bounds on neutrino masses, obtained in direct mass searches (see section 4.1),

$$m_{\nu_e} < O(10 \text{ eV}) \quad m_{\nu_\mu} < 160 \text{ keV} \quad m_{\nu_\tau} < 31 \text{ MeV} \quad . \tag{1.1}$$

The bound on m_{ν_e} is actually uncertain because unknown systematic effects are probably responsible for the negative experimentally measured $m^2_{\nu_e}$ values. Even if there is a combined bound of 5 eV with statistical confidence level of 95%, the above mentioned systematic effects bring the bound at about 10 eV. A preliminary result lowers the bound on m_{ν_τ} to 29 MeV. These bounds show that neutrinos are much lighter than their corresponding charged fermion, $m_e = 0.5$ MeV, $m_\mu = 105.6$ MeV and $m_\tau = 1.77$ GeV, and the other members of their respective family.

Thus neutrinos introduce an intra-familial hierarchy problem: why neutrinos are much lighter than the other members of each family. In the SM this last hierarchy is obtained by avoiding producing neutrino masses through the mechanism that gives origin to all the other masses, namely the vacuum expectation value of the standard scalar Higgs field (see section 3.1). Thus neutrinos are exactly massless in the SM. However this is a rather ad-hoc choice. No basic physical principle insures the masslessness of neutrinos (as is the case for the masslessness of the photon, insured by gauge invariance).

A large body of experimental and observational efforts are devoted right now to elucidate the mystery of neutrino masses. These are direct mass searches, neutrinoless double beta decay ($\beta\beta0\nu$) (or neutrinoless plus a boson ($\beta\beta0\nu J$) decay experiments), oscillation experiments in reactors and accelerators, solar neutrino observations, atmospheric neutrino observations and supernova neutrino observations.

While direct mass searches do not rely on any other possible neutrino property besides its mass, $\beta\beta0\nu$ requires neutrinos to be Majorana particles. Majorana proposed in 1937 that neutrinos, contrary to their charged family companions, can be their own antiparticle (see section 2). Particle and antiparticle carry opposite charges of any conserved lepton number. So if neutrinos are Majorana fermions, lepton number is not conserved. This would induce $\beta\beta0\nu$ where lepton number is violated by two units. While double beta decay with emission of two neutrinos has been observed, $\beta\beta0\nu$ has not. This places an upper bound on an effective Majorana electron neutrino mass (see section 4.2)

$$\langle m_{\nu_e} \rangle \lesssim 1 \text{ eV} \quad . \tag{1.2}$$

If neutrinos are massive, there would be mixings between neutrino flavours (see section 3.1). One of the most striking manifestations of neutrino mixing would be neutrino oscillations (see section 6). These appear because of the difference between weak interaction neutrino eigenstates and mass eigenstates, if neutrinos are massive. When a neutrino is produced it is necessarily in an interaction eigenstate, whose different mass eigenstate components propagate differently, according to their mass. Thus, after some time the propagating neutrino becomes a linear combination of interaction eigenstates, different from the initial one. Actually, because neutrino oscillations are an interference effect, they are sensitive to a combination of mixing angles and mass square differences, Δm^2. Oscillations may reveal masses much smaller than those testable through direct searches. Oscillations could explain the so-called solar neutrino problem and/or the atmospheric neutrino problem, and are actively being looked for at dozens of experiments at present.

The solar neutrino problem consists of the deficit of observed neutrinos emitted from the sun with respect to the theoretically expected amount. It has been with us for more than 30 yr, but for a long time it was based on only one experiment, led by R. Davis. Several other experiments have now also observed solar neutrinos. One of them, Kamiokande, actually became the first neutrino telescope when it succeeded in identifying the sun in the sky through its neutrino-image. The results of the four experiments that have observed solar neutrinos so far, even if still inconclusive, support the existence of a solar neutrino deficit, and suggest that the solution lies in new neutrino physics (as opposed to a modification of our standard solar model). This is a very exciting field of research and several new experiments that are under way will allow us to understand fully the solar neutrino problem within the next few years. One of the favourite explanations of this problem involves matter induced oscillations, with very small neutrino mass differences, $\Delta m^2 \simeq 10^{-6}$ eV^2 (see section 7).

Atmospheric neutrinos produced by cosmic rays hitting the earth atmosphere show a deficit of ν_μ relative to ν_e with respect to the expected ratio. This deficit could be explained if part of the ν_μ oscillated into ν_τ (oscillations into ν_e or a sterile neutrino are not allowed, see section 8) within the atmosphere, before reaching the surface of the earth. This would require $\Delta m^2 \simeq 10^{-2}$ eV^2 (see section 8). New long-baseline oscillations experiments are being proposed to test experimentally this hypothesis (see section 6). The baselines of these experiments are actually amazingly long. For example, neutrinos produced at CERN, in Switzerland, could be detected at the Gran Sasso Laboratory, in Italy.

Neutrinos are not only important in particle physics but also in astrophysics (section 9) and cosmology (section 5). Neutrino emissions are an important, and sometimes dominant, energy loss mechanism in stars. So much so that neutrino properties can be constrained on the basis of the unacceptable changes they would introduce in the evolution of stars (section 9). Nowhere is the neutrino energy loss more striking than in a supernova explosion in which 99% of the energy released goes into neutrinos. This is the only case in which the matter densities achieved are so high that neutrinos are temporarily trapped. The observation of 19 of the neutrinos emitted by the supernova SN1987A inaugurated neutrino astronomy outside the solar system, and brought about an amazing amount of information on neutrinos. For example, a bound on the ν_e mass was obtained, $m_{\nu_e} < 23$ eV. We would need the observation of a supernova in our galaxy (as opposed to SN1987A that happened in one of the Milky Way satellite galaxies, the Large Magellanic Cloud) to be able to say something about (even measure) the masses of the other neutrinos (if they are larger than about 25 eV with the experiments under construction at present).

Turning to cosmology (section 5), the theory of nucleosynthesis as well as the dark matter problem and its related issue of structure formation in the universe, would be greatly affected by new neutrino properties. The primordial nucleosynthesis of light elements (D + ^{3}He, ^{4}He and ^{7}Li) is one of the fundamental pieces of evidence on which the Big Bang model relies. Some neutrino properties, such as a finite mass, lifetime or magnetic moment may considerably change the outcome of nucleosynthesis and, in fact, this is used at present to constrain neutrino properties. Most of the mass content of the universe, 90 to 99%, is in a form of matter that does not emit or absorb light in any observable way. This is the dark matter (DM). Neutrinos of mass in the range of a few eV to a few tens of eV could be an important component of the DM. Structure formation arguments (namely the formation of galaxies and clusters of galaxies) seem to require these light neutrinos not to constitute the bulk of the DM. These neutrinos would be hot DM (HDM, i.e. relativistic when galaxies should start forming, namely at a temperature of approximately 1 keV) while structure formation prefers the bulk of the DM to be cold (CDM). However, CDM does not seem to account by itself for all the observations, and an admixture of something else seems to be required. This could be some neutrinos as HDM, with mass of a few eV, or heavier unstable neutrinos with a tuned combination of masses and lifetimes.

In this brief introduction we see that many important effects in physics, astrophysics, and cosmology depend on neutrino properties, all associated with their mass. We have not yet mentioned one of the major motivations that particle physicists have to find neutrino masses. This motivation has to do with the triumph and the tragedy of the standard model of elementary particles, both joined in the fact that it works extremely well. So well that at this point any sign of failure and a consequent indication to go beyond this model will be more welcome than a new confirmation. This is what any non-zero neutrino mass would be. A potentially rich window towards physics beyond the standard model. Moreover, the expectation of this opening towards new physics is intense at present because the solar and atmospheric neutrino problems seem to be giving hints of non-zero masses, that will be confirmed or rejected in the near future.

This review is organized in the following manner. We start by explaining the different neutrino masses corresponding to different type of neutrinos, Dirac or Majorana, in section 2. In section 3 we discuss the main elementary particle models for neutrino masses, starting with an explanation of the origin of the masslessness of neutrinos in the SM. We present then the main types of proposed extensions of the SM that incorporate neutrino masses (plain Dirac masses, left-handed neutrino Majorana masses, see-saw models) and their distinctive phenomenological consequences.

In section 4 we describe the status of direct mass searches and Majorana mass searches in neutrinoless double beta decays. In section 5 we go over the many cosmological implications of, and constraints on, neutrino properties, mainly masses and lifetimes. Sections 6, 7 and 8 review neutrino oscillations, the solar neutrino problem and the atmospheric neutrino problem, their implications for neutrino masses and the current and future experiments with which these problems will be clarified in the near future. We explain oscillations in vacuum in section 6 and oscillations in matter in section 7 (as well as other possible solutions to the solar neutrino problem). Section 9 describes stars, mainly SN1987A, as a laboratory for neutrino physics and summarizes the main bounds they impose on neutrino masses, lifetimes and magnetic and electric moments. A few concluding remarks follow.

2. Types of Possible Neutrino Masses

Because the electroweak interactions of the Standard Model (SM) violate parity P (and charge-conjugation C) maximally, they distinguish fermions of different chirality. These are eigenstates of the Dirac matrix γ_5, with eigenvalue $+1$ for right-handed chirality and -1 for left-handed chirality. Chirality eigenstates are called Weyl spinors. Starting from a Dirac spinor ψ, that has four independent complex components, we obtain two orthogonal two-component Weyl spinors of definite left and right-handed chirality, ψ_L and ψ_R, by means of the projectors P_L and P_R,

$$\psi_L = P_L\psi \equiv (\frac{1-\gamma_5}{2})\psi \quad , \quad \psi_R = P_R\psi \equiv (\frac{1+\gamma_5}{2})\psi \quad . \tag{2.1}$$

The property $\gamma_5^2 = 1$ insures that P_L and P_R are, in fact, projectors onto orthogonal components, i.e. $P_L^2 = 1$, $P_R^2 = 1$ and $P_R P_L = P_L P_R = 0$. The decomposition,

$$\psi = \psi_L + \psi_R \tag{2.2}$$

is invariant under the homogeneous Lorentz group (i.e. the continuous transformations of the Lorentz group, rotations and boosts), because ψ_L and ψ_R transform as separate irreducible representations. However, a discrete parity (P) transformation transforms left and right-handed Weyl spinors into each other. This is why a P-invariant theory must treat both chiralities identically (equal couplings) and conversely, when only one chirality is present, the violation of parity is maximal. The experimentally determined $V-A$ (vector minus axial vector) coupling in charged current weak interactions means that only left-handed fermions interact with the gauge bosons of the group $SU_L(2)$ and P-violation is maximal. For example, the leptonic charged current term in the Lagrangian of the SM is

$$\mathcal{L}^{CC} = \frac{g}{\sqrt{2}}\sum_\ell \bar{\ell}\gamma^\mu\frac{(1-\gamma_5)}{2}\nu_\ell W_\mu^- + h.c. = \frac{g}{\sqrt{2}}J_\ell^{-\mu}W_\mu^- + h.c. \quad . \tag{2.3}$$

Here g is the weak coupling constant, $\ell = e, \mu, \tau$ are the charged leptons of the known generations, ν_ℓ are the associated neutrinos and W_μ^- is the charged heavy gauge boson. Notice that

$$\frac{1}{2}\overline{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell = \overline{\ell}\gamma^\mu P_L\nu_\ell = \overline{\ell_L}\gamma^\mu\nu_{\ell L}, \tag{2.4}$$

because γ_5 anticommutes with all the other Dirac matrices $\gamma_\mu = \gamma_0, \gamma_1, \gamma_2, \gamma_3$, i.e. $\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$, and, thus $\overline{\psi_R} = \overline{\psi}P_L$, $\overline{\psi_L} = \overline{\psi}P_R$.

A Weyl spinor can describe only free massless particles, for which chirality coincides with helicity (the spin projection in the direction of motion), that is a good quantum number. This is so because mass terms mix chiralities. Thus, if a fermion is massive, the helicity eigenstates, the eigenstates of a freely propagating fermion, do not coincide with the chirality eigenstates, the eigenstates of weak interactions. A massive fermion can be either a Dirac or a Majorana particle, that are described respectively by a Dirac and a Majorana spinor.

A Dirac spinor, such as the one describing an electron, has four independent complex components, corresponding to particle and antiparticle, both of left- and right-handed helicity. It was actually Dirac's equation for the electron that led to the concept of particles and antiparticles and to the definition of charge-conjugation. Antiparticle spinors are obtained from particle spinors through charge-conjugation, under which $\psi \to \eta_c \psi^c$, with η_c a phase and

$$\psi^c = C\overline{\psi}^T = C\gamma_0\psi^*. \tag{2.5}$$

Here C is a unitary matrix that is defined by the property $C^{-1}\gamma_\mu C = -\gamma_\mu^T$. If we now apply P_L or P_R to the Dirac spinor and its charge-conjugate spinor, we see that under the physical charge conjugation operation, that should preserve chirality* (and ignoring the phase η_c hereafter), $\psi_L \to (\psi^c)_L$ and $\psi_R \to (\psi^c)_R$ (see, e.g., Langacker 1981). Hence, under this operation, a Dirac particle in a given state of momentum and helicity is changed into its antiparticle in the same state of momentum and helicity.

Notice that one does not obtain the charge-conjugate of the Weyl spinor components of a Dirac spinor by applying the conjugation operation (2.5) directly to them. In fact, the operations of conjugation (2.5) and projection onto chirality components do not commute, $(\psi_L)^c \equiv C\overline{\psi_L}^T$ is a R-handed spinor, it is the right component of ψ^c in (2.5), $(\psi_L)^c = (\psi^c)_R$. Hence, in this operation the chirality is reversed, while, as discussed above, under charge conjugation $\psi_L \to (\psi^c)_L = (\psi_R)^c$ (thus, an interaction containing only L-handed fields violates charge-conjugation maximally). Many authors then understandably avoid calling $(\psi_L)^c$ the charge-conjugate of ψ_L. They call it simply "conjugate" (see e.g. Mohapatra and Pal 1991), or even loosely "CP-partner" (see e.g. Langacker 1981 and 1992), because of the additional change of chirality, even if it is not the CP-conjugate**. We will then call c-conjugation the operation defined in (2.5) and we call ψ^c the c-conjugate of ψ for any kind of field ψ, Dirac or Weyl (or later Majorana). The operation c-conjugation coincides, up to a phase, with charge-conjugation C for a Dirac field (and for a Majorana field) but not for a Weyl spinor component of a Dirac field. It is

* It should preserve helicity, that is the actual physical property of a particle, but thinking of the limit in which chirality and helicity coincide, it is obvious that chirality must also be preserved.

** Actually CP conjugation is a different operation, involving an additional multiplication by γ_0 and a change of $\vec{x}$ into $-\vec{x}$, $\psi^{CP}(t,\vec{x}) = \eta_{CP}\gamma_0\psi^c(t,-\vec{x})$, where η_{CP} is a phase.

also trivial to check that using $(\psi_L)^c = (\psi^c)_R$ and $(\psi_R)^c = (\psi^c)_L$ or using the C-transformation $\psi_L \to (\psi^c)_L$ and $\psi_R \to (\psi^c)_R$ yields the same result when applied to $\psi_L + \psi_R$ (we only exchange which component of ψ^c we call the conjugate of which component of ψ).

In the language of relativistic quantum field theory, a Weyl field, say ψ_L, can annihilate a left-handed (L) particle or create a right-handed (R) antiparticle, while $(\psi^c)_R = C\overline{\psi_L}^T$, can annihilate a R-antiparticle or create a L-particle. For ψ_R the roles of right and left are exchanged. Notice that ψ_L and $(\psi^c)_R$ are not independent fields, they represent the same two degrees of freedom. The same is valid for ψ_R and $(\psi^c)_L$. For Dirac fermions ψ_L and ψ_L^c represent different degrees of freedom but for Majorana fermions they coincide. Since the fields ψ_L and $(\psi^c)_L$ must have opposite values of all additive quantum numbers, Majorana fermions cannot carry any conserved charge.

Electrons e and positrons e^c are clearly different particles, since they have opposite electric charges. However, we have examples of neutral particles, such as the π^0, that coincide with their antiparticles. Majorana (Majorana 1937) first proposed that a neutral fermion could have this property. While a Dirac fermion is always different from its antiparticle, a Majorana fermion is such that $(\psi^M)^c = \psi^M$, or actually,

$$(\psi^M)^c = e^{i\Theta}\psi^M \tag{2.6}$$

since a phase, $e^{i\Theta}$ (called sometimes Majorana creation phase factor, Kayser 1984), can always be incorporated in the definition of the fermion ψ^M. Instead of four independent components, a Majorana spinor has only two, since particle and antiparticle coincide.

Is ν^c different or identical to ν?

Only neutrinos of left-handed chirality and antineutrinos of right -handed chirality (present in the hermitian conjugated terms of (2.3)), ν_L and $(\nu^c)_R$, interact in the SM. If they exist, the components of opposite chirality, ν_R and $(\nu^c)_L$, are "inert" since they do not participate in weak interactions. If neutrinos are massless they are described by Weyl spinors, chirality and helicity coincide and we can never produce neutrinos and antineutrinos of the same helicity in weak processes. "Neutrino" and "antineutrino" are just names at this point, for states that interact differently due to their different helicity. For massive neutrinos we can change the helicity by a boost that inverts the momentum. We can then produce neutrinos and antineutrinos of the same helicity and in principle compare them. If they interact differently, neutrinos are Dirac particles, if they are identical, neutrinos are Majorana particles (for a more detailed explanation, see e.g. Kayser 1985). Although Majorana and Dirac neutrinos have different properties, the differences vanish with the neutrino mass. The only feasible experiment that may actually prove the Majorana nature of neutrinos is the neutrinoless double beta decay (see section 4.2).

A Dirac mass term, of the form

$$-\mathcal{L}_{\text{mass}} = m^D \overline{\psi}\psi = m^D \overline{(\psi_L + \psi_R)}(\psi_L + \psi_R) = m^D(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L) \tag{2.7}$$

mixes two different Weyl spinors, of opposite chirality. This is the type of mass generated in the SM. Thus, the exclusion in the SM of the ν_R (and $(\nu^c)_L$) components insures ad-hoc that neutrinos are massless. A Majorana mass term can be written with only one Weyl spinor, say ψ_L, and its c-conjugate $(\psi_L)^c = C\overline{(\psi_L)}^{\,T} = (\psi^c)_R$,

$$-\mathcal{L}_{\text{mass}} = \frac{1}{2} m^M \left[\overline{(\psi_L)^c}\psi_L + \overline{\psi}_L (\psi_L)^c \right] = \frac{1}{2} m^M \overline{(\psi_L + (\psi_L)^c)}(\psi_L + (\psi_L)^c) \ . \tag{2.8}$$

The mass eigenstate field ψ^M,

$$\psi^M \equiv \psi_L + (\psi_L)^c = \psi_L + (\psi^c)_R \ . \tag{2.9}$$

has the canonical mass term $\frac{1}{2} m^M \overline{\psi^M} \psi^M$ and is a Majorana field with creation phase $e^{i\theta} = +1$, since $(\psi^M)^c = \psi^M$ (notice that $((\psi_L)^c)^c = \psi_L$). The field

$$\widetilde{\psi}^M = \psi_L - (\psi_L)^c \tag{2.10}$$

is also a Majorana field, as defined in (2.6), with the phase $e^{i\theta} = -1$, i.e. $(\widetilde{\psi}^M)^c = -\widetilde{\psi}^M$. Clearly $\widetilde{\psi}^M$ and ψ^M describe the same degrees of freedom. It is straightforward to check that if we were to use $\widetilde{\psi}^M$ instead of ψ^M in (2.8), the sign of the mass term (2.8) would be reversed.

Notice that a Majorana mass term carries two units of fermion number since $\overline{(\psi_L)^c}$ has the same fermion number as ψ_L. A Dirac mass term has instead zero fermion number, since $\overline{\psi}_R$ and ψ_L have opposite fermion numbers. Thus, Majorana mass terms are forbidden if the fermion number, the lepton number in the case of neutrinos, is conserved.

When both Majorana and Dirac masses are present, in general the massive fermions are Majorana particles, and fermion number is not conserved. The most general mass matrix using the Weyl spinors ν_L and ν_R is:

$$\begin{aligned} -\mathcal{L}_{\rm mass} = & m_D \; (\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L) + \\ & + \frac{1}{2} m_L^M \big(\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L\big) + \frac{1}{2} m_R^M \big(\overline{(\nu_R)^c}\nu_R + \overline{\nu_R}(\nu_R)^c\big) \quad . \end{aligned} \tag{2.11}$$

In matricial form it becomes (using $\overline{\nu_L}\nu_R = \overline{(\nu_R)^c}(\nu_L)^c$)

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}\big(\overline{\nu_L} \;\; \overline{(\nu_R)^c}\big) \begin{pmatrix} m_L^M & m_D \\ m_D^T & m_R^M \end{pmatrix} \begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + h.c. \quad . \tag{2.12}$$

This expression is also valid for an arbitrary number of flavours, just taking $\nu_L^T = (\nu_{eL}, \; \nu_{\mu L}, \ldots)$ and similarly for ν_R. Due to the anticommutation properties of the fermion fields, the mass matrix in (2.12) turns out to be symmetric (see e.g. Bilenky and Petcov 1987). We recall that a symmetric matrix M can generally be brought to a diagonal matrix M_d with positive entries, by means of a unitary transformation U, i.e. $U^T M U = M_d$.

With one neutrino species, m_L^M, m_R^M and m_D are numbers (thus, m_D^T=m_D). Let us consider this case first. When $m_L^M = 0$ and $m_R^M = 0$, the mass eigenstate is a Dirac fermion $(\nu_L + \nu_R)$ with mass m^D.

