CLIC Note 244
17 August 1994

Measurements for Adjusting BNS Damping in CLIC

Walter Wuensch

Introduction

Two types of beam measurements are required for the trajectory correction algorithms currently used in CLIC main linac beam dynamics simulations: beam position measurements for linac alignment and emittance measurements for adjusting radio frequency quadrupole (RFQ) strengths to set the correct level of BNS damping [1].

The resolution and precision needed for beam position monitors and details of their placement within the linac have been well studied [1] and appropriate beam position monitors (BPM)s for CLIC are under development and test [2].

Attempts to match beam simulation with hardware for emittance measurements are less well advanced although there are some ideas about how to make the measurement [3]. The necessary performance of the emittance measurement has not been established in a way in which can guide the design of hardware. An approximate emittance measurement may be sufficient. It is also not clear how many measurements are needed along the length of the linac. Most simulation results use a very high density of emittance measurements. Attempts to use five per linac have led to puzzling results [4]

In the first half of this paper an attempt is made to qualitatively understand how the beam degrades when BNS damping is misapplied. In this way a measurement scheme can be developed that is sensitive to the essential features of the beam when RFQ strengths are wrong. Some idea of the number of measurements which are needed along the length of the linac can be found from the analysis as well.

In the second half of the paper a device for an approximate emittance measurement is proposed. It is based on the BPM which is being developed for CLIC and is not destructive to the beam.

Beam Dynamics Related to BNS Damping

The motion of a bunch subject to smooth focusing and a transverse wakefield is given by [5],

$$X''(z,s)+\frac{U'(z,s)}{U(z,s)}X'(z,s)+k^2(z,s)X(z,s)=\frac{1}{U(z,s)}\int_{-\infty}^{z}\rho(z')W_\perp^\delta(z-z')X(z',s)dz'$$

where X is the transverse displacement, z is the position along the bunch (from the head back) and s is the distance along the linac, k is the betatron wave number, ρ is the charge density of the bunch, W is the transverse wakefield and U is the energy of the bunch.

Instead of attacking the equation directly, a number of special cases will be considered to try to understand the general behaviour of the solutions. Other (usually more sophisticated) efforts have been made to solve the above equation under various approximations [6,7,8].

Throughout the following analysis bunches will be assumed to be short compared to the wavelength of the main deflecting mode and wakefields will be assumed to be linear.

$$W_\perp^\delta(z-z') = (z-z')W_0,$$

No acceleration will be considered,

$$U'=0.$$

The first approximation is to consider a bunch made up of two discrete charges located at z=0 and z=z_0 and which have the same k ,

$$\rho(z) = \rho_0\delta(z) + \rho_0\delta(z-z_0),$$

$$k(z,s) = k.$$

The first bunch sees no wakefield so just executes harmonic motion. An initial displacement of 1 gives the motion,

$$X(0,s) = e^{iks}.$$

The second charge sees a wake,

$$\rho_0 W_0 z_0 X(0,s),$$

so has an equation of motion,

$$X''(z,s) + k^2 X(s,z) = \frac{\rho_0 W_0 z_0}{U} e^{iks}.$$

The inhomogeneous solution has the form,

$$X = \alpha s e^{iks}$$

If we add the homogenous solution and impose the same initial conditions as on the first charge , the solution of the motion is,

$$X(z_0,s) = \left(1 - i\frac{\rho_0 W_0 z_0}{2kU}s\right)e^{iks}.$$

In the limit of large s, there is a linear growth in the amplitude of the second charge which is 90° out of phase with the driving term - the first charge. This is in contrast to the quadratic growth in amplitude one would have if the applied force were in the direction of the velocity. Unfortunately the nonlinearity of the equation of motion prohibits the summation of the two bunch motion to the give the solution for 3 or more bunches.

