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SUPERSYMMETRIC FIELD THEORY

FROM

SUPERMATRIX MODELS

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ABSTRACT

We show that the continuum limit of one-dimensional $\mathcal{N} = 2$ supersymmetric matrix models can be described by a two-dimensional interacting field theory of a massless boson and two chiral fermions. We interpret this field theory as a two-dimensional $\mathcal{N} = 1$ supersymmetric theory of two chiral superfields, in which one of the chiral superfields has a non-trivial vacuum expectation value.

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To use $d = 1$ matrix models [1, 2] for the purpose of understanding non-perturbative effects in superstring theory, it is essential to first construct the complete two-dimensional effective Lagrangian for the associated $d = 2$ superstring theory. We present the effective Lagrangian and discuss its properties. Our presentation is based on [3]. More details and explicit calculations are included there.

A class of $d = 1$, $\mathcal{N} = 2$ supermatrix models may be defined using a matrix superfield,

$$\Phi_{ij} = M_{ij}(t) + i\theta_1 \Psi_{1ij}(t) + i\theta_2 \Psi_{2ij} + i\theta_1\theta_2 F_{ij}(t), \tag{1}$$

where θ_1 and θ_2 are real anticommuting parameters, M_{ij} and F_{ij} are $N \times N$ bosonic Hermitian matrices and Ψ_{1ij} and Ψ_{2ij} are $N \times N$ fermionic Hermitian matrices. A manifestly invariant Lagrangian,

$$L = \int d\theta_1 d\theta_2 \left\{ \frac{1}{2} Tr D_1 \Phi D_2 \Phi + iW(\Phi) \right\}, \tag{2}$$

can be written, using Φ and the covariant derivatives

$$D_I = \frac{\partial}{\partial \theta_I} + i\theta_I \frac{\partial}{\partial t}, \quad I = 1, 2. \tag{3}$$

The superpotential W is a real polynomial in Φ,

$$W(\Phi) = \sum_n b_n Tr \Phi^n. \tag{4}$$

In terms of the component functions, Lagrangian (2) is the following

$$\begin{aligned} L &= \sum_{ij} \left\{ \frac{1}{2}(\dot{M}_{ij}\dot{M}_{ji} + F_{ij}F_{ji}) + \frac{\partial W(M)}{\partial M_{ij}} F_{ij} \right\} \\ &\quad - \frac{i}{2}\sum_{ij}(\Psi_{1ij}\dot{\Psi}_{1ji} + \Psi_{2ij}\dot{\Psi}_{2ji}) - i\sum_{ijkl} \Psi_{1ij} \frac{\partial^2 W(M)}{\partial M_{ij} \partial M_{kl}} \Psi_{2kl}. \end{aligned} \tag{5}$$

The supersymmetry transformations of the component functions are

$$\begin{aligned} \delta M_{ij} &= i\eta^1 \Psi_{1ij} + i\eta^2 \Psi_{2ij} \\ \delta \Psi_{1ij} &= \eta^1 \dot{M}_{ij} + \eta^2 F_{ij} \\ \delta \Psi_{2ij} &= \eta^2 \dot{M}_{ij} - \eta^1 F_{ij} \\ \delta F_{ij} &= i\eta^2 \dot{\Psi}_{1ij} - i\eta^1 \dot{\Psi}_{2ij}, \end{aligned} \tag{6}$$

where η^1 and η^2 are anticommuting constants.

The classical theory possesses, in addition to supersymmetry, a global $U(N)$ symmetry. To verify this fact note that Φ_{ij} remains a Hermitian matrix of superfields under the transformation $\Phi \to \mathcal{U}^\dagger \Phi \mathcal{U}$, where $\mathcal{U}$ is an arbitrary $N \times N$ matrix of complex numbers. The Lagrangian is invariant under such a transformation, provided that $\mathcal{U} \in U(N)$.

