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EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN - SL DIVISION

CERN SL/94-85 (AP) / sue 9443

Calculation of TMC Threshold in the Presence of Beam-Beam Force

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Abstract

The effect of the residual beam-beam force on the horizontal mode coupling threshold of LEP at injection is calculated based on a two particle head-tail model and a linear beam-beam force mode. The wake fields induced by the head particles in different bunches are evaluated locally at the 120 RF copper cavities and the calculation results of the threshold for 4 x4 and 8 x 8 bunches agree quite well with experimental results. Chromatic effects and threshold currents in different beam-beam schemes are also discussed.

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Geneva, Switzerland

12 October, 1994

SL/Div Reps

1 Introduction

In LEP, the transverse head-tail instability is found to be the dominant limit to the bunch current especially at injection energy. The vertical head-tail mode coupling of m=0 and m=-1 makes the threshold current at injection energy in a single bunch to be about 0.6 mA for a Q_s=0.08 and a bunch length of 20 mm[1]. Under the same conditions, the horizontal TMC threshold can never be reached, since its value would be about 0.7 mA. However, in case of counter rotating beams, the above situation changes.

The bunch intensity obtained with counter-rotating beams in LEP is always found to be less than the single beam bunch intensity. This suggests that the reduction is mainly due to the residual beam-beam interaction at the crossing points and the wake force created at the RF cavities by the counter-rotating bunches.

Furthermore, we observed the fact from the Q-meter that the horizontal TMC can be enhanced evidently by the counter-rotating beams. The horizontal TMC threshold can even be lower than the vertical one as shown by the pictures in the Appendix. This makes it preferable to study the horizontal TMC when counter-rotating beams are presented at injection energy.

Some theoretical works and measurements on the influence of residual beam-beam force on the head-tail threshold have been published [2][3]. However, simulation is needed to give a more realistic, quantitative explanation of this mechanism. In order to get reasonable computing time when simulating this complex system of many counter-rotating bunches, a two particle model is employed for the description of the head-tail coupling. We treat the beam-beam interaction as a linear force. The impedance of LEP mainly comes from the RF cavities and the bellows. However, with a bunch length above one centimeter, the contribution from RF cavities dominates the impedance, especially in the horizontal plane, so that only the wake fields generated at the RF cavities are taken into account.

1.1 Head-tail Model

A simple 2 particle model[4] is applied to calculate transverse head-tail mode coupling. For a bunch circulating in a storage ring, two super particles with opposite synchrotron phase are used to represent the head and the tail of the bunch. The motion of the "tail" particle therefore couples with the wake force created by the "head" particle. The two coupled oscillators evolve according to:

$$\begin{aligned} x_1''(s) + K_x(s)x_1 &= \delta_p(s)g(\tau)x_2 \\ x_2''(s) + K_x(s)x_2 &= \delta_p(s)g(-\tau)x_1 \end{aligned} \tag{1}$$

$g(\tau)$ represents the wake field strength with unit of m^{-1}. $\delta_p(s)$ is the periodical δ function.

1.2 Impedance and Wake Field Model

The LEP impedance comes mainly from the RF cavities and bellows[1]. There are 120 5-cell copper cavities in LEP ring which contribute to a transverse broad-band impedance with resonance frequency of about 2 GHz. For a bunch length of about 2 cm, the impedance contribution of the bellows is small in the horizontal plane. Therefore, the impedance can be approximated by a single broad-band impedance representing the RF cavities, for a bunch length larger than 1 cm.

In the above situation, the transverse wake field produced by a particle passing through an RF cavity can be represented by a damped oscillator:

$$g(\tau) = \begin{cases} 0 & (\tau \leq 0) \\ W_0 e^{-\lambda\tau} sin(\omega_z \tau) & (\tau > 0) \end{cases} \tag{2}$$

$\tau = \sigma_s \sin(\omega_s t)$ is the longitudinal position difference between the "head" particle and the "tail" particle. $\lambda > 0$ is the damping constant of the wake field. ω_z is the frequency of the wake field, σ_s and ω_s are the bunch length and synchrotron oscillation frequency.

This kind of wake is hence short range and vanishes behind a single bunch. It should be valid for LEP since the bunch spacing is quite large even in the case of colliding 8×8 bunches.

1.3 Beam-Beam Model

During injection at 20 GeV, the counter-rotating beams are separated vertically at the 8 interaction points by the electrostatic separators in the 4×4 scheme. In the 8×8 scheme, the beams are separated horizontally by the pretzel in the additional crossing points. If the oscillation amplitude of the particles is small with respect to the separation, we can approximate the beam-beam force by a linear one. When the bunch m meets bunch n at a crossing point of longitudinal position s=s_{mn}, the horizontal beam-beam force is focusing if they are separated vertically or defocusing if they are separated horizontally, and hence can be expressed by:

$$\begin{array}{lcl} x''_{m,i}(s) + K_x(s)x_{m,i} & = & -\sum_{j=1}^{2} \frac{1}{2} K_{mn}(x_{m,i} - x_{n,j})\delta_p(s - s_{mn}) \\ x''_{n,i}(s) + K_x(s)x_{n,i} & = & -\sum_{j=1}^{2} \frac{1}{2} K_{mn}(x_{n,i} - x_{m,j})\delta_p(s - s_{mn}) \\ i = 1,2 & & \end{array} \tag{3}$$

Here $x_{m,i}$ (i=1,2) are the horizontal coordinates of the two (super) particles in the m-th bunch, and $x_{n,i}$ (i=1,2) are the horizontal coordinates of the particles in the opposite bunch. K_{mn} is the beam-beam strength at the crossing point when the bunch m meets bunch n.

2 Equation of Motion and Simulation Method

Since the RF cavities are distributed around the ring, occupying a range in which the horizontal betatron phase varies considerably, the wake force can not be approximated by one or two localized kicks. Hence the wake fields are evaluated separately in each RF cavity. The same reason exists for the beam-beam force at different crossing points. Consequently, all the wake forces and beam-beam forces are evaluated locally.

