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ABOUT ENTROPY AND THERMALIZATION – A MINIWORKSHOP PERSPECTIVE [1]

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Abstract

We present a summary and perspective view of the Miniworkshop on "*Entropy and Thermalization*" in strong interactions (convener J. Rafelski), which was part of the NATO Advanced Research Workshop on "*Hot Hadronic Matter*" that took place in Divonne, 27 June to 1 July 1994.

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ABOUT ENTROPY AND THERMALIZATION – A MINIWORKSHOP PERSPECTIVE

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INTRODUCTION

Our intention is to summarize the main ideas brought forth in this miniworkshop on "*Entropy and Thermalization*" in strong interactions at high energy. In particular, some aspects and differing views introduced during the round-table discussion, which are not otherwise represented in these Proceedings, will be reported on here.

Anticipating our conclusions, there can be no doubt that there exists at present a rich diversity (if not confusion) of concepts related to the measure of entropy to characterize high-multiplicity events in high-energy reactions. This points to the fact that the fundamental problem concerning the characteristic of these reactions is far from being solved. In terms of a field theory it is still not at all clear how to precisely and most economically quantify their very complex multiparticle or many degrees of freedom aspects and relate them to experiments. Rather, the general impression is that, at best, one begins to see the scope of the problem and the first and still quite conventional approaches to describe the essential *disorder* of hot and dense hadronic matter, the *lack of information* on high-order correlations of various kinds, and the *dynamical complexity* of the underlying QCD fields. All of these may be encoded in corresponding measures of entropy. There is a generally shared feeling of the potential richness of collective phenomena hiding in strong interactions at high energy, in particular of heavy nuclei, but attempts to find an adequate formal description to uncover them from experimental findings have mostly been modest.

In the following we try to spread the good news that our present subject is part of one of the major scientific issues of our time, i.e. the measure and understanding of disorder vs. order, which can be observed to be in rapid development in several active parts of science, and of physics in particular. We proceed to present some still rather divergent opinions about the essential features of entropy as well as various first steps in the analysis of entropy production in strong interactions.

CAN WE MAKE SENSE OF DISORDER, LACK OF INFORMATION OR DYNAMICAL COMPLEXITY? [3]

How should we assess the structures of systems that can exhibit *disordered behaviour* in addition to apparently rather simple coherent and logically structured evolutions? By now there are many ideas about how to approach this problem. Here we consider *entropy* as one useful quantitative measure.

Sometimes apparently coherent motions such as sound waves depend on random microscopic behaviour. In other cases a true quantum coherence is essential. Intensity interferometry is rooted in a field ensemble of Gaussian random variables in the most common examples.

Curiously, physicists imagine entropy to be a gross thermodynamical measure, determined by an integration procedure required to convert heat transfer to a perfect differential. Chemists often understand entropy. And computer people have a digital and perhaps better feeling for this concept. Yet they must all be integrated into a single framework covering such diverse aspects as quantum limits of computation and Gödel's theorem [2]. The problem is to define and find the *algorithm* that produces the least computational needs. Yet it must be capable of capturing the

[3] For a related earlier discussion see Ref. [1].

essential qualities of disordered behaviour and complex structures. Surprisingly, topology nowadays does not (yet?) play any essential role in the field of strong interactions, which is governed by the QCD Lagrangian methodology.

Historically, Boltzmann's genius stands out as the beacon of this subject. A key paradox here is related to *Liouville's theorem*, wherein the many-particle entropy is conserved, contrary to what anybody knows to be the basic issue regarding entropy.

Then, there is *von Neumann's* definition of the *quantum entropy* in terms of the density matrix. This is also a conserved quantity, and it therefore seems useless at first sight. However, the natural way out is to consider the von Neumann entropy of intrinsically *open systems* (cf. below), which generally is not a constant of motion. This forms the basis of recent work by one of us reported on here [3].