For small s the wakefield introduces a phase shift

$$\Delta\varphi = -\frac{\rho_0 W_0 z_0}{2kU}$$

BNS damping cancels this phase shift by introducing a phase shift caused by a slightly different k for the second bunch.

$$k' = k + \frac{\rho_0 W_0 z_0}{2kU}$$

The motion of a continuous bunch is now considered. The equation of motion of a bunch with uniform charge density ρ_0 from z=0 to z=z_0 is

$$X''(z,s) + k^2(z,s)X(z,s) = \frac{\rho_0 W_0}{U}\int_0^z (z-z')X(z',s)dz'.$$

Through the RFQs the CLIC beam will be subject to a linear variation in focusing strength,

$$k(z,s) = k_0\left(1 + \frac{\Delta k}{k_0}\frac{z}{z_0}\right),$$

with $\Delta k/k \approx .05$.

In order to gain insight in the evolution of the bunch when RFQs are slightly too strong, the motion with wakefields turned off is considered. This is the limiting case of too strong RFQs. A smooth and continuous variation from coherent bunch motion - perfect BNS - to too strong RFQs is assumed.

With no wakefield each point along the bunch will execute harmonic motion with a continuously varying frequency due to the focusing strength spread. A straight beam which enters the linac with unit displacement evolves according to the function,

$$X(z,s) = e^{iks} e^{i(\Delta k s)\frac{z}{z_0}}$$

Thus the shape of the beam is modulated by an ever increasing spatial frequency,

$$\frac{\Delta k}{z_0}s$$

Here one can draw the first conclusions of the evolution of a bunch with finite length when RFQs are mistuned. For example the displacement of the beam will evolve to a half of a cosine after k/(2Δk) betatron cycles - about 10 cycles for CLIC and a full cosine after 20 cycles.

The opposite limiting case, the equation of motion of a uniform bunch with no focusing strength spread is given by the equation of motion,

$$X''(z,s)+k_0^2X(z,s)=\frac{\rho_0W_0}{U}\int_0^z(z-z')X(z',s)dz'.$$

Taking the Fourier Transform with respect to the variable s with transform pairs,

$$X(z,s)\Leftrightarrow x(z,t),$$

$$(k_0^2-t^2)x(z,t)=\frac{\rho_0W_0}{U}\int_0^z(z-z')x(z',t)dz'$$

This integral equation is a homogeneous Volterra equation which can be changed to a second order differential equation by differentiating twice with respect to z,

$$\left(\frac{k_0^2-t^2}{1}\frac{U}{\rho_0W_0}\right)\frac{\partial^2x}{\partial z^2}=x.$$

This has the solution,

$$x(z,t)=e^{z\sqrt{\frac{k_0^2-t^2}{1}\frac{\rho_0W_0}{U}}}.$$

Transforming back to the original functions,

$$X(z,s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{z\sqrt{\frac{k_0^2-t^2}{1}\frac{\rho_0W_0}{U}}}e^{ist}dt.$$

Unfortunately the author is stuck here. The inverse transform is probably not solvable in closed form, so approximate solutions must be investigated.

The behaviour of a beam over short distances with wakefields and no energy spread must be essentially the opposite of that for energy spread and no wakefields as the two effects compensate each other in BNS damping. This is of course only approximately true as the wakefield alone eventually causes a blow up in amplitude (as was shown in the two charge approximation) and focusing spread alone involves no amplitude growth.

From all these arguments the beam shape evolves to a fraction of a cosine when BNS damping is not applied correctly in either limit. A device which measures the average slope of the beam, the tilt, could be used for determined the level of BNS damping needed for a particular machine set up and thus for adjusting RFQ strengths. However such a device would not be sensitive to the ever increasing spatial frequency the beam would pick up if it evolved uncorrected beyond a half cosine. Thus a tilt measurement should be installed at least once every 10 betatron cycles - there are about 50 betatron cycles in a 250 GeV CLIC linac.

The measurement periodicity is however probably determined by effects not included in the smooth focusing model. The bunch must be controlled closely to limit uncorrectable effects like filamentation and the tolerance is probably passed before the beam shape has evolved beyond a quarter sine wave. This periodicity constraint can only be determined from beam simulations and is beyond the scope of this paper.

