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CERN-TH.7422/94

CALCULATING F_2^p AT SMALL x AND LARGE Q^2

Richard D. Ball[†] and Stefano Forte[‡]

Theory Division, CERN,
CH-1211 Genève 23, Switzerland.

arXiv:hep-ph/9409374v1 21 Sep 1994

Abstract

We show that the double asymptotic scaling of the HERA structure function data is consistent with pre-HERA data at larger x, soft pomeron behaviour at small x and a sensible starting scale Q_0. We can thus actually calculate F_2^p at small x and large Q^2 by evolving up perturbatively at two loops, without any fitting.

Talk presented at "QCD94", Montpellier, July 1994,
to be published in the proceedings (Nucl. Phys. B (Proc. Suppl.))

CERN-TH.7422/94
September 1994

[†] On leave from a Royal Society University Research Fellowship.

[‡] On leave from INFN, Sezione di Torino, Italy.

Perturbative QCD makes a definite prediction [1] for the shape of the rise of the singlet part of the structure function F_2 at large Q^2 and small x, provided only that the starting distribution is not too singular. This prediction is most striking when expressed as a double asymptotic scaling in the two scaling variables [2]

$$\sigma \equiv \sqrt{\ln \tfrac{x_0}{x} \ln \tfrac{t}{t_0}}, \qquad \rho \equiv \sqrt{\ln \tfrac{x_0}{x} / \ln \tfrac{t}{t_0}}, \tag{1}$$

where $t \equiv \ln(Q^2/\Lambda^2)$. When F_2 is rescaled by a multiplicative factor

$$R_F \equiv N\sigma^{1/2}\rho e^{-2\gamma\sigma+\delta\sigma/\rho}, \tag{2}$$

it should scale in both σ and ρ as they become large, since the growth predicted in [1] has been scaled out. The parameter γ which controls the rate of growth of F_2 is simply related to the leading coefficient of the beta function (in fact $\gamma \equiv \sqrt{12/\beta_0}$), while δ is an anomalous dimension. Recent data from HERA [3] are in remarkably good agreement with this double scaling prediction, to the extent that it may be used to directly determine β_0 by measuring the slope of the rise[4]. It thus now becomes imperative to compute higher order corrections: double scaling violations.

The original calculation in [1] expanded the one loop anomalous dimensions around their leading singularity, which should dominate the evolution at large Q^2 and small x if there is no corresponding singularity in the input distribution. In [2] we showed that this approximation turns the Altarelli-Parisi equation for the gluon distribution $xg(x;t)$ into a two-dimensional wave equation with light-cone variables $\ln(x_0/x)$ and $\ln(t/t_0)$. The (unstable) propagation of soft boundary conditions (in particular $g(x;t_0) \sim x^{-1}$ as $x \to 0$, the intercept of the soft pomeron being close to unity) into the interior of the light-cone then produces the generic rise in F_2. Asymptotically the details of the boundary conditions are washed out, and only their overall normalization is left. The simplest approximation is thus sufficient to capture the essential physics at small x: F_2 rises because of the instabilty of gluons to gluon emission via the triple-gluon vertex.

Of course the success of this simple picture still leaves some important questions unanswered. In particular, is the double asymptotic scaling behaviour consistent with the structure functions measured (pre-HERA) at larger values of x? Furthermore, how good is the leading singularity approximation to the splitting function, and how large are the two loop corrections? In other words, it would be useful to compute the normalization and sub-asymptotic corrections to double scaling, not only to explain why double scaling works

so well, but also to refine the comparison with the increasingly precise experimental data. Post-asymptotic corrections due to singularities in yet higher orders of perturbation theory, or to higher twist corrections from parton recombination effects, are discussed briefly in [5]; they will not be considered here.

To compute the normalization and sub-asymptotic corrections numerically, we adopt the following four step algorithm:

(a) Take a set of parton distributions Δ which has been fitted to pre-HERA data with $Q^2 \gtrsim 4\text{GeV}^2$, $x \gtrsim 10^{-2}$. For illustration we will use here the MRS S0′, D0′ and D-′ distributions [6].