The equations of motion then become

$$
\begin{aligned}
x''_{m,1}(s) + K_x(s)x_{m,1} &= \textstyle\sum_{r=1}^{N_r} \delta_p(s - s_{mr})g(\tau_r)x_{m,2} \\
&\quad - \textstyle\sum_{n=1}^{N_b} \sum_{j=1}^{2} \delta_p(s - s_{mn})\frac{1}{2}K_{mn}(x_{m,1} - x_{n,j}) \\
x''_{m,2}(s) + K_x(s)x_{m,2} &= \textstyle\sum_{r=1}^{N_r} \delta_p(s - s_{mr})g(-\tau_r)x_{m,1} \\
&\quad - \textstyle\sum_{n=1}^{N_b} \sum_{j=1}^{2} \delta_p(s - s_{mn})\frac{1}{2}K_{mn}(x_{m,2} - x_{n,j}) \\
x''_{n,1}(s) + K_x(s)x_{n,1} &= \textstyle\sum_{r=1}^{N_r} \delta_p(s - s_{nr})g(\tau_r)x_{n,2} \\
&\quad - \textstyle\sum_{m=1}^{N_b} \sum_{j=1}^{2} \delta_p(s - s_{mn})\frac{1}{2}K_{mn}(x_{n,1} - x_{m,j}) \\
x''_{n,2}(s) + K_x(s)x_{n,2} &= \textstyle\sum_{r=1}^{N_r} \delta_p(s - s_{nr})g(-\tau_r)x_{n,1} \\
&\quad - \textstyle\sum_{m=1}^{N_b} \sum_{j=1}^{2} \delta_p(s - s_{mn})\frac{1}{2}K_{mn}(x_{n,2} - x_{m,j})
\end{aligned}
\tag{4}
$$

Here s_{mr} and s_{nr} are the positions where bunch m and bunch n passe the r-th RF cavity, and s_{mn} is the position where the m-th bunch meets with the n-th bunch. N_r is the total number of RF cavities on the ring and N_b is the total number of bunches per beam. τ_r is the longitudinal position difference of the "head" particle and the "tail" particle at the r-th RF cavity.

The coherent synchrotron oscillation of the bunch center is ignored so that the longitudinal position of each bunch in the ring is the same after each revolution. Then the transverse oscillation of the "head" and "tail" particles can be evaluated turn after turn.

A bunch circulating in the ring, passes drift spaces, magnets, RF cavities and beam-beam crossing points in the same order turn by turn. Thus, it is reasonable to consider the elements of one turn a special "transfer line" for a bunch. Different bunches have different transfer lines since they may be located at different positions in the ring at the same moment, and some of them will even circulate in the opposite direction.

All the bunches are then positioned at the beginning of their respective transfer lines, which are chosen as the respective beam-beam crossing points. The bunches as well as their "head" and "tail" particles start their motion simultaneously and will re-enter their respective transfer lines when they exit. The coordinates of all the particles at the exit of their transfer lines turn after turn are then recorded for further analysis.

We calculate the coordinate evolution in the normalized phase space:

$$
\left\{
\begin{array}{lcl}
\psi_x & = & \int_0^s \beta_x^{-1} ds \\
X(\psi_x) & = & \beta_x^{-1/2} x(s) \\
X'(\psi_x) & = & \beta_x^{-1/2}(\alpha_x x(s) + \beta_x x'(s))
\end{array}
\right.
\tag{5}
$$

For a lattice by the phase advance ψ_{x21}, the transfer matrix is then simply a rotation

$$
\left\{
\begin{array}{lcl}
X^{(2)} & = & X^{(1)} cos(\psi_{x21}) + X'^{(1)} sin(\psi_{x21}) \\
X'^{(2)} & = & -X^{(1)} sin(\psi_{x21}) + X'^{(1)} cos(\psi_{x21})
\end{array}
\right.
\tag{6}
$$

Here $X^{(1)}$ and $X'^{(1)}$ are the horizontal coordinates of a bunch at the beginning, $X^{(2)}$ and $X'^{(2)}$ at the end of the lattice.

In an RF cavity, we assume the head particle experiences no force while the tail particle gets a kick. Then for the tail particle,

$$\begin{cases} X_t^{(2)} &= X_t^{(1)} \\ X_t'^{(2)} &= X_t'^{(1)} + \beta_x^r g(\tau_r) X_h \end{cases} \tag{7}$$

Where $X_t^{(1)}$ and $X_t'^{(1)}$ are the coordinates of the tail particle before the wake kick at the RF cavity and $X_t^{(2)}$ and $X_t'^{(2)}$ are the coordinates after the wake kick. X_h is the horizontal displacement of the head particle at the RF cavity which is supposed unchanged during passing the RF cavity. β_x^r is the horizontal betatron function at the RF cavity.

At a crossing point where bunch m meets bunch n, we have for head and tail particles,

$$\begin{cases} X_{mh}^{(2)} &= X_{mh}^{(1)} \\ X_{mh}'^{(2)} &= X_{mh}'^{(1)} - \beta_x^c K_{mn}(X_{mh}^{(1)} - \frac{X_{nh}^{(1)}+X_{nt}^{(1)}}{2}) \\ X_{mt}^{(2)} &= X_{mt}^{(1)} \\ X_{mt}'^{(2)} &= X_{mt}'^{(1)} - \beta_x^c K_{mn}(X_{mt}^{(1)} - \frac{X_{nh}^{(1)}+X_{nt}^{(1)}}{2}) \\ X_{nh}^{(2)} &= X_{nh}^{(1)} \\ X_{nh}'^{(2)} &= X_{nh}'^{(1)} - \beta_x^c K_{mn}(X_{nh}^{(1)} - \frac{X_{mh}^{(1)}+X_{mt}^{(1)}}{2}) \\ X_{nt}^{(2)} &= X_{nt}^{(1)} \\ X_{nt}'^{(2)} &= X_{nt}'^{(1)} - \beta_x^c K_{mn}(X_{nt}^{(1)} - \frac{X_{mh}^{(1)}+X_{mt}^{(1)}}{2}) \end{cases} \tag{8}$$

Here, $X_{mh}^{(1)}$ and $X_{mh}'^{(1)}$ are the coordinates of the head particle in bunch m before, $X_{mh}^{(2)}$ and $X_{mh}'^{(2)}$ after crossing, $X_{mt}^{(1)}$, $X_{mt}'^{(1)}$, $X_{mt}^{(2)}$ and $X_{mt}'^{(2)}$ are the respective coordinates of the "tail" particle in bunch m. K_{mn} is the residual beam-beam force strength at the crossing and β_x^c is the betatron function at this crossing point.