One may also approach the problem in a different way. This is known as *coarse graining*, unfortunately an all-encompassing term, one variant known as the random-phase approximation in the initial conditions. Another attempt to give it a precise meaning in terms of relevant time scales in the context of particle production by an external field is made in Ref. [4]. However, there seems to be no general foundation for such procedures, which are introduced studying individual cases. More surprising is that coarse graining does not distinguish the "direction" of time [5], even though the entropy has to increase in the process of coarse graining. Again, this seems to be open to debate and we only want to mention the idea that string theory may provide a natural coarse graining by having to integrate out unobservable modes and consequently may alter the fundamental quantum mechanical Schrödinger equation in a way that embodies an "arrow of time" [6].

It is well known that Boltzmann's H-theorem was derived from his famous kinetic equation, based on a single-particle phase-space distribution. By now it is clear that there are many extensions of his approach having to do with higher-order correlations and computational complexity among others, i.e. an infinite (quantum) hierarchy of coupled equations and cellular automata, respectively. Very little is known about how the latter or the former BBGKY (and analogous Schwinger-Dyson) hierarchies can be cast into more intuitively comprehensible schemes. This concerns the description, for example, of such striking phenomena as turbulence or multiparticle hadronization processes, which are "understood" to some extent on the basis of "simple" phenomenological equations.

In particle physics we try to calculate the S-matrix. From this we calculate only probabilities of certain events. Usually the most useful formulation is for the so-called *inclusive differential cross sections*, for which selected particles in phase space are collected, while all others are averaged over. Then, a sequence of probabilities can be constructed and a hierarchy of entropies follows rigorously, which are in agreement with quantum theory despite their classical appearance. We define, for example, the sequence of *inclusive probability densities*:

$$\rho_1 = \frac{1}{\sigma}\frac{d\sigma}{d\Gamma_1} \ , \quad \rho_2 = \frac{1}{\sigma}\frac{d^2\sigma}{d\Gamma_1 d\Gamma_2} \ , \quad \rho_3 = \frac{1}{\sigma}\frac{d^3\sigma}{d\Gamma_1 d\Gamma_2 d\Gamma_3} \ , \tag{1}$$

etc., where Γ_i denotes an appropriate phase-space variable. Now, *information entropies* are generally defined by:

$$\begin{aligned} S(|B) &= -\sum_A P(A|B) \ln P(A|B) \ , \\ S(|AB) &= -\sum_C P(C|AB) \ln P(C|AB) \ , \\ S(|ABC) &= -\sum_D P(D|ABC) \ln P(D|ABC) \ , \end{aligned} \tag{2}$$

etc. Here $P(D|ABC)$ denotes the conditional probability of finding the value D of an observable keeping values A, B, C of other observables fixed, and $\sum_D P(D|ABC) \propto \rho_3$, for example. To be truly inclusive, each of the sets of variables $-$ $\{A, B\}$, $\{A, B, C\}$, etc. $-$ has to be understood to be complete; the notation is meant to indicate especially the increasing number of exclusive variables. These entropies are obviously closely related to experiment. They can be reformulated in terms of *correlation entropies* analogous to cumulants [7], which systematically remove irrelevant lower- order contributions. These correlations vanish when any variable becomes statistically independent of another. Previous definitions of higher-order information entropy miss this point: there, for independent distributions leading to additive entropies, noise potentially obscures the signal of true correlations.

Finally, *all probabilistic entities can be reconstructed from the hierarchy of correlations using generating functional techniques, and individual events can be modelled* *by sampling from the probabilities.*