In this case the diagonalization of the matrix of (2.12) yields two chiral eigenstates $\psi_{L\pm} \equiv (\nu_L \pm (\nu_R)^c)/\sqrt{2}$, with equal and opposite mass eigenvalues, m_D and $-m_D$. In terms of the associated Majorana fields $\psi_\pm \equiv \psi_{L\pm} + (\psi_{L\pm})^c$, that are self-conjugate by construction, the mass term reads $-2\mathcal{L}_{mass} = m_D\overline{\psi}_+\psi_+^c - m_D\overline{\psi}_-\psi_-^c$. The negative sign in the mass can however be absorbed into a redefinition of the corresponding Majorana eigenstates, taking them to be $\psi_1 = \psi_+$ and $\psi_2 = \psi_{L-} - (\psi_{L-})^c$, and this last will consequently transform under conjugation as the field $\widetilde{\psi}^M$ discussed above. Notice that the original sign of the mass, call it λ_m, becomes now the Majorana creation phase (the phase in (2.6)) of the mass eigenstate field with positive mass. We also note that Majorana particles, unlike Dirac ones, have a well defined intrinsic CP parity, $\tilde{\eta}_{CP}$*. This is so because CP transforms a particle field into the antiparticle field and while for Dirac neutrinos there is the freedom to redefine the phase of the

* $\tilde{\eta}_{CP} = e^{i\theta}\eta_{CP} = \pm i$, where η_{CP} is the phase of the CP-transformation (see footnote in previous page) and $e^{i\theta}$ is the Majorana creation phase.

antiparticle field to absorb the CP phase, this is not possible in the Majorana case. It is always possible to chose the neutrino fields, as we did in our example, so that $\tilde{\eta}_{CP} = \lambda_m i$ (see e.g. Kayser 1988 and 1984 and Bilenky and Petcov 1987).

The linear combination of degenerate eigenstates $\nu = (\psi_1 + \psi_2)/\sqrt{2}$ is a Dirac neutrino, $\nu = (\nu_L + \nu_R)$, and $\nu^c = (\psi_1 - \psi_2)/\sqrt{2})$. Thus, a (four-component) Dirac field consists of two (two-component) degenerate Majorana fields with opposite values of λ_m (or, in terms of particle states, a Dirac state can be viewed as the sum of two degenerate Majorana states with opposite CP-parities). The phase λ_m is irrelevant for freely propagating neutrinos, but it will appear in the charged currents involving the $(\nu_L)^c$ field, affecting for instance the neutrinoless double beta decay (see section 4.2).

These phases appear generically when dealing with Majorana neutrinos. The reason is the following. When CP is conserved, arbitrary phases in the original fields can be chosen so that the mass matrix is real. Because it is also symmetric, and hence hermitian, one is tempted to diagonalize it by means of an orthogonal (not just unitary) transformation O. The price to pay is, then, the possible appearance of negative eigenvalues for self-conjugate fields or antiself-conjugate fields, if one wants positive masses (as in the simple example of two degenerate eigenstates presented above). An alternative procedure would be to insist in having self-conjugate fields with positive masses, at the expense of having a non-real transformation matrix. This is obtained by just replacing the $\psi_{nL} = \sum O_{nl}\nu_{lL}$ mass eigenstates with negative masses (here n labels mass eigenstates and l interaction eigenstates) by the states $\psi'_{nL} = i\psi_{nL} = \sum (iO_{nl})\nu_{lL}$. The Majorana states $\psi'_n = \psi'_{nL} + (\psi'_{nL})^c$ will be self-conjugate and have positive masses, but their mixing matrix (and their coupling to the charged gauge bosons) will involve the non-real matrix $U_{nl} = iO_{nl}$. These different approaches are of course generalizable to the CP violating case.

Going back to (2.12), when either m_L^M or m_R^M or both are non zero, the mass eigenvectors are two Majorana spinors with different mass. When the Majorana masses are small compared to the Dirac masses, the resulting eigenstate is a "pseudo-" Dirac neutrino (Wolfenstein 1981), in which the two Majorana fields are not exactly degenerate but almost so.

In order to incorporate the three known generations, we take each of the spinors to be a vector in generation space, $\nu_L = (\nu_{e_L}, \nu_{\mu_L}, \nu_{\tau_L})$, etc. and m^D, m_L^M and m_L^M to be matrices. If there are as many right-handed neutrino species as left-handed ones, the sub-matrices in (2.12) are 3×3. However, it may be that ν_R are entirely absent, so only m_L^M exists, or that the number of ν_R is different from three. In the case of several generations there is a special type of Dirac neutrino that may appear as a mass eigenstate, the ZKM type (Zeldovich 1952, Konopinski and Mahmoud 1953). Usually a Dirac neutrino left-handed component is "active" (it interacts weakly) while its right-handed component is "sterile" (or inert). Both components of a ZKM neutrino are active. The right handed component is the antiparticle of the left-handed neutrino of a different generation, e.g. $\nu = \nu_{eL} + (\nu_{\mu L})^c$. The ZKM neutrino appears when a linear combination of flavour lepton numbers different that the usual total lepton number ($L = L_e + L_\mu + L_\tau$ leading to a usual Dirac neutrino) is conserved, in the example it is $L_e - L_\mu$. This type of neutrino requires a complicated Higgs structure not necessary for usual Dirac neutrinos.

A Dirac neutrino has, in general, a magnetic dipole moment, and may also have an electric dipole moment (that violates CP). So, in principle, a torque exerted by an external magnetic or electric field, $\vec{B}$ or $\vec{E}$, can change its chirality (since dipole moment interactions couple Weyl spinors of opposite chiralities). Majorana neutrinos can only have "transition" dipole moments, (Schechter and Valle 1981 and 1982, Kayser 1982, Nieves 1982), which in a $\vec{B}$ or $\vec{E}$ field cause the neutrino not only to change chirality, but also to change flavour (i.e. a neutrino of one generation is changed into another different neutrino of a different generation), for example $\nu_{\mu L} \rightarrow (\nu_e^c)_R$. Transition moments can exist for Dirac neutrinos too, and they imply also the radiative decay of the heavier neutrino into the lighter one and a photon.

3. Main Elementary Particle Models for Neutrino masses.

3.1. The Standard Model (SM)

In the SM all charged fermions, quarks and leptons, get Dirac masses through the Higgs mechanism that breaks spontaneously the $SU_L(2) \times U_Y(1)$ electroweak gauge symmetry into $U_{e.m.}(1)$ of electromagnetism, giving mass to three of the four gauge vector bosons that mediate electroweak interaction, the W^+, W^-, and Z^0, while only the photon stays massless. The absence of the ν_R prevents the appearance of a Dirac mass for the neutrinos. The accidental global symmetry $U_{B-L}(1)$ corresponding to the conservation of $B-L$, Baryon number minus Lepton number, prevents the appearance of Majorana masses for neutrinos. Thus neutrinos are massless in the SM.

All masses in the SM, are proportional to the vacuum expectation value (VEV) of a complex $SU_L(2)$ doublet scalar field ϕ and its conjugate $\tilde{\phi} = i\sigma^2\phi^*$ (where σ^2 is the second Pauli matrix), that is also a doublet (because the representations of $SU(2)$ are real),

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} \quad , \quad \tilde{\phi} = \begin{pmatrix} \phi^+ \\ -\phi^{0*} \end{pmatrix} \quad . \tag{3.1}$$

A potential energy for ϕ is introduced, that has its minimum at

$$\langle \phi \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \quad . \tag{3.2}$$

Since the component ϕ^0 that has non-zero VEV is neutral, $U_{e.m.}(1)$ is preserved as a good gauge symmetry. Given the masses of the W and Z particles, one obtains

$$v \simeq 250 \text{ GeV} \ . \tag{3.3}$$

The matter fields are 15 Weyl spinors, repeated twice at larger masses yielding three generations or families (see the table 1). The quarks q and leptons ℓ of the different generations are distinguished by their "flavour". There are three flavours of charged leptons, the electron e, the muon μ, and the τ, and their corresponding neutrinos ν_e, ν_μ, ν_τ. Quarks carry also "colours", the quantum numbers of strong interactions. There are three colours for each quark flavour, since quarks belong to the fundamental representation of $SU_{\rm C}(3)$. In the table 1 the parentheses show the representations of $SU_C(3)$ and $SU_L(2)$ (both indicated by their dimensionality), and the value of the weak hypercharge Y, for each set of fields.

The left handed fermions f_L (q_L and ℓ_L) have weak isospin $T_L = 1/2$ (T_3 is $+1/2$ and $-1/2$ for the upper and lower components). The right handed fermions f_R, have $T_L = 0$, (thus $T_{3L} = 0$). The charge Q of each field is given by $Q = T_{3L} + Y$. The charges of the particles in each generation sum to zero, $\Sigma_f Q = 0$. This insures that anomalies cancel within each family and, thus, the SM symmetry is preserved at the quantum level.

Table 1. Matter fields in the Standard Model. Here, $i = 1, 2, 3$ is the colour index.

Quarks	Leptons
$q_L :\ (3, 2, 1/6)$	$\ell_L :\ (1, 2, -1/2)$
$\begin{pmatrix} u \\ d \end{pmatrix}_{Li}, \begin{pmatrix} c \\ s \end{pmatrix}_{Li}, \begin{pmatrix} t \\ b \end{pmatrix}_{Li}$	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_{L}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_{L}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_{L}$
$u_R :\ (3, 1, 2/3)$	
$u_{Ri}\ ,\ c_{Ri}\ ,\ t_{Ri}$	
$d_R :\ (3, 1, -1/3)$	$e_R :\ (1, 1, -1)$
$d_{R_i}\ ,\ s_{Ri}\ ,\ b_{Ri}$	$e_R\ ,\ \mu_R\ ,\ \tau_R$

The $SU_L(2) \times U_Y(1)$ assignments of f_L and f_R make it impossible to have gauge invariant (i.e. gauge singlet) mass terms, i.e.

$$m_f \overline{f_L} f_R \quad , \tag{3.4}$$

in the Lagrangian. Notice that these Dirac mass terms have a weak isospin $T_L = 1/2$, thus they need to be coupled to a weak doublet to form a singlet. In fact, the fermion masses come from gauge invariant Yukawa type couplings with the doublet scalar $\phi = (1, 2, -1/2)$,

$$-\mathcal{L}_{\text{Yukawa}} = \sum_{\alpha,\ \beta} \left[(\lambda_u)_{\alpha\beta}\ \overline{q_{L\alpha}} \phi u_{R\beta} + (\lambda_d)_{\alpha\beta}\ \overline{q_{L\alpha}} \tilde{\phi} d_{R\beta +} + (\lambda_\ell)_{\alpha\beta}\ \overline{\ell_{L\alpha}} \tilde{\phi} e_{R\beta} \right] + h.c. \quad , \tag{3.5}$$

where $\alpha, \beta = 1, 2, 3$ are generation indices and $(\lambda_f)_{\alpha\beta}$ are coupling constants. Replacing ϕ by its VEV (3.2), one obtains the fermion mass matrices

$$(m_f)_{\alpha\beta} = (\lambda_f)_{\alpha\beta} \frac{v}{\sqrt{2}} \quad . \tag{3.6}$$

In the SM, the couplings $(\lambda_f)_{\alpha\beta}$ are arbitrary complex numbers, thus all fermion masses are free parameters. As we mentioned before, it is only the absence of the right-handed neutrinos $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$, that prevents the appearance of Dirac neutrino masses through Yukawa couplings with the Higgs field.

Therefore, in the SM neutrinos are massless because there are no right-handed neutrinos ν_R and there are no other Higgs bosons besides the doublet (that could generate Majorana masses). We could entertain the idea that even in this case neutrinos could get masses, Majorana masses, due to higher order corrections. This is not possible in the SM due to the accidental conservation of $B-L$. Actually B and the separate flavour lepton numbers L_e, L_μ and L_τ are accidental symmetries, but they are all anomalous symmetries. Only the combination $B-L$ is anomaly-free and hence it is a good global symmetry of the SM. This symmetry prevents the appearance of Majorana mass terms also beyond the tree level, because they would violate L and, therefore, $B-L$, by two units.

We can see now that models of non-zero neutrino masses necessarily add to the SM either fermions, typically ν_R, or bosons, or both, and when the masses are of the Majorana type, they introduce a violation of $B-L$, either explicit or spontaneous. We can also see here that if neutrinos are massive, the leptonic charged currents in terms of mass eigenstates include a mixing matrix K_ℓ equivalent to the Cabibbo-Kobayashi-Maskawa (Cabibbo 1963, Kobayashi and Maskawa 1973) mixing matrix K_q present in the weak charged currents of quarks. The mass matrices (3.6) must be diagonalized, thus quarks and charged leptons are expressed in the basis of mass eigenstates

$$u_L \rightarrow U_L^{u\dagger} u_L \quad , \quad d_L \rightarrow U_L^{d\dagger} d_L \quad , \quad e_L \rightarrow U_L^{\ell\dagger} e_L \quad . \tag{3.7}$$

Because neutrinos are massless in the SM, U_L^ℓ is absorbed in the redefinition of the neutrinos so that one obtains the coupling in (2.3) and (3.8) where $K_\ell = 1$, while the matrix $K_q = U_L^u U_L^{d\dagger}$ remains in the charged current of quarks (we ignore colour indices),

$$J^{+\mu} = (\bar{u}\ \bar{c}\ \bar{t})_L\ \gamma^\mu K_q \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L + (\bar{\nu}_e\ \bar{\nu}_\mu\ \bar{\nu}_\tau)_L \gamma^\mu \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L \tag{3.8}$$

Neutrino mass terms in the Lagrangian are always written in the basis of "current eigenstates" (or interaction or flavour eigenstates), that are those that diagonalize the lepton weak charged current with the charged lepton mass eigenstates (as in (2.3) and (3.8)), namely ν_e, ν_μ, ν_τ. Moreover charged lepton current eigenstates are chosen to coincide with the charged mass eigenstates so that $U_L^\ell = 1$. Then, when neutrinos are massive, current neutrino eigenstates differ from mass eigenstates by a mixing matrix

$K_\ell = U_L^\nu$. Thus in (3.8)

$$(\nu_L^{\mathrm{current}})^{\mathrm{T}} = (\nu_e\ \nu_\mu\ \nu_\tau)_L = (\nu_L^{\mathrm{mass}})^{\mathrm{T}} K_\ell^* = (\nu_1\ \nu_2\ \nu_3\ ...)_L K_\ell^* \quad . \tag{3.9}$$

where the dots indicate that there could be more than three neutrino mass eigenstates (either heavier than $\simeq 45$ GeV or mainly consisting of inert neutrinos).

3.2. Plain Dirac Masses

This possibility corresponds to having only m_D non-zero in the general mass matrix of (2.12). Right-handed neutrinos can be easily added to the SM, as singlets of the gauge group $SU_C(3) \times SU(2) \times U_Y(1)$, $\nu_R : (1, 1, 0)$. Since they are inert, they do not contribute to anomalies and the renormalizability of the model is not affected. We could then add neutrino Yukawa couplings to (3.5) (similar to the up-quark terms),

$$\sum_{\alpha\beta} (\lambda_\nu)_{\alpha,\beta}\ \overline{\ell_{L\alpha}}\ \phi\ \nu_{R\beta} \quad , \tag{3.10}$$

that would yield Dirac neutrino masses as (3.6), $(m_\nu)_{\alpha\beta} = (\lambda_\nu)_{\alpha\beta}\ v/\sqrt{2}$. This is the simplest way to get non-zero neutrino masses. However, this mechanism provides no explanation for the smallness of neutrino masses with respect to the other fermions of the same generation. All we could say is that $(\lambda_\nu)_{\alpha\beta}$, being arbitrary as all the other Yukawa couplings, happen to be much smaller than the others. Other mechanisms provide some insight into this question.

3.3. Models Without ν_R

Counting only with left-handed Weyl spinors, ν_L, we can only have the Majorana mass terms of m_L^M in (2.12). These terms have weak isospin $T_L = 1$. Thus, renormalizable interactions at tree level give origin to these masses (through a Higgs Mechanism) only if a triplet Higgs field $\Delta = (\Delta^{++}, \Delta^{+}, \Delta^{0})$ is added to the SM, with Yukawa couplings

$$g_{\alpha\beta} [\overline{(\ell_{\alpha L})^c}\ \vec{\sigma}\ \ell_{\beta L}] \cdot \vec{\Delta} + h.c. \quad . \tag{3.11}$$

Here $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices, $\ell_{\alpha L} = (\nu_{\alpha L},\ e_{\alpha L})$ are lepton doublets, thus $(\ell_{\alpha L})^c = ((e_{\alpha L})^c, -(\nu_{\alpha L})^c)$, since $(\ell_{\alpha L})^c$ is also the SU(2) conjugate of $\ell_{\alpha L}$ (as

$\tilde{\phi}$ is the conjugate of ϕ in (3.1)), α, β are generation indices, $\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$ and $\Delta^0 = (\Delta_1 + i\Delta_2)/\sqrt{2}$, $\Delta^+ = \Delta_3$ and $\Delta^{++} = (\Delta_1 - i\Delta_2)/\sqrt{2}$. These Yukawa couplings contain terms

$$g_{\alpha\beta}\ \sqrt{2}\ \overline{(\nu_{\alpha L})^c}\nu_{\beta L}\Delta^0 + \cdots \tag{3.12}$$

that yield Majorana masses proportional to the VEV $\langle\Delta^0\rangle = v_T/\sqrt{2}$,

$$(m_L^M)_{\alpha\beta} = g_{\alpha\beta} v_T \ . \tag{3.13}$$

Here the smallness of the neutrino mass depends on choosing ad-hoc a small v_T, $v_T \ll v$.

When Majorana masses are present in a model there is a choice between explicit or spontaneous L violation. Moreover L (actually we should speak of $B-L$, because L is anomalous) may be a gauge or a global symmetry. If L is a global symmetry, its spontaneous breaking would generate a Nambu-Goldstone boson, that is called Majoron.

The lepton number of Δ is $L_\Delta = -2$, as defined by (3.11). Therefore, if L is conserved in the Lagrangian, $\langle\Delta^0\rangle \neq 0$ violates L-conservation spontaneously and a Goldstone boson results. This is the basic mechanism of the triplet Majoron model (Gelmini and Roncadelli 1981), now experimentally rejected (see below).

Given the matter content of the SM (table 1), with no ν_R, there are only two other scalars, besides the standard doublet ϕ and the triplet Δ, that can have $SU_L(2)\times U_Y(1)$ invariant Yukawa couplings. These are, a singly charged singlet scalar field h^+, first proposed by Zee (Zee 1980), that could couple to lepton doublets

$$f_{\alpha\beta}\overline{(\ell_{\alpha L})^c}\ell_{\beta L}h^+ = f_{\alpha\beta}(\overline{(e^-_{\alpha L})^c}\nu_{\beta L} - \overline{(\nu_{\alpha L})^c}e^-_{\beta L})h^+ \ , \tag{3.14}$$

and a doubly charged singlet scalar h^{++}, that could couple to lepton singlets

$$f'_{\alpha\beta}\overline{(e_{\alpha L})^c}\ (e_{\beta R})h^{++} \ . \tag{3.15}$$

Here, the couplings $f_{\alpha\beta}$ and $f'_{\alpha\beta}$ must be antisymmetric in α and β due to Fermi statistics (α, β denote generations). Thus h^+ and h^{++} couple only to two leptons of different families. Neither h^+ nor h^{++} can have a non-zero VEV (otherwise electromagnetism would be a spontaneously broken symmetry). Thus L violation has to be introduced in the Higgs potential, by adding at least a second doublet Higgs field, and Majorana masses for neutrinos are obtained through radiative corrections (Zee 1980). While the Zee model, based on h^+, does not include the h^{++} field, models with h^{++} do require h^+ (Babu 1988). In both models neutrino masses are small because they are generated through radiative corrections, and a distinctive feature is the presence of a neutrino much lighter than the other two.

Some L-violating physics beyond the SM, at a large mass scale Λ, may induce ν_L Majorana masses through effective (non-renormalizable) couplings. The leading non-renormalizable L-violating operator that can be written with the fields of the SM is of dimension four (Barbieri *et al* 1980)

$$\mathcal{L}_{\rm eff} = C_{\alpha\beta}\left[\overline{(\ell_{\alpha L})^c}\vec{\sigma}\ell_{\beta \rm L}\right]\frac{(\phi^T\sigma_2\vec{\sigma}\phi)}{\Lambda} \quad . \tag{3.16}$$

When the doublet ϕ gets a VEV (3.2), (3.16) gives Majorana masses

$$(m_L^M)_{\alpha\beta} \simeq \frac{C_{\alpha\beta}v}{\Lambda} \ . \tag{3.17}$$

For $\Lambda \gg v$ one naturally obtains small neutrino masses. As we will see below, in see-saw models Λ is the ν_R Majorana mass scale, $\Lambda \simeq m_R^M$.

We may not need new physics to have terms as (3.16). We know that gravity is not incorporated into the SM and it may not respect any global symmetry (for example, global charges disappear when falling into a black hole). Quantum gravity effects may violate L. In this case $\Lambda \simeq M_{\rm Planck} \simeq 10^{19}$ GeV and the resulting masses are $m_L^M \simeq C10^{-5}$ eV.

3.4. See-Saw Models

The "see-saw" mechanism (Yanagida 1979, Gell-Mann *et al* 1979) consists of making one particle light at the expense of making another heavy. It assumes a hierarchy in the values of the different elements of the mass matrix in (2.12), namely

$$m_R^M = M \gg m_D \gg m_L^M = \mu \ , \tag{3.18}$$

with μ either zero or negligible.