We restrict our attention to the sector of the theory that is a singlet under the global $U(N)$ symmetry. After eliminating the auxiliary fields using their equations of motion, the singlet sector can be described in terms of the eigenvalues λ_i of the bosonic matrix M, and their fermionic superpartners χ_i. χ_i are the diagonal elements of the matrix $\chi = U\Psi U^\dagger$, where U is the matrix used to diagonalize M. Note that U diagonalizes M, but that χ is not diagonal. The Lagrangian for the singlet sector is given by [4, 3]

$$\begin{aligned} L &= \sum_i \left\{ \frac{1}{2}\dot{\lambda}_i^2 - \frac{1}{2}\left(\frac{\partial W}{\partial \lambda_i}\right)^2 - \frac{\partial \mathcal{W}}{\partial \lambda_i}\frac{\partial W}{\partial \lambda_i} - \frac{1}{2}\left(\frac{\partial \mathcal{W}}{\partial \lambda_i}\right)^2 - \frac{i}{2}(\bar{\chi}_i \dot{\chi}_i - \dot{\bar{\chi}}_i \chi_i) \right\} \\ & \quad - \sum_{ij} \left\{ \frac{\partial^2 W}{\partial \lambda_i \partial \lambda_j} \bar{\chi}_i \chi_j + \frac{\partial^2 \mathcal{W}}{\partial \lambda_i \partial \lambda_j} \bar{\chi}_i \chi_j \right\}. \end{aligned} \tag{7}$$

The induced superpotential,

$$\mathcal{W} = -\sum_{j \neq i} \ln|\lambda_i - \lambda_j|, \tag{8}$$

represents a repulsive interaction between the bosonic eigenvalues.

In preparation for taking the continuum limit, it is useful to change variables, thus defining three collective fields,

$$\begin{aligned} \varphi(x,t) &= \sum_i \Theta(x - \lambda_i(t)) \\ \psi(x,t) &= -\sum_i \delta(x - \lambda_i(t))\chi_i(t) \\ \bar{\psi}(x,t) &= -\sum_i \delta(x - \lambda_i(t))\bar{\chi}_i(t). \end{aligned} \tag{9}$$

Note that (9) is nothing but a change of variables. It does not increase or decrease the number of dynamical variables. In terms of the collective fields,

$$L = \int dx \left\{ \frac{\dot{\varphi}^2}{2\varphi'} - \frac{1}{2}\varphi' W'(x)^2 + \frac{W''(x)}{\varphi'}\bar{\psi}\psi \right.$$

$$
\begin{aligned}
&-\frac{1}{2\varphi'}(\bar{\psi}\dot{\bar{\psi}}+\dot{\bar{\psi}}\bar{\psi})+\frac{i}{2}\frac{\dot{\varphi}}{\varphi'^2}(\bar{\psi}\psi'-\bar{\psi}'\psi)\bigg\}\\
&+\frac{1}{3}\fint dxdydz\frac{\varphi'(x)\varphi'(y)\varphi'(z)}{(x-y)(x-z)},\\
&+\fint dxdy\frac{\varphi'(x)\varphi'(y)}{(x-y)}W'(x)\\
&+\fint\frac{1}{(x-y)}\bigg\{\bar{\psi}(x)\psi'(y)-\frac{\varphi''(y)}{\varphi'(x)}\bar{\psi}(x)\psi(x)\bigg\}.
\end{aligned}
\tag{10}
$$

The measure of the path integral has, of course, to be changed accordingly. We will not do that explicitly here, since we will be interested in regions of parameter space in which the measure takes a simple form.

The continuum limit of bosonic matrix models is known to be a two-dimensional field theory [5, 6]. We expect, therefore, that the continuum limit of supermatrix models is a two-dimensional field theory as well. However, we will see that the number of fields that survive in the continuum limit is larger in the supersymmetric case. Taking the continuum limit consists of a few separate and independent steps which supply the original supermatrix models with additional information and should be considered as part the definition of the theory. At each step some choices have to be made, each determines essential properties of the resulting models. It is at this juncture that the field content and specific background are chosen. Previously, some attempts were made, with varying degree of success, to obtain the correct two-dimensional continuum field theory [4, 7–10]. The first step, necessary to ensure that the number of dynamical variables is enough to describe a two-dimensional field theory is simply

