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On Measuring the Kinetic Energy of the Heavy Quark Inside B Mesons

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Abstract

We discuss how one can determine the average kinetic energy of the heavy quark inside heavy mesons from differential distributions in semileptonic B decays. A new, the so-called third, sum rule for the $b \to c$ transition is derived in the small velocity (SV) limit. Using this sum rule and the measured momentum dependence of the $B \to D^*$ transition (the slope of the Isgur-Wise function), we obtain a new lower bound on the parameter $\mu_\pi^2 = (2M_B)^{-1}\langle B|\bar{b}(i\vec{D})^2 b|B\rangle$ proportional to the average kinetic energy of the b quark inside a B meson. Existing data suggest $\mu_\pi^2 > 0.4$ GeV^2 and (from the "optical" sum rule) $\overline{\Lambda} > 500$ MeV, albeit with some numerical uncertainties.

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1. It has been shown in two recent papers [1, 2] how the operator product expansion (OPE) allows one to derive various useful sum rules for heavy flavour transitions in the small velocity (SV) limit [3]. Non-perturbative corrections are incorporated into the theoretical side of the sum rules in the form of an expansion in inverse powers of the heavy quark mass. In Ref. [2] the so-called first sum rule at zero recoil was obtained, which was then used for estimating the deviation of the $B \to D^*$ transition form factor from unity at zero recoil to order $\mathcal{O}(\Lambda^2_{QCD})$. Another sum rule analysed in Ref.[1] yields a field-theoretic proof of the inequality

$$\mu_\pi^2 > \mu_G^2 \,, \tag{1}$$

where μ_π^2 and μ_G^2 are related to the kinetic energy and the chromomagnetic operators,

$$\mu_\pi^2 = \frac{1}{2M_B}\langle B|\bar{b}(i\vec{D})^2 b|B\rangle, \;\; \mu_G^2 = \frac{1}{2M_B}\langle B|\bar{b}(i/2)\sigma G b|B\rangle. \tag{2}$$

(This inequality had previously been obtained within a quantum-mechanical approach [4, 5].) In this paper we exploit similar ideas to obtain a new sum rule in the SV limit, which relates μ_π^2 to the expectation value of the square of the excitation energy of the final hadronic state X_c in $B \to l\nu X_c$ transitions. At present the corresponding inclusive differential distribution has not been measured yet. Instead we use the measured slope of the Isgur-Wise function as extracted from $B \to D^* l\nu$ decays near zero recoil to get a lower bound on μ_π^2, without any reference to μ_G^2. The resulting bound turns out to be numerically close to that of Eq. (1).

2. The general method for deriving sum rules in the SV limit is presented in Ref. [1]. Here we restate only some basic points, primarily to introduce the relevant notations. The OPE is applied to the transition operator [6, 7]

$$\hat{T}_{ab}(q) = i\int d^4x\, e^{iqx}\, T\{j_a^\dagger(x) j_b(0)\}\,, \tag{3}$$

where j_a denotes a current of the type $\bar{c}\Gamma_a b$ with an arbitrary Dirac matrix Γ_a; q is the momentum carried by the lepton pair. The average of $\hat{T}_{ab}$ over the heavy hadron state H_b with momentum p_{H_b} represents a forward scattering amplitude (the so-called hadronic tensor),

$$h_{ab}(p_{H_b}, q) = \frac{1}{2M_{H_b}}\langle H_b|\hat{T}_{ab}|H_b\rangle \;. \tag{4}$$

The observable distributions are expressed through structure functions w_{ab}: $w_{ab} = (1/i)\, disc\; h_{ab}$. The hadronic tensor can be decomposed in terms of the possible covariants [7] (their number depends on the Lorentz structure of the currents) with coefficients h_i. In the case of vector and axial-vector currents we deal with the functions h_i^{VV}, h_i^{AA} and h_i^{VA}, $i = 1, ..., 5$, introduced in Ref. [8]. In the HQET limit

[9] – when one neglects $1/m_{b,c}$ corrections –, the hadronic tensor h_{ab} is defined by a single invariant function h for any matrix Γ_a in the current j_a, namely:

$$h_{ab} \;=\; C_{ab} h\,, \quad C_{ab} \;=\; Tr\left[\frac{1+\not v_1}{2}\overline{\Gamma}_a \frac{1+\not v_2}{2}\Gamma_b\right]\,. \tag{5}$$