(b) Evolve Δ backwards to a starting scale Q_0. This scale should be chosen sufficiently low that it is reasonable to match to soft non-perturbative behaviour there, but not so low that perturbation theory has become meaningless. Here we choose $Q_0 = 1\text{GeV}$, as in [2].

(c) Remove the (unphysical) small-x tails of the distributions (that is those parts with $x \lesssim 10^{-2}$, and replace them the conventional expectation from Regge theory. In particular the glue and sea distributions should be given soft tails, $xg(x, t_0) \sim x^{-0.08}$, to match the intercept of the (nonperturbative) soft pomeron. This gives new distributions, which we will call $(\Delta)0$.

(d) The new distributions $(\Delta)0$ are now evolved back up to high Q^2, where they can be compared to experimental data (at low Q^2 such comparison would be difficult because of contamination by higher twist corrections). At large-x they will (by construction) be indistinguishable from the original distributions Δ. At small-x they should now exhibit double scaling, but with a definite normalization and sub-asymptotic corrections. They can thus be compared directly to the HERA data.

The results of this procedure[1] are shown in Fig. 1 (the distribution (D0′)0; (S0′)0 is very similar) and Fig. 2 (the distribution (D-′)0), presented as σ and ρ scaling plots; they should be compared with the scaling plots in [2,4].[2] The original MRS distributions D0′ and D-′ are also shown (dotted; see also [5]). The figures largely speak for themselves. The normalization of the HERA data [3] is correctly reproduced, with no free parameters (unless Q_0 were to be considered as such): this is not a fit to the HERA data! The subleading and two loop corrections do not spoil double scaling: although the slope of the

[1] The parameters of the three new sets of distributions functions (S0′)0, (D0′)0 and (D-′)0 may be obtained by email from `rball@surya11.cern.ch`.

[2] Note that in the ρ plots most of the data points now have $\sigma \gg 1.1$.

	χ^2	Best fit Normalization		% mom.
		χ^2	Rel. Norm.	in glue
DAS[2]		107		33%
(S0′)0	115 [309]	105 [230]	1.04 [1.13]	40% [44%]
(D0′)0	126 [263]	106 [213]	1.06 [1.10]	41% [44%]
(D-′)0	114 [323]	112 [185]	0.98 [0.87]	38% [43%]

Table.

σ-plots is now a little lower (2.09 ± 0.35 for (D0′)0, 1.92 ± 0.35 for (D-′)0) than at leading order (where it was 2.4; the data have a slope of 2.37 ± 0.16), there is now a slight rise in the ρ plot at large ρ.

To show quantitatively how well the new distributions account for the data, we also give a table of values of χ^2 (for all 115 data points with $x < 0.1$). In the third column we show how the χ^2 falls if the normalization of the distributions is left free. For comparison we give in square brackets similar statistics for the original MRS distributions [6], none of which fit the HERA data because they do not exhibit double scaling (S0′ and D0′ because Q_0 was too large, D-′ because it incorporates the power-like singular growth inspired by the Lipatov 'hard pomeron').[3] The more recent H and A distributions [7], which achieve a fit to the HERA data by introducing two new free parameters (basically parameterizing a small admixture of a power rise at small-x, thus roughly interpolating between D0′and D-′) have for comparison a χ^2 of 100. From the calculations presented here it should be clear that similar (perhaps better) results could have been obtained with no new parameters simply by dropping Q_0^2 from 4GeV^2 to 1GeV^2, and there taking a soft initial distribution of the same form as D0′: indeed if such an approach had been originally taken in [6], an extremely accurate prediction for F_2^p at small x would have resulted.

If using perturbation theory at scales as low as 1GeV makes one uncomfortable, one could instead incorporate the expected asymptotic behaviour [1] by hand into a starting distribution at 4GeV^2; it really does not matter how the double scaling is produced, provided it obtains throughout the HERA kinematic region. After all, the most striking feature of the small-x data [3] is the precocious onset of double asymptotic scaling [2]. But if one evolves at two loops from a soft distribution at 1GeV, one can also generate both the correct normalization and subasymptotic corrections.

[3] The GRV distributions [8] also fail; they implicitly incorparate double scaling, but overshoot the data because their starting scale is much too low.