$$N \to \infty. \tag{11}$$

It is useful to think about N as the cutoff, in momentum space, of the theory. Then a regularization procedure has to be chosen to ensure that all terms in the Lagrangian are finite as the cutoff is taken to infinity. We use

$$\fint_{-\infty}^{+\infty} dx\frac{\phi(x)}{x-a} = \pm i\pi\phi(a) \tag{12}$$

to define our regularization scheme. In addition, some dependence of the coupling parameters in the superpotential W, on the cutoff N, has to be chosen, placing the theory within a specific universality class. On general grounds, $W = NW(\frac{x}{\sqrt{N}})$. Our choice is the following

$$\begin{aligned} W(x) &= \sqrt{N} c_1 x + \frac{1}{6} \frac{c_3}{\sqrt{N}} x^3 + \cdots \\ & c_1 c_3 < 0. \end{aligned} \tag{13}$$

The terms denoted by $\cdots$ are of higher power in $x/\sqrt{N}$ and do not change the universality class for non-vanishing $c_i,\ i = 1, 3$. For completeness we list some useful expressions for the derivatives of W

$$\begin{aligned} W'(x) &= \sqrt{N} c_1 + \frac{1}{2} \frac{c_3}{\sqrt{N}} x^2 + \cdots, \\ W'(x)^2 &= N c_1^2 + c_1 c_3 x^2 + \frac{1}{3} \frac{c_1 c_4}{\sqrt{N}} x^3 + \cdots, \\ W''(x) &= \frac{c_3}{\sqrt{N}} x + \cdots. \end{aligned} \tag{14}$$

The result of applying all the steps above to the Lagrangian (10), taking (14) into account, is the continuum Lagrangian

$$\begin{aligned} L = \int dx \Bigg\{ & \frac{\dot{\varphi}^2}{2\varphi'} \pm \frac{\pi^2}{6} \varphi'^3 + \frac{1}{2} \omega^2 x^2 \varphi' \\ & - \frac{i}{2\varphi'} (\psi_1 \dot{\psi}_1 + \psi_2 \dot{\psi}_2) \pm \frac{i\pi}{2} \psi_1 \psi_1' \pm \frac{i\pi}{2} \psi_2 \psi_2' \\ & + \frac{i}{2} \frac{\dot{\varphi}}{\varphi'^2} (\psi_1 \psi_1' + \psi_2 \psi_2') \Bigg\}. \end{aligned} \tag{15}$$

Note that there are still three ambiguous signs in the previous Lagrangian, related to the sign ambiguity in (12). For the first sign we choose a minus sign, corresponding to Minkowski spacetime. Our choice is a minus sign for the second and a plus sign for the third. This choice determines the chiralities of the fermions.

A classical static solution of the equations of motion derived from (15) is given by

$$\begin{aligned} \psi_{10} &= 0 \\ \psi_{20} &= 0 \\ \varphi_0' &= \frac{1}{\pi} \sqrt{\omega^2 x^2 - 1/g}. \end{aligned} \tag{16}$$

We expand around that classical solution

$$
\begin{aligned}
\varphi &= \varphi_0(x) + \frac{1}{\sqrt{\pi}}\zeta \\
\psi_+ &= \frac{2^{1/4}}{\sqrt{\pi}}\psi_1 \\
\psi_- &= \frac{2^{1/4}}{\sqrt{\pi}}\psi_2
\end{aligned}
\tag{17}
$$

and change coordinates,

$$
\begin{aligned}
\tau'(x) &= \frac{1}{\pi}(\varphi_0'(x))^{-1} \\
&= \frac{1}{\sqrt{\omega^2 x^2 - 1/g}},
\end{aligned}
\tag{18}
$$