Consider first the case of just one generation. Then, the mass matrix has one heavy eigenvector, mainly consisting of the inert ν_R, $N \simeq [\nu_R + (\nu_R)^c] + (m_D/M)[\nu_L + (\nu_L)^c]$, and one light eigenvector, mainly consisting of the active Weyl spinor ν_L, $\nu \simeq [\nu_L - (\nu_L)^c] + (m_D/M)[\nu_R - (\nu_R)^c]$, with masses

$$m_N \simeq M, \qquad m_\nu \simeq \frac{m_D^2}{M} \ll m_D \ . \tag{3.19}$$

Thus in the see-saw mechanism the larger M the lighter is the light neutrino, explaining the intra-familial hierarchy $m_\nu \ll m_D$, while the Dirac neutrino masses are naturally similar to the Dirac masses of the other fermions of the same generation.

If μ is not negligible, the light mass eigenvalue becomes*

$$m_\nu \simeq \left| \mu - \frac{m_D^2}{M} \right| \tag{3.20}$$

Thus, unless $\mu < m_D^2/M$ one loses the natural explanation of the smallness of m_ν.

In the case of three generations, each fermion in (2.12) becomes a vector and M, m_D and μ become matrices. Heavy (N) and light (ν) eigenvectors are found diagonalizing the matrices

$$m_N = M, \quad m_\nu = \mu - m_D M^{-1} m_D^T \ . \tag{3.21}$$

Here, for the heavy masses we kept only the leading term. In the usual see-saw models there are as many right- as left-handed neutrinos, M, m_D and μ are 3×3 matrices (with μ negligible or zero (3.8)) and the symmetric 6×6 mass matrix has six eigenvectors, (usually) three light and three heavy, with masses

$$\frac{m_{D1}^2}{M_1} \ , \quad \frac{m_{D2}^2}{M_2} \ , \quad \frac{m_{D3}^2}{M_3} \ , \quad M_1, M_2, M_3 \ , \tag{3.22}$$

where M_i, i=1,2,3, are the eigenvalues of the matrix M.

There are many different versions of this mechanism. For example, the Dirac masses m_{Di} in the eigenvalues (3.22) could be similar to the up-quark masses (usual in Grand Unified models), or to the charged lepton masses of the same generation (or even something different). The neutrino mass hierarchy depends on this choice since $m_t/m_c \simeq 100$ while $m_\tau/m_\mu \simeq 17$. In a "quadratic see-saw", the three heavy masses

* When this eigenvalue is negative before taking the modulus, the minus sign is incorporated in the definition of the light eigenstate, that becomes anti-selfconjugate (see section2), as is the case of ν above.

M_i are similar, i.e. $M_i \simeq M$ for i = 1,2,3. Consequently, the hierarchy of light neutrino masses is that of m_{Di}^2, i.e. $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} \simeq m_u^2 : m_c^2 : m_t^2$ (or $m_e^2 : m_\mu^2 : m_\tau^2$). In a "linear see-saw", the hierarchy of the heavy masses M_i coincides with that of m_{Di}, $M_1 : M_2 : M_3 \simeq m_u : m_c : m_t$ (for example, M_i generated through loops in a SO(10) model, Witten 1990). As a consequence, also the ratio of light neutrino masses is linear in m_D. Vastly different scales are possible for M, depending on the model, from 1 TeV or lower (for example, in left-right symmetric models), to 10^{16} GeV or higher (in GUT's). Only in the simplest GUT models the leptonic mixings coincide with those of quarks, $K_\ell = K_q$ (up to renormalization effects) in (3.8) and (3.9), but one usually also obtains incorrect relations of mass ratios (namely $m_e/m_\mu = m_d/m_s$). Otherwise the mixing angles are entirely model dependent.

There are many other less usual possibilities for see-saw mechanisms. For example, in the "incomplete see-saw" (Johnson *et al* 1986, 1987, Glashow 1991), a 3×3 matrix M but of rank 2, i.e. with one zero eigenvalue, yields a Dirac neutrino mass eigenstate of mass $\simeq m_D$, besides two heavy and two light neutrinos, of masses $\simeq M$ and $\simeq m_D^2/M$ respectively, as before.

3.5. Majoron Models

When a global continuous symmetry is spontaneously broken, a zero mass boson, called Nambu-Goldstone boson, appears for every broken generator. One may wonder if such massless bosons would lead to new gravitational or Coulomb-like r^{-1} potentials, against which there are very stringent upper limits. This is not the case. Goldstone bosons generate only spin-dependent r^{-3} non-relativistic long-range potentials for which bounds are much less restrictive (Gelmini *et al* 1983).

Majorons are the Goldstone bosons associated with the spontaneous breaking of a leptonic global symmetry, at a scale V, that usually also generates Majorana neutrino masses (thus the name Majoron), $m_\nu \simeq g_{eff} V$. The effective coupling g_{eff} is also roughly the coupling of neutrinos with the Majoron, J. These couplings provide the most important phenomenological consequences characteristic of these models. In general, given a certain range of masses m_ν, for small values of V (typically $V < v \simeq 100$ GeV), the couplings g_{eff} are large, neutrinos may decay ($\nu_h \rightarrow \nu_l J$) and/or annihilate ($\nu\nu \rightarrow JJ$) or interconvert ($\nu\nu \rightarrow \nu'\nu'$) very fast (so neutrinos in the early Universe could not remain to be the Dark Matter). Besides, the Majoron could be emitted in neutrinoless double beta decays at a level that could be observable soon and Majorons may play a role in energy loss mechanisms in stars or in the collapse of a supernova. If instead, given a certain range of masses m_ν, V is large (typically $V > v \simeq 100$ GeV), then g_{eff} is relatively small and the only important consequence of Majorons is, usually, that neutrinos may decay with much longer lifetimes than before, shorter or longer than the lifetime of the universe (so that, for example, neutrinos could be a relevant part of the Dark Matter).

Majoron models are almost unique in allowing neutrino masses in the range between $\simeq 100$ eV and a few GeV, otherwise forbidden due to cosmological arguments, through neutrino decays or annihilations into invisible modes in the early universe. Another important consequence of Majoron models is that the standard Higgs particle may be "invisible" itself through decaying dominantly into the invisible channel $H \to JJ$ (Schrock and Suzuki 1892, see for example Lopez-Fernandez *et al* 1993 or Brahmachari *et al* 1993 and references therein).

The main properties of these models are determined by the weak isospin of the Majoron. The singlet Majoron model (Chikashige, Mohapatra and Peccei 1980, 1981), adds to the SM right-handed neutrinos and a singlet Higgs field σ coupled to them,

$$h_{\alpha\beta}\overline{(\nu_{\alpha R})^c}\nu_{\beta R}\sigma + h.c. \ . \tag{3.23}$$

The VEV $\langle\sigma\rangle \neq 0$, generates Majorana masses for the ν_R, $M_{\alpha\beta} \simeq h_{\alpha\beta}\langle\sigma\rangle$. Light neutrino eigenstates ν_α result from the see-saw mechanism ((2.12) with $m_L^M = 0$), if $\langle\sigma\rangle$ is large enough, $\langle\sigma\rangle \gtrsim v$. Because L is chosen to be an exact symmetry of the Lagrangian and (3.23) defines a non-zero lepton number for σ, $L_\sigma = -2$, L is spontaneously broken. The Majoron in this model is $J = \sqrt{2}Im(\sigma)$, thus

$$\sigma = \frac{1}{\sqrt{2}}[V + \rho + iJ] \ . \tag{3.24}$$

Thus, the Majoron couple to the light neutrinos only through the small admixture of ν_R, of order (m_D/M), that they contain. Consequently, the decay rate of a heavier light neutrino ν_h into a lighter one ν_l, is

$$\Gamma(\nu_h \to \nu_l J) \lesssim (\frac{m_D}{M})^4 m_{\nu_h} \quad . \tag{3.25}$$

Actually, in the simplest form of the singlet model presented here, the leading $(m_D/M)^2$ terms in the amplitude can be rotated away (Schechter *et al* 1982), so that $\Gamma \simeq (m_D/M)^8 m_{\nu_h}$. Thus ν lifetimes are large with respect to the lifetime of the universe. This is not true in more complicated versions of the model (Gelmini and Valle 1984, Jungman and Luty 1991, Babu 1991, Glashow 1991).

Besides its effects on neutrinos, a singlet Majoron is practically invisible. It has very small couplings to charged fermions, only through neutrino loops, and, because the field σ is a gauge singlet, J is not coupled to the Z^0 boson, thus it does not contribute to its invisible decay modes.

Non-singlet Majorons have been rejected by the LEP bound on the effective number of neutrinos, $N_\nu < 2.983 \pm 0.025$ (Review of Particle Properties 1994). The reason is that in these models there are additional contributions to the invisible width of the Z^0-boson that would count as extra light neutrinos. The new decay mode is $Z \to \rho J$, where ρ is a light boson associated to the Majoron (usually ρ is the real part of the same combination of fields of which J is the imaginary part, as in (3.24)). Because non-singlet Majoron models are less "invisible" than singlet Majorons, the scale of L-violation V is phenomenologically required to be $V \ll v \simeq 100$ GeV, and in this case $m_\rho \ll V$, and ρ can be emitted in a Z^0 decay.

The models rejected are the triplet Majoron (Gelmini and Roncadelli 1981, Georgi *et al* 1981), the doublet Majoron (Bertolini and Santamaria 1988) and supersymmetric Majoron models where it is the left-handed scalar neutrino $\tilde{\nu}_L$ (the supersymmetric partner of ν_L) VEV that violates L spontaneously, $V = \langle \tilde{\nu}_L \rangle$ (Aulakh and Mohapatra 1983, Ross and Valle 1985). In these three models the additional Z^0 width, $\Gamma(Z^0 \to \rho J)$, equals 2, 0.5 and 0.5 respectively of its partial width into a light neutrino species, $\Gamma(Z^0 \to \nu_\alpha \nu_\alpha)$, while only a few percent is still allowed by the LEP results. Therefore viable Majorons are singlet (coupled or not to ν_R) or mostly singlet (even if mixed with non-singlets). Examples are the original CMP singlet model (Chikashige, Mohapatra and Peccei 1980, 1981), the singlet-triplet or "invisible triplet" (Choi *et al* 1989, Choi and Santamaria 1991, D'Ambrosio and Gelmini 1987) and supersymmetric models where the right-handed sneutrino (an electroweak singlet as its supersymmetric partner ν_R) VEV breaks L spontaneously, $V = \langle \tilde{\nu}_R \rangle$ (Masiero and Valle 1990).

The "invisible" triplet model preserves many of the characteristics of the original triplet Majoron. This is a variation of the triplet model presented in section 3.2, which yields a Majoron if L is only spontaneously violated by $\langle \Delta^0 \rangle \simeq v_T \neq 0$. The triplet Majoron is mainly $J \simeq Im\Delta^0$, thus it couples at tree level to SM neutrinos. In the "invisible" triplet a singlet σ (not coupled to fermions) is added, whose VEV $\langle\sigma\rangle \simeq v_S \neq 0$ also breaks L. If $v_S > v_T$ the Majoron is now mostly singlet, $J \simeq Im(\sigma + (v_T/v_S)\Delta^0)$, and ρ is the real part of the same combination. Thus now $\Gamma(Z \to \rho J) \sim (v_T/v_S)^4$ is reduced to an acceptable level.

Singlet models tend to have large V (and thus small $g_{\rm eff}$). However, several singlet and mixed models have been proposed (Berezhiani *et al* 1992, Burgess and Cline 1993, 1994, Carone 1993) in which $g_{\rm eff}$ is large (and V small), mainly motivated by the possibility of finding Majoron emission in neutrinoless double beta decay, i.e. having $(g_{\rm eff})_{\nu_e \nu_e J}$ not much smaller than its experimental upper bound of 0.7×10^{-4} (Moe 1994, see section 4.2).

Also models in which a leptonic global symmetry group G is spontaneously broken into a conserved lepton number $\widetilde{L}$, $G \to U_{\widetilde{L}}(1)$, have been considered, and the ensuing Goldstone bosons are also called Majorons many times. In this case the Majorons carry the conserved $\widetilde{L}$ number. $\widetilde{L}$ may be a non-orthodox lepton number (such as $L_e + L_\mu - L_\tau$, for example. The existence of this (almost) conserved lepton number produces the appearance of at least one (pseudo) Dirac neutrino mass eigenstate. Models of this type were produced for the "17 keV neutrino" and to explain a possible signature for neutrinoless double beta decay with emission of a boson (see later).

Finally, interesting models result when an explicit L-breaking is present in Majoron models, transforming the Majoron into a massive pseudo-Golstone boson. The explicit breaking may be due to quantum gravity (Akhmedov, Berezhiani and Senjanovic' 1992, Akhmedov *et al* 1993b, Cline *et al* 1993), although one may protect the global L-symmetry with a gauge symmetry (gauge symmetries are respected by gravity, Rothstein *et al* 1993) or by other mechanisms (Lusignoli *et al* 1990). In this models, Majorons could even be the dark matter in the universe (Akhmedov *et al* 1993a, Rothstein *et al* 1993, Berezinsky and Valle 1993).

A brief comment on familons is in order. Familons F are the Goldstone bosons associated with the spontaneous breaking of an inter-familial global symmetry (Reiss 1982, Wilczek 1982, Gelmini, Nussinov and Yanagida 1983). The main difference between Majorons and familons is that Majorons have usually much larger couplings to neutrinos than to other fermions, allowing for faster neutrino decays, for example. Since the inter-familial symmetry rotates in principle whole families together, or at least whole lepton doublets together (in only leptonic inter-familial symmetries) the same coupling allowing $\nu_\mu \to \nu_e F$ or $\nu_\tau \to \nu_e F$ would allow $\mu \to eF$ or $\tau \to eF$. From experimental bounds in these last modes one gets large lower bounds on the breaking scale, of order 10^{10} GeV and 10^7 GeV respectively (D'Ambrosio and Gelmini 1987).

3.6. Models with Gauged Lepton-Number

The simplest models of this type are left-right symmetric models, where right and left handed fermions are assumed to play identical roles, consequently there is a ν_R for every ν_L (see for example Mohapatra and Pal 1991). The maximal symmetry of these models is $SU_L(2) \times SU_R(2) \times U(1) \times P$, where P is a sort of parity that exchanges left and right indices (P-parity insures also that the coupling constants of the two $SU(2)$ groups coincide). In the simplest versions of these models there are only three Higgs fields, two triplets $\Delta_R \equiv (1,3,2)$ and $\Delta_L \equiv (3,1,2)$ and the usual doublet $\phi \equiv (2,2,0)$. The VEV $\langle\Delta_R\rangle \neq 0$ gives masses to the right gauge bosons Z_R, $W_R^\pm$ and Majorana masses to the ν_R of the same order M, breaking the symmetry into the SM. In these versions, $\langle\Delta_L\rangle \simeq \lambda v^2/\langle\Delta_R\rangle$ results from the minimization of the potential (Mohapatra and Senjanovic 1981) and the mixed mass matrix in (2.12) is

$$\begin{pmatrix} f\langle\Delta_L\rangle & m_D \\ m_D & f\langle\Delta_R\rangle \end{pmatrix} \quad . \tag{3.26}$$

where $m_D \sim v$ and λ, f are combinations of coupling constants. Consequently, the light neutrino masses are, (see (3.18)),

$$m_\nu = f\langle\Delta_L\rangle - \frac{m_D f^{-1} m_D^T}{\langle\Delta_R\rangle} \tag{3.27}$$

and, unless λ is extremely small, the first term dominates. These models have been used, adding an inter-familial (or "horizontal") symmetry, to obtain three almost degenerate mass eigenstates, whose splittings are due to the see-saw mechanism (in models that try to account for most of the present hints for non-zero neutrino masses, namely accommodating solar and atmospheric neutrino deficits, hot dark matter and giving rise to neutrinoless double beta decay close to the present bounds).

By complicating the left-right models a bit, P may be broken at a larger scale than the rest of the group and $\langle\Delta_L\rangle$ becomes negligible. Then a usual see-saw mechanism provides small neutrino masses. Phenomenologically $M > 1$ TeV, due to the bound on additional Z bosons and on W_R bosons. With $M \simeq 1$ TeV one obtains the hierarchy of neutrino masses $\nu_e : \nu_\mu : \nu_\tau \simeq$ eV: keV: MeV.

The solution to the solar neutrino problem may require much smaller masses, (see section 7) 10^{-6} eV:10^{-3} eV:1 eV, pointing to much larger ν_R Majorana masses, $M \simeq 10^{10} - 10^{12}$ GeV, precisely in the range of the intermediate mass scales in $SO(10)$ Grand Unified models, necessary to obtain the unification of the SM coupling constants after the precision measurements at LEP (Mohapatra and Parida 1992, Babu and Mohapatra 1993). These precise measurements of the coupling constants at the electroweak energy scale showed that these couplings do not converge simultaneously to a single value at a large energy scale, the Grand Unification scale M_U, (as necessary in Grand Unified models, Langacker 1981, Mohapatra 1986) in non-supersymmetric models with one stage of unification. Thus either supersymmetric models or non-supersymmetric models with several stages of partial unification are indicated by these measurements (see for example Langacker and Polonsky 1992).

The natural Grand Unified symmetry that incorporates left-right models is $SO(10)$. $SO(10)$ predicts the existence of the ν_R by incorporating all Weyl spinors of each generation into a 16-dimensional multiplet. In models with intermediate mass scales, $SO(10)$ breaks into the SM in two stages. The first, at the scale $M_U \simeq 10^{16}$ GeV, brings the symmetry to a left-right symmetric group, that breaks at the intermediate scale M_I into the SM. In the intermediate stage there is a discrete symmetry, called D-parity, that has the same role of the P-parity mentioned above. D-parity needs to be broken at a scale larger than M_I to obtain a see-saw mechanism with $M \simeq M_I$, otherwise a solution like (3.27) is obtained (Caldwell and Mohapatra 1994, Ioannisian and Valle 1994, Bamert and Burgess 1994, Joshipura 1994, Lee and Mohapatra 1994). In $SO(10)$ models, m_D are similar to the masses of up-quarks.

Supersymmetric $SO(10)$ also yields unification of the three coupling constants of the SM and in this case a see-saw model with $M_V \simeq 10^{16}$ GeV produces masses 10^{-11} eV:10^{-8} eV:10^{-3} eV. But also in supersymmetric $SO(10)$ intermediate scales can be "gravity induced" (Cvetic and Langacker 1992). There are still many other possibilities, using other grand unified groups like E_6, technicolour, etc..

4. Neutrino Mass Searches

4.1. Direct Searches of Neutrino Mass

These searches are based on kinematical arguments and assume only finite neutrino masses. (For a detailed review on direct searches until 1988, see Robertson and Knapp 1988). Indirect searches require additional criteria such as Majorana masses in neutrinoless double beta decay and large enough mixing angles in oscillation experiments (that measure actually not masses, but mass differences). Neutrinos were originally proposed by Pauli in 1932, to account for the continuous spectrum of the emitted electrons in weak nuclear β-decays, $(A, Z) \rightarrow (A, Z+1)\ e^- \nu_e^c$. If only e^- were emitted, their spectrum would be a δ-function with energy equal to the Q-value, $M_Z - M_{Z+1}$, the difference in mass between the initial and final nuclei. The minimum energy of ν_e^c is its mass, m_{ν_e} (CPT insures ν_{eL} and $(\nu_{eL})^c$ have the same mass). Thus, the upper end-point E_o of the electron spectrum $E_0 = M_{Z+1} - M_Z - m_{\nu_e}$, and what is more relevant experimentally, the curvature of the spectrum near the end-point, are sensitive to m_{ν_e}. The best constraints on m_{ν_e} come from the shape of the spectrum of electrons emitted in the Tritium decay, $^3\mathrm{H} \rightarrow {}^3\mathrm{He}\ e^- \nu_e^c$, because the low Q-value of this decay, $E_o = 18.58$ keV, allows to notice the effect in the spectrum of relatively smaller m_{ν_e}. The first bound using this method, $m_{\nu_e} < 1$ keV, was obtained as early as 1948 (Curran *et al* 1949), and by the 1970's the limit was already 55 eV (Bergkvist 1972). The claim of a positive detection in 1980 by Lyubimov *et al* originated many new experiments attempting to verify this result (for a review see Holzschuh 1992). Six of these experiments (see table 2) have now upper limits that rule out the revised Moscow claim of 17 eV $< m_{\nu_e} <$ 40 eV (Boris *et al* 1987).

A striking feature of these experiments can be seen in the second column of the table 2, namely they all find negative m_{ν_e} values. This means the experiments find the e^- spectra near the end-point deformed with opposite curvature with respect to what a neutrino mass would cause. They use then a statistical analysis method (prescribed by the Particle Data Group 1986) for dealing with a non-physical result, to take into account only positive $m^2_{\nu_e}$ and obtain the quoted bounds on m_{ν_e}. However, the negative $m^2_{\nu_e}$ values are large enough that statistical fluctuations as a cause have a very low probability (about 1%). It seems at this point that the systematics of these experiments are not well understood (Wilkerson 1993). Thus, even if nominally the combined results of the first five entries of the table 2 imply $m_{\nu_e} < 5$ eV at $95\% C.L.$, due to the systematic uncertainties it is hard to claim more than an upper limit of about 10 eV. The still unpublished new result from Troitsk involves a new kind of systematics and its low negative $m^2_{\nu_e}$ seems promising.

Table 2. Present upper bounds on m_{ν_e}.

Experiment	$m^2_{\nu_e} \pm \sigma_{stat} \pm \sigma_{syst}[\mathrm{eV}^2]$	95 % CL limit on m_{ν_e} [eV]	Reference
Los Alamos	$-147 \pm 68 \pm 41$	9.3	Robertson *et al* 1991
INS, Tokyo	$-65 \pm 85 \pm 65$	13	Kawakami *et al* 1991
Zurich	$-24 \pm 48 \pm 61$	11	Holzschuh *et al* 1992
Mainz	$-39 \pm 34 \pm 15$	7.2	Weinheimer *et al* 1993
Livermore	$-130 \pm 20 \pm 15$	8	Robertson 1994
Troitsk	-18 ± 6	4.5	Belesev *et al* 1994

The best bounds on m_{ν_μ} come from the measurement of the momentum of μ produced in the decay of pions at rest, $\pi \to \mu\nu_\mu$ (Abela *et al* 1984, Jeckelmann *et al* 1986). After several recent objections (Wilkerson 1993) have been cleared up, the present bound is $m_{\nu_\mu} < 160$ keV at 90% C. L. (Assamagan *et al* 1994).