Acknowledgements: We are particularly grateful to R.K. Ellis for a copy of his efficient two-loop evolution code. We would also like to thank S. Catani, F. Hautmann, Z. Kunszt, J. Kwiecinski, P.V. Landshoff, R.G. Roberts and D.A. Ross for discussions.

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Figure Captions

Fig. 1. Double scaling plots of $R_F F_2^p$ vs. a) σ and b) ρ. The data are taken from ref.[3], and the curves are those of the new parton distributions (D0′)0. The old MRS prediction D0′ is shown dotted.

Fig. 2. As fig. 1, but with the curves now corresponding to (D-′)0 and D-′(dotted).

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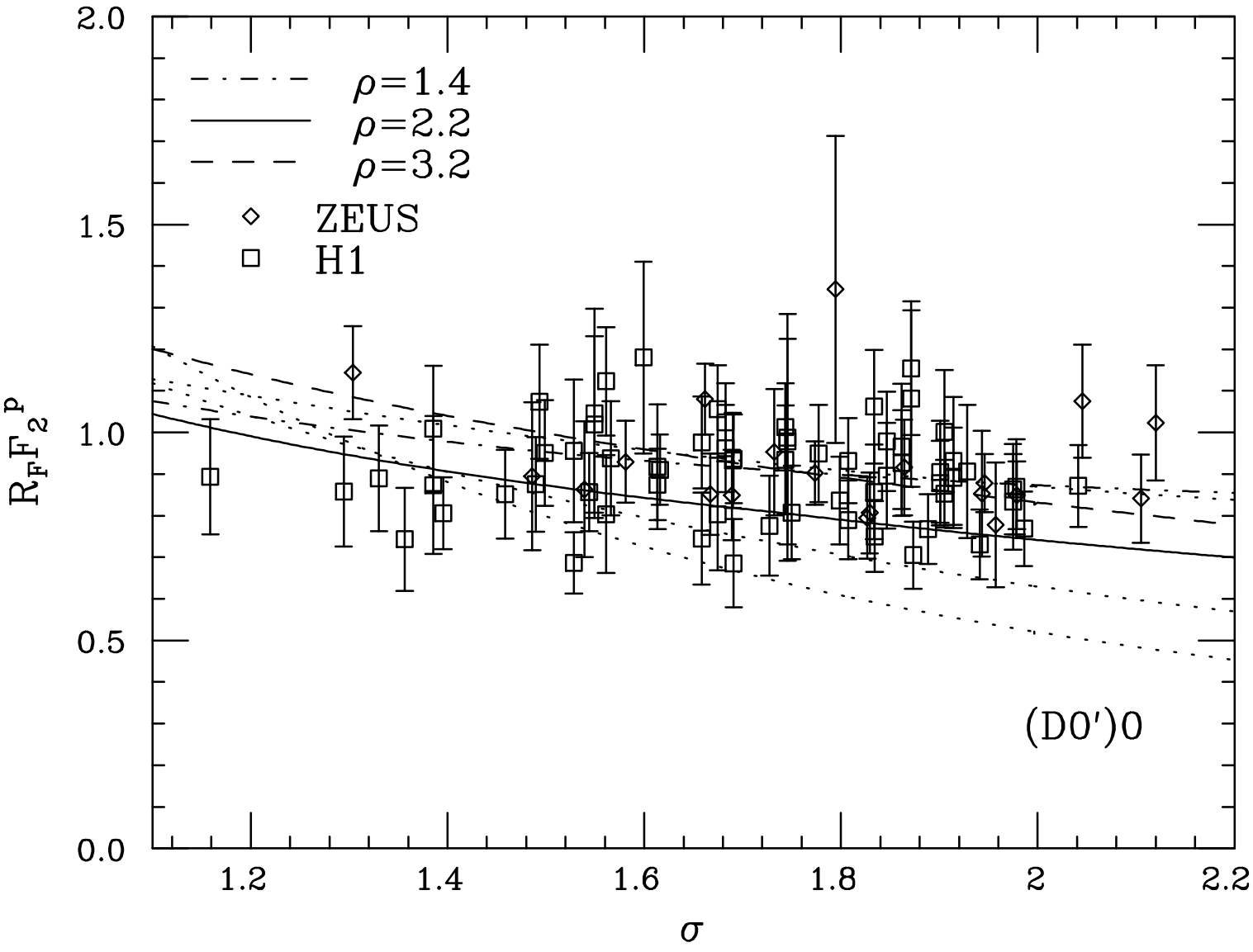
2.0
1.5
1.0
0.5
0.0
1.2
1.4
1.6
1.8
2
2.2
σ
$R_F F_2^p$
ρ=1.4
ρ=2.2
ρ=3.2
ZEUS
H1
(D0′)0