Next, in order to eliminate one source of confusion, we argue that the above introduced *information entropy is identical to von Neumann's entropy if evaluated for a suitably defined open quantum system.* To see this, we recall Eqs. (1) defining the inclusive densities or, rather, consider the associated probabilities ($\equiv$ density $\times$ flux factor). The same physics of a scattering experiment, for example, can be described in a somewhat unconventional way by calculating a partial trace (with the exclusive variables kept fixed) of the time-evolved density matrix of the total system and integrating over time from $-\infty$ to $+\infty$; here the initial condition has to be specified according to the in-state of the scattering reaction. This defines a *time-independent density submatrix*, which can be diagonalized in exclusive variables by a unitary transformation (provided these variables correspond to quantum mechanical observables of the system). Applying the formal results from Sect. 2 of Ref. [8], we conclude that the resulting matrix elements are indeed the *probabilities* to find the corresponding values of observables of the subsystem defined by the exclusive variables. The "inclusive variables" over which one averages or which are integrated out by calculating partial traces, respectively, automatically constitute the *environment* that complements the *open subsystem* in the total closed system. Thus, formally, if we calculate either information entropies according to Eqs. (2) or the corresponding *von Neumann entropies*,

$$\begin{aligned} S_{v.N.}(|B) &= -\mathrm{Tr}_A\ \hat{\rho}(A|B)\ \ln\hat{\rho}(A|B)\ , \\ S_{v.N.}(|AB) &= -\mathrm{Tr}_C\ \hat{\rho}(C|AB)\ \ln\hat{\rho}(C|AB)\ , \end{aligned} \tag{3}$$

etc., we obtain the same result. Here the notation parallels the one in eq. (2), with $\mathrm{Tr}_A\hat{\rho}(A|B) \equiv \hat{\rho}(B)$ denoting a density submatrix with its elements defined on the space associated with observable B, etc. In general, the judicious choice of the exclusive variables is dictated as much by the physical system under consideration as the meaningful separation of subsystem and environment, which is studied for strong interactions in Ref. [8]. Both approaches give a quantum mechanically precise meaning to the term "*coarse graining*" by consistently eliminating either inclusive variables or environment degrees of freedom.

Several further points can be made. One is, What is the connection, if any, with *thermodynamics*? Although not necessary, it is always an interesting limit to consider. The merit of this limit, if justified, is that strongly time-dependent dynamical details become irrelevant in a stationary equilibrium state, concealing our ignorance of the true situation. We remind the reader of the elegance of Landau's application of relativistic fluid mechanics to multiparticle production.

The main point is, What information is obtained, and what does it tell us about nature? Despite impressive advances in the precision of experimental data, the conceptual framework for the description of *multihadron production* is still deficient. Can anything of fundamental value come out of the incredibly complicated evolution of hadronic and nuclear collisions being analysed with theoretical tools, which were shaped by experience with few-body final states?

From the moment analysis of multiparticle correlations, we can see interesting and strong effects. Recent studies of Bose-Einstein correlations suggest a new and interesting direction. Apart from this, one can imagine that resonance decays account for the main component of the correlation data. This is not very fundamental. At relativistic energies we must sift through enormous data sets in which it is not clear that much of interest has happened. One of the fascinating regularities is the omnipresent negative binomial count distribution, and the associated "linked pair" structure of the cumulant correlations studied by one of us (see Ref. [7] and references therein).

As far as entropy is concerned, the mathematical concept related to probability theory has an intrinsic validity not based on a particular set of variables. However, the variables used to define the probabilities themselves deserve more exploratory thinking in order to reveal some essence of complex dynamical behaviour. The subtleties of the behaviour of systems with many degrees of freedom can defeat the methodology of S-matrix formulations, and standard perturbative calculations are doomed to fail if non-linearities are important, such as in semi-classical Yang-Mills fields [9] and hadronizing QCD systems.

To conclude, we are still awaiting specific answers to the question posed in the title of this section. It appears challenging to study these problems of a rather general nature and of importance beyond the physics of strong interactions.

ASPECTS OF ENTROPY IN STRONG INTERACTIONS

Several members of the round table presented short contributions highlighting their respective views on entropy and related attempts to understand the complex irreversible behaviour in high-energy collisions.

R. Omnès asked the basic question, Given von Neumann's definition of quantum entropy, $S = -\mathrm{Tr}\hat{\rho}\ln\hat{\rho}$, which is a constant of motion for an isolated system, what is the S that increases? He argued that the answer is given by *decoherence theory* in ordinary quantum mechanics and provided an outline thereof. This work is documented in depth, for example, in the review articles [10].