The ν_τ has only been observed as missing energy in τ-decays, contrary to the other two neutrinos that have also been observed by producing their corresponding charged lepton in interactions with matter. Rare multi-particle semileptonic decays of the τ provide the best bounds on ν_τ (Weinstein and Stroynowski). The ARGUS collaboration (Albrech *et al* 1992) obtained $m_{\nu_\tau} < 31$ MeV at 95%C.L., with 20 events, studying the reaction $\tau^- \to 5\pi\ \nu_\tau$. The CLEO collaboration (Cinabro *et al* 1993), with a much larger sample of 113 events, obtained $m_{\nu_\tau} < 32.6$ MeV at 95%C.L.. There is already a preliminary upper bound from Beijing of 29 MeV at 95%C.L. (Darriulat 1994). Improvements on this method are not expected to lower the m_{ν_τ} bound to better than about 10 MeV.

Besides these laboratory searches, several cosmological and astrophysical arguments considered later in this review (sections 5 and 9) provide additional direct bounds on neutrino masses.

Here we have called m_{ν_e}, m_{ν_μ} and m_{ν_τ} the three mass eigenvalues (more properly called m_1, m_2 and m_3) of the mass eigenstates ν_1, ν_2 and ν_3, and we assume ν_1 consists mainly of ν_e, ν_2 of ν_μ and ν_3 of ν_τ. In recent years evidences for a 17 keV neutrino seen in nuclear beta decay spectra were claimed by various experiments, giving origin to a quite intense experimental and theoretical activity (for a review see, for example, Gelmini, Nussinov and Peccei 1992). This neutrino was seen as a shoulder in the e^- spectrum, at 17 keV of the end-point, indicating that ν_e was composed mainly of a very light (or massless) neutrino mass eigenstate with an admixture of order 1% of the heavy neutrino mass eigenstate. The existence of this shoulder has been definitely rejected in several recent experiments (Hime 1993, Bonvicini 1993).

4.2. Double Beta Decay

Neutrinoless double beta decay ($\beta\beta 0\nu$) is a process that provides a very sensitive probe of Majorana neutrino masses. Unlike the ordinary double beta decay with emission of two neutrinos ($\beta\beta 2\nu$), that is just an allowed but rare transition at second order in the weak interactions $(A, Z) \to (A, Z+2) + 2e^- + 2\bar{\nu}_e$, the $\beta\beta 0\nu$ transition $(A, Z) \to (A, Z+2) + 2e^-$ is a process that violates lepton number by two units and hence requires a departure from the SM.

The simplest scenario in which the $\beta\beta 0\nu$ decay takes place is precisely in the presence of Majorana neutrino masses, that convert the $(\nu_{eL})^c$, emitted in association with one of the electrons, into a ν_{eL} that can thus be absorbed in the second vertex. Although the required chirality flip introduces a strong suppression in the amplitude (with respect to that of the $\beta\beta 2\nu$), the neutrinoless decay has also a larger available phase space. Furthermore, it has the much cleaner experimental signature of producing a single peak in the spectrum of the sum of the two electron energies. The $\beta\beta 2\nu$ decay leads instead to a continuous spectrum that is hard to identify above the experimental radioactive background. These considerations also imply that the chances of observing a $\beta\beta$ signal increase if elements with large Q-values are used, because of the significant reduction of background at large energies. Another obvious requirement is that the single β decay of the isotope be absent or strongly suppressed.

Since the typical lifetimes of $\beta\beta$ emitters are 10^{19}–10^{24} yr, the first indirect experimental evidence for this process came from geochemical searches, studying elements for which the daughter nucleus is a noble gas (and hence naturally absent in solid materials). Looking for instance for ^{130}Xe and ^{128}Xe in Te rich rocks or for ^{82}Kr coming from ^{82}Se, the double beta decays of these Te and Se isotopes were established. These experiments however cannot distinguish between the 2ν and 0ν channels, with the possible exception of the ratio of ^{128}Te and ^{130}Te lifetimes (Bernatowicz *et al* 1992), due to the similarity of the corresponding matrix elements and the difference of phase space ratios in both types of decays. A somewhat similar radiochemical study of a ^{238}U artificial sample revealed the presence of ^{238}Pu atoms produced by $\beta\beta$ decay after the original purification (Turkevich *et al* 1991).

In the last few years, due to the significant improvements in background reduction, detector technology and the use of large amounts of isotopically enriched samples, the $\beta\beta 2\nu$ process was directly observed in the isotopes ^{82}Se (Elliot *et al* 1992), ^{76}Ge (Balysh *et al* 1994, Avignone *et al* 1991), ^{100}Mo (Elliot *et al* 1991, Dassie *et al* 1994, Ejiri *et al* 1994), ^{116}Cd and ^{150}Nd (for recent experimental reviews see Moe 1993 and Morales 1992). The measured lifetimes are also in satisfactory agreement with recent theoretical calculations (Wu *et al* 1991). This has given renewed impetus to this field.

Many experiments are also devoted to pursue the more interesting search of the neutrinoless mode. The non observation at present of any peak at the endpoint of the two electron spectrum implies lower bounds on the lifetime of the $\beta\beta 0\nu$ decay, $T^{0\nu}_{1/2}$. The Heidelberg-Moscow 90% C.L. constraint $T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 1.9\times 10^{24}$ yr (Maier 1994), provides the most restrictive constraint on the effective Majorana e-neutrino mass (see below), $\langle m_{\nu_e}\rangle < 1.1$ eV (the precise bound slightly depends on the theoretical matrix element calculation followed). Other bounds have been obtained for instance from $T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 3.4\times 10^{23}$ yr, which implies $\langle m_{\nu_e}\rangle < 2.8$ eV (Vuilleumier *et al* 1993). In the near future, the experiments with ^{76}Ge, and probably also those with ^{136}Xe, ^{100}Mo and ^{116}Cd, are expected to reach a sensitivity down to $\langle m_{\nu_e}\rangle \sim 0.1$ eV.

If the electron neutrino is a Majorana mass eigenstate, the effective e-neutrino mass $\langle m_{\nu_e}\rangle$ which appears in the transition amplitude is just m_{ν_e}. If the electron neutrino is instead a linear combination of several mass eigenstates, $\nu_e = \sum U_{em}\nu_m$, one has

$$\langle m_{\nu_e}\rangle = \left| \sum_m U^2_{em} m_{\nu_m} \right| , \tag{4.1}$$

where we use self-conjugate Majorana fields ν_m with positive masses m_{ν_m}. The matrix

U_{em} is in general complex. If CP is conserved, the arbitrary phases in the fields can be chosen so that the matrix U becomes real, but in so doing, if the Majorana fields are taken to be self-conjugate their masses may not be all positive, i.e. $m_{\nu_m} = \lambda_m |m_{\nu_m}|$, with $\lambda_m = \pm 1$. The effective mass can be written in the CP conserving case as

$$\langle m_{\nu_e} \rangle = \left| \sum_m \lambda_m |U_{em}^2| |m_{\nu_m}| \right| . \tag{4.2}$$

We recall that $\tilde{\eta}_{CP} = \lambda_m i$ is the intrinsic CP parity of the Majorana neutrino ν_m (see section 2, Wolfenstein 1981, Kayser 1984, 1988).

Equation (4.2) shows that cancellations can occur between different states in $\langle m_{\nu_e} \rangle$. If this happens, the actual neutrino masses can be all larger than the bound on $\langle m_{\nu_e} \rangle$ implied by the non-observation of $\beta\beta 0\nu$. In particular, a Dirac ν_e can be viewed as the limiting case of two degenerate Majorana states with opposite CP parities (see section 2), for which the cancellation is complete and no neutrinoless decay takes place. This is of course also expected from the absence of lepton number violation in this case. If CP is violated in the neutrino sector, there will be unremovable phases in the matrix U and hence cancellations can occur even among states with equal CP parities.

Eq. (4.1) is actually valid if all the ν_m states appearing in ν_e are light ($m_{\nu_m} < 10$ MeV), since otherwise the nuclear matrix elements involving the heavy neutrino propagator are strongly suppressed. Hence, very heavy components have an additional suppression besides that of the small U_{em}^2 factor.

In addition to the "mass mechanism" just discussed, the $\beta\beta 0\nu$ can also take place in left–right symmetric theories. In this case, the presence of ν_R states, right–handed charged currents as well as a larger Higgs sector combine to give rise to the neutrinoless decays in several possible ways (for a review see e.g. Vergados 1986, Tomoda 1991).

Other interesting extensions of the SM affecting $\beta\beta$ processes are the Majoron models (see section 3.5). Here the coupling of the Majoron Goldstone boson (J) to neutrinos

$$\mathcal{L} = -\frac{(g_{\text{eff}})_{\nu_e \nu_e J}}{\sqrt{2}} \;\; \overline{\nu_{eL}} (\nu_e^c)_R J + h.c. \tag{4.3}$$

produces the required chirality flip in the exchanged neutrino line by the emission of the neutral Majoron*. Due to the energy carried away by the undetectable J,

* Since LEP has excluded both triplet and doublet Majorons, this effective coupling can only be with singlet or mixed Majorons and could arise from a ν_e mixing with singlet neutrinos (see section 3.5).

the electron spectrum is no longer a peak. $\beta\beta 0\nu J$ is however still concentrated at larger energies than the $\beta\beta 2\nu$ spectrum. The non-observation of this decay has then resulted in significant constraints on the coupling $(g_{\rm eff})_{\nu_e\nu_e J}$. In particular, the best direct bound $g_{eff} < 7 \times 10^{-5}$ comes from ^{150}Nd (Moe 1994), even if the lifetime limit is weaker than that from Ge (Beck *et al* 1993), due to the favorable matrix element and phase space factors of the first.

5. Neutrinos in Cosmology

Cosmological and astrophysical arguments are a complement to laboratory experiments as a probe of neutrino physics and actually provide some of the most restrictive constraints on certain neutrino properties.

The possible cosmological consequences of neutrinos, and in particular of their non-vanishing masses, started to be explored in the seventies. Bounds on the masses from the contribution of stable neutrinos to the density of the universe were derived first. Then, bounds on unstable neutrinos were obtained from the effects of the decay products. These cosmological tests derived for neutrinos are now routinely applied to any new proposed particle to limit its lifetime, mass, cosmological density and decay modes.

The hot Bing Bang, the standard model of cosmology, establishes that the universe is expanding from a state of extremely high temperature and density. The moment the expansion started is taken as the origin of the lifetime of the universe t, which is now determined to be $t_o \simeq 1.3$ to 1.7×10^{10} yr (from the oldest globular clusters, Demarque, Deliyannis and Sarajedini 1991 give 14-17 Gyr, Renzini 1993 gives 13-15 ± 3 Gyr). Notice that in cosmology the subscript zero denotes present values. The Big Bang model is based on three major empirical pieces of evidence, namely: the Hubble expansion, the cosmic blackbody microwave background radiation, CMB, and the relative abundance of the light elements (up to^{7}Li). These pieces of evidence are very difficult to explain with a model different than the Big Bang (Peebles *et al* 1991).

The Hubble parameter provides the proportionality between the velocity of recession of far away objects v, and their relative distance d, v $= H$d. Its present value, the Hubble constant, is known up to a factor of two, $H_o = 100h$ km/sec Mpc, with $0.4 \lesssim h \lesssim 1$, where a parsec is $pc = 3.26$ light years (see e.g. Jacoby *et al* 1992; Fukugita, Hogan and Peebles 1993; Scott *et al* 1994).

The CMB was produced at $t_{\rm rec} \simeq 3 \times 10^5$ yr, the recombination epoch, when atoms became stable and replaced ions and electrons in a plasma as the constituents of the universe. The Cosmic Background Explorer (COBE) satellite (Mather *et al* 1994) confirmed that the CMB has a blackbody spectrum, and made the most accurate measurement of its temperature, $T_o = 2.726 \pm 0.010$ K (95% C.L.). Thus we know with great accuracy the number density of relic photons (the CMB photons are the most abundant in the universe, by several orders of magnitude, see e.g. Kolb and Turner 1990, p. 143) $n_\gamma = (2\zeta(3)/\pi^2)T_o^3 = 411$ cm^{-3}.

The cosmological abundance of ^{4}He and of the trace elements D, ^{3}He and ^{7}Li are well accounted for in terms of nuclear reactions that occurred at $t \simeq 10^{-2} - 10^2$ sec, the nucleosynthesis epoch, when the photon temperature was $T \simeq 10 - 0.1$ MeV (necessarily below the binding energy of the light nuclei). This is the earliest available proof of the consistency of the Big Bang model. The relative amount of the primordial light elements synthesized is very sensitive to the relative abundance of baryons (protons and neutrons) over photons, $\eta \equiv n_B/n_\gamma$ (n is number density), and to the expansion rate (and, thus, to the energy density of the universe ρ, since $H \sim \sqrt{\rho}$) at the epoch of nucleosynthesis. Recent analysis (Walker *et al* 1991, Smith *et al* 1993) find that η must be in a small range $\eta = (2.8 - 4) \times 10^{-10}$ (95%C.L.) to fix three very different abundances: ^{4}He, D + ^{3}He and ^{7}Li. This implies a bound on the energy density of baryons, $\Omega_B = \rho_B/\rho_c$, through the relation $\eta = 2.68 \times 10^{-8}\Omega_B h^2$, which is $0.01 \leq \Omega_B \leq 0.015$. It is convenient to express energy densities ρ in units of the critical density $\rho_c = 3H_o^2/8\pi G = 10.5h^2$ keV cm^{-3} (defined to be such that for $\rho_{\rm T0TAL} \leq \rho_c$ the universe will expand forever and for $\rho_{\rm TOTAL} > \rho_c$ the universe will eventually recollapse), i.e. $\Omega \equiv \rho/\rho_c$. Attempts to avoid the nucleosynthesis upper bound on Ω_B have been largely unsuccessful so far (Malaney and Mathews 1993). These include, inhomogeneities in the baryon density (Applegate, Hogan and Scherrer 1987), late decaying particles (Dimopoulos *et al* 1988) and generating entropy after nucleosynthesis (Bartlett and Hall 1991). We mention later a new idea involving a decaying ν_τ (Gyuk and Turner 1994).

An increase in the expansion rate of the universe leads to an earlier freeze out of the weak interactions that determine the ratio of neutrons over protons and, consequently, to a larger value of this ratio and to overproduction of ^{4}He (since as a first approximation all neutrons end up in ^{4}He nuclei). Thus, the observational upper bound on the primordial ^{4}He abundance provides an upper bound on the total ρ at the moment of nucleosynthesis, that is expressed in terms of the allowed number of light neutrino families, $N_\nu \lesssim 3 + \delta N_\nu$. Walker *et al* 1991 found $\delta N_\nu = 0.3$, but this number changes often, with the incorporation of new measurements of the cosmic abundances of elements*. Even if the total width of the Z^o measured at LEP insures that $N_\nu = 3$, the nucleosynthesis bound is still useful. This is because it limits any extra contribution beyond those of three left handed relativistic neutrinos to ρ during nucleosynthesis, to be equivalent at most to δN_ν of a neutrino species.

Both CMB and nucleosynthesis provide a plethora of bounds on neutrinos, some of which we will mention below. Other bounds are provided by the present cosmic energy density and by structure formation in the universe (namely the formation of galaxies and galaxy aggregates).

May be the most important cosmological constraint on stable neutrinos is the mass bound. Light neutrinos ($m_\nu < 1$ MeV) with SM interactions are kept in equilibrium with charged leptons and photons in the cosmic plasma (due to weak interactions) until a temperature $T \simeq 1$ MeV. At these temperatures the rate of weak processes becomes smaller than the expansion rate of the universe and neutrinos "decouple" or "freeze out". Their number (per comoving volume, i.e. a volume increasing due to the Hubble expansion) becomes constant afterwards. Taking into account that the number of photons is increased when e^+e^- annihilate (at $T \lesssim m_e$, when e^+e^- can no longer be formed), due to entropy conservation, the present number density of neutrinos (plus antineutrinos) per species is $n_{\nu_i} = (3/11)n_\gamma = 102\ \mathrm{cm}^{-3}$.

If neutrinos are massive, their energy density is $\rho_{\nu_i} = m_{\nu_i} n_{\nu_i}$, therefore (Gerstein

* Loopholes of this bound exist but they require rare specific neutrino properties, such as excess lepton numbers of the order of the photon number density n_γ, namely $(n_{\nu_i} - n_{\nu_i^c})/n_\gamma \simeq O(1)$, (while one expects this leptonic asymmetry to be of the same order of magnitude of the baryonic asymmetry $\eta \simeq O(10^{-10})$ (Olive *et al* 1991), or a heavy tau neutrino with mass in the MeV range, decaying with a short lifetime ($\lesssim 30$ sec) into ν_μ or, better, ν_e and other invisible particles (a Majoron, for example), as mentioned later in this section (Kawasaki *et al* 1994 and Dodelson, Gyuk and Turner 1994a).

and Zeldovich 1972, Cowsik and McClelland 1972)

$$\Omega_\nu h^2 = \sum_{\mathrm{i}=1}^{3} \frac{m_{\nu_\mathrm{i}}}{92 \text{ eV}} \quad . \tag{5.1}$$

This bound assumes that only left-handed neutrinos are ever in equilibrium in the primordial plasma (what is correct, even in the case of Dirac neutrinos for Dirac masses below a few keV, see e.g. Kolb *et al* 1991). The best upper bound on $\Omega_o h^2$ does not come from estimates of Ω_o and h, but from t_o. Due to the relation between t_o, H_o and Ω_o (that depends on the nature of the content of the universe: matter, radiation or a cosmological constant, namely vacuum energy), a lower bound on the age of the universe translates into an upper bound on $\Omega_o h^2$. Thus, $t_o \gtrsim 1.3 \times 10^{10}$ yr provides the bound $\Omega_o h^2 \lesssim 0.4$ for a matter dominated universe (and zero cosmological constant, see Kolb and Turner 1990, figure 3.3), thus $\sum_{\mathrm{i}=1}^{3} m_{\nu_\mathrm{i}} \lesssim 37$ eV. Only for small values of h, $0.4 \leq h \leq 0.5$, it can be $0.25 \lesssim \Omega_o h^2 \lesssim 0.4$ (see the same figure). Thus, if the more popular lower bound of $h \simeq 0.5$ is taken then $\Omega_o h^2 \lesssim 0.25$ and consequently, $\sum_{\mathrm{i}=1}^{3} m_{\nu_\mathrm{i}} \lesssim 23$ eV. The presence of a cosmological constant Λ would relax these constraints a bit (because $\Omega_o h^2$ can be larger given the same t_o, see e.g. Kolb and Turner 1990). Neutrinos with mass close to these upper bounds could dominate the mass density and provide, therefore, the Dark Matter (DM).

It is by now well established that the dominant form of matter in the universe is only detectable through its gravitational effects (and, because of this, it is called Dark Matter). Most measurements of the DM at large scales obtain $\Omega_{DM} \simeq 0.2$ to 0.3, but some obtain close to 1 (see e.g. Kolb and Turner 1990, Peebles 1993). Because all other known contributions to Ω_o are much smaller, the total amount of DM in the universe is responsible for the ultimate fate of our universe, expansion forever or recollapse (if $\Omega_o \leq 1$ or $\Omega_o > 1$, respectively). The nature of the DM is one of the most important entirely open questions in physics. DM candidates are classified as cold or hot according to the galaxy formation scenarios derived from them. Galaxies are assumed to be formed through gravitational instability, from tiny inhomogeneities in the energy density ($\delta\rho/\rho < 10^{-4}$ at recombination), that leave an imprint in the CMB, recently found by COBE. Very different mechanisms result if the DM is relativistic (hot DM, HDM) or non-relativistic (cold DM, CDM) when galaxy size inhomogeneities could first start collapsing at $T \simeq 1$ keV. Massive neutrinos (with

$m < 1$ keV) could be HDM. Simulations of structure formation with pure HDM fail to fit the data, because galaxies form too late (by fragmentation of the much larger structures that form first).

Pure CDM accounts for the bulk of the known data, but seems to fail in detail. The COBE measurements of anisotropies in the CMB provide a measurement of density inhomogeneities (Smoot *et al* 1992, Gorski et al 1994). In the context of models with $\Omega = 1$ and primordial scale-invariant fluctuations (the simplest assumptions), once the normalization given by COBE is imposed on the spectrum of density fluctuations predicted by pure CDM at large scales, the spectrum has too much power on smaller scales, namely the scales of galaxy clusters (Wright *et al* 1994). A possible acceptable solution explored at present is mixed DM (MDM or HCDM). Recent simulations suggest an admixture of $\Omega_\nu \simeq 0.30$ of HDM in neutrinos in a universe dominated by CDM (Wright *et al* 1994, Nolthenius *et al* 1993, Bonometto *et al* 1993, Klypin *et al* 1994). These simulations use $\Omega_o = 1$, and $h = 0.5$, thus requiring $\sum_{i=1}^{3} m_{\nu_i} \simeq 7$ eV.

Up to now we mentioned neutrinos lighter than 1 MeV. Neutrinos with $m_\nu \gtrsim 1$ MeV would decouple while they are non-relativistic and their density is thus reduced by a Boltzman factor $\Omega_\nu \sim \exp(-m_\nu/T)$. In this case the bound in the energy density $\Omega_o h^2$ is satisfied for $m_\nu \gtrsim$ few GeV (Lee and Weinberg 1972, Hut 1977, Sato and Kobayashi 1977, Vysotsky, Dolgov and Zeldovich 1977). The mass bounds differ for a Dirac or a Majorana neutrino, they are higher for the latter (Kolb and Olive 1986, see also Kolb and Turner 1990). Only a 4^{th} generation neutrino could be that heavy, but the LEP bound excludes its existence unless $m_\nu > M_Z/2 \simeq 45$ GeV. These heavy neutrinos would only have a small density $\Omega_\nu < 10^{-2}$.