Fig. 1a

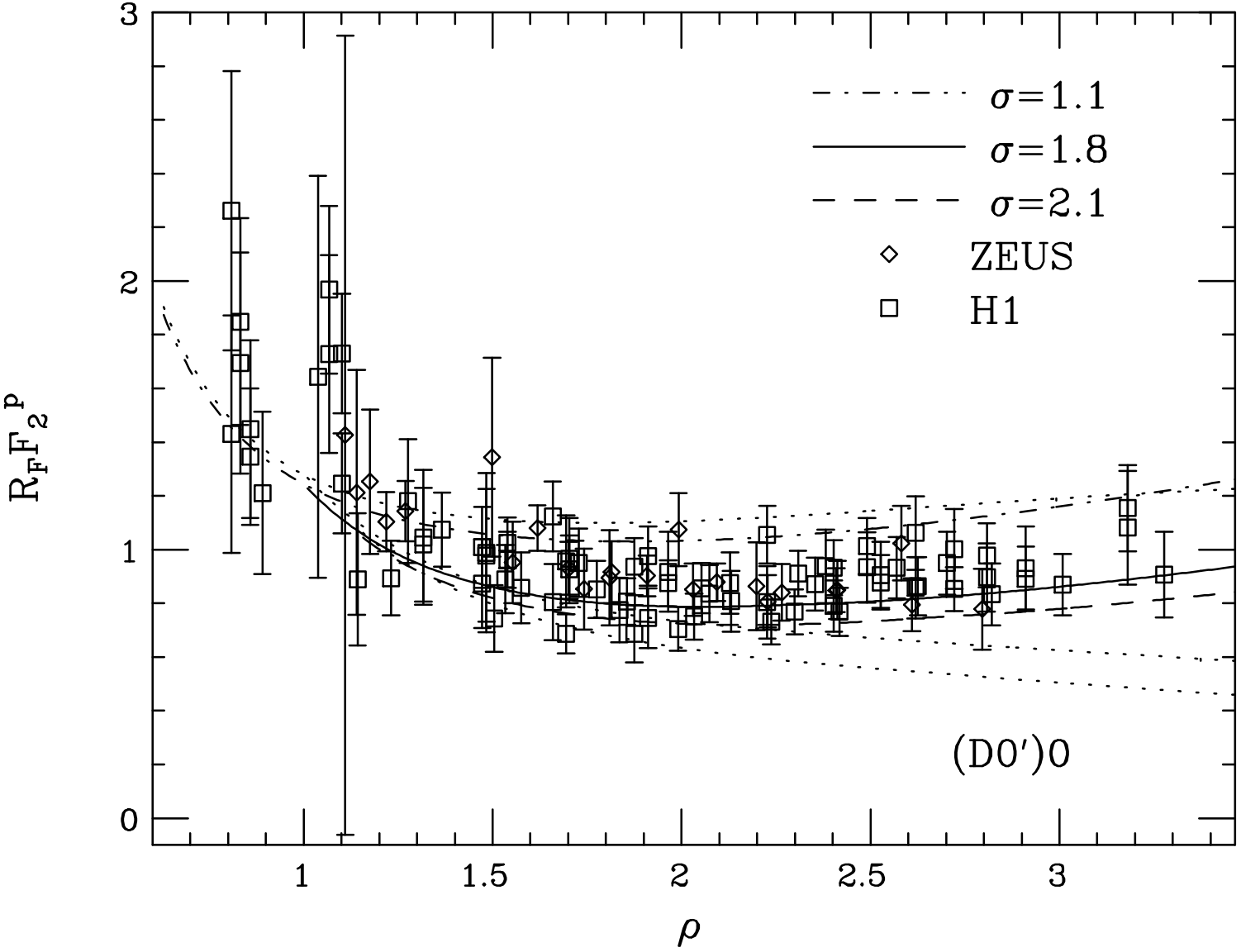

Fig. 1b

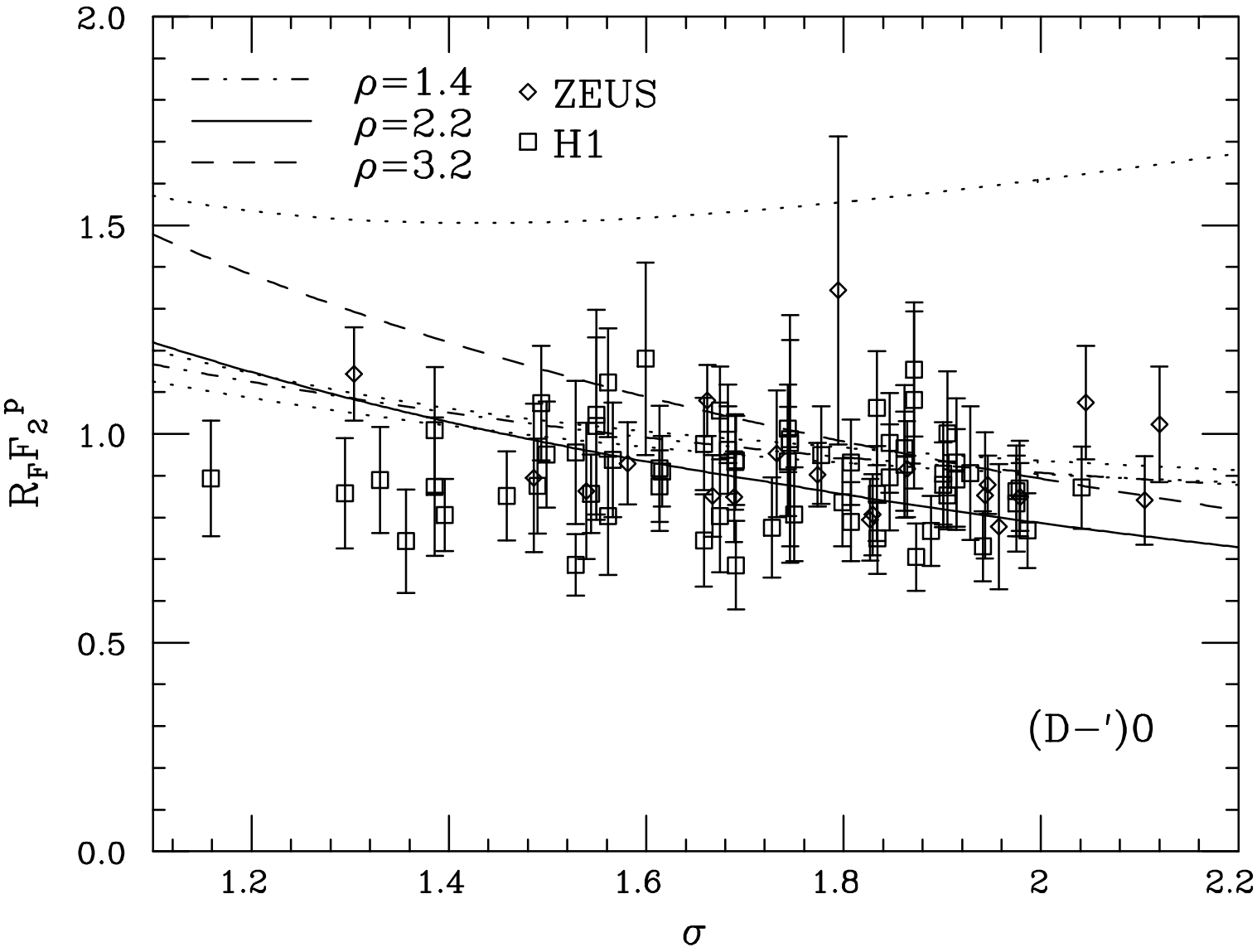
2.0
1.5
1.0
0.5
0.0
1.2
1.4
1.6
1.8
2
2.2
$R_F F_2^p$
σ
ρ=1.4
ρ=2.2
ρ=3.2
ZEUS
H1
(D−′)0