Furthermore $|\beta_x^c K_{mn}| = |K|$ is imposed for each crossing point so that the residual beam-beam force at different crossing points results in the same amount of tune shift.

3 Fourier Analysis and Calibration

3.1 Fourier transformation

Given the initial coordinates of all particles in the phase space for different bunches, the phase plots will rotate with steady angular speed $\omega_\beta = Q_x \omega_0$ as long as there is no wake force or beam-beam force. Here, ω_0 is the revolution frequency. When a particle passes through an RF cavity or a crossing point, only the slope of motion is assumed to have a sudden change. As a result, we can obtain the positions of all particles varing with time t. For instance, we may get the positions of two particles in the same bunch:

$$\begin{aligned} X_1 &= X_1(t) \\ X_2 &= X_2(t) \end{aligned} \tag{9}$$

The Fourier transformation is then applied to the center of mass position of the bunch, i.e.

$$A(\omega) = \int_0^{NT_0} [\frac{X_1(t) + X_2(t)}{2}] e^{i\omega t} dt \tag{10}$$

Where N is the number of turns of the bunch and T_0 is the revolution period.

The mode spectrum strength at frequency ω is $|A(\omega)|$. It is found that the spectrum position and relative strength indicated by $|A(\omega)|$ will be independent from initial coordinates if N is large enough, for instance, N=3000.

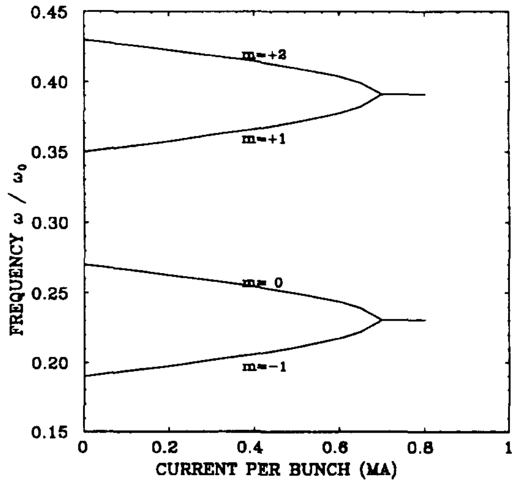

Figure 1(a) Calibration: I_{th}=0.7mA for Single Bunch Horizontal Mode Coupling

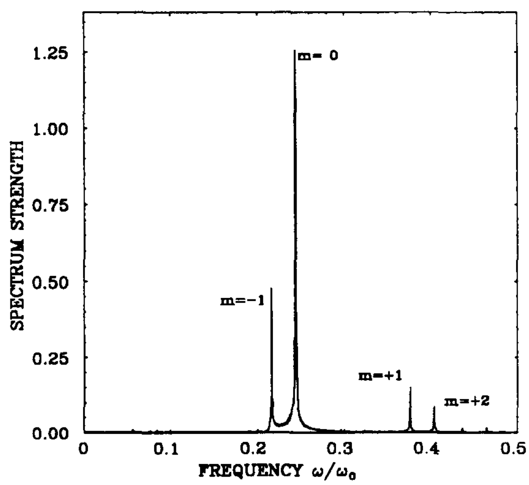

Figure 1(b) Single Bunch at 0.6mA

3.2 Calibration

The injection lattice parameters of LEP are used for the calculation. Some main parameters are: Q_x=90.27, Q_s=0.08, $\lambda=1.2\times10^{10}s^{-1}$ and f_z=2.0 GHz.

The parameter W_0 in the wake function $g(\tau)$ is adjusted so that the horizontal TMC threshold current is 0.7 mA for a single bunch. The beam-beam force strength is also adjusted such that it leads to the same beam-beam tune shift of 0.0015 per crossing for a bunch current of 0.45 mA, which corresponds to the measured coherent tune shifts. We thus have,

$$\begin{aligned} W_0 &= 7.57129 \times 10^{-4} I \\ K &= \begin{cases} 2.22222 \times 10^{-2} I & \textit{vertical separation} \\ -2.22222 \times 10^{-2} I & \textit{horizontal separation} \end{cases} \end{aligned} \tag{11}$$

Where K is dimensionless, W_0 and I are with units of m^{-1} and mA respectively.

Figure 1(a) shows the effect of calibration to the threshold current of TMC between m=0 and m=-1 modes for a single bunch. Figure 1(b) is the Fourier analysed spectrum of the center of mass of a single bunch oscillation at the current of 0.6 mA. Figure 2(a) shows the dependence of the residual beam-beam tune shift on bunch current in the absence of wake force for a single interaction point. It is linear since the beam-beam force is linear. Figure 2(b) shows the σ and π modes for the 1 × 1 bunches for a bunch current of 0.7 mA.