These constraints apply to neutrinos with only standard interactions (i.e. to neutrinos that are non-standard only because they have a non-zero mass). Neutrino masses could be in the forbidden range mentioned above, namely 30 eV $< m_\nu <$ few GeV, if they have interactions beyond the SM that allow for faster annihilation or decay.

Unstable neutrinos with $m < 1$ MeV whose relativistic decay products dominate the energy density of the universe until the present must have a lifetime

$$\tau \leq (92 \text{ eV}/m_\nu)^2 (\Omega_o h^2)^2 t_o \quad , \tag{5.2}$$

to insure that the energy density of the decay products, Ω_{DP}, is not too large, i.e.

$\Omega_{DP} \leq \Omega_o$ (Dicus, Kolb and Teplitz 1977, Pal 1983, Kolb 1986)*. The bound in (5.2) and the corresponding bound for heavier masses are shown in figure 1 with a continuous contour line. Since neutrinos and photons are almost in equal numbers, the neutrino energy density $\rho_\nu = n_\nu m_\nu$ becomes dominant over the radiation density $\rho_{\rm rad} \simeq n_\gamma T$ as soon as $m_\nu > T$, i.e. neutrinos matter-dominate the energy density of the universe as soon as they become non relativistic (actually at T$\simeq 0.1 m_\nu$). Thus, if neutrinos decay while non-relativistic into relativistic decay products, i.e. $\rho_\nu = \rho_{DP}$ at $t = \tau_\nu$, these products radiation-dominate the universe. Because in (5.2) the universe is assumed to be radiation dominated until the present, the constraint $t_o > 1 \times 10^{10}$ yr (1.3 $\times 10^{10}$ yr) requires $\Omega_o h^2 < 0.3$ (0.1). If the universe is assumed to become dominated by matter at some time after the neutrino decay, the bound on the lifetime is more restrictive than (5.2) (and the bound in figure 1)**. In fact, the massive neutrino has to decay earlier for the energy of their decay products to have a longer time to decrease, as T^4, and become subdominant before the present with respect to a matter density component that decreases slower, as T^3. This is actually required by structure formation arguments. Because the growth of density fluctuations is suppressed in the period of radiation domination of the decay products, this period should finish at most when structure in ordinary matter could start forming, i.e. at recombination $t_{\rm rec} \lesssim 10^{-5} t_o$, when atoms become stable. This argument replaces t_o by $t_{\rm rec}$ in (5.2), thus yielding a bound more stringent by a factor 10^{-5} (Steigman and Turner 1985)***. This bound is shown for $m < 1$ MeV in figure 1 with a dashed contour line.

A neutrino with mass in the keV to MeV range, with a lifetime close to the lower limit just obtained, could not only be harmless but actually can help in the structure formation. It could actually be what CDM needs to find perfect agreement with data (White, Gelmini and Silk 1994, Bardeen, Bond and Efstathiou 1987, Bond and Efstathiou 1991). The main effect of the decaying neutrino is to delay the onset of the

* This bound is obtained with a simple calculation that assumes all neutrinos decay suddenly at t=τ. A proper calculation shows that this assumption overestimates the Ω_{DP} by about 15% (Turner 1985). Thus the bound (5.2) should be actually higher by a factor of 1.3.

** Notice this is in contradiction with the equivalent bounds derived in Kolb and Turner 1990 and presented in their figure 5.4, that we believe are not correct.

*** For a more accurate version of this bound based on numerical calculations and using the present constraints from large scale structure and CMB see White, Gelmini and Silk 1994.

matter domination by the CDM, by adding to the radiation density the contribution of the (relativistic) decay products. Also a heavier neutrino, a ν_τ with mass between 1 and 10 MeV could do it (Dodelson, Gyuk and Turner 1994b), provided the lifetime is $\lesssim 10^2$ sec to avoid excessively perturbing nucleosynthesis (Dodelson, Gyuk and Turner 1994a, Kawasaki *et al* 1994)

In fact, a non-relativistic neutrino present during nucleosynthesis, necessarily a tau-neutrino, may contribute more to ρ than a relativistic species. Therefore, the bound on the additional number of relativistic neutrinos mentioned above, $\delta N_\nu < 0.3$, forbids the mass ranges 0.2 MeV−33 MeV (for a Dirac neutrino) or 0.4 MeV−30 MeV (for a Majorana neutrino), if the neutrino is present during nucleosynthesis, i.e. for $\tau > 10^2$ sec (Dodelson, Gyuk and Turner 1994a, Kawasaki *et al* 1994, Dolgov, Kainulainen and Rothstein 1994, Kolb and Scherrer 1992, Kolb *et al* 1991). This bound is shown in figure 1 with a dot-dashed contour line. These nucleosynthesis bound combined with the laboratory upper limits (31 MeV, see section 4) nearly exclude* a ν_τ more massive than about 0.4 MeV, if $\tau > 10^2$ sec (and if the ν_τ does not have additional annihilation channels besides those of the SM, so that its relic density is the one computed with standard interactions).

For shorter lifetimes the bounds depend on the decay channel. Radiative decays, i.e. into photons or e^+e^-, are excluded for all allowed neutrino masses, as can be seen in figure 2. Bounds come from not allowing distortions of the CMB (this excludes the gray region in figure 2, taken from Kolb and Turner 1990) and from excessive entropy generation or the dissociation by the decay products of the synthesized nuclei after nucleosynthesis. Also stringent bounds come from astrophysics (section 9). Bounds obtained from the supernova SN1987A exclude the hatched region in figure 2.

The decay modes available are $\nu_\tau \to 3\nu'$ and $\nu_\tau \to \nu_e \phi$. The mode $\nu_\tau \to \nu_e \phi$, has been studied for all masses and lifetimes. For very short lifetimes ϕ is brought into equilibrium by decays and inverse decays and its contribution to ρ during nucleosynthesis becomes unacceptably large (Kawasaki *et al* 1994).

Let us mention two curious properties of these massive, fast decaying tau neutrinos. A ν_τ with mass $m >$1 MeV and short lifetimes $\tau < 10$ sec, that decays into another neutrino (and other sterile particles), could decrease the amount of ^{4}He produced thus

* The preliminary bound from Beijing of 29 MeV, if confirmed, would already have closed the gap between both limits.

actually loosening the nucleosynthesis bound on new light species. Available analyses differ slightly in the range at stake (Dodelson, Gyuk and Turner 1994a, Kawasaki *et al* 1994). This possibility could become particularly useful if the observational data would lead to a lower ^{4}He abundance than now assumed (reducing the allowed number of effective neutrino families to less than three). The second property refers to a ν_τ with mass (20–30) MeV decaying into ν_e (and other invisible particles) with a lifetime of (200–1000) sec, that has a relic density smaller than that of a standard neutrino (i.e. this neutrino must have larger than standard annihilation cross sections in the early universe). In this case the bound on Ω_B from nucleosynthesis could be loosen (Gyuk and Turner 1994). Moreover there is the possibility we already mentioned of a massive short-lived neutrino of $m > 1$ MeV helping in structure formation in the early universe (Dodelson, Gyuk and Turner 1994b). Rejecting experimentally a ν_τ mass larger than 1 MeV would eliminate these possibilities.

As already mentioned, radiative decays ($\nu \to \nu'\gamma$, $\nu \to \nu' e^+e^-$) cannot help in evading the cosmological mass bound (see figure 2), i.e. cannot have shorter lifetimes than those excluded in figure 1. Only decays into neutral weakly interacting particles are allowed, namely $\nu \to 3\nu'$ and $\nu \to \nu'\phi$. It is difficult to find extensions of the SM in which the first decay is fast enough while avoiding conflicts with cosmological as well as laboratory bounds and suppressing the related forbidden radiative decays (for a review see e.g. Mohapatra and Pal 1991). The mode $\nu \to \nu'\phi$, where ϕ is a Goldstone boson, seems the most promising, and finding a neutrino mass in the forbidden cosmological range would strongly suggest the existence of Goldstone bosons.

If neutrinos are "cosmologically stable", i.e. $\tau \gg t_0$, the radiative decay of a massive neutrino is not excluded by the above mentioned bounds. A decaying DM neutrino with $m_\nu \simeq O(10\text{ eV})$ would however produce a monochromatic UV line (with $E_\gamma = m_\nu/2$ if the daughter neutrino is massless) that could be observable with satellite detectors. A neutrino of $m_\nu \simeq 28$ eV and $\tau \simeq 10^{23}$ sec has even been proposed to account for the observed ionization of the galactic and intergalactic hydrogen (De Rújula and Glashow 1980, Sciama 1990). Although this lifetime is much shorter than those resulting in the SM with the addition of Dirac neutrino masses, it is surprising that it is just at the right value in models in which both masses and radiative decays (through a dipole transition) are generated by loop effects (Roulet and Tommasini 1991, Gabbiani *et al* 1991).

The nucleosynthesis bound on the effective number of relativistic neutrinos δN_ν applies also to any new mechanisms that could bring sterile particles into the primordial plasma, such as active-sterile neutrino mixings (see sections 7 and 8) or a Dirac neutrino mass. With respect to Dirac masses, recall from section 2 that if neutrinos are Dirac particles, there exist right-handed chirality neutrinos (ν_R, and also left-handed chirality $(\nu^c)_L$) that in the SM do not have weak interactions. Since the physical states of a neutrino are helicity states, and helicity and chirality do not coincide for massive neutrinos, all four helicity states have weak interactions. However, the admixture of the "wrong" chirality (the interacting one) in a right-handed helicity neutrino is of order m/E (for $m < E$, E is the neutrino energy), thus their interactions are suppressed by a factor $(m/E)^2$ with respect to those of a left handed helicity neutrino. An upper bound on the abundance of right-handed helicity neutrinos (given by δN_ν in this case) translates into an upper bound on their reaction rates and thus on the Dirac mass. If $m < 0.3$ MeV the right-handed neutrinos decouple before the quark-hadron QCD phase transition, in which there is a large increase of the number of still interacting particles. Thus the equilibrium number of right-handed neutrinos becomes relatively very small (Fuller and Malaney 1991, Enqvist and Uibo 1993). However, there are also out of equilibrium processes in which right handed neutrinos are produced, such as pion decays (Lam and Ng 1991). Dolgov, Kainulainen and Rothstein (1994) find $m_{\nu_\mu} \lesssim 170$ keV $[(\delta N_\nu - 0.10)/0.20]^{1/2}$ and $m_{\nu_\tau} \lesssim 210$ keV$[(\delta N_\nu - 0.10)/0.20]^{1/2}$, if the temperature of the quark-hadron phase transition is $T_{QCD} = 100$ MeV. These bounds are anyhow less restrictive than the bound derived from the supernova SN1987A, that forbids Dirac masses from $O(10$ keV$)$ to 1 MeV (see section 9). Notice that if the right handed neutrino would be trapped inside the supernova by unknown interactions beyond the SM (Babu, Mohapatra and Rothstein 1991) thus avoiding the SN1987A bound (based on energy loss due to the escape of right handed neutrinos), the same interactions would bring them into equilibrium during nucleosynthesis (leading to an unacceptable δN_ν=1). This conclusion is difficult to obviate (Babu, Mohapatra and Rothstein 1992).

Let us mention last an upper bound on Majorana neutrino masses due to the persistence of a baryon asymmetry generated at temperatures higher than the electroweak phase transition, as most baryon generation models assume. The L violating processes associated with the Majorana mass in conjunction with $B + L$

violating non-perturbative processes in the SM would erase any existing B asymmetry unless the Majorana masses are small enough. A stringent upper bound of 0.1 eV was formerly derived, but recent re-considerations of this bound have loosen it considerably to around 10 keV (see Cline, Kainulainen and Olive 1993b).

6. Neutrino oscillations

6.1. Neutrino mixing

The phenomenon of neutrino oscillations (Pontecorvo 1958), appears because the neutrino current (or interaction or flavour) eigenstates ν_α, $\alpha = e, \mu, \tau$ (namely, the neutrino states produced in a weak decay in association with a given charged lepton flavour, see (3.8)) are generally superpositions of different neutrino mass eigenstates ν_i, $i = 1, 2, 3$,

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \ . \tag{6.1}$$

Here the unitary matrix $U = K_\ell^\dagger$, where K_ℓ is the leptonic analog of the Cabibbo-Kobayashi-Maskawa quark matrix K_q (see (3.8) and (3.9)). Due to the dissimilar propagation of the neutrino mass eigenstates, the flavour content of the propagating neutrino ν changes with time. Since the neutrinos are usually detected by means of charged current processes, sensitive to the neutrino flavour, this oscillating behaviour may be observable. Moreover this is an interference effect, sensitive therefore to very small neutrino mass differences.

6.2. Oscillations in vacuum

To study the ν oscillations, assume that a ν_α flavour eigenstate (namely ν_e, ν_μ or ν_τ) is produced at $x = 0$, $t = 0$. Although the ν states are wave-packets with a certain spread in momentum, it is sufficient to consider just the plane wave solutions, since limitations due to the finite coherence length of the wave-packets in general do not affect the cases of interest (Nussinov 1976, Kayser 1981). We know that the space-time dependence of a free mass eigenstate $\nu_i(t)$ of momentum p_i and energy $E_i = \sqrt{p_i^2 + m_i^2}$ is

$$\nu_i(t) = \exp[i(p_i x - E_i t)] \quad \nu_i. \tag{6.2}$$

It is convenient to take the propagating neutrinos with a common definite momentum p (the same conclusion result if we take e.g. a common energy and different momenta (Winter 1981) or the wave-packets themselves), so that for the ultra-relativistic neutrinos $E_i \simeq p + m_i^2/(2E)$ and the neutrino state is then

$$\nu(t) = \exp[ip(x-t)] \sum_i U_{\alpha i} \mathrm{exp}\left(-i\frac{m_i^2}{2E}t\right) \ \nu_i \ . \tag{6.3}$$

The probability with which the neutrino produced as ν_α is converted into ν_β after travelling a distance $x = t$ results

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) = |\langle \nu(t)|\nu_\beta\rangle|^2 = & \left|\sum_i U_{\alpha i} \mathrm{exp}\left(-i\frac{m_i^2}{2E}t\right) U_{\beta i}^*\right|^2 \\ =& \mathrm{Re} \sum_{i,j} U_{\alpha i} U_{\alpha j}^* U_{\beta i}^* U_{\beta j} \mathrm{exp}\left[-i\frac{m_i^2 - m_j^2}{2E}x\right]. \end{aligned} \tag{6.4}$$

We see that in order to have $P(\nu_\alpha \to \nu_\beta) \neq \delta_{\alpha\beta}$ it is necessary that at least two neutrinos be non-degenerate.

These probabilities satisfy some important relations. CPT invariance implies $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$, while for two flavour mixing, and only if CP is conserved also for mixing with three or more flavours, one has $P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha)$ (Cabibbo 1978), because these probabilities are related by the replacement $U \leftrightarrow U^*$ in eq. (6.4). These relations are valid when neutrinos propagate in the vacuum. When interactions with matter affect the neutrino propagation they may no longer hold because the medium itself is generally not symmetric under CP and CPT. Finally, the unitarity of U implies the probability conservation relation

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sum_{\beta \neq \alpha} P(\nu_\alpha \to \nu_\beta). \tag{6.5}$$

We will hereafter consider the case of mixing between just two neutrino flavours, ν_α and ν_β,

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} . \tag{6.6}$$

To be more general, ν_β can be taken here as one of the three known neutrinos or eventually as a new light singlet neutrino ν_s not participating in the weak interactions (sterile species). For a sterile neutrino the oscillation formalism is still valid but the predictions for observations by both charged and neutral current processes are affected.

The probability of neutrino conversion then results

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \ \sin^2 \frac{\Delta m^2}{4E} x \quad , \tag{6.7}$$

where $\Delta m^2 \equiv m_2^2 - m_1^2$. Clearly the probability of observing the same flavour in the neutrino beam is just $P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$. We see that these probabilities oscillate with an amplitude proportional to the neutrino mixing factor $\sin^2 2\theta$. Noting that

$$\frac{4E}{\Delta m^2} = 0.8 \, \mathrm{m} \frac{E \ [\mathrm{MeV}]}{\Delta m^2 \ [\mathrm{eV}^2]} \quad , \tag{6.8}$$

we also see that the oscillation length is macroscopic for the typical energies of reactor ($1-10$ MeV) or accelerator ($10^{-2}-10^2$ GeV) neutrinos, at least for the cosmologically allowed values of the masses of stable neutrinos ($< 10^2$ eV).

When considering an experiment, it is important that the neutrino source is not monochromatic but instead has in general a broad energy spectrum. Furthermore, both the region of neutrino production and the detector have finite sizes. These facts make the oscillatory term in the conversion probability to be usually averaged out, leading just to $P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta/2$. However, if Δm^2 is so small that the oscillation length becomes much larger than the baseline d between the production point and the detector, i.e. $\Delta m^2 \ [eV^2] \ll E \ [MeV]/d \ [m]$, the oscillations do not have enough time to develop and no effect can be seen. Clearly going to larger baselines allows to test smaller Δm^2 values, as long as the reduction in the ν flux ($\propto d^{-2}$) does not become the limiting factor. Actually, there is intense activity at present to develop long-baseline ($10 - 10^4$ km) experiments using accelerator neutrinos, to increase the Δm^2 sensitivity.

The main sources of artificial neutrinos used in oscillation experiments are:

- $\bar{\nu}_e$ from β^- decays of fission products in reactors;
- Fluxes of ν_μ, $\bar{\nu}_\mu$ and ν_e, in comparable amounts and with energies of tens of MeV, from decays of stopped π^+ at low energy accelerators;
- Beams of ν_μ or $\bar{\nu}_\mu$, with a small contamination at the percent level of ν_e and $\bar{\nu}_e$, from decays in flight of π and K produced in high energy accelerators.

In addition to the experiments using artificial sources, studies of ν oscillations can be performed using natural sources of neutrinos, such as atmospheric neutrinos produced by cosmic rays in the upper atmosphere or solar neutrinos produced in the fusion reactions inside the sun. In table 3 we show the typical energies, the distances involved and the minimum Δm^2 values leading to testable oscillations, for the different kinds of neutrino sources just mentioned.

Table 3. Neutrino energies E, baseline distances d and minimum testable mass-square differences Δm^2 for reactor, accelerator (existing short-baselines and proposed long-baselines), atmospheric and solar neutrinos.

	reactor	accelerator		atmospheric	solar
		short-base.	long-base.		
E [MeV]	$\lesssim 10$	$30-10^5$		10^3	$\lesssim 14$
d [m]	$10-300$	10^2-10^3	10^4-10^7	10^4-10^7	10^{11}
Δm^2 [eV^2]	10^{-2}	10^{-1}	10^{-4}	10^{-4}	10^{-11}

As we will see in sections 7 and 8, studies of solar and atmospheric neutrinos provide some indications in favour of neutrino oscillations. On the contrary, the searches of oscillations using reactor and accelerator neutrinos have provided up to the present no compelling evidence for oscillations, therefore excluding the ranges of Δm^2 and $\sin^2 2\theta$ explored, as we turn now to discuss.

6.3. Oscillation experiments

There are essentially two strategies for detecting neutrino oscillations starting from a beam of neutrinos of a given flavour ν_α:
i) to measure the survival probability $P(\nu_\alpha \to \nu_\alpha)$ looking for an eventual reduction in the ν_α flux, i.e. the so called *disappearance* experiments;
ii) to try to directly observe in the detector the interactions due to a different neutrino flavour, in the so called *appearance* experiments.

Usually a disappearance experiment has two detectors (or one that can be moved), and the comparison of the measurements nearer to the source with those farther away from it are used to search for any possible reduction in the ν_α flux due to oscillations. This experimental set-up implies that there is not only a minimum testable value of Δm^2 (corresponding to oscillation lengths much larger than the distance from the source to the far detector), but also a maximum testable Δm^2, since for large enough mass differences oscillations would have already been averaged in the nearby detector.

Appearance experiments look appealing because only a few events above background are enough to establish an oscillation pattern, while large statistics are required for a meaningful signal in a disappearance experiment. There are however situations in which an appearance experiment cannot be performed, like in reactor experiments, where the CC interactions of the μ or τ neutrinos are kinematically forbidden, or for the study of oscillations into sterile species, that only show up in a disappearance experiment.

6.4. Present situation

In figure 3 we show the constraints obtained on Δm^2 and $\sin^2 2\theta$, under the assumption of two flavour mixing, arising from the unsuccessful searches of oscillations of the type $\nu_e - \nu_\mu$ (figure 3*a*) and $\nu_\mu - \nu_\tau$ (figure 3*b*). We have only depicted the experiments giving the stronger constraints although several more exist. The bound are slightly relaxed if mixing among three flavours is allowed (Blümer and Kleinknecht 1985).

For oscillations of e-neutrinos, the best bounds on Δm^2 result from reactor experiments because of their small energies. The $\bar{\nu}_e$ are here detected by the inverse β reaction $\bar{\nu}_e p \to e^+ n$. Since these are disappearance experiments, the resulting bounds apply to oscillations into any type of neutrinos, and they actually represent the best experimental constraints on ν_e-ν_τ oscillations. At the Goesgen reactor in Switzerland (Zacek *et al* 1986) the fluxes measured at three distances (37.9, 45.9 and 64.7 m) were compared among themselves and in addition they were compared to the flux expected from the knowledge of the ν source and of the detector efficiency (this explains why the excluded region extends also to large Δm^2 values). The experiment of the Kurchatov group (Vidyakin *et al* 1990), in the site of three nuclear reactors in Moscow, used one detector at 57 m from the first and second reactors and 231 m from the third. These comparatively large distances allowed them to exclude down to $\Delta m^2 \simeq 8 \times 10^{-3}$ (for maximum mixing) from the comparison of the rates measured with all the different possible combinations of reactors on and off.