Here $\overline{\Gamma}_a = \gamma_0 \Gamma_a^\dagger \gamma_0$ and $v_{1\mu}$, $v_{2\mu}$ are 4-velocities of the initial and final hadrons,

$$v_{1\mu} \;=\; \frac{(p_{H_b})_\mu}{M_{H_b}}\;, \quad v_{2\mu} \;=\; \frac{(p_{H_b}-q)_\mu}{M_{H_c}}\;, \tag{6}$$

(M_{H_b} and M_{H_c} can be substituted by m_b and m_c, respectively, to leading order). The function h depends on two scalar invariants available in the process, namely $(v_1 q)$ and q^2. In what follows we will assume the hadron H_b to be at rest; the first invariant then reduces to q_0. Moreover, in studying the transitions $b \to c$ at zero recoil or in the small velocity (SV) limit, it is convenient to employ directly the space-like momentum transfer $\vec{q}^{\,2} = (v_1 q)^2 - q^2$ as the second argument of h. More specifically, for vector and axial-vector currents one has to leading order:

$$h = \frac{h_1^{AA}}{1+v_1 v_2} = \frac{1}{2}\frac{m_c}{m_b} h_2^{AA} = -m_c h_5^{AA};$$

$$h = \frac{h_1^{VV}}{1-v_1 v_2} = -\frac{1}{2}\frac{m_c}{m_b} h_2^{VV} = m_c h_5^{VV}; \qquad h = -m_c h_3^{VA}.$$

The functions h_i not listed here vanish in this approximation. The expressions for all h_i up to order $1/m_b^2$ when the factorization (5) is broken can be found in [8].

The factorization of $h_{\alpha\beta}$ into a universal kinematical structure multiplied by a single hadronic function h in general ceases to be valid in higher orders in $1/m_{c,b}$. Yet it still holds for those corrections that are relevant for the third sum rule to be derived below. We will explain this point shortly.

Since it does not matter which hadronic function we deal with – they all lead to one and the same third sum rule – we will use h_1^{AA} in our derivation. Thus, we consider the transitions of the B meson induced by the axial-vector current $A_\mu = \bar{c}\gamma_\mu\gamma_5 b$. To isolate h_1^{AA} one considers the spatial components of the axial current generating the transitions of the B meson to D^* and the corresponding higher excitations. In [2] sum rules at zero recoil ($\vec{q}=0$) were obtained; here we will work at small, but non-vanishing values of $|\vec{q}|$. The terms $\mathcal{O}(\vec{q}^{\,2})$ will be kept while those of higher order in $|\vec{q}|$ will be neglected. First we consider $h_1^{AA}(q_0,\vec{q})$ in the complex q_0 plane ($\vec{q}$ is assumed to be fixed, and $\Lambda_{QCD} \ll |\vec{q}| \ll M_D$) and shift q_0 by introducing the quantity

$$\epsilon = q_{0max} - q_0,\; q_{0max} = M_B - E_{D^*},\;\; E_{D^*} = M_{D^*} + \frac{\vec{q}^{\,2}}{2M_{D^*}}. \tag{7}$$

The physical cut is characterized by ϵ real and positive. The imaginary part of h_1 is given by the "elastic" contribution of D^* plus inelastic excitations. For what follows it is crucial that all these contributions are *positive-definite*.

Negative ϵ describe the region below the cut, and the amplitude h_1^{AA} can be computed – and it actually was [8, 10, 11] – as an expansion in $\Lambda_{QCD}/m_{c,b}$. For our purposes it is sufficient to limit ourselves to the corrections of first and second order in Λ_{QCD}. This is exactly the approximation adopted in [8, 10, 11], and expressions obtained there will be used below.