The underlying reasoning, already implicitly alluded to in the above discussion of an *open subsystem* and its *environment*, is the following: Consider a complex dynamical system with many degrees of freedom (e.g. $N \approx 10^{23}$), such as a piece of solid matter. Select suitable collective observables, such as the centre-of-mass coordinates or momenta, etc. Then, split the object ideally into two interacting systems, C and E, defined by the collective ("exclusive") and the complementing ("inclusive") environment degrees of freedom, respectively. The Hamiltonian splits accordingly:

$$H \;=\; H_C + H_E + H_{CE} \quad , \tag{4}$$

where the last term is responsible for energy exchange between C and E ("*dissipation*"). Starting with an initial state that is, for example, a coherent superposition of states representing the piece of matter located at x_1 and x_2, respectively, and the environment in its ground state,

$$|\psi\rangle \;=\; a|x_1\rangle_C|0\rangle_E + b|x_2\rangle_C|0\rangle_E \quad , \tag{5}$$

one can show that, for suitable model interactions, the environment picks up very rapidly (due to excessively small energy denominators) a little excitation energy through H_{CE}. Most important, however, the relevant collective subsystem density matrix obtained by tracing over the environment degrees of freedom,

$$\begin{aligned}\hat{\rho}_C(t) &\equiv \mathrm{Tr}_E\ \hat{\rho}(t) = \mathrm{Tr}_E\ |\psi(t)\rangle\langle\psi(t)| \\ &\approx |c_1(t)||x_1(t)\rangle\langle x_1(t)| + |c_2(t)||x_2(t)\rangle\langle x_2(t)| \ , \end{aligned} \tag{6}$$

where $|c_1|+|c_2| = 1$, becomes essentially diagonal on the same short time scale. This is *environment-induced decoherence.* As far as the collective variables are concerned, a *pure zero-entropy initial state* (chosen only for simplicity) has become a *mixed state with non-zero von Neumann entropy,*

$$S_{v.N.} = -\mathrm{Tr}_C\ \hat{\rho}_C \ln \hat{\rho}_C = -(|c_1| \ln |c_1| + |c_2| \ln |c_2|) \ . \tag{7}$$

This effect, *mutatis mutandis*, is believed to be almost universal, although explicit calculations are restricted to the class of models that can eventually be represented by Gaussian path integrals. Some pertinent questions are: *How complete* is the decoherence effect? *How fast* is it? Under which conditions is the resulting *entropy production irreversible*? Answers from explicit calculations can be given, for example, for the particular quantum mechanical systems studied in Refs. [3, 8], which are constructed with an eye on the relativistic quantum field theory extensions of decoherence theory to be applied to strong interactions [8, 11].

Note that, in the above example of a solid piece of matter, the microscopic environment (mostly phonons) and the macroscopic collective degrees of freedom are known and rather clearly separated. In general, this will not be the case. For strongly interacting hadronic systems and high-energy collisions, in particular, the main stumbling block preventing a deeper understanding is precisely that we know very little about which are *the* relevant degrees of freedom among an infinity of others. Single-particle observables do not seem to provide a clue.

M. Danos addressed the fundamental question of *irreversibility in quantum physics*, which in a sense is even prior to our discussion of entropy. He presented a provocative point of view, which we quote directly with minor changes [12]:

One of the key points in dissipation in quantum physics is the observation that time-reversal-invariant states have probability measure zero. – Generally, the physical states of a system do not exhibit the symmetries of the Hamiltonian. This is so also for the time-reversal symmetry. Since the Hamiltonian itself is time-reversal-invariant, time-reversal-invariant states must exist, and indeed they do. Only they have "measure zero". – Rather than providing the mathematical derivation of this result [13], a more physical explanation is given here. Take as the simplest possible example a two-channel system. In a physical state there will be an incoming wave in one channel, say channel 1, and outgoing waves in both channels. The wave function (in the asymptotic region) will be