To verify that the beam tilt is a sensitive measure of RFQ tune beam tracking simulation results are considered. An example [9] of beam displacements at the end of a sector when RFQ strengths are varied is shown in figure 1. α is the ratio of the RFQ focusing strength to the static quadrupole focusing strength. Corresponding emittances are shown in figure 2.

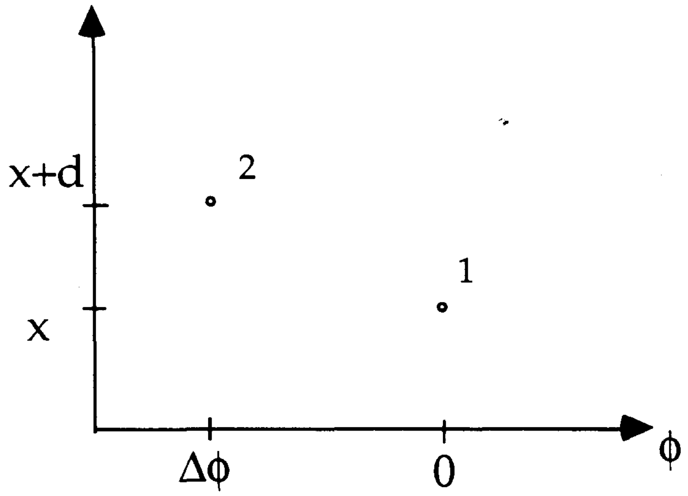
One can see that the beam tilts gradually as the RFQs are misadjusted. Also by eye the least tilt also corresponds to the best emittance. This means that even with a full emittance measurement mainly tilt is being adjusted and not the narrower features. The narrower features are thought to originate from earlier in the linac - the 4th of 5 sectors is being adjusted in this case.

II Excitation of a resonant cavity with a tilted beam

The device which will now be considered to measure beam tilt is a standard CLIC beam position monitor. CLIC beam position monitors are TM_{110} resonant cavities. Tilt is measured as natural result of the finite phase extension of bunches. A tilted bunch which is very short compared to the wavelength of the TM_{110} mode does not excite the mode when center of charge passes through the center of the cavity because the effects of head and tail exactly cancel. If the bunch has a finite phase extension then there is no longer an exact cancellation because the fields excited by the head evolve before the tail arrives. The effect can be used for CLIC because the beam is .17 mm long (1 σ) which corresponds to 6° of 30 GHz phase.

The tilt signal manifests itself as a minimum voltage that the beam produces when it is scanned through the center of the cavity. The minimum voltage signal is 90° out of phase with the signal for large beam offsets.

This effect can be clearly seen if the bunch is approximated by two charges of charge 1/2, separated by a phase $\Delta\phi$ and a transverse offset d,



The voltages induced in the cavity by the two charges are,

$$V_1 = \frac{1}{2} x e^{i0} \qquad V_2 = \frac{1}{2}(x+d)e^{i\Delta\varphi}$$

The total voltage is,

$$V_{total} = \frac{1}{2}\left(x + (x+d)e^{i\Delta\varphi}\right)$$

The square of the magnitude is,

$$V_{tot}V_{tot}^* = \frac{1}{4}\left(x^2 + (x+d)^2 + 2x(x+d)\cos(\Delta\varphi)\right)$$

A minimum occurs when,

$$\frac{\partial V_{tot}V_{tot}^*}{\partial x} = (2x+d)\left(1+\cos(\Delta\varphi)\right) = 0$$

The minimum occurs when the center of charge is on axis,

$$x = -\frac{d}{2}$$

The minimum voltage is,

$$V_{min} = \frac{d}{2}\sqrt{\frac{1-\cos(\Delta\varphi)}{2}}.$$

Approximating the CLIC bunch by two charges .2 mm apart the minimum voltage is,

$$V_{min} \approx \frac{d}{30}$$

This means that the BPM is 1/30th as sensitive to tilt as it is to center of mass displacement. This may seem to be hopeless but the measurement performance limit for tilt is resolution whereas for beam position it is precision.