to obtain

$$
\begin{aligned}
L = & \int d\tau \Big\{ \frac{1}{2}(\dot{\zeta}^2 - \zeta'^2) - \frac{i}{\sqrt{2}}(\psi_+\dot{\psi}_+ - \psi_+\psi_+') - \frac{i}{\sqrt{2}}(\psi_-\dot{\psi}_- + \psi_-\psi_-') \\
& - \frac{1}{2}\frac{\mathcal{g}(\tau)\dot{\zeta}^2\zeta'}{1 + \mathcal{g}(\tau)\zeta'} - \frac{1}{6}\mathcal{g}(\tau)\zeta'^3 \\
& + \frac{i}{\sqrt{2}}\frac{\mathcal{g}(\tau)\zeta'}{1 + \mathcal{g}(\tau)\zeta'}(\psi_+\dot{\psi}_+ + \psi_-\dot{\psi}_-) \\
& + \frac{i}{\sqrt{2}}\frac{\mathcal{g}(\tau)\dot{\zeta}}{(1 + \mathcal{g}(\tau)\zeta')^2}(\psi_+\psi_+' + \psi_-\psi_-') \Big\} + \frac{1}{3}\int d\tau \frac{1}{\mathcal{g}(\tau)^2}.
\end{aligned}
\tag{19}
$$

The coupling parameter of the theory varies in space

$$
\mathcal{g}(\tau) = 4\sqrt{\pi} g \frac{\frac{1}{\kappa}e^{-2\omega(\tau - \tau_0)}}{(1 - \frac{1}{\kappa}e^{-2\omega(\tau-\tau_0)})^2}. \tag{20}
$$

We are now in a position to take stock of the field content of the theory. Looking at the quadratic terms in the first line of Eq.(19), we observe that the theory contains one massless bosonic field ζ, and two chiral massless fermions $\psi_\pm$. The chiralities of the fermions are determined by the choice of signs in (15). If we choose them as we did they have opposite chiralities and the field content can be fitted within a chiral superfield of a $(1,1)$ two-dimensional supersymmetry.

The Lagrangian (19) is not supersymmetric. It is not even Poincare invariant. Motivated by the expected relation to string theory, and based on our experience in interpreting the bosonic theory [12, 11], we interpret it as follows. We assume that the theory really started out as a two-dimensional supersymmetric theory, containing two superfields, Φ_1 and Φ_2. The superfield Φ_2 obtains a non-trivial vacuum expectation value (VEV). The VEV breaks Poincare invariance as well as supersymmetry. Our task then becomes to reconstruct the original theory as best as we can. As will become obvious, it is not possible to reconstruct the theory completely. We can, however, capture enough of its features to make the reconstruction an interesting enterprise.

Of the two chiral superfields of $(1,1)$ supersymmetry,

$$\begin{aligned} \Phi_1 &= \zeta + i\theta^+\psi_+ + i\theta^-\psi_- + i\theta^+\theta^- Z \\ \Phi_2 &= \alpha + i\theta^+\chi_+ + i\theta^-\chi_- + i\theta^+\theta^- A, \end{aligned} \tag{21}$$

it is Φ_1 that contains the degrees of freedom in the original Lagrangian. It is straightforward to write a manifestly invariant kinetic term for Φ_1,

$$\begin{aligned} \mathcal{L}_{01}^{(eff)} &= \int d\theta^+ d\theta^- D_+\Phi_1 D_-\Phi_1 \\ &= \frac{1}{2}(\dot{\zeta}^2 - \zeta'^2) - i\psi_+\partial_-\psi_+ - i\psi_-\partial_+\psi_- + Z^2. \end{aligned} \tag{22}$$

Doing the same for the other superfield is a little bit more involved procedure. We expect the superfield Φ_2 to acquire a non-trivial VEV. Based on our experience in the interpretation of the bosonic theory and motivated by the expected relation with other formulations of two-dimensional superstring theory, we expect the components of Φ_2 to obtain the following VEV

$$\begin{aligned} <\alpha> &= e^{-\omega|\tau-\tau_0|} \\ <\chi_\pm> &= 0 \\ <A> &= 0. \end{aligned} \tag{23}$$

Furthermore, we impose that the most singular term in the Lagrangian has a $1/\alpha^4$ dependence, in agreement with classical string theory. We now desire a manifestly supersymmetric Lagrangian that (i) has (23) as a solution to its equations of motion, (ii) when the VEV, (23), of Φ_2 is substituted into the Lagrangian, it reduces to the constant term in (19), and