$$\Psi \;=\; \mathrm{e}^{-ik_1x} + a\mathrm{e}^{ik_1x} + b\mathrm{e}^{ik_2x} \quad , \tag{8}$$

where ${k_2}^2 = {k_1}^2 - 2mB$ (*), with B the inelasticity of channel 2. The time-reversed wave function is $\Phi = \Psi^*$, which has amplitude- and phase-related incoming waves in both channels, and an outgoing wave in channel 1. To achieve that form the energy matching of Eq. (*) must be fulfilled *exactly*. To actually construct this wave would require an infinite set-up time as a consequence of the time-energy uncertainty relation.

Hence, even though it is easy to write down an expression for the time-reversed state of a physical state for any system, it is principally impossible to *actually construct* such states. Then, the superposition of a state and its time-reversed partner forming a time-reversal-invariant state becomes equally impossible. Hence, such states cannot exist in nature.

Unfortunately, we are unable to recall the spirit of the subsequent lively discussion with the audience, which expressed doubts about the validity of quantum mechanics, questions about the existence of pure states and the relevance of infinite numbers of degrees of freedom, and several others.

R. Weiner turned the attention of those present to problems closer related to experimental observations. Namely, attempts to understand results of correlation measurements and Hanbury-Brown-Twiss type interferometry with secondary hadronic particles in terms of the space-time and internal structure of their sources. This subject is covered in various ways by his and others' contributions to these Proceedings, in particular those to the Miniworkshop on "*Multiparticle Dynamics*". We mention some interesting points concerning our present subject.

Consider a system, which is completely characterized by its density matrix $\hat{\rho}$, in terms of *coherent states*, $a_k|\alpha_k\rangle = \alpha_k|\alpha_k\rangle$, where a_k = annihilation operator for mode k. Then, omitting the (non-trivial) sum over modes (due to overcompleteness),

$$\hat{\rho} \; = \; \int d^2\alpha \; P(\alpha)|\alpha\rangle\langle\alpha| \quad . \tag{9}$$

Quantum statistics allows the weight function to range between the extremes of a coherent distribution and a chaotic one with $P_c(\alpha) = \delta^2(\alpha - \alpha_0)$ and $P_{ch}(\alpha) = \pi^{-1}\bar{n}^{-1}\exp(-|\alpha|^2/\bar{n})$, respectively. It can be shown that a *chaotic distribution is necessary and sufficient to maximize the von Neumann entropy* [14].

Now, the simplest usually measured *Bose-Einstein correlations* are defined by

$$C_2(k_1,k_2) \; = \; \frac{\rho_2(k_1,k_2)}{\rho_1(k_1)\rho_1(k_2)} \; = \; \frac{\mathrm{Tr}\;\hat{\rho}I_1I_2}{\mathrm{Tr}(\hat{\rho}I_1)\mathrm{Tr}(\hat{\rho}I_2)} \quad , \tag{10}$$

where the "intensities" are given by number operators, $I_j = I(k_j) = a^+_{k_j}a_{k_j}$, and the densities follow from Eqs. (1) with $\Gamma = k$. Thus, in principle, Bose-Einstein correlations measure the density matrix of the system and provide a test for randomization. The very existence of non-trivial correlations, i.e. $C_2(k_1,k_2) > 1$, which is observed in a wide range of experiments, is *evidence for a (partial) randomization and non-vanishing entropy.* However, unfortunately this aspect is usually taken more or less for granted and, so far, empirical parametrizations of these correlations are only employed to derive source geometry and lifetime parameters.

U. Heinz, finally, recalled the apparently strongly disordered outgoing state in a heavy-ion collision. Then, Why is there a $\beta = T^{-1}$ characterizing the exponential slope of major parts of the spectra of secondaries? There are partial answers to this decades-old question, but one still feels a lack of understanding: T being a Lagrange multiplier for the variational problem to *maximize S at constant energy*, one asks oneself, Where does the measured large value of this entropy come from? Given that a local thermodynamic equilibrium description of high energy density matter in a collision can be justified, which in turn is still only partly understood [15], the entropy in terms of particle multiplicities is an important additional piece of information to distinguish various phenomenological equations of state [16, 17].