Although accelerator experiments have not reached such low values of Δm^2, $\nu_\mu \leftrightarrow \nu_e$ appearance experiments have achieved sensitivities to $\sin^2 2\theta$ values much smaller ($\sim 3\times 10^{-3}$) than those reached at reactors ($\gtrsim 10^{-1}$). The best bounds on $\sin^2 2\theta$ are from the BNL experiments E776 (Borodovsky *et al* 1992) and E734 (Ahrluder *et al* 1985) and from the Serpukhov experiment SKAT (Ammosov *et al* 1988). Also shown in figure 3a are the constraints from the Los Alamos Meson Physics Facility (LAMPF) (Durkin *et al* 1988) and the bubble chamber BEBC at CERN (Angelini *et al* 1986).

Regarding the $\nu_\mu - \nu_\tau$ oscillations, searches looking for ν_μ disappearance were performed at Fermilab CCFR experiment and by the CDHS (Dydak *et al* 1984) and CHARM (Bergsma *et al* 1984) collaborations at CERN. The best constraints on $\sin^2 2\theta_{\mu\tau}$ were obtained by the Fermilab E531 experiment (Ushida *et al* 1986) with an emulsion detector sensitive to the appearance of a τ track. Recently, also the CHARM II collaboration (Gruwé *et al* 1993) was able to constrain the ν_τ appearance by means of the study of the $\tau\to\pi\nu_\tau$ decay mode in their fine-grained calorimeter. These results are shown in figure 3b.

There is also an important astrophysical constraint on $\nu_e - \nu_{\mu,\tau}$ oscillations, to be discussed in section 9, arising from the non-disruption of r-processes in supernovae, that excludes Δm^2 values bigger than ~ 4 eV^2 for $\sin^2 2\theta \gtrsim 10^{-5}$.

6.5. Future prospects

While one may say that in the past oscillation experiments have been performed just looking blindly into the attainable ranges of Δm^2 and $\sin^2 2\theta$, the situation looks quite different for the future. In fact, there are two main physics issues that motivate the forthcoming experiments.

First, there is the dark matter problem, that may be accounted for, at least partially, if the heavier of the three neutrinos has a mass in the range between a few eV and a few tens of eV (see section 5). This is most likely a ν_τ, and an analogy with the mixing in the quark sector would suggest that its most significant mixing is with ν_μ (the nearest generation). Thus, this argument has strongly encouraged $\nu_\mu \to \nu_\tau$

appearance searches sensitive to tiny mixing angles for $\Delta m^2 \gtrsim$ eV^2 (Harari 1989). Two such experiments, CHORUS and NOMAD (di Lella 1993), are now running at CERN. They are expected to take data in the period 1994–1995, and to be sensitive down to $\sin^2 2\theta_{\mu\tau} \sim 3 \times 10^{-3}$. CHORUS (de Jong *et al* 1993) has an emulsion target with scintillating fibers to assist in the location of the τ tracks while NOMAD selects kinematically the τ decays based on the correlation between the resulting hadronic shower and the missing p_T carried by the produced ν_τ. An approved proposal (P803) related to the main injector project at Fermilab, using 1 ton of emulsion target, will extend the sensitivity to ν_τ appearance down to $\sin^2 2\theta \sim 6 \times 10^{-5}$ by the end of the decade (Kodama 1990).

The second motivation comes from the observations performed with atmospheric neutrinos that, as will be discussed in the section 8, may suggest a mixing between ν_μ and ν_τ (or ν_e) with $\sin^2 2\theta \gtrsim 0.4$ and Δm^2 somewhere in between 10^{-1} and 10^{-3} eV^2. The need to clarify this issue has encouraged the realization of long-baseline experiments, where a very intense ν beam is pointed towards a large far away detector (Pantaleone 1990, Bernstein and Parke 1991). Several proposals of this kind have been studied, including sending a beam from: Fermilab main injector to SOUDAN (baseline 710 km) (Allison *et al* 1991); BNL to three new Cerenkov detectors at 1, 3 and 20 km (Mann and Murtagh 1993); CERN SPS to Gran Sasso ICARUS detector (730 km) (Revol 1993) or to NESTOR (1680 km) (Resvanis 1993); CERN to Superkamiokande (8750 km) (Rubbia 1993); KEK to Superkamiokande (250 km) (Nishikawa 1992).

Since some of these experiments have detectors with the ability to measure separately neutral and charged current events, a strategy somehow in between the usual appearance and disappearance technics can be used to study oscillations among active neutrino species. This strategy consists in using the NC events as a measure of the total flux (including the flavours resulting from the oscillations) and the CC events to measure the original flavour content. Another similar approach is to use the through-going muons produced in the rock outside the detector as a measure of the ν_μ content of the beam and the events contained inside the detector (NC + CC) as an indication of the total ν flux.

If the atmospheric ν problem were due to oscillations in the $\nu_e - \nu_\mu$ channel (a possibility, however, that seems now excluded, see section 7), these may also be studied using reactor $\bar{\nu}_e$ and large detectors at distances $\gtrsim 1$ km. There is a proposal to use the San Onofre reactor in California with a 12 ton detector at a distance of 1 km from the reactor (phase I) and a kton detector at a distance of 10-15 km (phase II) that will cover all the relevant range. Another proposal is based on the use of the Perry reactor and a kton detector at the IMB site that is 13 km apart. Finally, there is a proposal to use the Chooz reactor in France with a detector 1 km away.

Besides the proposed detectors just discussed, there are some ongoing experiments that will provide results in the near future. The third phase of the oscillation experiments at the Bugey reactor in France may slightly improve the sensitivity of the Goesgen experiment. The KARMEN detector at the spallation source ISIS in the Rutherford Appleton Laboratory (Drexlin *et al* 1990) is studying $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ appearance. A very important improvement in the $\nu_\mu \to \nu_e$ oscillations will be achieved with the Liquid Scintillator Neutrino Detector (LSND) at LAMPF (Whitehouse *et al* 1991).

The approximate sensitivities expected in the experiments discussed in this section are shown in figures 4a ($\nu_e \to \nu_\mu$) and 4b ($\nu_\mu \to \nu_\tau$), together with the region excluded at present (continuous lines). Also indicated are the regions relevant for the dark matter problem and for oscillations of atmospheric neutrinos.

7. Solar Neutrinos and Oscillations in Matter

7.1. The Solar Neutrino Problem

The most important natural source of low-energy neutrinos ($E_\nu \leq 18$ MeV) reaching the earth, except at the moment of a galactic supernova explosion, is our own sun. In fact, 98% of the solar energy production arises from the fusion chain that converts four protons into a ^{4}He nucleus, two positrons (by charge conservation) and two ν_e (by lepton number conservation), releasing about 27 MeV and with a maximum ν energy of 0.42 MeV (for these so called *pp* neutrinos) *. The proton-proton chain also gives rise to the production of the heavier nuclei Li, Be and B, although with negligible

* Since the solar energy flux at the earth is $\simeq 8.5 \times 10^{11}$ MeV cm^{-2} s^{-1}, the integrated *pp* neutrino flux must be $\simeq 6 \times 10^{10}$ cm^{-2} s^{-1}.

effects on the overall energy production. The ^{8}B decays give rise to an important continuum neutrino spectrum extending up to $E \simeq 14$ MeV, while the ^{7}Be produces, by electron capture, two neutrino lines at energies 0.86 MeV (90%) and 0.38 MeV (10%). Another line at 1.44 MeV arises from $p + e + p \to d + \nu$ (pep) while the $^3\mathrm{He} + p$ (hep) reaction gives a small continuous spectrum up to 18.7 MeV. Finally, in the CNO (carbon, nitrogen and oxygen) chain, essentially a ^{12}C catalyzes the fusion of 4 protons into ^{4}He to produce the remaining 2% of the solar energy output. The decays of the ^{13}O, ^{13}N and ^{17}F produced in the intermediate steps lead to other continuous neutrino spectra. All this is shown in figure 5 (Bahcall and Ulrich 1988).

Although the pp neutrino flux is well established theoretically, the other ones can depend significantly on the solar modelling and the input data used in it. For instance, the ^{8}B and ^{7}Be neutrino fluxes are sensitive functions of the solar core temperature T_c

$$\Phi(^8\mathrm{B}) \propto T_c^{18} \ , \quad \Phi(^7\mathrm{Be}) \propto T_c^{8} \tag{7.1}$$

(while $\Phi(pp) \propto T_c^{-1.2}$, actually decreases with increasing T_c due to the conservation of the total solar luminosity). Hence, a change on the radiative opacities (namely the cross-sections per gram) that determine the energy transport in the sun, can affect these fluxes significantly. The opacities themselves depend on the abundances of heavy elements and in particular on that of Fe, that contributes sizably to the number of free electrons in the plasma. Making different estimates of the latter affect the ^{8}B neutrino predictions by as much as 10%*. Another source of uncertainties comes from the nuclear cross sections involved. In particular, the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ cross section is known at the energies that are relevant in the sun from an extrapolation of the values measured in the laboratory at higher energies, and this procedure may also cause discrepancies among different ^{8}B neutrino flux estimates (see e.g. Dar and Shaviv 1994).

Solar neutrinos have been observed by four experiments up to now, with the thresholds indicated in figure 5. The pioneering Cl radiochemical experiment (Davis *et al* 1990), based on the reaction $^{37}\mathrm{Cl} + \nu_e \to ^{37}\mathrm{Ar} + e$ ($E_\nu^{thr} = 0.81$ MeV), has detected solar neutrinos for more than 20 years. It is sensitive to the ^{8}B as well as the

* Another possible way of modifying the predictions of T_c is to consider non-standard solar models, e.g. invoking large magnetic fields in the solar core, significant turbulence, etc.

^{7}Be, CNO and *pep* neutrinos. The Kamiokande water Cerenkov detector (Hirata *et al* 1991a-b) has been able to observe in real time the νe scattering, having an analysis threshold of 7 MeV that makes this device sensitive only to the ^{8}B neutrinos. The directional information of the events makes this detector the first 'neutrino telescope' able to obtain a 'picture' of an extraterrestrial neutrino source (the sun). Finally, there are two radiochemical gallium experiments, SAGE at Baksan (Abazov *et al* 1991) and GALLEX at Gran Sasso (Anselmann *et al* 1994). They use the reaction $^{71}\mathrm{Ga} + \nu_{\mathrm{e}} \to ^{71}\mathrm{Ge} + \mathrm{e}$, whose low threshold ($E_\nu^{thr}$=0.23 MeV) makes them sensitive to the *pp* neutrinos in addition to the more energetic ^{7}Be, ^{8}B and CNO ones. In table 4 we show the rates measured in these four experiments, as well as the theoretical predictions based on the standard solar models of Bahcall and Pinsonneault (1992) and Turck-Chièze and Lopes (1993).

Table 4. Solar neutrino measurements and theoretical expectations within the Standard Solar Model of Bahcall and Pinsonneault (1992), SSM-BP, and Turck-Chièze and Lopes (1993) SSM-TCL.

Experiment	Measurement	SSM-BP	SSM-TCL
^{37}Cl [SNU]	2.23 ± 0.23 [a]	8 ± 1	6.4 ± 1.4
Kamioka $\left(\frac{\mathrm{Observed}}{\mathrm{SSM-BP}}\right)$	$0.50 \pm 0.04 \pm 0.06$ [b]	1 ± 0.14	0.77 ± 0.17
GALLEX [SNU]	$79 \pm 10 \pm 6$ [c]	131.5^{+7}_{-6}	122.5 ± 7
SAGE [SNU]	73^{+18+5}_{-16-7} [d]	"	"

[a] Davis 1992, [b] Suzuki 1993, [c] Anselmann *et al* 1994, [d] Abdurashitov *et al* 1994.

The most intriguing feature of these four measurements is that they all give values significantly below the theoretical predictions. This constitutes the so called solar neutrino problem. The explanation of this deficit calls either for modifications in the solar models or for new properties of the neutrinos, such as mixings or magnetic moments, that could convert the electron neutrinos into less detectable species in their journey from the center of the sun to the earth (radiochemical experiments are only sensitive to ν_e, while the $\nu - e$ cross section relevant for Cerenkov detection is 7 times smaller for $\nu_{\mu,\tau}$ than for ν_e, and vanishes for sterile species). Also important is that the observed Ga rates are not less than the ~ 70 SNU expected (reliably) from the *pp* neutrinos alone, since this would otherwise exclude any explanation not invoking new neutrino properties. However, if all experimental results are taken at face value, one should note that an explanation based on a reduction of the core temperature is not possible, since it would reduce much more the ^{8}B flux than the ^{7}Be one, contrarily to what results from the comparison of Kamiokande with Cl data.

Turning to the explanation in terms of new neutrino properties, the simplest one is to invoke oscillations in vacuum of the ν_e (Gribov and Pontecorvo 1967). In view of the typical ν energies ($\sim$ MeV) and the earth-sun distance (1 A.U. $\simeq 1.5 \times 10^{11}$ m), values of $\Delta m^2 \gtrsim 10^{-11}$ eV^2 are required for a significant ν_e suppression (see eq. (6.10) and (6.11)). However, for $\Delta m^2 \gtrsim 10^{-10}$ eV^2 oscillations are completely averaged. In this case, the reduction factor results the same in all experiments, and it is larger than 1/2 for the mixing among two flavours. It is only for 10^{-11} eV$^2 < \Delta m^2 < 10^{-10}$ eV^2 that the oscillation length is of the order of 1 A.U. so that neutrinos of different energies are converted in different amounts. With this 'just so' oscillations (Barger, Phillips and Whisnant 1981, Glashow and Krauss 1987), that also require sizeable mixing angles, it becomes then possible to fit simultaneously all three types of experiments (Barger *et al* 1992, Krastev and Petcov 1992).

A more beautiful possibility is that of explaining the solar neutrino problem using the enhancement of the oscillations induced by the effects of the solar medium, that we now turn to discuss.

7.2. Neutrino Oscillations in Matter

When neutrinos propagate through matter, a crucial differentiation among the neutrino flavours appears due to the fact that ν_e (unlike ν_μ or ν_τ), have charged current interactions due to W exchange with the electrons. As noted by Wolfenstein (1978), these interactions can affect the pattern of neutrino oscillations and the effect can be significantly enhanced (Mikheyev and Smirnov 1985) when a resonance crossing takes place, in the so called MSW effect (Bethe 1986, Rosen and Gelb 1986, Parke 1986, Haxton 1986).

The charged current interaction can be written as $H_{CC} = (4G_F/\sqrt{2})J_\ell^{+\mu} J_{\ell\mu}^{+\dagger}$ (see (2.3) and (3.8)) for energies much lower than the W boson mass. Because there are electrons in normal matter but not muons or taus, we are interested only in the e

ν_e interactions. After a Fierz rearrangement to write them as a product of charged conserving currents, we get

$$H_{CC} = \frac{G_F}{\sqrt{2}} \bar{e}\gamma^\mu(1-\gamma_5)e\bar{\nu}_e\gamma_\mu(1-\gamma_5)\nu_e \quad . \tag{7.2}$$

For an unpolarized medium at rest the only non-vanishing component of the electron current is the temporal part of the vector current, that is nothing but the electron density N_e. Hence, an 'effective' potential energy $\langle H_{CC}\rangle \equiv \langle e\nu|H_{CC}|e\nu\rangle \simeq \sqrt{2}G_F N_e$ will affect the propagation phase of the ν_e*. Although the neutral currents are also present, they can be omitted, since they affect in equal amounts all active flavours giving no net effect to the neutrino oscillations (they are relevant however for oscillations into sterile species).

For definiteness we will consider the case of mixing between two active neutrinos, ν_e and ν_α ($\alpha = \mu$ or τ)

$$\begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} = R_\theta \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \quad , \quad \text{with} \quad R_\theta = \begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix} \tag{7.3}$$

($s\theta \equiv \sin\theta$, etc.). The evolution of the flavour content along the neutrino path can be simply obtained from the equation

$$i\frac{d}{dx}\begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} = \frac{1}{2E}\mathbf{M}^2 \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} . \tag{7.4}$$

The matrix $\mathbf{M}^2$ is

$$\mathbf{M}^2 = \frac{1}{2}\left[R_\theta \begin{pmatrix} -\Delta m^2 & 0 \\ 0 & \Delta m^2 \end{pmatrix} R_\theta^T + 2E\begin{pmatrix} \langle H_{CC}\rangle & 0 \\ 0 & -\langle H_{CC}\rangle \end{pmatrix}\right] \tag{7.5}$$

The first term in the r.h.s. is the usual one, already appearing in vacuum oscillations, and the second term arises from the $\nu_e - e$ coherent forward scattering. We have subtracted a piece proportional to the identity matrix (what changes just a common phase) to put $\mathbf{M}^2$ in a more symmetric form.

It proves very convenient to define the matter eigenstates as

$$\begin{pmatrix} \nu_m^1 \\ \nu_m^2 \end{pmatrix} = R_{\theta_m}^T \begin{pmatrix} \nu_e \\ \nu_\alpha \end{pmatrix} , \tag{7.6}$$

* This can also be thought of as inducing an index of refraction for ν_e.

where R_{θ_m} diagonalizes the matrix $\mathbf{M}^2$, i.e.

$$R_{\theta_m}^T \mathbf{M}^2 R_{\theta_m} = \frac{1}{2} \begin{pmatrix} -\Delta_m & 0 \\ 0 & \Delta_m \end{pmatrix}. \tag{7.7}$$

Here $\Delta_m = \Delta m^2 \sqrt{(a - c2\theta)^2 + s^2 2\theta}$ with $a = 2E\langle H_{CC}\rangle / \Delta m^2$. The matter mixing angle θ_m entering in R_{θ_m} satisfies

$$s^2 2\theta_m = \frac{s^2 2\theta}{(c2\theta - a)^2 + s^2 2\theta}. \tag{7.8}$$

Even if the vacuum mixing is very small, there will then be maximum mixing in matter ($\theta_m = \pi/4$) in the 'resonance region' corresponding to an electron density such that $a = c2\theta$. In matter with varying density, the width of the resonance corresponds to the densities for which $|a - c2\theta| = |s2\theta|$. It is important that if we were considering antineutrinos, the sign of $\langle H_{CC}\rangle$ would be reversed and no resonance would appear (for $\Delta m^2 > 0$). Hence, the presence of electrons in the medium (and no positrons) results in very different oscillation patterns for neutrinos and antineutrinos.

The ν_m evolution is determined by

$$i\frac{d}{dx} \begin{pmatrix} \nu_m^1 \\ \nu_m^2 \end{pmatrix} = \begin{pmatrix} -\Delta_m/4E & -i\frac{d}{dx}\theta_m \\ i\frac{d}{dx}\theta_m & \Delta_m/4E \end{pmatrix} \begin{pmatrix} \nu_m^1 \\ \nu_m^2 \end{pmatrix} \quad , \tag{7.9}$$

with

$$\frac{d\theta_m}{dx} = \frac{1}{2} \frac{s2\theta}{(a - c2\theta)^2 + s^2 2\theta} \frac{da}{dx} \quad . \tag{7.10}$$

We see that the matter states are the propagation eigenstates in a medium with constant density ($\mathrm{d}\theta_m/\mathrm{d}x \simeq 0$), since no transitions between them can occur in this case. When the density of the medium varies along the neutrino path, transitions between matter mass eigenstates are induced by a nonzero $d\theta_m/dx$. This is the case in the sun, where the densities fall nearly exponentially in the radial direction. The transitions between matter mass eigenstates are usually negligible unless the neutrinos are near the resonance layer, for which the diagonal elements in (7.9) are minimum and $d\theta_m/dx$ is enhanced. If $P = |\langle \nu_m^1 | \nu_m^2 \rangle|^2$ is the probability of $\nu_m^1 \to \nu_m^2$ conversion in the resonance crossing, the averaged probability to detect an electron neutrino that has crossed a resonance is

$$P_{\nu_e\nu_e} = \frac{1}{2} + \left(\frac{1}{2} - P\right) c2\theta \times c2\theta_m \quad , \tag{7.11}$$

where θ_m is the matter mixing angle corresponding to the point where the ν_e was produced, while θ is the vacuum angle*.

Under the assumption that the electron density varies linearly in the resonance layer, the probability of level crossing at resonance is found to be

$$P = e^{-\pi\gamma/2}, \tag{7.12}$$

where the adiabaticity parameter γ is

$$\gamma \equiv \frac{\Delta_m}{4E|d\theta_m/dx|} = \frac{\Delta m^2}{2E}\frac{s^2 2\theta}{c2\theta}\frac{1}{|d\ln N_e/dx|_r} \tag{7.13}$$

(the sub-index r stands for the resonance value).

In the adiabatic case, i.e. $\gamma \gg 1$, the off-diagonal terms in (7.9) can be neglected even at resonance, so that $P = 0$ and the probability of having a ν_e after adiabatic conversion is $P_{\nu_e\nu_e} = (1 + c2\theta \times c2\theta_m)/2$. What happens is that if the production point is at densities larger than the resonance value, a ν_e is mainly the ν_m^2 eigenstate ($\theta_m \simeq \pi/2$) and remains so during the adiabatic propagation. Thus, it becomes essentially ν_α when it comes out from the sun. An almost complete flavour conversion results. The resonance condition $a = c2\theta$ can be written as

$$\Delta m^2 c2\theta \simeq \frac{E}{10\ \text{MeV}}\frac{\rho}{\rho_0}10^{-4}\text{eV}^2\ , \tag{7.14}$$

where $\rho_0 \simeq 150\ \text{gr/cm}^3$ is the central density of the sun and we have used $N_e = Y_e\rho/m_p$ (with $Y_e = N_e/(N_n + N_p) \sim 0.7 - 0.8$ in the sun). We then see that the solar neutrinos will meet a resonance only for $\Delta m^2 \lesssim 10^{-4}$ eV^2.

