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ON A POSSIBLE SOLUTION FOR THE POLONYI PROBLEM IN STRING COSMOLOGY.†

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ABSTRACT

We establish the main features of homogeneous and isotropic dilaton, metric and Yang-Mills configurations in a cosmological framework. We identify a new energy exchange term between the dilaton and the Yang-Mills field which may lead to a possible solution of the Polonyi problem in 4-dimensional string models.

String theory is the best candidate advanced so far to make General Relativity compatible with quantum mechanics and unify all the fundamental interactions of nature. However, this unification takes place at very high energy, presumably at the Planck scale, and it is, therefore, particularly relevant to study the salient features of this theory in a cosmological context, hoping to be able to observe some of its implications [1]. Four-dimensional string vacua emerging, for instance, from heterotic string theories, correspond to N=1 non-minimal supergravity and super Yang-Mills models. The four-dimensional low-energy bosonic action arising from string theory is, at lowest order in α', the string expansion parameter, given by

$$S_B = \int d^4x\sqrt{-g}\left\{-\frac{R}{2k^2} + 2(\partial\phi)^2 - e^{-2k\phi}Tr\left(F_{\mu\nu}F^{\mu\nu}\right) + 4V(\phi)\right\}, \tag{1}$$

where $k^2 = 8\pi M_P^{-2}$, M_P being the Planck mass and we allow for a dilaton potential, $V(\phi)$. The field strength $F^a_{\mu\nu}$ corresponds to the one of a Yang-Mills theory with a gauge group G, which is a subgroup of $E_8 \times E_8$ or Spin$(32)/Z_2$. We have set the antisymmetric tensor field $H_{\mu\nu\lambda}$ to zero and dropped the $F^a_{\mu\nu}\tilde{F}^{\mu\nu a}$ term.

As we are interested in a cosmological setting, we shall focus on homogeneous and isotropic field configurations on a spatially flat spacetime. The most general metric is then given by

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega_3^2, \tag{2}$$

where N(t) and a(t) are respectively the lapse function and the scale factor.

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As for the gauge field, we consider for simplicity the gauge group G=SO(3); most of our conclusions, however, will be independent of this choice. An homogeneous and isotropic ansatz, up to a gauge transformation, for the gauge potential is the following [2,3]

$$A_\mu(t)dx^\mu = \sum_{a,b,c=1}^{3} \frac{\chi_0(t)}{4} T_{ab}\epsilon_{acb}dx^c, \tag{3}$$

$\chi_0(t)$ being an arbitrary function of time and T_{ab} the generators of SO(3).

Introducing the ansätze (2) and (3) into the action (1) leads, after integrating over R^3 and dividing by the infinite volume of its orbits, to the effective action from which our considerations about the Polonyi problem will follow

$$S_{eff} = -\int_{t_1}^{t_2} dt \left\{ -\frac{3\dot{a}^2 a}{k^2 N} + \frac{3a}{N} e^{-2k\phi} \left[\frac{\dot{\chi}_0^2}{2} - \frac{N^2}{a^2}\frac{\chi_0^4}{8} \right] + \frac{2a^3}{N}\dot{\phi}^2 - 4a^3 N V(\phi) \right\}, \tag{4}$$

where the dots denote time derivatives.

Let us consider the so-called entropy crisis and Polonyi problems associated with the Einstein-Yang-Mills-Dilaton (EYMD) system. The former difficulty concerns the dilution of the baryon asymmetry generated prior to ϕ conversion into radiation. The entropy crisis problem can be solved by regenerating the baryon asymmetry after ϕ decay, as discussed in Ref. 4, considering models in which the Affleck-Dine mechanism can be implemented to generate an $\mathcal{O}(1)$ baryon asymmetry and then allow for its dilution via ϕ decay.

In models where the dilaton mass is very small, such that its lifetime is greater than the age of the Universe ($\Gamma_\phi^{-1} \geq t_U \approx 10^{60}\ M_P^{-1}$), one may encounter the Polonyi problem, i.e. ρ_ϕ dominates the energy density of the Universe at present. This problem exists in various N=1 supergravity models with one or more chiral superfields and even non-minimal models as well as in string models. A necessary requirement to avoid the problem is that, at the time when ϕ becomes non-relativistic, i.e. $H(t_{NR}) = m$, the ratio of its energy density to the one of radiation satisfies [5]

$$\epsilon = \frac{\rho_\phi(t_{NR})}{\rho_{\chi_0}(t_{NR})} \lesssim 10^{-8}. \tag{5}$$

Notice that, since the condition $\Gamma_\phi^{-1} \geq t_U$ implies $m \leq 10^{-20} M_P$, which falls outside the mass interval for which inflation takes place (see below), we have to assume that, in models where this problem occurs, some field other than the dilaton will drive inflation and be responsible for reheating. Initially and until ϕ becomes non-relativistic, $\rho_\phi \simeq \rho_{\chi_0} \simeq \frac{1}{2}m^2\phi_*^2$, implying that $\epsilon = \mathcal{O}(1)$ (see e.g. Ref. 5). Hence, any mechanism for draining ϕ energy into radiation has to be quite effective in order to be able to help to avoid the Polonyi problem. The mechanism we propose is related to the fact that our construction does allow us to describe radiation through the field χ_o rather than treating it as a macroscopic fluid, a fact which has an immediate bearing on the issue of energy exchange between the dilaton and the Yang-Mills field. In fact, it is then easy to show, working out the equations of motion resulting from the variation of the effective action (4), that

$$\dot{\rho}_\phi = -3H\dot{\phi}^2 - \frac{1}{2}ke^{-2k\phi}\zeta_{\chi_o}\dot{\phi}, \tag{6}$$