WHAT DO NEXT?

Instead of providing *the* answer to Lenin's question for our present subject, we discuss the contributions to this miniworkshop, with more details to be found elsewhere in these Proceedings.

The most recent and surprising results of the most conservative approach to entropy in heavy-ion collisions, in the sense that it has already a history of four decades, were described by **J. Letessier** [16]. It is based on the hypothesis of the formation of a *fireball*, i.e. a space-time region of hot and dense hadronic matter with approximately thermal properties in the centre of mass of these reactions. Its total energy E and baryon number B are assumed to be fixed and their internal properties (particle content, phase-space distributions, etc.) to be determined consistently by a *kinetic equilibrium temperature* T and *fugacities* $\lambda_i = \exp \mu_i/T$ of the constituents (as well as additional parameters). The major advantage of this model is its conceptual and technical *simplicity*, in principle, which allows direct comparison with experimental results on multiplicities. Here the measured final state *specific entropy* S/B is employed to discriminate between different equations of state. It is argued that a *hot hadronic gas scenario is unable to fit all available data from the 200 GeV A CERN experiments, whereas incorporating a temporarily existing quark-gluon plasma gives a satisfactory description* ("too good to be true") [16].

The success of this approach crucially depends on the assumptions of (local) thermal equilibrium joined with a simple "macroscopic" collective expansion of the fireball. One further important *consistency* check should be to study two-particle correlations in precisely the same model, as well as single-particle spectra [18]. Finally, it is stated that $\approx 70\%$ *of the measured entropy must be produced already during the pre-thermal phase*, the study of which thus becomes crucial also for the understanding of the fireball model parameters.

The above and the following model share the uncertainty ("flexibility") about details of the *hadronization* from the parton phase. According to folklore, there is essentially *no entropy production* during the (non-equilibrium) phase transition.

This is important for the interpretation, e.g. in a thermal model, of

$$\frac{dS}{dy} \;=\; c_{qg}\frac{dN_{qg}}{dy}|_{b=0} \;\propto\; \frac{dN_\pi}{dy}|_{b=0} \;\;, \tag{11}$$

which is a typical relation between the entropy density calculated in a partonic model, see below, and the observed hadron multiplicity.

A study of the early space-time evolution was presented by **K. Geiger** [15] employing a *parton cascade* approach to simulate quark and gluon transport during hadronic or nuclear collisions. This probabilistic scheme is based on state-of-the-art *perturbative QCD* cross sections, which are employed similarly as in simulations of Boltzmann equations including collision terms. Partons are sampled from measured *structure functions* and *propagated classically* in accordance with Altarelli-Parisi type equations. A justification of this procedure for multiple scatterings in extended dense systems, i.e. multiple inter-cascade interactions, seems rather difficult within the parton model, since it goes beyond proved factorization theorems.

Here the total *entropy* arises from three contributions,

$$S \;=\; S_{primary} + S_{secondary} + S_{hadronization} \;\;\; . \tag{12}$$

The *primary contribution*, which amounts to about 40% of the total, stems from the *decoherence process* that sets in once the incoming hadronic wave functions are perturbed by initial-state QCD interactions. This is the dynamical origin of the structure functions, which has recently been addressed in Refs. [3, 8, 11]. Another way of stating this is by recalling the definition of structure functions as being related to *inclusive cross sections*; according to our discussion following Eq. (1), there must be an associated entropy. At present, this contribution *cannot* be calculated *ab initio* or in any quantitative model.

The *secondary contribution*, which accounts for practically all the rest of the produced entropy, is due to the *production of secondary partons* in elementary bremsstrahlung or scattering processes. This is essentially analogous to what happens in any kind of molecular dynamics simulation, namely a covering of the available classical single-particle phase space via scattering. Finally, the *hadronization contribution* is arguably considered to be small and taken into account by hadronization prescriptions based on universal parton-hadron duality and fitted, for example, to e^+e^- data.