It could be that in the resonance crossing the propagation becomes non-adiabatic (namely, $\gamma \lesssim 1$), making the flavour conversion less efficient. This happens when the vacuum mixing angle is small so that the resonance becomes very narrow. In the extreme non-adiabatic case ($\gamma \ll 1$), corresponding to*

$$\Delta m^2 s^2 2\theta \ll 2E\left|\frac{1}{N_e}\frac{dN_e}{dr}\right|_r \simeq 6\times 10^{-8}\left(\frac{E}{10\ \text{MeV}}\right)\text{eV}^2\ \ , \tag{7.15}$$

* This is obtained by projecting at the production point the ν_e into matter eigenstates, following then the outward propagation in the sun including an eventual jump between matter eigenstates at the resonance crossing, and using the fact that the flavour content of those states changes smoothly with electron density until they match the vacuum mass eigenstates when they leave the sun.

* For $r > 0.1R_\odot$, $N_e \propto \exp(-10.5r/R_\odot)$.

the ν flavour content is essentially unchanged in the resonance crossing and no enhancement in $P(\nu_e \to \nu_\alpha)$ is found.

The survival probabilities depend strongly on the neutrino energy, as is shown in figure 6, in which the effect of the conditions of crossing a resonance and that the transition not be extremely non-adiabatic are clearly seen. For instance, for $s2\theta = 0.1$ as in the figure, only the high energy neutrinos ($E \gtrsim 10$ MeV) would be suppressed if $\Delta m^2 \sim 10^{-4}$ eV^2 (adiabatic case) while instead for $\Delta m^2 \sim 10^{-6}$ eV^2 it would be mainly the pp neutrinos and not the high energy ones to be converted (non-adiabatic case).

An agreement with the deficit observed at the Davis experiment, with adiabatic neutrino evolution, is obtained in two regions of the $\Delta m^2 - s^2 2\theta$ plane. The first is for $\Delta m^2 \simeq 10^{-4}$ eV^2, if the resonant layer is not too narrow, i.e. $s2\theta \gtrsim 10^{-2}$. The second is the so-called "large angle solution", $s^2 2\theta \sim 0.7$ and 10^{-4} eV^2 $> \Delta m^2 > 10^{-8}$ eV^2, for which the MSW oscillations start approaching the large-mixing vacuum oscillations. A non-adiabatic solution exists for $\Delta m^2 s^2 2\theta \simeq 10^{-7.5}$ eV^2, closing a 'triangle' in the $\Delta m^2 - s^2 2\theta$ plane. Due to the different energy dependence of the reduction factor in the three regimes, experiments with other thresholds, or capable of measuring the neutrino spectra, can distinguish among the three solutions. The results of the Kamiokande detector in fact disfavor the $\Delta m^2 \simeq 10^{-4}$ eV^2 adiabatic solution, that would deplete too much the high energy ^{8}B neutrinos, while the gallium experiments disfavor Δm^2 values below 10^{-6} eV^2, that would result into an excessive conversion of pp neutrinos. The combination of all the experiments, then, points out to two possible solutions, a non-adiabatic one with $s^2 2\theta \sim 8 \times 10^{-3}$ or a large angle solution with $s^2 2\theta \sim 0.7$, both with $\Delta m^2 \sim 10^{-5}$ eV^2, as is shown in figure 7, taken from Hata and Langacker (1993b) (see also Hata and Langacker 1994).

For ν_e oscillations into sterile species ν_s, the solution is slightly different because the ν_s do not have NC interactions. Hence, the matter effects in the sun as well as the cross section for detection in Kamiokande are affected. As a result (Hata and Langacker 1994), the large angle solution (that would anyhow also conflict with nucleosynthesis (Shi *et al* 1993)) is absent for $\nu_e - \nu_s$ oscillations and the small angle solution is slightly shifted.

There are two interesting cases in which the neutrino propagation through the earth may be affected by matter effects. In these cases no MSW effect, namely the crossing of a resonance layer, really takes place since the density varies only slightly, from 3 to 5.5 gr/cm^3 in the mantle and from 10 to 13 gr/cm^3 in the core. Anyhow, oscillations of neutrinos that are resonant can be significantly enhanced since the matter mixing becomes maximal. The first is the case of the accelerator neutrinos in the planned long-baseline experiments and of the atmospheric neutrinos that pass through the earth before reaching the detectors. They would be resonant in the mantle if $\Delta m^2 \simeq 3 \times 10^{-3}$ eV^2 for energies $\simeq 10$ GeV. Hence, matter effects can extend the sensitivity to smaller $s^2 2\theta_{e\alpha}$ values in this mass range (Carlson 1986, Akhmedov, Lipari and Lusignoli 1993, Fiorentini and Ricci 1993). However, very long-baselines are required to see any effect because the wavelength of the oscillations in matter, $\lambda_m \equiv 4\pi E/\Delta_m$, becomes

$$\lambda_m \simeq \frac{3 \times 10^6 \mathrm{m}}{\mathrm{tg}2\theta(\rho/5 \text{ gr cm}^{-3})} \tag{7.16}$$

in the resonance (taking $Y_e = 0.5$). This is of the order of or larger than the radius of the earth. The second case in which matter effects in the earth are relevant is that of the ^{8}B neutrinos observed by Kamiokande ($E \simeq 10$ MeV). The resonance in the earth in this case is achieved for $\Delta m^2 \simeq 3 \times 10^{-6}$ eV^2, and can give rise to a day–night effect in this mass range if $s2\theta$ is not too small (if λ_m is not too large, Baltz and Weneser 1987, Bouchez *et al* 1986). The non-observation of this effect (Hirata *et al* 1991a) leads to the exclusion of the region shown in figure 7 (see also Hata and Langacker 1993a and references therein).

7.3. Other Solutions

Many alternative solutions have been proposed to explain the solar neutrino deficit. One of them is based on the interaction of the neutrinos with the magnetic fields of the sun, that, in the convective, zone increase in strength in periods of high activity (Cisneros 1971, Voloshin, Vysotsky and Okun 1986). The resulting effect would be to flip the neutrino chirality to produce either a sterile state (with Dirac type moments) or just a different active flavour (with Majorana type $|\Delta L| = 2$ transition moments). This solution became especially attractive to account for an anticorrelation with the solar activity of the rates, observed in the Davis experiment but not confirmed by Kamiokande.

The required magnetic moments are $\mu \simeq 10^{-11}\mu_B$ ($\mu_B \equiv e/2m_e$ is the Bohr magneton). Although these values are not in conflict with direct experimental bounds, $\mu < 1.08\times10^{-9}\mu_B$ (Review of Particle Properties 1994), they would result in excessive stellar cooling (Raffelt 1990), or conflict with SN1987A in the case of Dirac type transitions (see e.g. Barbieri and Mohapatra 1988), as we will see in section 9.

Another difficulty is that it is not easy to find models providing magnetic moments of the required size keeping the neutrino masses small, since a loop diagram contributing to μ gives a mass term when the photon is removed, unless some particular symmetries are invoked (Voloshin 1988). Furthermore, transition type spin precession is quenched when the mass difference between states is $\Delta m^2 \gtrsim 10^{-7}$ eV^2. However, matter effects could compensate the mass terms and lead to an enhanced 'spin-flavour' precession in a resonance crossing (Akhmedov 1988, Lim and Marciano 1988). Recent analysis of the experimental data in this context can be found in Krastev (1993), Akhmedov, Lanza and Petcov (1993) and Pulido (1993).

The usual MSW picture is also modified in the presence of other non-standard properties in the neutrino sector. For instance, if mixing with heavy singlet neutrino species occurs, the Z-mediated interactions can become non-universal in flavour. Then, it may even become possible to have a resonance for massless neutrinos, by compensating the effect of the charged-current with the neutral-current ν_e-interactions (Valle 1987). However, due to experimental constraints on the weak boson couplings, this may only be achievable in media such as neutron stars, but not in the sun.

More interesting for the solar neutrino problem are new flavour changing neutrino interactions that appear in supersymmetric or GUT extensions of the standard model. They may induce neutrino oscillations even for vanishingly small vacuum mixings (Roulet 1991), and their resonant amplification can result in sizeable effects, even for small (experimentally allowed) couplings. In this type of scenarios, the resonance conversion in the massless case may take place in the presence of large new diagonal couplings of ν_τ (Guzzo *et al* 1991) (for a general discussion see Barger *et al* (1991)). Finally, the possibility that non-universal gravitational interactions (or new flavour-dependent long range forces) affect the solar neutrino oscillations (Halprin and Leung 1991), and the possibility of matter induced neutrino decay into another neutrino and a Majoron in the sun (see e.g. Berezhiani and Rossi 1993) have also been considered.

7.4. *Future Prospects*

Several new solar neutrino detectors are in construction or under study at present. The Sudbury Neutrino Observatory in Canada (Aardsma *et al* 1987), using 1 kton of heavy water, will be able to detect, besides the scattering off electrons, the reactions $\nu_x d \to pn\nu_x$ and $\nu_e d \to ppe$. By observing the n capture, the deuterium disintegration ($E_\nu^{th} = 2.2$ MeV) will allow to measure the NC rates due to all active neutrino flavours. The inverse β decay will be useful to reconstruct the neutrino spectrum accurately above $\simeq$ 5 MeV. The comparison of the NC and CC rates can give a clear signal of oscillations, independently of any solar model input. The spectral shape can help to distinguish among different solutions. This experiment (starting in 1995) and the Superkamiokande water Cerenkov (Totsuka 1990), with 22 kton fiducial mass (starting in 1996), will be able to study with large statistics the ^{8}B neutrino fluxes.

The study of the Be line is the main objective of the Borexino experiment (Arpesella *et al* 1992). It consists of 100 ton fiducial mass of ultrapure liquid scintillator with extremely low radioactive background, in order to be able to detect the νe scattering in the window 0.25 MeV $< E <$ 0.8 MeV (90% of the signal coming from ^{7}Be). The events of larger energies will also permit to study the ^{8}B neutrino fluxes. A signal that can be useful is the annual modulation of the Be flux due to the eccentricity of the earth orbit. For 'just so' oscillations the modulation will differ from the simple r^{-2} behaviour. A counting test facility with 2 ton fiducial mass is now being built at Gran Sasso. Another experiment, Icarus, a liquid argon TPC to be installed at Gran Sasso (3 modules of 5 kton each), could allow to study the ^{8}B neutrinos measuring separately CC and NC events. A 3 ton prototype built at CERN has performed satisfactorily (Revol 1993). Two experiments to detect *pp* neutrinos are under study at present, both helium-based. Heron (Lanou, Maris and Seidel 1987) would detect the rotons produced by neutrino interactions in superfluid helium and Hellaz (Arzarello *et al* 1994) would consist of a helium gas TPC at high pressure.

These experiments, together with increased statistics in the gallium experiments, should firmly establish the nature of the solution to the solar neutrino problem, within the next ten years.

8. Atmospheric neutrinos

The cosmic ray nuclei that impinge on the atmosphere are stopped in the upper layers by collisions with the air nuclei. These interactions can produce pions and kaons that in turn decay giving rise to neutrinos (e.g. $\pi^+ \to \mu^+ + \nu_\mu$ and similarly for π^- and $K^\pm$). Also the muons so produced decay after loosing some energy and produce more neutrinos via $\mu \to \nu_\mu + e + \bar{\nu}_e$ (muons with energies below a few GeV decay before reaching the ground). The resulting neutrino fluxes on earth are quite significant, and were observed many years ago (Krishnaswamy 1971, Reines 1971) by looking at horizontal and upgoing muons produced in the rock surrounding deep underground detectors (downgoing muons are dominated by those directly produced high in the atmosphere by very energetic meson decays). In recent years, it became also possible to observe contained events ($E_\nu \leq 2$ GeV), produced directly by neutrino interactions inside large volume detectors (IMB, Kamiokande, Fréjus, NUSEX, SOUDAN).

Atmospheric neutrinos are now an important subject for all underground neutrino detectors. They also constitute a background for proton decay searches, and due to the long 'baseline' distances involved ($10-10^4$ km), they are of particular interest for ν oscillation studies.

Several theoretical computations of the resulting ν fluxes have been performed, both for the low energy neutrinos that lead to contained events (Barr, Gaisser and Stanev 1989, Lee and Koh 1990, Honda *et al* 1990, Kawasaki and Mizuta 1991, Bugaev and Naumov 1989) and for those of higher energies that produce throughgoing muons (Volkova 1980, Mitsui, Minorikawa and Komori 1986, Butkevich, Dedenko and Zheleznykh 1989). The different computations typically agree on the absolute values of the $\nu_e + \bar{\nu}_e$ and $\nu_\mu + \bar{\nu}_\mu$ fluxes only within 20-30%. The reason is that they are affected by the poor knowledge of the primary cosmic ray spectrum and composition, the uncertainties in the meson production cross section, the use of different calculational methods, etc. Also the resulting absolute values of muon fluxes, that could in principle be used to normalize the calculations, are poorly known experimentally. Hence, the measurements of the absolute neutrino fluxes can hardly give reliable information on neutrino oscillations.

This difficulty can be overcome by considering the ratio $R = (\nu_\mu + \bar{\nu}_\mu)/(\nu_e + \bar{\nu}_e)$ of contained events of the μ and e-type. Most uncertainties cancel in this ratio and, in fact, the different calculations agree to within better than 5% on the value of this quantity. One naively expects $R = 2$ at low energies, as would result from the meson decay chain presented above. Detailed evaluations of R lead to a value larger than 2 only by a few percent, for $E_\nu \lesssim 1.5$ GeV. When this ratio was measured some years ago by the Kamiokande collaboration (Hirata *et al* 1988), a significant discrepancy with the expected value was found. This discrepancy was later confirmed by the other Cerenkov detector IMB (Casper *et al* 1991, Becker-Szendy *et al* 1992b), and recently also by the SOUDAN II tracking calorimeter (Kafka 1993). No anomaly was found with the other detectors of this type, Fréjus (Berger *et al* 1989) and NUSEX (Aglietta *et al* 1989), that are, however, statistically less significant than the Cerenkov detectors. These experiments report the ratio of ratios

$$R(\mu/e) \equiv \frac{R_{obs}}{R_{MC}} \quad , \tag{8.1}$$

where R_{obs} is the observed ratio of μ to e-type events and R_{MC} the Monte Carlo results, that take into account also experimental cuts and detector details. The values for this quantity are given in the table 5.

Table 5. Ratio of observed and Monte Carlo ratios of μ-type to e-type contained neutrino events.

Experiment	$R(\mu/e) = R_{obs}/R_{MC}$	reference
Kamiokande	$0.60^{+0.07}_{-0.06} \pm 0.05^{a}$	(Hirata *et al* 1992)
	$0.67^{+0.08}_{-0.07} \pm 0.07^{b}$	(Fukuda *et al* 1994)
IMB	$0.54 \pm 0.05 \pm 0.12$	(Becker-Szendy 1992b)
Nusex	$0.99^{+0.35}_{-0.25}$	(Aglietta *et al* 1989)
Fréjus	$1.06 \pm 0.18 \pm 0.15^{c}$	(Berger *et al* 1989)
	$0.87 \pm 0.16 \pm 0.08^{d}$	
Soudan II	$0.69 \pm 0.19 \pm 0.09$	(Kafka 1993)

[a] Sub-GeV; [b] Multi-GeV [c] Vertex contained; [d] Fully contained.

In figure 8 we show the momentum distribution of e-type ($8a$) and μ-type ($8b$) events in the Kamiokande data (Beier *et al* 1992, Hirata *et al* 1992), compared to a

Monte Carlo using the neutrino fluxes from Lee and Koh (1990). Similar results were also obtained by IMB (Becker-Szendy *et al* 1992)).

The deficit of μ-type neutrinos (or excess of e-type, or both combined), found in some of the detectors, constitutes the so called atmospheric neutrino problem. If taken at face value, it would suggest neutrino oscillations of the type $\nu_\mu - \nu_\tau$, $\nu_\mu - \nu_e$ or $\nu_\mu - \nu_s$. These would require large mixing angles ($\sin^2 2\theta \gtrsim 0.4$) and, due to the typical ν energies (GeV) and large baselines ($\lesssim$ 12000 km) they would require $\Delta m^2 \gtrsim 10^{-3}$ eV^2.

Another feature of the data is that there is no evidence of zenith angle dependence on $R(\mu/e)$ at low energies, $E_\nu \leq$ GeV (Hirata *et al* 1992), suggesting that oscillations should have already settled at baselines $\simeq$ 30 km for these energies. However, a significant angular dependence has been observed in the recent analysis of multi-GeV events by Kamiokande (Fukuda *et al* 1994) which is consistent with the sub-GeV data in the scenario of neutrino oscillations, because of the increase of the oscillation length with energy. The values of Δm^2 and $\sin^2 2\theta$ consistent with the Kamiokande results are shown in figure 9 for the $\nu_\mu - \nu_e$ channel (9a) and $\nu_\mu - \nu_\tau$ channel (9b). Also shown are the bounds from reactors and accelerators and from the negative Fréjus result, that for the $\nu_\mu - \nu_e$ option exclude the parameter space allowed at present by Kamiokande. An explanation in terms of $\nu_\mu - \nu_s$ oscillations would be in conflict with constraints from primordial nucleosynthesis on the number of effective neutrino species at $T \simeq$ MeV mentioned in section 5, $N_\nu \lesssim 3.3$ (Walker 1993), since for the required values of Δm^2 and $\sin^2 2\theta$ the sterile species ν_s would be brought into equilibrium by the mixing itself (Dolgov 1981, Barbieri and Dolgov 1991, Kainulainen 1990, Enqvist *et al* 1992, Shi *et al* 1993) and hence lead to $N_\nu \simeq 4$.

Since neutrino oscillations depend strongly on the neutrino energy, another way to test the oscillation solution to the atmospheric neutrino problem is to compare the upward going muons stopping inside the detector (typical energies $E_\nu \sim 10$ GeV) with those going through the detector ($E_\nu \sim 100$ GeV). This method is independent of the overall normalization of the neutrino fluxes, but depends on its spectral shape. The analysis of the IMB data (Becker-Szendy *et al* 1992a, Frati *et al* 1993) shows no evidence of an anomaly, what excludes the region with $\Delta m^2 \sim 10^{-3} - 10^{-2}$ eV^2 (since $d \simeq 10^4$ km) and large mixing angles shown in figure 9b.

Also the IMB-1 collaboration (Bionta *et al* 1988) used the non observation of an anomaly between the upgoing and downgoing ν_μ ($E_\nu \sim$ GeV), to constrain $\nu_\mu - \nu_\tau$ oscillations with Δm^2 around 10^{-4} eV^2 (IMB-1 curve in figure 9*b*), for which only the ν_μ from the lower hemisphere would have oscillated sizably (and not those from above).

Finally, there have been attempts to directly compare the upward-going muons with the theoretical expectations to further constrain the mixing parameters (these fluxes are consistent with no oscillations (Becker-Szendy *et al* 1992a, Mori *et al* 1991, Boliev *et al* 1991)). The uncertainties in the absolute neutrino fluxes and on the quark structure functions entering the inelastic cross section for muon production, make however those analysis less reliable (Frati *et al* 1993). In conclusion, the atmospheric neutrino anomaly could be explained in terms of oscillations between ν_μ and ν_τ, with $\Delta m^2 \simeq 10^{-2}$ and $\sin^2 2\theta \simeq 0.5$.

For the future, the results of the Soudan II experiment will significantly improve since their preliminary data (based on the first 1.5 kt y) still have large statistical uncertainties. This is important because none of the previous tracking calorimeters (Fréjus and Nusex) observed an anomaly in the atmospheric neutrino data. Also relevant in this respect is the planned exposure of a Cerenkov detector to e and μ beams at KEK, to check for possible systematic effects due to the μ/e identification capability of these devices, that was never tested before. In addition, the new detectors Superkamiokande, SNO, Dumand, Amanda and NESTOR, that are under construction, will provide much more information on this issue, in particular from the detailed study of the zenith angle dependence of the ν fluxes (Doncheski, Halzen and Stelzer 1992) as well as the other methods mentioned before. As we discussed in the section 6, also the proposed long-baseline oscillation experiments can help to reliably settle the issue of whether the atmospheric neutrinos oscillate or not.

9. Neutrinos from Supernovae and Other Stars

Neutrinos play an important role in stellar evolution. Neutrinos produced in the interior of stars can just stream out, except at the large densities of collapsing supernovae, where they are trapped for some time. While in ordinary main-sequence stars, such as our sun, the photon luminosity is larger than the neutrino luminosity, for stars whose central temperature is higher than 10 keV (O and Si burning stars and beyond) neutrino emission is the main form of energy loss. Thus, stars can test any non-standard neutrino property that increases the neutrino energy loss rate from stars or the neutrino energy transfer inside stars, thus changing their standard course of evolution (for a review see Raffelt 1990a). For example, an increase of energy loss accelerates the evolution of stars, shortening their lifetimes.

Nowhere the effect of neutrino cooling is more dramatic than in a supernova explosion. The observation of the neutrino burst from the supernova SN1987A provided a confirmation of the theoretical understanding of the stages of the gravitational collapse of a type II (core collapse) supernova (see e.g. Burrows 1990 or Hillebrandt and Hoflich 1989). Most of the energy liberated in the collapse of the iron core of the progenitor star into a neutron star, i.e. $\simeq 3 \times 10^{53}$ ergs, is emitted in neutrinos in about 10 sec. While the collapse is well understood, the actual explosion which ejects the outer layers of the original star is not. Because only $\simeq 1\%$ of the total energy liberated in the collapse goes into the explosion, relatively very small effects have to be well understood to account for it theoretically. Therefore, supernovae explosions are very difficult to obtain in simulations. This difficulty is sometimes called the "supernova problem". Neutrino properties that could transfer some of the neutrino energy to the outer layers of the star have been repeatedly advocated as a solution to this problem.