A tilt measurement is made by scanning a BPM so the beam passes through the center of the BPM to find the minimum voltage. The scan can be made moving the cavity with a piezo electric crystal (PZT) since the movement resolution must be small but does not need to be calibrated. The measurement would be fast as a scan taking 20 steps would only require .01 sec.

We can now return to the beam simulation results shown in figure 1 to estimate the signals that can be expected from a BPM tilt measurement. The signal strength is determined by approximating the bunch by 40 slices, referring displacements to the center of charge, then adding the complex voltages induced by the slices . Figure 3 shows the values determined from the data in figure 1. Clearly the resolution of the BPM must be in the .05 μm range or below.

The dynamic range of the detection electronics must be large and the beam deflections limited when making a tilt measurement. Other than this the limit to resolution due to the BPM signal processing system should be minor.

A more serious limit to the performance of the BPM for a tilt measurement is the alignment of the BPM with respect to the trajectory of the bunch. A skew of the BPM results in an out of phase excitation exactly like the tilt measurement. The excitation depends on the length of the BPM, 3.332 mm, rather than the bunch length. A BPM a quarter wavelength long, 2.5 mm, would have a skew to displacement sensitivity ratio of $1/\sqrt{2}$ (as opposed to 1/30 for beam tilt). To keep the resolution to below .1 μm the BPM must be aligned to about the same value over its active length of a few mm This corresponds to an angle of .1 mrad. Since it is probably not possible to pre-align tilt monitors to this precision, the tilt monitors should be mounted on a pair of PZTs and nulled iteratively by displacing and changing the angle. This will increase measurement time by roughly a square, so 400 points or 0.3 sec. The tilt monitors could also be aligned using a special low charge beam (less affected by wakefields) as a reference.

A shorter cavity would have a lower skew to displacement sensitivity ratio. The improvement is limited because field lines will always extend into the beam pipes every cavity must have. The beam pipe diameter could be reduced below the current 4 mm but this means of improvement is limited by an unacceptable increase in wakefields.

Conclusions

A beam tilt measurement is appropriate for adjusting the level of BNS damping in the CLIC main linac.

CLIC BPMs mounted on PZTs may be sensitive enough to fulfil the role of a tilt monitor. Such a tilt monitor would be relatively simple and inexpensive so could be installed as often as needed. The measurement is not destructive so some trajectory corrections could be made in parallel.

Beam simulations must confirm that the tilt monitor sensitivity is good enough.

Parallelism of the tilt monitor to beam trajectory seems to be the most critical issue for the sensitivity. This issue is general to any emittance or emittance approximating measurement which uses a detection element of finite length to measure a bunch of finite length.

Acknowledgements

The author wishes to thank G. Guignard, G. Parisi, J. Sladen, W. Schnell and I. Wilson for their help.

References

[1] G. Guignard, CLIC note 215 and references contained within.

[2] J. P. H. Sladen, I Wilson, W. Wuensch, CLIC note 189.

[3] B. Montague, Minutes of the CLIC meeting held on 16 October 1992.

[4] G. Parisi, Minutes of the CLIC meeting held on 3 March 1994.

[5] H. Henke, W. Schnell, CLIC note 22.

[6] K. Bane, IEEE Transactions on Nuclear Science, Vol NS-32, No. 5, October 1985.

[7] V.Balakin, A Novokhalsky, V. Smirnov, Proc. 12th Int. Conf on High Energy Accelerators, 1983.

[8] H. Henke, CLIC note 40.

[9] G. Guignard, G. Parisi private communication.