At the next stage we assume $\Lambda_{QCD} \ll |\epsilon| \ll m_{b,c}$. The amplitude h_1^{AA} is expanded in powers of Λ_{QCD}/ϵ and $\epsilon/m_{b,c}$. Polynomials in ϵ can be discarded since they have no imaginary part. We are interested only in negative powers of ϵ. The coefficients in front of $1/\epsilon^n$ are related, through dispersion relations, to the integrals over the imaginary part of h_1^{AA} with weight functions proportional to the excitation energy to the power $n-1$. Thus, the first sum rule considered in Ref. [2] corresponds to $n=1$; the second sum rule (sometimes called optical or Voloshin's sum rule [12], see also [13, 14]) corresponds to $n=2$. The lower bound on μ_π^2 – our main aim in this work – stems from the third sum rule, i.e. we need to analyse the coefficient in front of $1/\epsilon^3$ in the expansion of h_1.

The $1/\epsilon$ expansion can be read off from Eq. (A.1) in Ref. [8]. One technical element of the derivation deserves a comment. The theoretical expression for the amplitude h_1^{AA} presented in [8] contains only the quark masses without any reference to the meson masses. It is then convenient to expand h_1^{AA} first in an auxiliary quantity,

$$\epsilon_q = m_b - E_c - q_0, \quad E_c = m_c + \frac{\vec{q}^2}{2m_c}\,. \tag{8}$$

Then, if necessary, we reexpress the expansion obtained in this way in terms of ϵ. The difference between ϵ_q and ϵ is $\mathcal{O}(\Lambda_{QCD}\cdot\vec{q}^2/m_{b,c}^2)$ and $\mathcal{O}(\Lambda_{QCD}^2/m_{b,c})$. It will be seen shortly that for our purposes this difference can simply be ignored in the third sum rule in the SV limit. It cannot be discarded, however, in the second sum rule. (The situation is quite different from what took place in the sum rules at zero recoil, see [1]: there the difference between ϵ and ϵ_q is absolutely essential for the $n\geq 2$ sum rules.)

The expression for h_1^{AA} in Eq. (A.1) in [8] has the form

$$-h_1^{AA} = [(m_b+m_c-q_0)+\mathcal{O}(\Lambda_{QCD}^2/m_b)]\frac{1}{z}+\mathcal{O}(\Lambda_{QCD}^2)\frac{1}{z^2}+\frac{4}{3}(m_b+m_c-q_0)\mu_\pi^2\vec{q}^2\frac{1}{z^3} \tag{9}$$

$$z = \epsilon_q(2E_c+\epsilon_q). \tag{10}$$

Notice the similarity of the coefficient in front of $1/z^3$ and the leading part of the coefficient in front of $1/z$. This is not accidental. The terms $1/z^3$ appear only in the expansion of the denominator $(m_bv-q)^2-2q\pi$ to second order in πq (see Ref. [8]) and, therefore, preserve the same universal factorization pointed out above in the HQET limit.

Expanding in $\epsilon_q/2E_c$ we observe that $1/z^n$ reduces to $1/\epsilon_q^n$, plus lower powers of $1/\epsilon_q$, plus a polynomial in ϵ_q. Next one eliminates ϵ_q in favour of ϵ. The term $1/z^3$ comes with a coefficient $\mu_\pi^2\cdot\vec{q}^2$; hence the difference between $1/\epsilon$ and $1/\epsilon_q$ is here of higher order and can be neglected. Likewise, to $\mathcal{O}(\Lambda^2_{QCD})$ one can replace $1/\epsilon_q$ by $1/\epsilon$ in $1/z^2$. As far as $1/z$ is concerned, we must reexpress $1/\epsilon_q$ in terms of $1/\epsilon$,

$$\frac{1}{\epsilon_q} = \frac{1}{\epsilon} + \frac{(\epsilon - \epsilon_q)}{\epsilon^2} + ... \tag{11}$$

The next terms in Eq. (11) are irrelevant since they lead to corrections of higher order in Λ_{QCD} and/or $|\vec{q}|$. This observation is crucial, since it tells us that the $1/z$ part contributes only to the first and the second sum rules; it generates no $1/\epsilon^3$ terms. As a result h_1^{AA} has the form

$$-h_1^{AA} = \frac{1}{\epsilon}\left(1 - \frac{\vec{q}^2}{4m_c^2} + \mathcal{O}(\Lambda^2_{QCD}/m_c^2)\right) + \frac{1}{\epsilon^2}\left(\mathcal{O}(\Lambda^2_{QCD}/m_c) + \mathcal{O}(\Lambda_{QCD}\vec{q}^2/m_c^2)\right) +$$