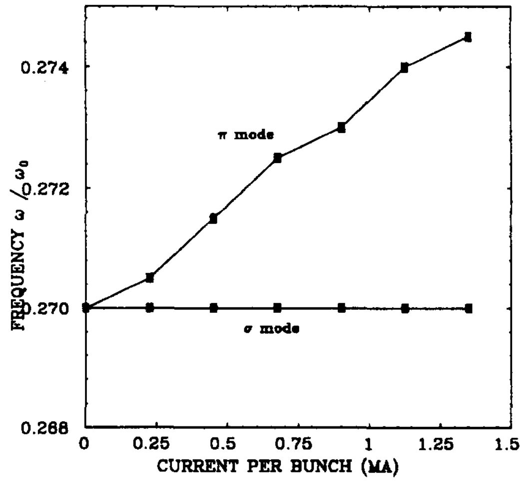

Figure 2(a) Beam–beam Force Calibration With Single Interaction Point

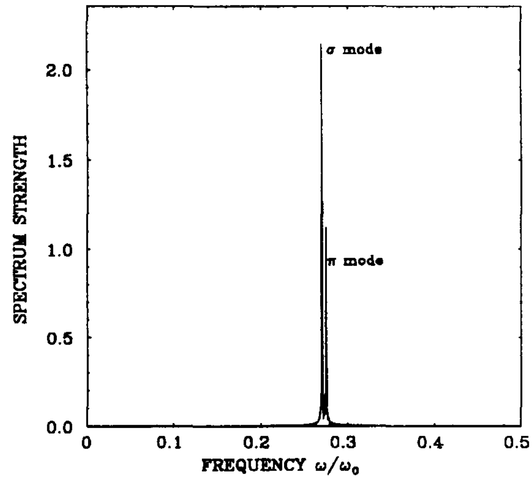

Figure 2(b) Spectrum of 1 x 1 bunch at 0.7 mA/bunch without Wake Force

4 Coherent Mode Analysis

4.1 Mode Classification

Since we are interested in coherent effects, we need only take into account the center of mass oscillations of the bunches in simulation. However, because there are many electron bunches, and an equal number of counter-rotating positron bunches, which couple through beam-beam forces at the crossing points and the wake fields at the RF cavities, there will be also many oscillation modes. These modes can be classified in three kinds according to the strength of the beam-beam force which they experience.

The first kind is the "σ mode" which represents the motion of the center of mass of all electron bunches added to that of the center of mass of all the positron bunches. This mode experiences no beam-beam force and is referred as the m=0 mode.

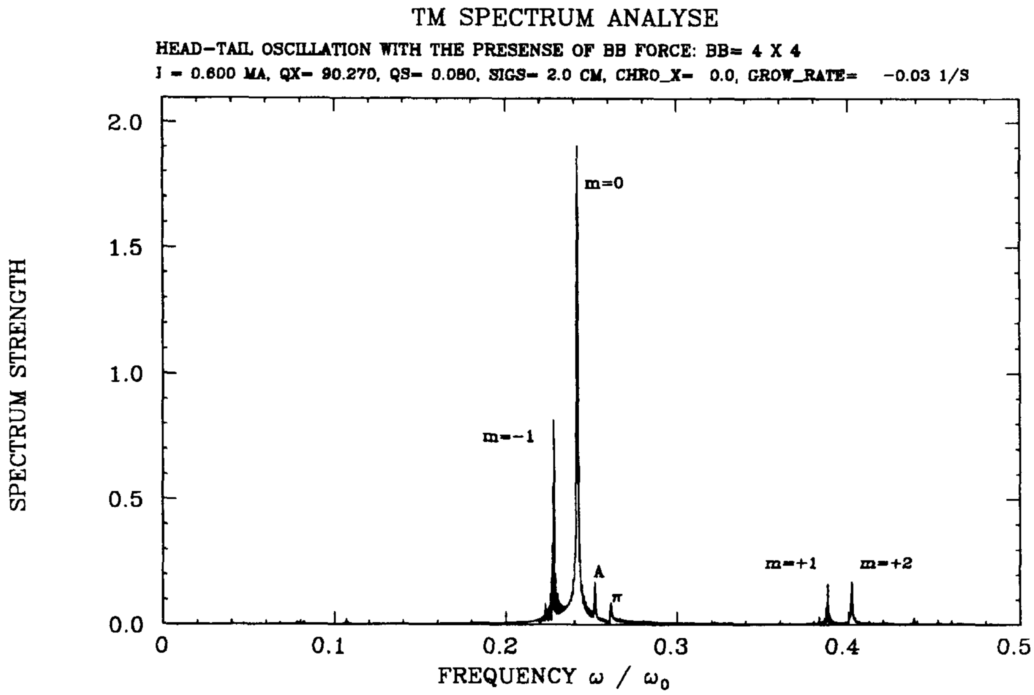

Figure 3 Mode Classification

The second kind is the "π mode" which represents the motion of the center of mass of all electron bunches minus that of the center of mass of all the positron bunches. This mode experiences the strongest beam-beam force.

The third kind of modes are intermediate modes with frequency between the σ and the π mode. These intermediate modes experience less beam-beam force than the π mode.

These three kinds of modes can be clearly found by simulation. Figure 3 shows the mode spectrum of the first electron bunch in the case of 4×4 bunches , with current of 0.6 mA for each bunch. We can see clearly in this figure the σ mode(m=0) and their three synchrotron sidebands m=-1 mode, m=+1 mode and m=+2 mode. The π mode and one of the intermediate modes (peak A) can also be found. Other intermediate modes are, however, too week to be seen.

4.2 Effect of Wake force on Beam-beam Tune Shift

The beam-beam tune shift, (for simplicity, defined as the shift of the π mode away from the σ mode), will no longer depend linearly on the bunch current when the wake forces are taken into account. The tune shift dependence on bunch current becomes quite nonlinear especially near the mode coupling threshold. Fortunately, when the bunch current is further away from the threshold, the tune shift dependence on the bunch current is rather linear, which makes the beam-beam force calibration (mentioned in Figure 2(a)) effective because for single bunch, 0.45 mA is far away from the threshold near 0.7mA.