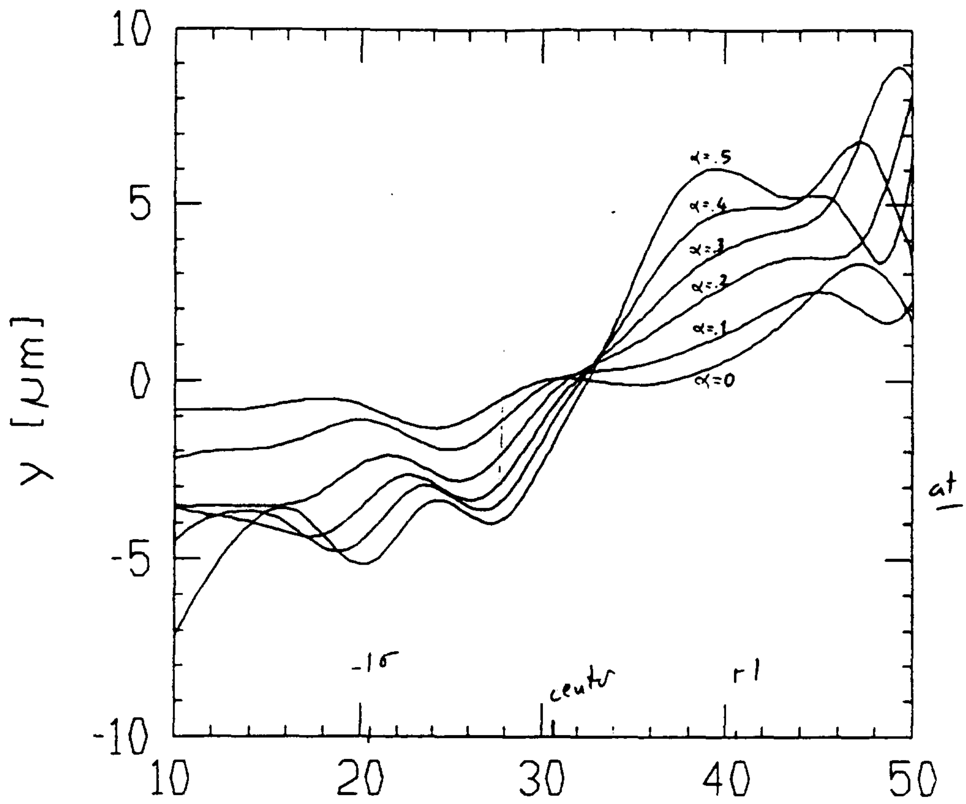


Figure 1: Beam displacement as a function of slice number for various values of α. The data is given for the end of the 4th sector of the linac in a focusing quadrupole.