The most interesting result in the present context is the *rapid saturation of entropy production* (together with a thermalization of parton spectra) on a time scale of 0.5 fm/c or less, with a value of the specific entropy per particle, $S/N \approx 4$, which is more or less the ideal parton gas value [15].

The *Schwinger mechanism*, i.e. e^+e^- creation in a time-independent homogeneous electric field, was studied by **J. Rau** [4] w.r.t. entropy production and irreversibility. The main result is that the *"relevant" entropy* defined here in terms of the single-particle ($e^\pm$) occupation numbers,

$$S_{rel}(t) \;=\; \sum_{all\ modes} \left\{ \tfrac{1}{2}n_- \ln \tfrac{1}{2}n_- + (1-\tfrac{1}{2}n_-)\ln(1-\tfrac{1}{2}n_-) + [\, n_- \leftrightarrow n_+ \,] \right\} \quad , \tag{13}$$

tends to increase. However, there are two essential time scales for the process: the *memory time*, $\tau_{mem} \approx (\hbar/m) + (m/qE)$, and the *production time*, $\tau_{prod} \approx (m/qE)\exp(\pi m^2/2\hbar qE)$. Depending on their relative size the process is essentially Markovian and *irreversible* (weak fields), leading to monotonically increasing S_{rel}, or else it shows important *memory effects* (strong fields), leading to oscillations of S_{rel} on the scale of $\tau_{mem} \approx \hbar/m$ [4]. These effects are analogous to what happens with the Boltzmann equation depending on the relative size of the time between collisions ("τ_{mem}") and the duration of a single collision ("τ_{prod}").

It seems important to realize how the "relevant" entropy here fits into our preceding discussion of coarse graining, inclusive variables, and open systems with their environments. Clearly, the entropy S_{rel} determined by occupation numbers is *relevant* w.r.t. experiments measuring single-particle observables. However, it corresponds to a *chosen cut in the space of observables* of the system. Thus, one deliberately discards information in an inclusive way, e.g. about relative phases of outgoing single-particle waves or higher-order correlations (n-point functions with $n > 2$), which amounts to a coarse graining and results in information entropy as before. In distinction, the considerations in Refs. [3, 6, 8, 10] are based on the observation that in some systems or theories (e.g. QCD) there is a *dynamical cut in the space of fundamental modes* of the system, which naturally separates it into an "observable" subsystem and its environment. In this case, the coarse graining is dictated by the complex system itself. Then, the *von Neumann entropy* related to all information available about the subsystem is *not* a constant of motion and is the relevant entropy. An *additional* coarse graining, such as a restriction to inclusive single-particle observables, may still be necessary for practical purposes.

The contribution by **H.-Th. Elze** [3] provides a simple introduction to the mechanism of *environment-induced quantum decoherence* (cf. the above discussion of R. Omnès' presentation) with a view towards strong interactions. In QCD a separation of non-perturbatively interacting, almost constant, field configurations, which can neither hadronize nor initiate hard scatterings, from the usual high-energy or far off-shell partons seems essential to attack the strong-coupling problem underlying entropy production in multiparticle processes.

We mention two particular results. Employing the *Schmidt decomposition* of the complex system density matrix, see Sect. 2 of Ref. [8], one finds that the von Neumann entropy for the subsystem always equals the one for its environment. Therefore, one may choose to *eliminate either the environment or the subsystem* degrees of freedom, whichever is simpler. Secondly, in the example of the inverted oscillator [3], which is partially chaotic in the classical limit, one observes an *exponentially growing entropy production*, which is governed essentially by the classical Lyapunov exponent. Here, the decoherence is induced by the coupling to the *vacuum fluctuations* of only one environment oscillator. Thus, under suitable conditions an extremely simple zero-temperature environment is sufficient to cause entropy production in the subsystem, which might be relevant in the following.

