CERN-TH.7407/94
EFI-94-41
UdeM-GPP-TH-94-08

ASSUMPTIONS UNDERLYING DETERMINATION OF A WEAK COUPLING PHASE

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ABSTRACT

We clarify the assumptions underlying the determination of the weak phase γ from a comparison of rates for charged B meson decays to $\pi\pi$ and πK.

CERN-TH.7407/94
August 1994

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In a recent paper [1] we showed that it is possible to obtain the weak phase γ by measurements of charged B-meson decay rates to $\pi^+ K^0$, $\pi^0 K^+$, $\pi^+\pi^0$, and their charge-conjugate states. One of the ingredients in this analysis was the observation that, in the limit in which annihilation diagrams are neglected, the decay $B^+ \to \pi^+ K^0$ is pure penguin. It has subsequently been suggested [2] that there is a hidden assumption in this result. What follows are some comments which explain the relation between our explicit assumption and the interpretation of Ref. [2]. We also wish to remind the reader about previous discussions of the potential of this method for determining the weak phase γ, as a function of the strong phase difference and of γ itself.

A diagrammatic approach, equivalent to an SU(3) decomposition of amplitudes, was adopted in Ref. [1]. Let q stand for d or s, and let unprimed and primed amplitudes correspond to $q = d,\ s$, respectively. The dominant amplitudes are expected to be (i) a color-favored "tree" contribution T or T' with subprocess $\bar{b} \to \bar{q} u \bar{u}$, (ii) a color-suppressed tree contribution C or C' with this same subprocess, and (iii) a penguin contribution P or P' with subprocess $\bar{b} \to \bar{q}$. (We omit reference to gluons or SU(3) singlet quark-antiquark pairs.) Other amplitudes, expected to be suppressed in comparison with these, are (iv) an annihilation subprocess A or A' involving $\bar{b}u \to \bar{q}u$, (v) an exchange subprocess E or E' involving $\bar{b}q \to \bar{u}u$, and (vi) a "penguin annihilation" subprocess PA or PA' involving $\bar{b}q \to$ [SU(3) singlet system].

The neglect of contributions (iv) – (vi) in comparison with (i) – (iii) was noted explicitly to be equivalent to the assumption that rescattering effects are unimportant. For example, a final state which can be reached through the annihilation diagram can also be reached through a tree diagram followed by a rescattering. Several tests of this hypothesis were proposed [1, 3]. One can expect such an assumption to lead to relations between final-state phases in different decay channels. Indeed, one such phase relation was noted to exist between $B \to \pi\pi$ and $B \to \pi K$ [4]. Another example was given in Ref. [2].

The fundamental process involved in the decays is $\bar{b} \to \bar{s} +$ (light quark – antiquark pair). Let us denote a decay amplitude by A if it involves the light quark-antiquark pair with isospin 1, and B if the pair has isospin zero. We can decompose amplitudes for $B \to \pi K$ charge states into isospin amplitudes $A_{3/2}$, $A_{1/2}$, and $B_{1/2}$, where the subscript denotes the total isospin of the final state [5]. We also quote the decomposition into amplitudes associated with graphs:

$$A(B^0 \to \pi^- K^+) = A_{3/2} + A_{1/2} - B_{1/2} = -(T' + P') \quad , \qquad (1)$$

$$\sqrt{2} A(B^0 \to \pi^0 K^0) = 2A_{3/2} - A_{1/2} + B_{1/2} = -C' + P' \quad , \qquad (2)$$

$$A(B^+ \to \pi^+ K^0) = A_{3/2} + A_{1/2} + B_{1/2} = P' + A' \quad , \qquad (3)$$

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = 2A_{3/2} - A_{1/2} - B_{1/2} = -(T' + C' + P' + A') \quad . \qquad (4)$$

Solving for the respective amplitudes, we find

$$A_{3/2} = -\frac{C' + T'}{3} \quad , \tag{5}$$

$$A_{1/2} = \frac{2C' - T' + 3A'}{6} \quad , \tag{6}$$

$$B_{1/2} = P' + \frac{T' + A'}{2} \quad . \tag{7}$$

The point raised in Ref. [2] is that the $I = 3/2$ combination

$$A(\pi^+ K^0) + \sqrt{2} A(\pi^0 K^+) = 3A_{3/2} = -(T' + C') \equiv \sqrt{2}\hat{A} e^{i\gamma} e^{i\delta_3} \tag{8}$$

and the $I = 1/2$ tree contribution to the combination

$$[2A(\pi^+ K^0) - \sqrt{2} A(\pi^0 K^+)]_{tree} = 3(A_{1/2} + B_{1/2})_{tree}$$
$$= +(T' + C') \equiv \sqrt{2}\hat{C} e^{i\gamma} e^{i\delta_1} \tag{9}$$

should have the same strong final-state phases $\delta_1 = \delta_3$ if their sum,

$$[3A(\pi^+ K^0)]_{tree} = \sqrt{2}\hat{A} e^{i\gamma} e^{i\delta_3} + \sqrt{2}\hat{C} e^{i\gamma} e^{i\delta_1} \tag{10}$$

is to vanish.

In the graphical description of Ref. [1], this is automatically the case, since the amplitude $A_{3/2}$ and the tree contribution to the combination $A_{1/2} + B_{1/2}$ are both proportional to $T' + C'$. Thus, the equivalence of the strong final-state phases, $\delta_1 = \delta_3$, is not a hidden assumption, but is rather a direct consequence of our assumption that the annihilation diagrams are negligible. If the annihilation amplitude A' is neglected, the only remaining contribution to Eq. (3) is then P', as stated.

It is noted in Ref. [2] that the method could provide a powerful constraint on the weak phase γ even if only an upper limit on CP violation is obtained from $B^\pm \to \pi^0 K^\pm$. Indeed, it was stressed in Ref. [1] that γ can be determined even without observing a CP asymmetry, similar to the method proposed in Ref. [6]. Here too, the precision of determining this angle is highest for $\gamma = \pi/2$. The range of strong phases for which γ can be measured with a given precision is maximal for this value of the angle [7].

We thank L. Wolfenstein for fruitful discussions. M. G. wishes to thank the CERN Theory Division and J. R. wishes to thank the Fermilab Theory Group for their warm hospitality. This work was supported in part by the United States – Israel Binational Science Foundation under Research Grant Agreement 90-00483/3, by the German-Israeli Foundation for Scientific Research and Development, by the Fund for Promotion of Research at the Technion, by the United States Department of Energy under Contract No. DE FG02 90ER40560, and by the N. S. E. R. C. of Canada and les Fonds F. C. A. R. du Québec.

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