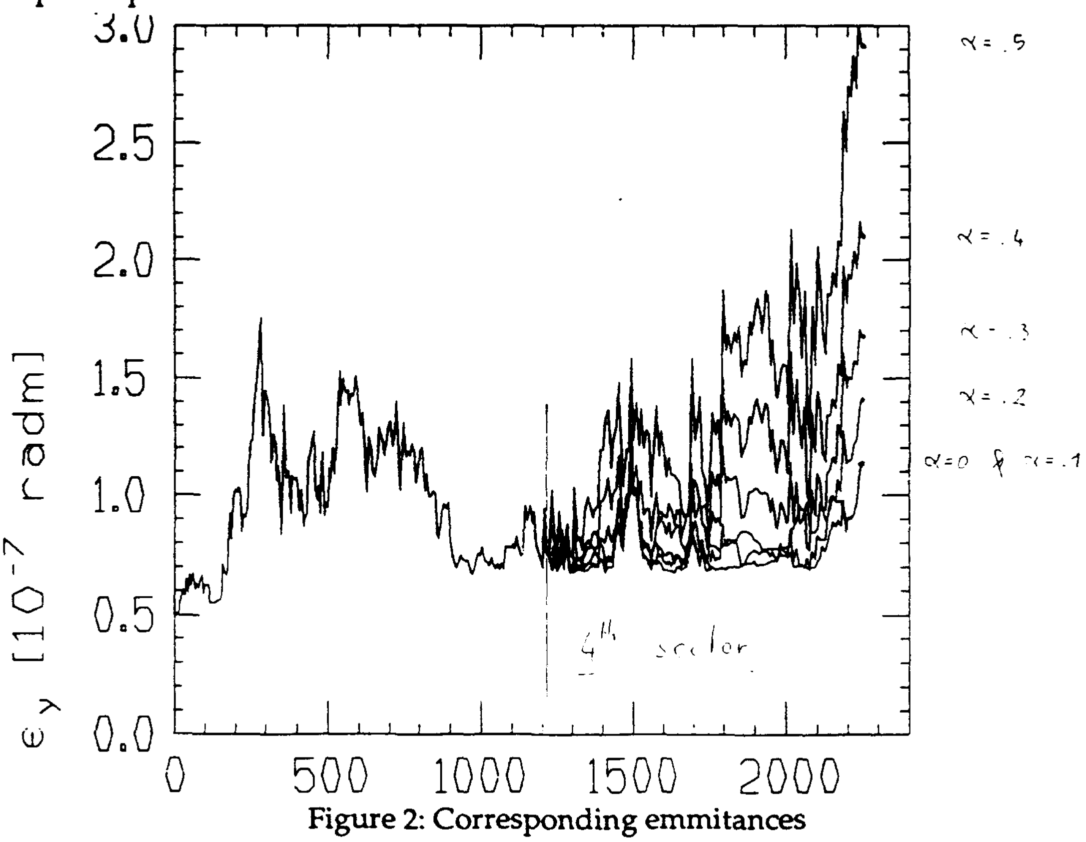


Figure 2: Corresponding emmitances

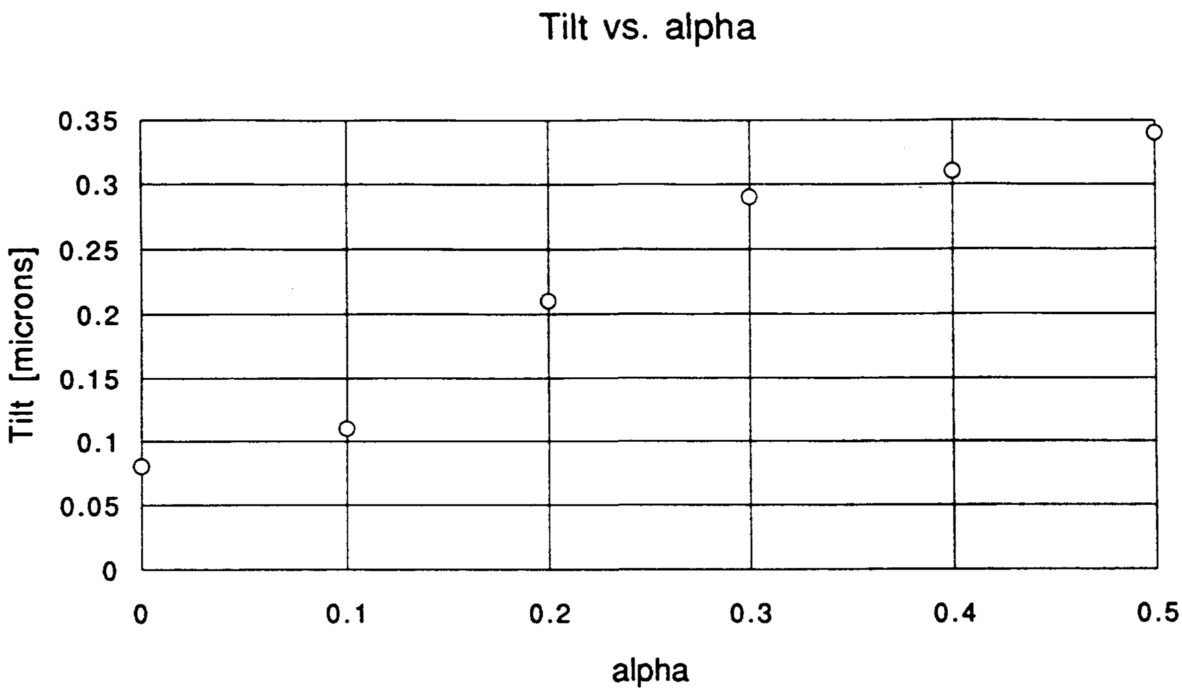


Figure 3: Beam tilt monitor response as a function of α.