$$\frac{1}{\epsilon^3}\frac{\mu_\pi^2}{3}\frac{\vec{q}^2}{m_c^2} + polynomial\,, \tag{12}$$

where only the terms $\mathcal{O}(\vec{q}^2)$ are kept. We do not discuss perturbative corrections either. Writing out the dispersion relation in ϵ,

$$-h_1^{AA}(\epsilon, \vec{q}^2) = \frac{1}{2\pi}\int d\tilde{\epsilon}\,\frac{w_1^{AA}(\tilde{\epsilon}, \vec{q}^2)}{\epsilon - \tilde{\epsilon}} =$$

$$\frac{1}{\epsilon}\cdot\frac{1}{2\pi}\int d\tilde{\epsilon}\, w_1^{AA}(\tilde{\epsilon}, \vec{q}^2) + \frac{1}{\epsilon^2}\cdot\frac{1}{2\pi}\int d\tilde{\epsilon}\,\tilde{\epsilon}\, w_1^{AA}(\tilde{\epsilon}, \vec{q}^2) + \frac{1}{\epsilon^3}\cdot\frac{1}{2\pi}\int d\tilde{\epsilon}\,\tilde{\epsilon}^2 w_1^{AA}(\tilde{\epsilon}, \vec{q}^2) + ... \tag{13}$$

and expanding it in $1/\epsilon$ we get the sum rules by equating the coefficients in front of $1/\epsilon^n$. Here $w_1^{AA} = 2\,Im\,h_1^{AA}$.

3. Now we discuss the phenomenological side of the sum rule. The structure function w_1^{AA} is non-vanishing for positive ϵ,

$$w_1^{AA}(\epsilon) = \sum_{i=0}^{\infty}\frac{|F_{B\to i}|^2}{2E_i}\,2\pi\delta(\epsilon - \delta_i), \tag{14}$$

where the sum runs over all possible final hadronic states: the term with $i = 0$ corresponds to the "elastic" transition $B \to D^*$, while $i = 1, 2, \ldots$ represent excited states with energies $E_i = M_i + \vec{q}^2/(2M_i)$. Strictly speaking, $|F_{B\to i}|^2$ does not present the square of a form factor; rather it is the contribution to the given structure function coming from the multiplet of degenerate states, which includes summation over spin states as well. In the particular example considered, D is not produced in the elastic transition, so that in the elastic part one needs to sum only over the polarization of D^*. Therefore, the term "form factor" for $F_{B\to i}$ is rather symbolic; $|F_{B\to i}|^2$ depends on $\vec{q}$. Moreover, δ_i in Eq. (14) is the excitation energy (including the corresponding kinetic energy),

$$\delta_i = E_i - E_{D^*}.$$

For the elastic transition δ_0 vanishes, of course.

The dispersion representation (13) and (12) lead to the following sum rule for the second moment of w_1^{AA} (the coefficient in front of $1/\epsilon^3$, the third sum rule in the nomenclature of Ref. [1]):

$$\frac{1}{2\pi}\int d\epsilon \epsilon^2 w_1^{AA}(\epsilon) = \sum_{i=1}^{\infty} \frac{|F_{B\to i}|^2}{2E_i}\delta_i^2 = \frac{1}{3}\mu_\pi^2 \frac{\vec{q}^2}{m_c^2}. \tag{15}$$

A few remarks regarding Eq. (15) are in order here. First of all, since $\delta_0 = 0$, the elastic contribution drops out on the left-hand side, and the sum actually starts from the first excitation. Secondly, since all δ_i^2 are of order Λ_{QCD}^2, we need to know $F_{B\to 2}$, $F_{B\to 3}$, etc., only to zeroth order in Λ_{QCD}. To this order all transition form factors to the excited states are proportional to $\vec{q}$, i.e.

$$|F_{B\to i}|^2 \propto \vec{q}^2. \tag{16}$$

(The transitions to P-wave states are actually relevant, see [15] for further details.) Moreover, due to Eq. (16) we can neglect the $\mathcal{O}(\vec{q}^2)$ term in δ_i; thus in Eq. (15):

$$\delta_i = M_i - M_{D^*}.$$

Thirdly, m_c^{-2} on the right-hand side can be replaced, to the accuracy desired, by $(M_{D^*})^{-2}$ or by the mass of any excited state.