(iii) the most singular term in the Lagrangian has a $1/\alpha^4$ dependence. A solution with the desired properties is given by

$$\begin{aligned}
\mathcal{L}_{02}^{(eff)} &= \int d\theta^+ d\theta^- \left\{ F_1(\Phi_2) D_+\Phi_2 D_-\Phi_2 - \frac{1}{\omega^2} F_2(\Phi_2)\partial_- D_+\Phi_2 \partial_+ D_-\Phi_2 \right\} \\
&= F_1(\alpha)\partial_+\alpha\partial_-\alpha - \frac{1}{\omega^2}F_2(\alpha)(\partial_+\partial_-\alpha)^2 \\
&\quad + F_1(\alpha)A^2 - \frac{1}{\omega^2}F_2(\alpha)\partial_+ A\partial_- A \\
&\quad - iF_1(\alpha)\chi_+\partial_-\chi_+ - \frac{i}{\omega^2}F_2(\alpha)\partial_-\chi_+\partial_+\partial_-\chi_+ \\
&\quad - iF_1(\alpha)\chi_-\partial_+\chi_- - \frac{i}{\omega^2}F_2(\alpha)\partial_+\chi_-\partial_-\partial_+\chi_-,
\end{aligned} \tag{24}$$

where

$$\begin{aligned}
F_1(\Phi_2) &= -\frac{1}{48\pi\kappa\omega^2 g^2}\left(\frac{11}{5}\frac{\kappa^3}{\Phi_2^6} - \frac{28}{3}\frac{\kappa^2}{\Phi_2^4} + 18\frac{\kappa}{\Phi_2^2} - 4 + \frac{5}{3}\frac{\Phi_2^2}{\kappa}\right) \\
F_2(\Phi_2) &= -\frac{1}{48\pi\kappa\omega^2 g^2}\left(-\frac{2}{5}\frac{\kappa^3}{\Phi_2^6} + \frac{8}{3}\frac{\kappa^2}{\Phi_2^4} - 12\frac{\kappa}{\Phi_2^2} - 8 + \frac{2}{3}\frac{\Phi_2^2}{\kappa}\right).
\end{aligned} \tag{25}$$

This Lagrangian indeed has (23) as a solution of its equation of motion. Obviously, (24) is not the unique Lagrangian with the desired properties. However, it is the Lagrangian with the least number of terms. We therefore choose to present it. The fact that we were able to find any solution to our requirements is not at all trivial.

So far, we were able to construct a manifestly supersymmetric theory, to lowest order in the coupling $g(\tau)$, using the two superfields and their covariant derivatives. Amazingly enough, there exist a manifestly supersymmetric Lagrangian that reduces to the full non-linear interacting two-dimensional field theory. The details of the derivation are given in [3]. We give the final result here,

$$\begin{aligned}
\mathcal{L}^{(eff)} = \int d\theta_+ d\theta_- \Big\{ & D_+\Phi_1 D_-\Phi_1 \\
& + F_1(\Phi_2)D_+\Phi_2 D_-\Phi_2 - \frac{1}{\omega^2}F_2(\Phi_2)\partial_- D_+\Phi_2\partial_+ D_-\Phi_2 \\
& - \frac{f(\Phi_2)}{\omega^3\Phi_2^3}\frac{\partial_{(+}\Phi_1\partial_{-)}\Phi_2\partial_{[+}\Phi_1\partial_{-]}\Phi_2}{1+\frac{f(\Phi_2)}{\omega\Phi_2}\partial_{(+}\Phi_1\partial_{-)}\Phi_2} D_{(+}\Phi_1 D_{-)}\Phi_2 \\
& + \frac{1}{3}\frac{f(\Phi_2)}{\omega^5\Phi_2^5}(\partial_{[+}\Phi_1\partial_{-]}\Phi_2)^3 D_+\Phi_2 D_-\Phi_2
\end{aligned}$$