$$\dot{\rho}_{\chi_0} = -4H\rho_{\chi_0} + 6k\frac{\dot{\chi}_0^2}{a^2}\dot{\phi}. \tag{7}$$

where $H = \dot{a}/a$ and $\rho_{\chi_o} = 3\left[\frac{\dot{\chi}_0^2}{2a^2} + \frac{\chi_0^4}{8a^4}\right], \zeta_{\chi_0} = 3\left[\frac{\dot{\chi}_0^2}{2a^2} - \frac{\chi_0^4}{8a^4}\right]$ and $\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$. The new and somewhat surprising feature of the above equations is the appearance of terms proportional to $\dot{\phi}$. The origin of these terms is ultimately related with the coupling of the dilaton to the kinetic energy terms of other fields. Let us then estimate the efficiency of the terms proportional to $\dot{\phi}$ in Eqs. (6) and (7). We get for ϵ

$$\epsilon \simeq \frac{1 - 2\Delta/m^2\phi_*^2}{1 + 2\Delta'/m^2\phi_*^2}, \tag{8}$$

where $\phi_* \simeq \phi(t_{RN}) \approx M_P$ and Δ, Δ' are the integrated contributions of the last two terms of Eqs. (6) and (7), respectively, over the time interval $(t_i,\ t_{NR})$. One expects that $\Delta \simeq \Delta'$. Demanding ϵ to satisfy condition (5) implies that the ratio $\alpha \equiv \frac{2\Delta}{m^2\phi_*^2}$ has to be fairly close to 1. From the equations of motion resulting from the variation of action (4), it is easy to see that effective energy exchange occurs during the period where $H \simeq 2k\dot{\phi}$ [1]. Assuming that this relation holds after inflation and, furthermore, that $\zeta_{\chi_0} \sim a^{-4}$ and $a(t) = a_R\left(\frac{t}{t_R}\right)^{1/2}$, we obtain

$$\Delta \simeq \frac{t_R^2}{a_R^4}(t_i^{-2} - t_{NR}^{-2}), \tag{9}$$

where the index R refers to the time when the inflaton decays. Hence, in order to get $\alpha = \mathcal{O}(1)$, we must have, if $t_{NR} \gg t_i$

$$t_i \simeq \frac{1}{mM_P}\frac{t_R}{a_R^2}. \tag{10}$$

For typical values of the relevant parameters, e.g. $t_i \simeq 10^{10}M_P^{-1}$, $t_R \gtrsim 10^{30}M_P^{-1}$ and $a_R \gtrsim 10^{30}M_P^{-1}$, we see that the dilaton mass is required to be exceedingly small, $m \leq 10^{-40}M_P$. Since solving the Polonyi problem requires α to be very close to 1, it is clear, from our estimate, that this can be achieved provided the energy exchange is effectively maintained over a sufficiently long time. Actually, energy exchange via terms proportional to $\dot{\phi}$ occurs also when coupling the dilaton to bosons and fermions through $e^{-2k\phi}\mathcal{L}_{matter}$ (see e.g. Ref. 6), which will then contribute to further draining of ϕ energy. Other contributions to this process would occur if we had chosen a larger gauge group as, besides $\chi_0(t)$, another multiplet of fields would appear in the effective action [2,3] leading to extra energy exchange terms.

Notice that, when discussing a very light or massless dilaton, one has to deal with the implications of the fact that coupling constants and masses are dilaton dependent and the ensued problems, such as cosmological variation of the fine structure constant as well as other coupling constants and violations of the weak equivalence principle. The study of the cosmological evolution of the Einstein-Matter-Dilaton system indicates that the inclusion of non-perturbative string loop effects is crucial to render consistency with the experimental data if the dilaton is massless [7]. The impact of the string loop effects in the EYMD system is to change the Yang-Mills-Dilaton coupling to $B(\phi)F^a_{\mu\nu}F^{\mu\nu a}$, where $B(\phi) = e^{-2k\phi} + c_0 + c_1 e^{2k\phi} + ...$ and $c_0,\ c_1, ...$ are constants. Hence, in what concerns the Polonyi problem we have been discussing, the extra terms may either weaken or reinforce our previous conclusions regarding a possible solution to this problem, depending on the value and more crucially the sign of the constants c_i.

Finally, we briefly describe our results concerning the inflationary solutions of the model. As shown in Ref. 6, where radiation is treated as a fluid, one obtains chaotic inflationary solutions driven by the dilaton for $V(\phi) = \frac{1}{2}m^2(\phi - \phi_0)^2$, with $10^{-8}M_P < m < 10^{-6}M_P$ and $\phi_0 \sim M_P$, a result which remains valid if we add a quartic term to the potential. Although the dilaton potential, which has its origin is non-perturbative effects such as gaugino condensation and a possible v.e.v. for the antisymmetric tensor field, has a more complicated structure, it is reassuring to see that it is possible to obtain inflationary solutions in simple cases. We have checked that, with our field treatment of radiation, inflationary solutions do exist and inflation with more than 65 e-foldings requires that the initial value of the ϕ field is such that $\phi_i \gtrsim 4M_P$ [1,3,6] and, actually, these correspond to most of the trajectories, with a probability $1 - (m/M_P)^2$. Inflation is therefore a fairly general feature of expanding models with $V(\phi) = \frac{1}{2}m^2(\phi - \phi_0)^2$, for $\phi - \phi_0 > 0$ and where the initial value of ϕ satisfies the condition $\phi_i \gtrsim 4M_P$. In fact, this initial condition is indeed shown to be favoured, as follows from the study of the solutions of the Wheeler-DeWitt equation for the EYMD system in the minisuperspace approximation[8].

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