After all these simplifications the third sum rule takes the form

$$\sum_{i=1}^{\infty} \frac{|F_{B\to i}|^2}{2M_i}(M_i - M_{D^*})^2 = \frac{1}{3}\mu_\pi^2 \vec{v}^2, \qquad \vec{v} = \frac{\vec{q}}{M} \tag{17}$$

(it does not matter which particular mass, M_{D^*} or M_i, stands in the denominator).

The next steps are rather obvious. The lower bound on μ_π^2 is a consequence of positivity of all individual contributions in the left-hand side of Eq. (17). Indeed, let us rewrite it as follows:

$$\frac{1}{3}\mu_\pi^2 = \delta_1^2 \cdot \sum_{i=1}^{\infty} \frac{|F_{B\to i}|^2}{2M_i\vec{v}^2} + \sum_{i=2}^{\infty} \frac{|F_{B\to i}|^2}{2M_i\vec{v}^2}(\delta_i^2 - \delta_1^2). \tag{18}$$

The second term is evidently positive. The first sum can be found, in turn, by using the Bjorken sum rule [16]. This sum rule relates the sum over the P-wave states in the brackets to the $\vec{q}^2$ dependence of the "elastic" $B \to D^*$ transition (the slope of the Isgur-Wise function [17]).

4. It is instructive to briefly reiterate the derivation of the Bjorken sum rule, which, as explained above, is needed only to zeroth order in Λ_{QCD}. Equating the coefficients of $1/\epsilon$ in Eqs. (12) and (13) one immediately finds

$$\frac{1}{2\pi}\int d\epsilon\, w_1^{AA}(\epsilon) = \sum_{i=0}^{\infty} \frac{|F_{B\to i}|^2}{2E_i} = 1 - \frac{\vec{v}^2}{4}\,. \tag{19}$$

The elastic part here can be parameterized in terms of the Isgur-Wise function $\xi(v_1 v_2)$ [17, 18]. The $B \to D^*$ transition has the form

$$\langle D^*(v_2)|A_\mu|B(v_1)\rangle = \sqrt{M_B M_{D^*}}\;[\epsilon_\mu(1+v_1v_2) - (\epsilon v_1)v_{2\mu}]\,\xi(v_1v_2)\,,$$

where $v_{1,2}$ are the four-velocities. This means that

$$(2E_{D^*})^{-1}|F_{B\to D^*}|^2 = \frac{M_{D^*}}{E_{D^*}}\left(\frac{1+v_1v_2}{2}\right)^2 |\xi(v_1v_2)|^2 \approx 1 - \rho^2\vec{v}^2\,. \qquad (20)$$

Here ρ^2 is the slope parameter [16],

$$\xi(v_1v_2) = 1 - \rho^2(v_1v_2 - 1) + ... = 1 - \rho^2\frac{\vec{v}^2}{2} + ..., \qquad (21)$$

where we have used the fact that ξ at zero recoil is unity [3].

Notice that although we discuss the Bjorken sum rule for the axial current, it can actually be derived for an arbitrary current $j_a = \bar{b}\Gamma_a c$. To leading order in $1/m_{b,c}$ the universal factorization (5) takes place for the structure functions $w_{ab} = 2Im\,h_{ab}$. Moreover, the sum over any HQET degenerate multiplet of states gives

$$\frac{1}{2M_B}\sum_i \langle B|j_a^\dagger|H_c^i\rangle\langle H_c^i|j_b|B\rangle \;=\; C_{ab}M_{H_c}\frac{1+v_1v_2}{2}|\xi_{H_c}(v_1v_2)|^2\,, \qquad (22)$$

where ξ_{H_c} is the Isgur-Wise function for the H_c multiplet.