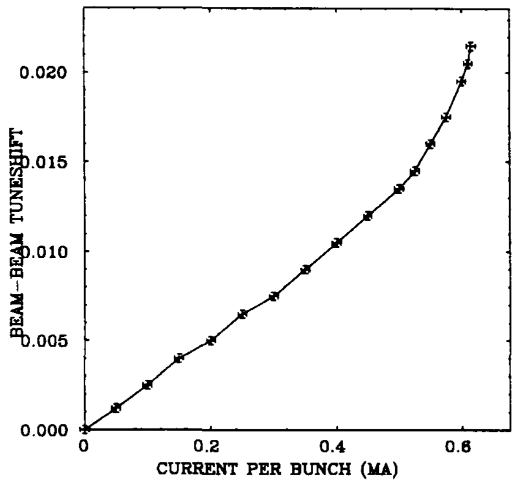

Figure 4 Calculated B–B Tuneshift of 4 X 4 Bunches

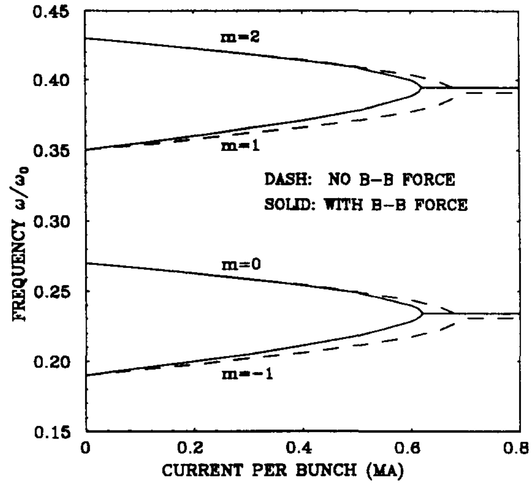

Figure 5 Additional Shift of m=−1 And m=1 Mode Due to B–B force with 4 X 4 Bunches

Figure 4 shows the nonlinear effect of wake force on the tune shift in the case of 4 × 4 bunches. It is evident that the π mode shifts quickly away from the σ mode when the threshold is approaching. This indicates that the beam-beam interaction is dramatically enhanced by the wake force near mode coupling.

4.3 Effect of Beam-beam Force on the m=±1 Mode

As mentioned before, the m=0 mode (or σ mode) is not affected by the beam-beam force. However, the beam-beam force does enhance the shift of the m=±1 mode for instance, as described by an earlier paper[3].

This effect is found to be nonlinear even when the bunch current is quite away from the threshold. Figure 5 compares the mode evolutions in the case of 4 × 4 bunches with and without the residual beam-beam force. It is quite obvious that the residual beam-beam force reduces the mode coupling threshold.

4.4 Synchrotron Sidebands of π Mode

Simulation of multi-bunch evolution in the presence of residual beam-beam forces shows that the intermediate modes and the π mode split from the σ mode. It is more surprising to find that also some modes split from m=±1 and m=2 modes. Since m=±1 and m=2 modes are the synchrotron sidebands of the m=0 mode, it is assumed that these modes are the

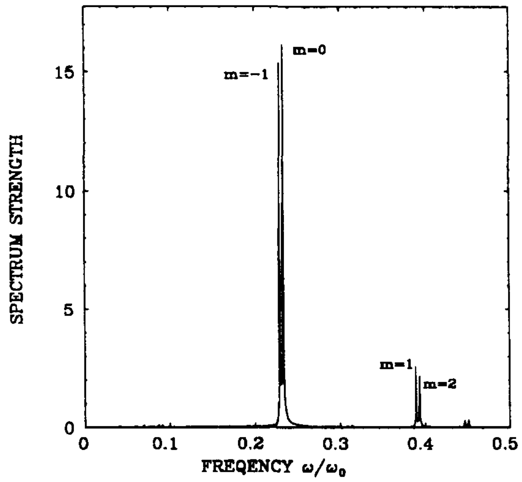

Figure 6(a) Spectrum of $X_{ce}+X_{cp}$ for 1 X 1 Bunch with Focusing B-B Force at 0.675 mA per Bunch

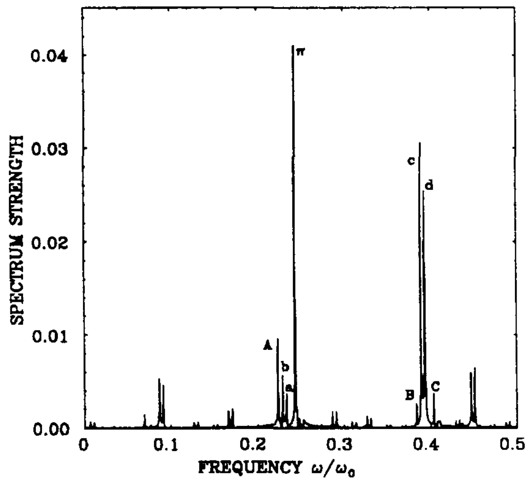

Figure 6(b) Spectrum of $X_{ce}-X_{cp}$ for 1 X 1 Bunch with Focusing B-B Force at 0.675 mA per Bunch

synchrotron sidebands of the correspondent intermediate modes and the π mode.

Because the π mode represents the strongest beam-beam force, special attention should be paid to it and its synchrotron sidebands. The mode coupling between the π mode and one of its synchrotron sidebands may also lead to bunch instability under certain circumstances as shown below.

Take the simplest 1×1 bunch case as an example, where no intermediate mode exists. This make it easier to distinguish the π mode and its synchrotron bands from other modes.

First, the two bunches are assumed to meet at the IP2 and IP6 of LEP where they are separated vertically. The residual beam-beam force is then focusing in the horizontal plane and will shift the π mode to a higher frequency than that of σ mode. The spectra of $X_{ce}+X_{cp}$ and X_{ce}-X_{cp} for bunch current of 0.675 mA (near threshold) are shown in Figures 6(a) and 6(b), respectively, with X_{ce} and X_{cp} correspond to the center of electron bunch and positron bunch. The $X_{ce}+X_{cp}$ motion cancels the beam-beam effect and we see m=0 mode and its three synchrotron sidebands. The X_{ce}-X_{cp} motion is affected most strongly by beam-beam force, and we see the π mode and its three synchrotron sidebands with peaks A(ω_π-ω_s), B($\omega_\pi+\omega_s$) and C($\omega_\pi+2\omega_s$). The m=0, m=-1, m=+1 and m=+2 mode are also coupled to this motion by the wake force. They correspond to peaks a,b,c and d. Other spectrum peaks are not well understood.