$$-\frac{f(\Phi_2)}{\omega^5\Phi_2^5}\frac{(\partial_{[+}\Phi_1\partial_{-]}\Phi_2)^2\partial_{(+}\Phi_1\partial_{-)}\Phi_2}{1+\frac{f(\Phi_2)}{\omega\Phi_2}\partial_{(+}\Phi_1\partial_{-)}\Phi_2}D_+\Phi_2D_-\Phi_2\Bigg\}, \tag{26}$$

where

$$f(\Phi_2) \;=\; 4\sqrt{\pi}g\frac{\frac{1}{\kappa}\Phi_2^2}{(1-\frac{1}{\kappa}\Phi_2^2)^2}. \tag{27}$$

In components, (26) is given by

$$\begin{aligned}
\mathcal{L}^{(eff)} \;=\; & +\partial_+\zeta\partial_-\zeta + Z^2 - i\psi_+\partial_-\psi_+ - i\psi_-\partial_+\psi_- \\
& +F_1(\alpha)\partial_+\alpha\partial_-\alpha - \frac{1}{\omega^2}F_2(\alpha)(\partial_+\partial_-\alpha)^2 \\
& +F_1(\alpha)A^2 - \frac{1}{\omega^2}F_2(\alpha)\partial_+A\partial_-A \\
& -iF_1(\alpha)\chi_+\partial_-\chi_+ - \frac{i}{\omega^2}F_2(\alpha)\partial_-\chi_+\partial_+\partial_-\chi_+ \\
& -iF_1(\alpha)\chi_-\partial_+\chi_- - \frac{i}{\omega^2}F_2(\alpha)\partial_+\chi_-\partial_-\partial_+\chi_- \\
& +\sum_n \mathcal{O}(\alpha^n\chi_+\chi_- + \alpha^{n-1}A\chi_+\chi_-). \\
& -\frac{1}{2}\frac{f(\alpha)\dot{\zeta}^2\zeta'}{1+f(\alpha)\zeta'} - \frac{1}{6}f(\alpha)\zeta'^3 \\
& +\frac{i}{\sqrt{2}}\frac{f(\alpha)\zeta'}{1+f(\alpha)\zeta'}(\psi_+\dot{\psi}_+ + \psi_-\dot{\psi}_-) + \frac{i}{\sqrt{2}}\frac{f(\alpha)\dot{\zeta}}{[1+f(\alpha)\zeta']^2}(\psi_+\psi_+' + \psi_-\psi_-') \\
& +\mathcal{O}\Bigg\{\partial\zeta(\psi\chi + \chi\chi + Z\psi\chi + Z\chi\chi + A\psi\psi + A\psi\chi) + \psi\psi\chi + \psi\chi\chi\Bigg\}.
\end{aligned} \tag{28}$$

As can be checked, the general solution of the equations of motion derived from (26) is the following,

$$\begin{aligned}
<\alpha> \;&=\; \exp\Bigg\{\omega[|t-t_0|\sinh\theta_0 - |\tau-\tau_0|\cosh\theta_0]\Bigg\} \\
<\zeta> \;&=\; constant \\
<\chi_\pm> \;&=\; \eta_0^\pm <\alpha> \\
<\psi_\pm> \;&=\; 0 \\
<A> \;&=\; 0 \\
<Z> \;&=\; 0.
\end{aligned} \tag{29}$$

If this solution is substituted back into (26) and the auxiliary fields are eliminated through their equations of motion, the result exactly reproduces (19).

The Lagrangian (26) has some interesting properties. First, the superfield Φ_1 has only derivative interactions, and so, in particular, has no superpotential. The interactions of the superfield Φ_2 always contain some derivatives, therefore the superfield Φ_2 has no superpotential as well. The coupling parameter of the theory is field-dependent. This is a typical situation in low-energy effective field theories of string theory. The overall coupling strength is determined by the parameter g, which is sometimes called the "string coupling constant". However, if Φ_2 has a space-dependent VEV, as in (23), the coupling strength varies in space-time and even blows up at some finite point, signalling the possible existence of new physical phenomena.

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