At $\vec{v}=0$ the sum rule (19) is trivially satisfied since at zero recoil all inelastic form factors vanish, and we are left with the elastic contribution, which reduces to unity. The term linear in $\vec{v}^2$ yields a relation between the slope of ξ and the inelastic contributions,

$$\rho^2 - \frac{1}{4} = \sum_{i=1}^{\infty}\frac{|F_{B\to i}|^2}{2M_i\vec{v}^2}\,. \qquad (23)$$

Let us recall that the ratio $|F_{B\to i}|^2/\vec{v}^2$ has a finite limit at zero recoil. Equation (23) is the Bjorken sum rule proper [16]. Let us add for completeness that in the notation of Ref. [15], where the P-wave inelastic contributions are written out explicitly, it takes the form

$$\rho^2 - \frac{1}{4} = \sum_{n=1}^{\infty}|\tau_{1/2}^{(n)}(1)|^2 + 2\sum_{n=1}^{\infty}|\tau_{3/2}^{(n)}(1)|^2$$

(for a simple derivation see Ref. [13]). From these expressions it follows, in particular, that $\rho^2 > 1/4$.

Combining Eq. (23) with Eq. (18), we finally arrive at

$$\mu_\pi^2 = 3\delta_1^2\left(\rho^2 - \frac{1}{4}\right) + 3\sum_{i=2}^{\infty}\frac{|F_{B\to i}|^2}{2M_i\vec{v}^2}(\delta_i^2 - \delta_1^2)\,, \quad \delta_i = M_i - M_{D^*}\,. \qquad (24)$$

Equation (24) is a direct $n = 3$ generalization of Voloshin's sum rule written for $n = 2$ [12], see also [1],

$$\overline{\Lambda} = 2\delta_1 \langle \rho^2 - \frac{1}{4} \rangle + 2 \sum_{i=2}^{\infty} \frac{|F_{B\to i}|^2}{2M_i \vec{v}^2} (\delta_i - \delta_1) \, . \tag{25}$$

Since the second term in Eq. (24) is positive we obtain the following, obvious inequality:

$$\mu_\pi^2 > 3\delta_1^2 \langle \rho^2 - \frac{1}{4} \rangle \tag{26}$$

(we recall that δ_1 here is the lowest excitation energy, $\delta_1 = M_1 - M_{D^*}$).

For a numerical estimate let us take that

$$\delta_1 \approx 500 MeV \tag{27}$$

and let us use for ρ^2 the central value of the measured slope [19] of the $B \to D^*$ form factor,

$$\rho^2 = 0.84 \pm 0.12 \pm 0.08. \tag{28}$$

Then we get

$$\mu_\pi^2 > 0.45 \; GeV^2. \tag{29}$$

With the same parameters the lower bound for $\overline{\Lambda}$ from Voloshin's sum rule is

$$\overline{\Lambda} > 590 \; MeV \, . \tag{30}$$

We will discuss shortly the numerical uncertainty in the lower bounds. Before this three comments are in order here regarding the sum rules presented above. First, the very same final results are obtained irrespective of what currents we start from, axial or vector, or a mixture of these two. The only difference is that, say, for the vector currents we would get M_D rather than M_{D^*} in the definition of δ_1. This difference is unimportant in the limit $m_{b,c} \to \infty$, of course. This remark brings us to the second point. In Eq. (26) all subleading $1/m_{b,c}$ terms have been omitted; these terms together with radiative corrections are the main source of the uncertainty in the lower bound (29). Finally, in the original sum rules the sum runs over all states including those which represent high-energy excitations described, in the sense of duality, by perturbative formulae (see Ref. [1] for more details). To get predictions for μ_π^2 and $\overline{\Lambda}$ normalized at a low (quark-mass-independent) scale μ one must truncate the sum over the excited states at $\delta_i \sim \mu$ and invoke duality between the perturbative corrections and the contributions of the excited states above μ.

In general, the sum rules at non-zero recoil get $\Lambda_{QCD}/m_{b,c}$ corrections, which depend on the particular choice of the weak current considered and can be sizeable. However, all corrections to the hadronic tensor h_{ab} start with terms explicitly proportional to $\Lambda_{QCD}^2/m_{b,c}^2$ [7, 20, 21], see Eq. (A.1) in Ref. [8]. The question is: Where do the linear corrections come from? A source of subleading corrections is quite obvious: they appear at the stage when one expresses ϵ_q in the theoretical formulae in terms of ϵ; since $M_B = m_b + \overline{\Lambda} + ...$ (and the same for the charmed quark), they contain linear terms. This does not affect, of course, the first sum rule ($n = 1$), and in this case the prediction starts from unity plus corrections at the level $\overline{\Lambda}^2/m_{b,c}^2$ [2].