It is evident that the TMC instability results from the coupling of m=0 and m=-1 modes. The approach of these two modes leads to a threshold of 0.677 mA, which is 3.3% lower than

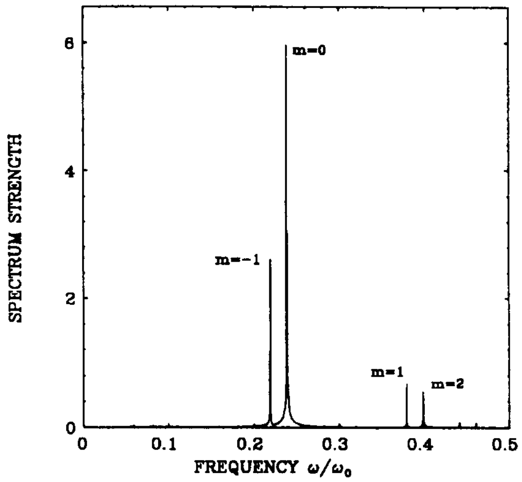

Figure 7(a) Spectrum of X_{ce}+X_{cp} for 1 X 1 Bunch with Defocusing B–B Force at 0.673 mA per Bunch

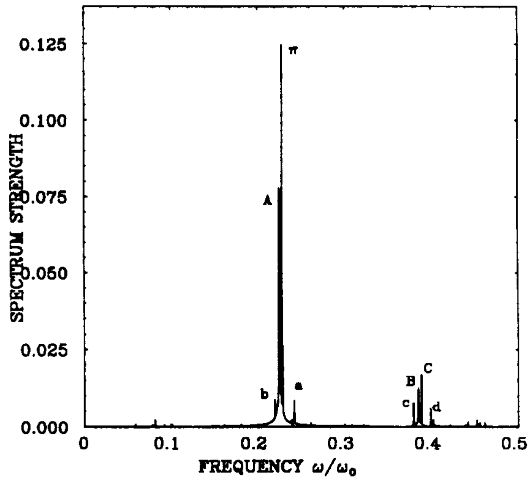

Figure 7(b) Spectrum of X_{ce}−X_{cp} for 1 X 1 buch with Defocusing B–B Force at 0.673 mA per Bunch

the single bunch threshold(0.7mA).

Now, we assume the two bunches cross at two pretzel separation crossing points. Since the two bunches are separated horizontally, the residual beam-beam force is then defocusing in the horizontal plane and will push the π mode to a lower frequency than that of the σ mode. The spectra of X_{ce}+X_{cp} and X_{ce}-X_{cp} with bunch currents of 0.673 mA (near threshold) are shown in Figures 7(a) and 7(b), respectively, with all peak labels the same as in Figures 6(a) and 6(b). The m=0 mode and its synchrotron sidebands are seen well in Figure 7(a) and no coupling between them seems to happen because they are quite far apart. However, in Figure 7(b) we find that coupling between the π mode and its synchrotron sidebands (ω_π-ω_s) is about to happen. It is found that the coupling between the π mode and its synchrotron sideband (ω_π-ω_s) which leads to the instability of the bunch happens at a threshold current of 0.674 mA, slightly lower than in the case of vertical separation discussed above.

It is worth to note that if the threshold comes from the vertical TMC, and if the beams are separated vertically, the threshold will be given by the coupling between the π mode and its synchrotron sideband ω_π-ω_s (instead of the coupling between m=0 and m=-1 mode as for horizontal TMC).

5 Threshold Currents Calculation

The threshold currents for different cases (1×1 bunch, 2×2 bunches etc) are obtained by comparing the growth rate of the center of mass oscillation of a bunch to the transverse damping rate which is about 3.75 s^{-1} (damping time corresponds to about 3000 turns with wigglers on) at 20 Gev. If the growth rate is larger than the damping rate, the bunch is expected to be unstable. The calculation of growth rates for different bunches shows they have the same growth rate near threshold. This is because all the bunches have equal intensity and their motions couple to each other. Practically, only the growth rate of the first electron bunch is calculated to judge whether the bunches are stable or unstable.

The normal separation scheme is used, i.e., vertically separated at all the 8 IPs of LEP and horizontally separated by the pretzel orbits at all other crossing points.

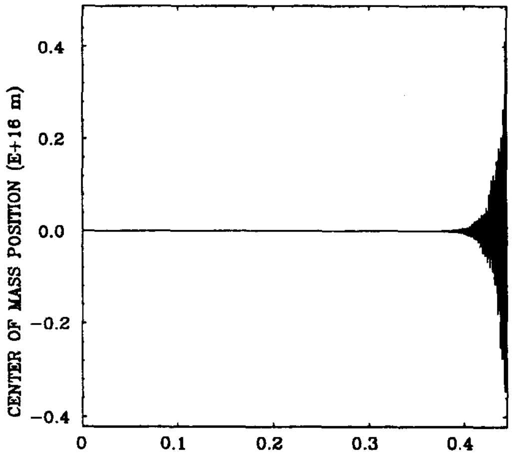

Figure 8(a) Center of Mass Oscillation Of Bunch 1 in the Case of 4 X 4 Bunches at 0.627 mA/bunch (Threshold + 0.001 mA)