A surprising wealth of information on neutrinos have been obtained from SN1987A, considering that only 19 neutrino events were observed (by two large water Cerenkov detectors, IMB, Bionta *et al* 1987 and Bratton *et al* 1988, and Kamiokande II, Hirata *et al* 1987 and 1988, through the reaction $\nu_e^c + p \rightarrow n + e^+$). Let us mention some of the bounds obtained (for a review see e.g. Raffelt 1990a and 1990b).

The simple arrival of (electron anti-) neutrinos from the SN1987A, provides a bound on any effect that could have removed them from the burst before arriving to earth, such as decay, scattering, or interaction with magnetic fields. The lower bound obtained on their lifetime is $\tau_{\nu_e} > 6 \times 10^5$ sec (m_{ν_e}/eV). Due to the possibility of a difference between current and mass eigenstate neutrinos this bound does not exclude that part of the electron neutrinos emitted in the sun could decay before arriving to the earth (Frieman, Haber and Freese 1988), explaining in this way the solar neutrino problem. However decaying neutrinos do not provide a satisfactory fit to the solar neutrino observations (Acker and Pakvasa 1994). The neutrinos arriving from SN1987A where not removed by scattering, for example, with a background of relic Majorons, thus the neutrino majoron coupling (see section 3) must be $(g_{\rm eff})_{\nu_e \nu_e J} < 10^{-3}$ (Kolb and Turner 1987). The arrival of neutrinos provides also a bound on their charge. If neutrinos would have a non-zero electric charge Q_{ν_e}, their trajectories would have been bent in the galactic magnetic field. The observation of the neutrino burst gives a limit $Q_{\nu_e} \lesssim 3 \times 10^{-15} e$ (Barbiellini and Cocconi 1987) (while the best limit from the neutrality of atoms is $Q_{\nu_e} \lesssim 10^{-21} e$, Bauman et al 1989).

An upper bound on m_{ν_e} is due to the (non) dispersion of the neutrino emission time. The neutrino signal was spread over $\simeq 10$ seconds and the emission time could not have been much longer. Massive neutrinos from the star arrived to earth with a delay time (time of arrival minus time of emission) $\Delta t = d/v \simeq d[1 + (1/2)(m_\nu/E_\nu)^2]$ where $d \simeq 50$ kpc$= 1.6 \times 10^5$ lyr is the distance travelled and E_ν is the neutrino energy. If all neutrinos (with equal mass m_ν) had been emitted at the same time, the more energetic ones should have arrived earlier. This is not what happened (energies and times were not so correlated). Thus, by increasing the assumed mass m_ν, the inferred dispersion of the emission times increases and for some mass value becomes unacceptably large. This argument yields the upper bound $m_{\nu_e} < 23$ eV (at 95% $C.L.$ ignoring systematic uncertainties, Loredo and Lamb 1989). This bound (as those of section 4.1) is independent of the Dirac or Majorana nature of the neutrinos. There are no bounds on the other flavour neutrino masses because the bulk of the events observed were ν_e^c, due to detection cross-section arguments.

The number and energy of the detected events correspond well with what is expected, leaving not much room for new sources of energy loss. Also a new mechanism for energy loss would have speed up the cooling, thus shortening the pulse with respect to what was seen. For example Majoron models (singlet or mostly singlet Majorons, the only viable ones, see section 3) are constrained by these argument, that reject a region in m_ν-V space, for V, the scale of spontaneous symmetry breaking, in the range few GeV $\lesssim V \lesssim$ 1 TeV (Choi and Santamaria 1990). If neutrinos are Dirac particles, their mostly sterile right-handed helicity components can be produced due to their m/E admixture in the left-handed chirality component (the interactive component). Subsequently they escape, because their cross sections are too small to trap them, increasing the cooling rate of the neutron star. A bound $m_\nu^D < O(10$ keV$)$ has been obtained (Raffelt and Seckel 1988, Burrows, Gandhi and Turner 1992, Mayle *et al* 1993 and references therein) from preventing a shortening of the duration of the observed cooling neutrino pulse. This bound applies only to neutrinos lighter than 1 MeV, that can be copiously produced, because the production of heavier neutrinos within the star is approximately suppressed by a Boltzman factor (Sigl and Turner 1994). The upper bound of O(10 keV) is hard to improve due to the difficulty in evaluating the effect of the very fast multiple scattering of nucleons inside the supernova on the emission of the right-handed neutrinos (Raffelt and Seckel 1991 and 1992). This difficulty affects most bounds (also on other particles) based on energy loss in SN1987A. Sterile right handed neutrinos could also have been produced due to new interactions with an effective coupling constant G_R (the equivalent of the usual Fermi constant G_F). The above cooling arguments yield $G_R \lesssim 10^{-4} G_F$ (Raffelt and Seckel 1988, 1991, 1992).

If neutrinos are Dirac particles and they are unstable, the ν_R that escaped from the inner, hotter, regions of the supernova, could decay into energetic ($E \simeq 100$ MeV) left-handed neutrinos that would have been seen as extra events if 1 keV $\lesssim m_\nu \lesssim 300$ keV and $10^{-9}\text{sec}(m/1\ \text{keV}) \lesssim \tau \lesssim 5 \times 10^{7}\text{sec}(m/1\ \text{keV})$, thus these ranges are forbidden for Dirac neutrinos (Dodelson, Frieman and Turner 1992). Following a similar argument, one finds a more dubious bound that would apply to both Dirac and Majorana neutrinos in the same mass range, emitted from the "neutrino sphere", the boundary of the region where neutrinos are temporarily trapped (so these are interacting neutrinos), if electron neutrinos are produced in the decay. Then, lifetimes 3×10^{5} sec (keV$/m$) $\lesssim \tau \lesssim 2 \times 10^{10}$ sec (keV$/m$) are rejected (Gelmini, Nussinov and Peccei 1992, Mohapatra and Nussinov 1992, Soares and Wolfenstein 1989). This bound is less meaningful because neutrinos are emitted from the neutrino sphere with energies of O(10 MeV), consequently the energy of the decay products is smaller than before, thus reducing the expected number of events and their energies, what makes these events more difficult to distinguish from background events. These bounds on lifetimes apply to invisible decay modes, in which the decay products do not include photons or e^+e^-, i.e. $\nu \to 3\nu'$ or $\nu \to \nu'\phi$, where ϕ is a Goldstone boson, a Majoron in most models (see section 3). Much more restrictive bounds apply to radiative decays, as shown below.

Turning now to the electromagnetic properties of neutrinos, a magnetic or electric dipole moment μ_ν would also allow for helicity flips into otherwise inert ν_R of Dirac neutrinos in the electromagnetic scattering of the trapped ν_L with charged particles. Thus, the same arguments on energy loss from SN1987A impose, for Dirac neutrinos only, $\mu_\nu < 10^{-12}\mu_B$, with the uncertainty mentioned above (Barbieri and Mohapatra 1988, Lattimer and Cooperstein 1988, Goldman et al 1988) for neutrinos that can be copiously produced in the supernova, namely $m \lesssim 1$ MeV. Here $\mu_B = e/2m_e$ is the Bohr magneton. Other bounds, valid for $m_\nu \lesssim 10$ keV, arise from energy loss due to the plasmon decay $\gamma \to \nu_i^c \nu_j$ in red-giants and in white dwarfs. These are respectively $\mu_\nu < 3 \times 10^{-12}\mu_B$ (Raffelt 1990c, Raffelt and Weiss 1992) and $\mu_\nu < 1 \times 10^{-11}\mu_B$ (Blinnikov 1988, Raffelt 1990a). These two bounds apply to both Dirac (direct or diagonal and transition) and Majorana (transition) neutrino dipole moments ('diagonal' and 'transition' refer to flavour space, see section 2). Note that these bounds would exclude the spin-flip solution to the solar neutrino problem (see section 7).

For comparison, let us mention here the cosmological bound based on the effective number of relativistic neutrinos during nucleosynthesis, $N_\nu < 3.4$ (Olive *et al* 1990, see section 5). It requires $\mu_\nu \lesssim 5 \times 10^{-11}\mu_B$ for Dirac neutrinos (Morgan 1981, Fukugita and Yazaki 1987), because otherwise the mostly inert right-handed helicity components could be brought in equilibrium in the early universe due to the electromagnetic interactions of left-handed neutrinos. On the other hand, the laboratory bounds for magnetic moments (Review of Particle Properties 1994 and references therein) are $\mu_{\nu_e} < 1.08 \times 10^{-9}\mu_B$, $\mu_{\nu_\mu} < 7.4 \times 10^{-10}\mu_B$ and $\mu_{\nu_\tau} < 5.4 \times 10^{-7}\mu_B$. This latter bound on μ_{ν_τ} applies only to a direct moment (Cooper-Sarkar *et al* 1992). Using the same data (Cooper-Sarkar *et al* 1985), Babu, Gould and Rothstein (1994) found a laboratory bound for transition moments $\mu_{\nu_\tau \mathrm{tran}} < 1.1 \times 10^{-9}(\mathrm{MeV}/m_{\nu_\tau})\mu_B$, that is more restrictive for an MeV tau neutrino. In the simplest extension of the SM, where only ν_R and Dirac masses m_ν are added (section 3.2), the predicted neutrino magnetic moment is much smaller than any of these bounds, $\mu_\nu \simeq 3 \times 10^{-19}(m_\nu/\mathrm{eV})\mu_B$ (Lee and Schrock 1977, Fujikawa and Shrock 1980).

The non-observation (by the Solar Maximum Mission Satellite, SMM) of a γ-ray burst in association with the SN1987A neutrino burst, forbids radiative neutrino decays $\nu \to \gamma\nu'$ with lifetimes $\tau_{\mathrm{rad}} > 2 \times 10^{15}$ sec (m_ν/eV) for $m_\nu < 20$ eV. For larger masses, the photons produced would be spread out in time and the upper bound weakens to $\tau_{\rm rad} \gtrsim 3\times10^{16}$ sec for 20 eV $\lesssim m_\nu \lesssim 100$ eV and $\tau_{\rm rad} > 0.8\times10^{18}({\rm eV}/m_\nu)$ for 100 eV $\lesssim m_\nu \lesssim$ 1 MeV (Kolb and Turner 1989, Raffelt 1990a and 1990b and references therein). The mass-lifetime region excluded by the SMM bounds is shown in the hatched area of figure 2. These bounds on $\tau_{\rm rad}$ translate into very restrictive bounds on transition moments, for all neutrino types and flavours (through the relation $\tau_{\rm rad} = 8\pi/{\mu_\nu}^2{m_\nu}^3$, valid for negligible final neutrino mass). These bounds are $\mu_{\nu{\rm tran}} < 1\times10^{-8}({\rm eV}/m_\nu)^2\mu_B$ for $m_\nu \lesssim$ 20 eV and $\mu_{\nu{\rm tran}} < 5\times10^{-10}(1\ {\rm eV}/m_\nu)\mu_B$ for 20 eV $\lesssim m_\nu \lesssim$ 1 MeV (Raffelt 1990a).

The mass vs. lifetime bounds for visible decay modes of neutrinos heavier than 1 MeV (only possible for ν_τ) have been recently reanalysed (Sigl and Turner 1994, Babu, Gould and Rothstein 1994). These neutrinos have another visible decay mode besides $\nu_\tau \to \nu'\gamma$, namely $\nu_\tau \to \nu' {\rm e}^+{\rm e}^-$. Even if this is the dominant decay, the SMM bound can be applied, because photons will be produced with branching ratio 10^{-3} through bremsstrahlung (Sigl and Turner 1994) or by other processes (Mohapatra, Nussinov and Zhang 1994). The SMM bound only applies if neutrinos decay outside the supernova, i.e. for $\tau > 100$ sec. For shorter lifetimes, the decay products would be trapped inside and bounds result from their effect on the supernova energetics. The combinations of both bounds exclude a massive ν_τ of lifetime 10^{-6} sec $\lesssim \tau \lesssim 10^8$ sec decaying into visible modes (Sigl and Turner 1994). The excluded region for $m_\nu > 1$ MeV taken from Sigl and Turner (1994) is included in the hatched area of figure 2. Laboratory bounds (Cooper-Sarkar *et al* 1985, Babu, Gould and Rothstein 1994) exclude these modes for even shorter lifetimes, $\tau \lesssim 0.1(m_\nu/{\rm MeV})$ sec *. So, if the dominant decay modes are visible, all lifetimes shorter than $\simeq 10^8$ sec are forbidden by the combination of SN1987 bounds and laboratory bounds. If we add the nucleosynthesis bounds (see section 5, Dodelson, Gyuk and Turner 1994, Kawasaki *et al* 1994) that forbid ν_τ heavier than $\simeq 0.3$ MeV if the lifetime is shorter than $\simeq 100$ sec, the combination of the three bounds reject a ν_τ heavier than 0.3 MeV that decays dominantly into visible modes for any lifetime.

The MSW effect (see section 7) could also happen within a supernova. The oscillation of ν_μ or ν_τ into ν_e outside the neutrino sphere could have important consequences. The reason is that the emitted ν_μ and ν_τ are more energetic than

* Very unlikely short lifetimes $\tau \lesssim 2\times10^{-12}(m_\nu/{\rm MeV})$ would be allowed only for the mode $\nu_\tau \to \nu_s\gamma$, where ν_s is a sterile neutrino.

the emitted ν_e. This is because ν_μ and ν_τ only interact through neutral currents with the surrounding matter. This means, they have a larger mean free path than ν_e (that also interact through charged currents) and they consequently emerge from deeper and, thus, hotter layers of the supernova core. Thus, if a large fraction of the ν_τ (or ν_μ) are transformed into energetic ν_e (that, as we just said, have larger interaction rates with the surrounding matter), the increased energy given to outer layers of the star may help explode them (Fuller et al 1992). However, the same flavour conversion may preclude nucleosynthesis through r-processes in supernovae. Heavy elements are believed to be synthesized through rapid neutron capture processes (r-processes) in the neutron-rich outer layers of an exploding supernova. The more energetic electron-neutrinos, resulting from a MSW conversion of outgoing ν_μ or ν_τ before reaching the region where r-processes should occur, would reduce the amount of neutrons in the material of the crucial layers (through $\nu_e + \mathrm{n} \to \mathrm{p} + \mathrm{e}^-$). The environment would then become proton-rich and this would stop r-processes entirely. Because there is no other known production site for the elements that should be produced though r-processes in supernovae, this effect must be avoided. This excludes the region of the $\Delta m^2 - \sin^2 2\theta$, for $\nu_e - \nu_\mu$ and $\nu_e - \nu_\tau$ oscillations, corresponding to approximately $\Delta m^2 \gtrsim 4$ eV^2 for $\sin^2 2\theta \gtrsim 10^{-5}$ (Qian *et al* 1993). This is an important region, because it corresponds to a ν_τ or ν_μ with a mass suited to be hot dark matter (between a few eV and a few tens of eV, see section 5), if ν_e is much lighter.

The IMB and Kamioka cerenkov detectors have shown the feasibility of observing the neutrinos from supernova explosions, and as we just discussed their few events allowed to obtain significant information on neutrino properties. In the future, with the construction of Superkamiokande and also especially of the SNO detector sensitive to neutral currents, the detection of a supernova within our galaxy (there are ~ 2 galactic supernova explosions per century!) may allow to constrain the masses of ν_μ and ν_τ if they are larger than $\simeq 25$ eV (Minakata and Nunokawa 1990, Seckel, Steigman and Walker 1991, Krauss et al 1993, Burrows, Klein and Gandhi 1992), and maybe even down to 15 eV with new type of detectors (Cline *et al* 1994). It may also become possible to detect the background of neutrinos from all past supernovae, that should be the dominant neutrino flux at energies just beyond those of the solar neutrinos (20–30 MeV).

Finally, let us mention the under-water (ice) neutrino telescopes under construction at present with energy thresholds between a few and 10 GeV. They will observe the high-energy neutrinos that should be emitted in comparable numbers with γ rays in the most energetic galactic and extragalactic processes, such as the shock acceleration in Active Galactic Nuclei. These experiments, DUMAND, AMANDA, Baikal and NESTOR, with an area of approximately 0.02 km^2, can be considered the prototypes for the final desirable full size experiments of 1 km^2 (for a review see Halzen 1993).

10. Concluding Remarks

The hints for non-zero neutrino masses in solar and atmospheric neutrinos, and the possibility of confirming or rejecting them in the near future, make the subject of neutrino masses a particularly exciting field of research at present. So much so that one could indulge in the actually premature exploration of the consequences of the confirmation of both. One could even wonder about their compatability with other possible desirable mass values, such as masses of order 1-10 eV for neutrinos to account for part of the Dark Matter in the Universe, or Majorana masses within the reach of present neutrinoless double beta decay experiments, i.e. $< m_{\nu_e} > \simeq 1$ eV.

The MSW solution to the solar neutrino problem would require $m^2_{\nu_i} - m^2_{\nu_e} \simeq O(10^{-6})$ eV^2, while the solution to the atmospheric neutrino deficit requires $m^2_{\nu_j} - m^2_{\nu_\mu} \simeq O(10^{-2})$ eV^2. An obvious simultaneous solution for both requires $m_{\nu_\tau} \simeq 10^{-1}$ eV, $m_{\nu_\mu} \simeq 10^{-3}$ eV and $m_{\nu_e} \ll m_{\nu_\mu}$, with $\nu_i = \nu_\mu$ and $\nu_j = \nu_\tau$. These mass values can be obtained in a see-saw model with a large right-neutrino Majorana mass $M \simeq 10^{11}$ GeV (see section 3).

Incorporating either the solar neutrino solution or the atmospheric neutrino solution with either DM neutrinos or $\langle m_{\nu_e} \rangle \simeq 1$ eV is easy in see-saw models with smaller values of M (see e.g. Peltoniemi, Tommasini and Valle 1993). A solution incorporating both the solar and the atmospheric neutrino solutions and either DM neutrinos or $\langle m_{\nu_e} \rangle \simeq 1$ eV requires quasi-degenerate neutrinos, where the mass differences necessary to account for the oscillations are much smaller than the masses themselves (Caldwell and Mohapatra 1993, Peltoniemi and Valle 1993). Models of this type, possibly with extra inert neutrinos, can even incorporate all four mass requirements mentioned. In a possible solution the three active neutrinos are almost degenerate. This can be obtained if the see-saw mechanism determines the mass differences but the left-neutrino Majorana masses give the masses themselves (see section 3).

The confirmation of a see-saw mechanism is by no means the only possible outcome of neutrino mass experiments in the near future. We may uncover the Majorana nature of neutrinos in future neutrinoless double beta decay experiments if the effective ν_e mass is not much smaller than 1 eV (section 4.2). Still, we may find neutrino masses in the range forbidden by the present energy density of the universe (in direct mass searches on with a galactic supernova explosion), what would be a strong indication of the existence of Goldstone bosons (typically singlet or mostly singlet Majorons, see section 3), necessary for the neutrinos to decay or annihilate fast in the early universe (section 5). Or even neutrinoless double beta decay with the emission of a boson, that would be an even more direct indication of Goldstone bosons. Even if these results are possible, they are unlikely. We do expect instead a resolution of the solar and atmospheric neutrino problems within the next decade. These are certainly exciting times for neutrino physics.

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Figure captions

Figure 1. Bounds on unstable neutrinos that apply on any decay mode. The gray area is rejected. The continuous contour shows the bound $\Omega_{DP} \leq \Omega_o$. The dashed contour shows the structure formation bound only for experimentally allowed masses. The dot-dashed contour shows the nucleosynthesis bound on the allowed number of effective extra neutrino species δN_ν (see section 5).

Figure 2. Bounds on unstable neutrinos whose main decay products include photons or e^+e^- pairs. The gray area, excluded by cosmological limits (distortions of the CBR and other photon backgrounds), is reproduced from the figure 5.6 of Kolb and Turner 1990. The hatched area is excluded by the non observation of photons in coincidence with the observed neutrino flux from the supernova SN1987A (see section 9).

Figure 3. Present constraints on $\Delta m^2 - \sin^2 2\theta$ from reactor and accelerator searches of $\nu_e - \nu_\mu$ oscillations (a) and for $\nu_\mu - \nu_\tau$ (b). For references see the text.

Figure 4. Sensitivity of future oscillation searches together with the present bounds and the regions suggested by atmospheric neutrinos and to account for the dark matter. Figure $4a$ is for $\nu_e - \nu_\mu$ oscillations while $4b$ is for $\nu_\mu - \nu_\tau$.

Figure 5. Standard solar neutrino spectrum (Bahcall and Ulrich 1988) and experimental thresholds for the existing detectors.

Figure 6. Survival neutrino probabilities vs. $E/\Delta m^2$ for $\sin 2\theta = 0.1$.

Figure 7. MSW contours for the different experiments and preferred values of $\Delta m^2 - \sin^2 2\theta$ (shadowed). Also shown is the region excluded by the non-observation of a day-night asymmetry by Kamiokande.

Figure 8. Spectrum of e-type and μ-type atmospheric neutrinos measured by Kamiokande. The histogram is the MC prediction using the fluxes of Lee and Koh.

Figure 9. Values of $\Delta m^2 - \sin^2 2\theta$ suggested by Kamiokande measurement of $R(\mu/e)$ atmospheric ratio and its zenith angle dependence, together with the bounds from accelerators and reactors for $\nu_\mu - \nu_e$ (a) and $\nu_\mu - \nu_\tau$ (b) oscillations. Also shown are

the regions excluded by other searches with atmospheric neutrinos: Fréjus value of R, IMB-1 up/down ν_μ, and IMB stopping/through-going muons.