5. We now proceed to a more careful discussion of the numerical uncertainties. The experimentally measured $B \to D^*$(unpolarized) $l\nu$ decay rate is expressed in terms of the Isgur-Wise function in the leading approximation, see Eq. (20). In this approximation the slope of the Isgur-Wise function is related to the $\vec{q}^2$ dependence of the $B \to D^*$ rate. It is clear that with $1/m_{b,c}$ and radiative corrections included the $\vec{q}^2$ dependence of the decay rate does not exactly coincide any more with the slope of the Isgur-Wise function. The corrections were estimated in the literature (see the review paper [22]). These estimates are consistent with the preliminary result of recent CLEO measurement [23]

$$\rho^2_{ff} = 1.01 \pm 0.15 \pm 0.09, \tag{31}$$

where form factors were extracted without use of heavy quark symmetry relations.

The difference between the values of ρ^2 in Eqs. (28), (31) is attributed to radiative and power corrections. We will use this difference to estimate possible corrections to the bounds (29, 30) as 20%.

Similar effects due to the finite mass of the c quark enter our lower bound implicitly, when we use the observed mass values of the excited charmed mesons. In the future these pre-asymptotic corrections can be isolated in a model-independent way once the masses of the beauty counterparts are measured. The most sizeable corrections are expected from the chromomagnetic interaction of the heavy quark spin inducing hyperfine splitting among the members of the heavy spin multiplets. In particular, $M_{D^*} - M_D \sim 140$ MeV. This effect is presumably accounted for by substituting the spin averaged masses for the ground S-wave states and for the P-wave excitations, rather than the actual masses of D, D^*, etc. We actually did this spin averaging. Another shift arises from the heavy quark kinetic energy term in the hadron mass. It is natural to expect its value to be smaller in the excited mesons than for the ground state. Therefore, the static limit of δ_1 is expected to be somewhat larger than the value of δ_1 experimentally observed for the actual charmed particles, but probably by not more than 50 MeV. We then use the value of δ_1 given by Eq. (27) as a very reasonable educated guess.

The lower bounds (29), (30) are seen to lie not very far from the estimates obtained earlier within QCD sum rules [24]

$$\mu_\pi^2 \sim 0.55\ GeV^2\ , \qquad \overline{\Lambda} \sim 450\ MeV\ . \tag{32}$$

Note that the lower bound on μ_π^2 in Eq. (29) is numerically close to the bound (1) derived recently in [4, 5].

Unfortunately, numerical uncertainties in all the above numbers prevent us from making a conclusive statement. Nevertheless, let us assume for a moment that future refined measurements and calculations of the subleading corrections in the third sum rule will confirm these values and establish the fact that the two inequalities in Eqs. (29), (30) are rather close to saturation. This would mean that the sum rules are actually saturated – to a reasonable degree of accuracy – by the contributions from the states with masses around $M_D + \delta_1$ generically called D^{**} in this context. To account for non-perturbative effects in $b \to c$ decays, one would then need only to consider one inelastic channel, "D^{**}". The higher excited states will be represented (in the sense of duality) by purely perturbative probabilities calculated in the free quark-gluon approximation. We actually consider such a situation as a most natural scenario in QCD. It is worth noting that the $D\pi$ contribution to the third sum rule is suppressed for soft pions, unlike the first sum rule where it was quite substantial [2]. The effect of the "hard" pion emission is well represented by some of the P-wave D^{**} resonances.

6. We have derived the third sum rule for the $b \to c$ transition in the SV limit and showed how one can use it to constrain the kinetic energy parameter μ_π^2 by using the data on $B \to D^*$. In principle it is quite conceivable that the full differential distribution in q_0 and $\vec{q}^2$ in the inclusive semileptonic B decays will be measured in the future. This measurement can then be immediately translated in the value of μ_π^2, one of the most important parameters of the heavy quark physics. The more one will learn about the decays to the excited states, the more accurate the determination of μ_π^2 will become.

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