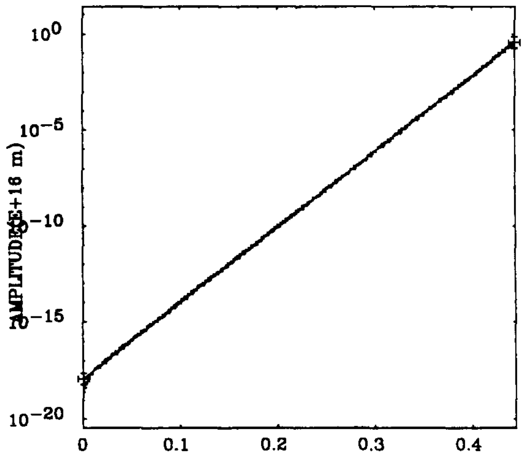

Figure 8(b) Growth Rate Fitting in Case of 4 X 4 Bunches with 0.627 mA per Bunch.
Growth Rate Fitted is 90.73 1/s

5.1 Growth Rate Fitting

The horizontal center of mass positions are recorded turn by turn. The maximum values are then taken out and used to fit the growth rate. This gives total growth rate of the center of mass oscillation instead of that of a single mode. However, when mode coupling happens and leads to instability, it corresponds to the growth rate of the coupled modes because they dominate the bunch oscillations.

Figure 8(a) and Figure 8(b) show the center of mass oscillation of the first electron

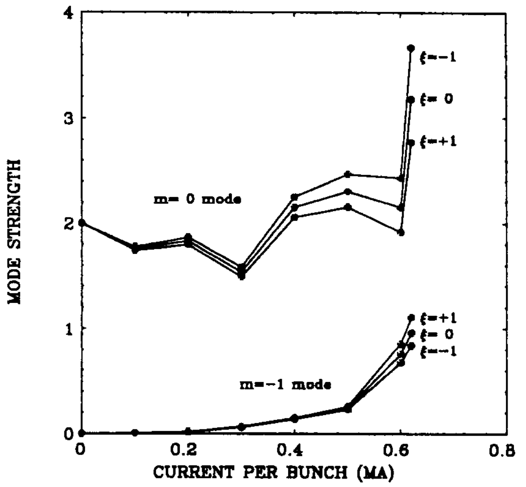

Figure 9(a) Chromatic Effect to Different Modes, m=0 Mode Is Damped By Positive Chromaticity and Antidamped By Negative Chromaticity

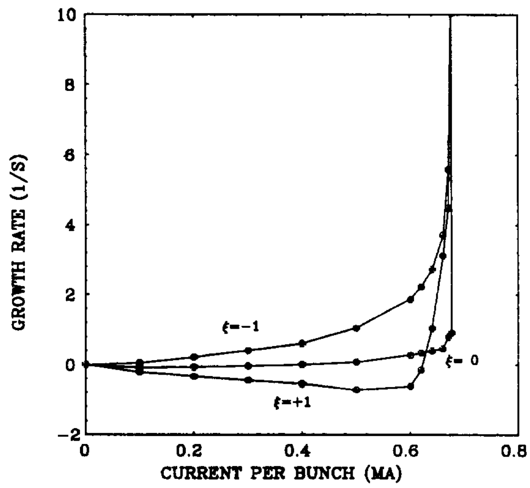

Figure 9(b) Chromatic Effect On Total Growth Rate. Positive Chromaticity Damps Dipole Motion Before Mode Coupling Happens

bunch and its growth rate for the case of 4 × 4 bunches with currents 0.627 mA/bunch, which exceeds the threshold by 0.001 mA.

5.2 Chromatic Effect

According to [4], chromatic effects modulate the phase rather than the amplitude of free betatron oscillations. We denote $\omega(\delta) = \omega_\beta + \omega_0\xi\delta$ as the modulated betatron frequency with $\delta=\Delta E/E_0$ the energy deviation, ω_0 the revolution frequency, ω_β the unperturbated betatron frequency and ξ the chromaticity. By applying δ=-$(d\tau/dt)/\alpha_p$, where α_p is the momentum compaction factor, we have the modulated betatron phase,

$$\begin{aligned} \psi_x(t) &= \int \omega(\delta)dt \\ &= \omega_\beta t + \omega_0\xi \int \delta dt \\ &= \psi_{x0}(t) - \tfrac{\xi\omega_0}{\alpha_p}\tau(t) \end{aligned} \tag{12}$$

Where ψ_{x0} is the unmodulated betatron phase. Since $\tau = \sigma_s \sin(\omega_s t)$ has been applied to the head-tail wake force calculation, it is again applied here to calculate the betatron phase modulation.

Figure 9(a) compares the chromatic effects on the modes strength of the m=0 and m=-1 modes for the 1 ×1 bunch case. It is evident that a positive chromaticity damps the m=0 mode and antidamps the m=-1 mode. A negative chromaticity damps the m=-1 mode but antidamps the m=0 mode. It is also found that a positive chromaticity damps the m=2 mode and a negative chromaticity antidamps the m=2 mode. Figure 9(b) shows the chromatic effect on the growth rate of the dipole motion. We can see the negative chromaticity leads to a slow blow up while the positive chromaticity provides a little damping effect on the dipole motion before mode coupling happens. Once the mode coupling happens, the growth rates join together because the mode coupling instability is so strong that the excitation or damping effect caused by chromaticity is submerged.

5.3 Threshold Currents

The calculated threshold currents for different cases and some experimental results[3] are summarized in table 1, where Δ = (I_{th}-0.7)/0.7 is the reduction factor calculated from the single bunch threshold, Δ_{exp} is the reduction factor measured in experiment.

Table 1. Threshold Currents For Different Cases

	ξ=-1		ξ= 0		ξ= 1		
Case	I_{th}(mA)	Δ	I_{th}(mA)	Δ	I_{th}(mA)	Δ	Δ_{exp}
1×1	.660	5.7%	.677	3.3%	.668	4.6%	
2×2	.655	6.4%	.668	4.5%	.662	5.4%	
4×4	.615	12.1%	.626	10.6%	.619	11.6 %	12%
8×8	.550	21.4%	.560	20.0%	.555	20.7%	(20-25)%

It can be seen that the calculated threshold currents agree quite well with the experiment results.