Fig. 2a

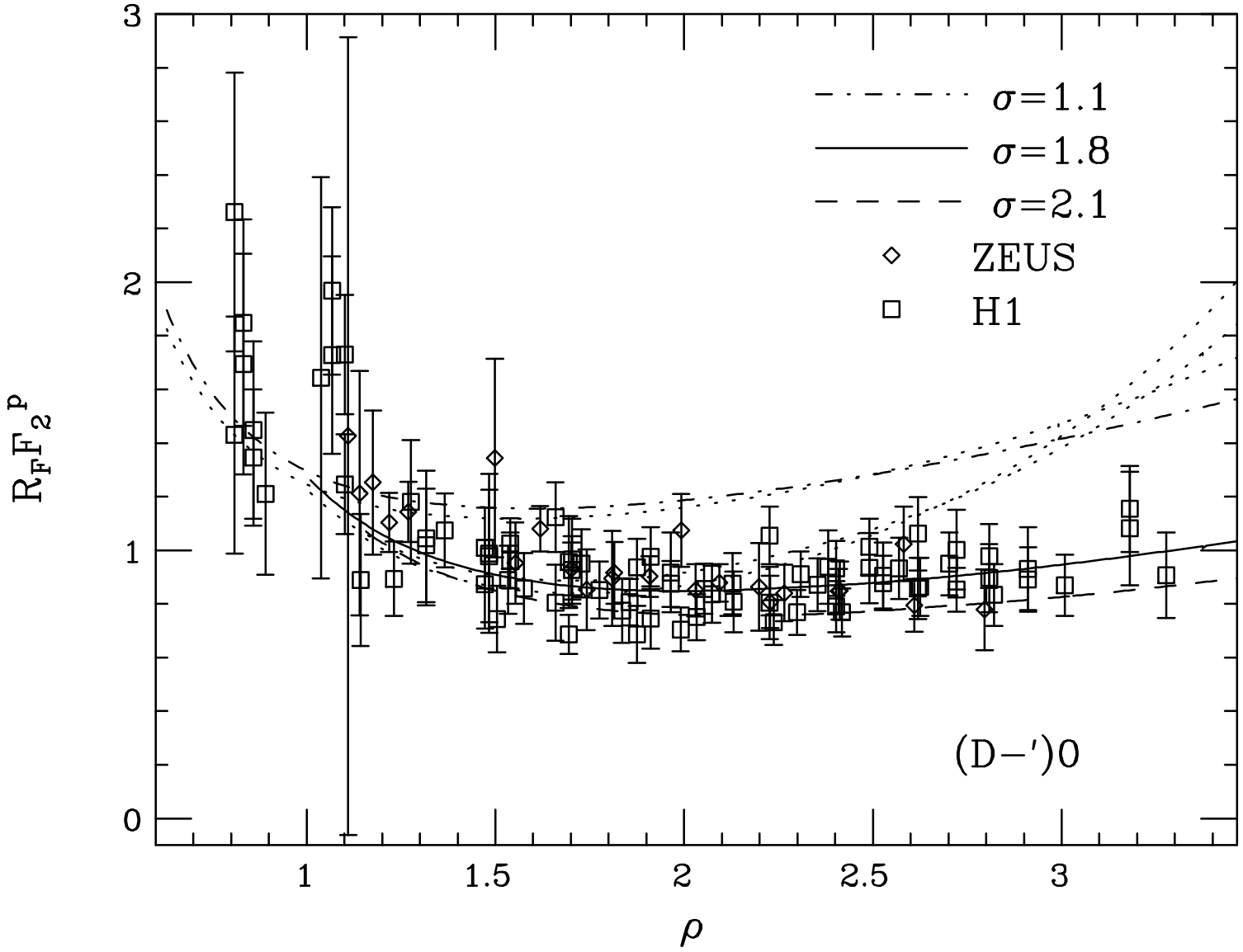

Fig. 2b