The work on *chaos and entropy production in classical Yang-Mills fields* reported by **B. Müller** [9] addresses the question of entropy production as being connected intimately to the problem of thermalization in strongly interacting systems. One studies the chaotic time evolution of classical Yang-Mills fields employing the lattice gauge theory discretization for the Hamiltonian equations of motion. Thus, it is shown that a random *ensemble of initial field configurations self-thermalizes* rapidly, i.e. the probability distribution of the magnetic plaquette energy evolves into an exponential Boltzmann distribution. Furthermore, the maximal Lyapunov exponent of the time-dependent classical system is demonstrated to yield the *damping rate of coloured collective (plasmon) excitations* $\propto g^2T$, which is calculated otherwise by finite-temperature QCD perturbation theory ($T \gg T_c$). Its inverse yields a *thermalization time*, which rapidly decreases with increasing T from $\tau_S^0 \approx 0.5$ fm/c at $T \approx 200$ MeV. Employing the complete Lyapunov spectrum, which presumably corresponds to including other unstable collective excitations at finite T, an even shorter thermalization time can be deduced from the rate of entropy growth, $\tau_S = \bar{S}_{equil}/\partial_t\bar{S}$. Herein, the relevant entropy is the *Kolmogorov-Sinai entropy* $\bar{S}$, which arises by a coarse graining of the classical phase space [9].

Two related points seem to deserve further study in order to fully understand these remarkable results. Firstly, where does the ensemble of initial field configurations come from? Following Refs. [3, 8], one is led to conjecture that the *environment of high-energy or far off-shell partons* and integrating out these ultraviolet degrees of freedom, respectively, result in the effectively *classical* initial conditions above. Secondly, are the strongly coupled Yang-Mills system under consideration and its evolution stable w.r.t. the ultraviolet quantum fluctuations? Here, *asymptotic freedom* may help to keep such stability, which is necessary in order to relate this approach to actual hadronic or nuclear collisions.

In conclusion, we hope to have raised or rephrased some interesting questions to stimulate further research on entropy and thermalization, particularly in strong interactions. We thank all participants of the Miniworkshop for sending copies of their presentations and, especially, J. Rafelski for the intellectual and organizational support without which it would not have happened.

References

[1] P. A. Carruthers, in Proc. NASI "Hot and Dense Nuclear Matter", Bodrum (Turkey), 1993, to be published by Plenum Press.

[2] R. Landauer, Phys. Today 45, No. 5 (1991) 23;

C. J. Chaitan, “Information, Randomness and Incompleteness”, World Scientific, Singapore, 1987.

[3] H.-Th. Elze, contribution to these Proceedings.

[4] J. Rau, contribution to these Proceedings.

[5] M. C. Mackey, “Time’s Arrow: The Origins of Thermodynamic Behaviour”, Springer Verlag, New York, 1992.

[6] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Phys. Lett. B293 (1992) 37; preprint CERN-TH.7195/94; and references therein;
M. Gell-Mann and J. B. Hartle, Phys. Rev. D47 (1993) 3345.

[7] P. A. Carruthers, Int. J. Mod. Phys. A4 (1989) 5587.

[8] H.-Th. Elze, preprint CERN-TH.7131/93 (hep-ph/9404215), to appear in Nucl. Phys. B.

[9] B. Müller, contribution to these Proceedings.

[10] W. H. Zurek, Phys. Today 44, No. 10 (1991) 36;
R. Omnès, Rev. Mod. Phys. 64 (1992) 339.

[11] H.-Th. Elze, preprint CERN-TH.7297/94 (hep-th/9406085), submitted to Phys. Rev. Lett.

[12] M. Danos, private communication.

[13] M. Danos, NIST Technical Note 1403 (1993).

[14] R. Weiner, private communication.

[15] K. Geiger, contribution to these Proceedings.

[16] J. Letessier, contribution to these Proceedings.

[17] A. Tounsi, contribution to these Proceedings.

[18] U. Heinz, contribution to these Proceedings.