6 Pretzel Scheme Investigation

Though LEP has pretzel scheme only in the case of 8×8 bunches, the program can investigate the pretzel scheme of 1×1, 2×2 and 4×4 bunches. The calculated results are listed in Table 2. Schemes H, V and A correspond to that of all crossing points separated horizontally, vertically and alternatively. The actual schemes are scheme A for 8 × 8 bunches and scheme V for the other cases. The results are for zero chromaticity. Non-zero chromaticity reduces all these thresholds.

Table 2. Threshold Currents of Pretzel and Non-pretzel Schemes

case	1×1			2×2			4×4			8×8		
scheme	V	A	H	V	A	H	V	A	H	V	A	H
I_{th}(mA)	.677	.700	.674	.668	.654	.653	.626	.617	.614	.582	.560	.559

The results indicate almost the same threshold for pretzel scheme (scheme A) and the scheme where all the beam-beam forces are defocusing (scheme H) except for the 1 × 1 bunch case, where beam-beam force cancels out in the pretzel scheme. The scheme with all beam-beam forces focusing (scheme V) always yields higher threshold values.

7 Summary

The TMC threshold values are calculated by using a two particle head-tail model and a linear beam-beam force model. The head-tail TMC instability is enhanced in the presence of the beam-beam force. If the wake strength W_0 is adjusted so that the threshold current for a single bunch is 0.7 mA, the threshold currents (with the zero chromaticity), will be reduced by 10.6% and 20.0% for the cases of 4×4 and 8×8 bunches. This agrees quite well with the experimental results.

The effect of chromaticity has also been investigated. A positive chromaticity provides some damping on the dipole motion before mode coupling happens, while a negative chromaticity leads to a slow blow-up of the dipole motion before mode coupling happens. For positive chromaticity the m=-1 mode is unstable before TMC is reached. The investigation of different beam-beam force schemes shows that the pretzel scheme has a lower threshold than schemes where the beam-beam force is focusing at all crossing points.

Ackowledgement

The authors would like to thank Bruno Zotter for his constant advice, discussions and friendship. One of the authors (Li) would like to thank all the members of AP group, particularly Jacques Gareyte, for their kind hospitality and help.

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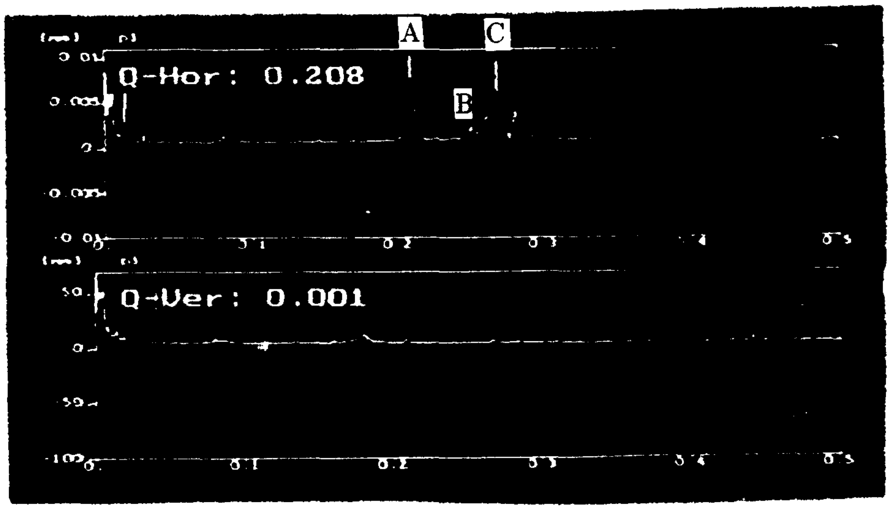
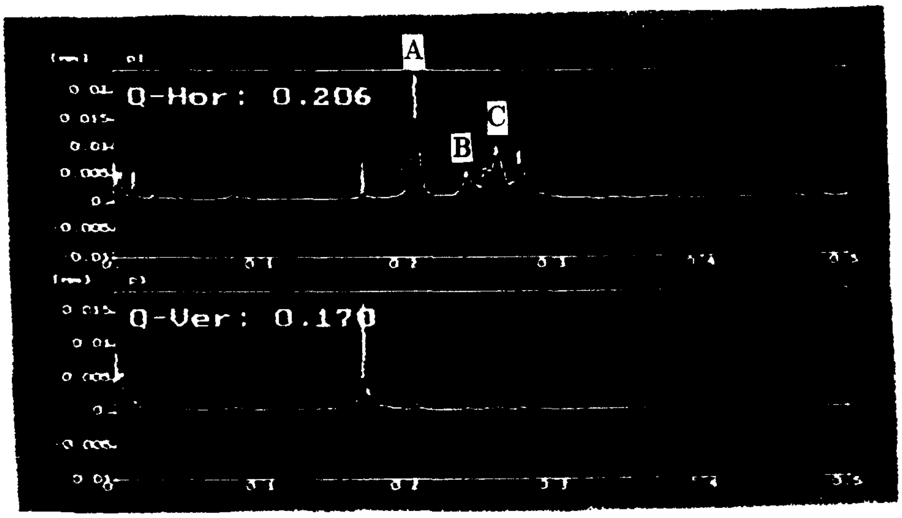
Appendix

Mode Spectra Observation

Here we show the spectra of 8 × 8 bunches at injection energy of LEP. The first and the second spectrum are taken at 0.280 and 0.330 mA/bunch, respectively. Here A is the m=-1 mode, B is a shifted π-mode (coming from the pretzel crossing) and C is the m=0 mode. Already for 0.330 mA/bunch, the distance between A and B is only 0.025 which is smaller than the distance between the vertical m=-1 and m=0 mode of that current(0.035).



a)Spectrum at 0.280 mA



b)Spectrum at 0.330 mA