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Beyond the MSSM

Fabio Zwirner[1]

Theory Division, CERN
CH-1211 Geneva 23, Switzerland

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Abstract

To increase the predictivity of the Minimal Supersymmetric Standard Model (MSSM), one needs to go to an underlying, more fundamental theory, where at least some of the many MSSM parameters can be determined by symmetries or by dynamics. Progress may come from four-dimensional superstring solutions and their effective supergravities. Summarizing some recent work [1–3], we introduce a class of 'large-hierarchy-compatible' (LHC) models that could naturally embed a stable hierarchy $m_Z \lesssim m_{3/2} \ll M_{\rm P}$. We discuss how in LHC models one may determine: 1) the explicit mass terms of the MSSM, as functions of the gravitino mass; 2) the scales of gauge and supersymmetry breaking, m_Z and $m_{3/2}$; 3) the heavy-fermion masses.

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1. Introduction

The gauge hierarchy problem of the Standard Model (SM) is related to the existence of quadratically divergent one-loop corrections to the effective potential, proportional to

$$\mathrm{Str}\, \mathcal{M}^2(\phi) \equiv \sum_i (-1)^{2J_i}(2J_i+1) m_i^2(\phi)\,, \tag{1}$$

where ϕ is the classical Higgs field and the index i runs over the states of the model, with field-dependent squared masses $m_i^2(\phi)$ and spins J_i. Correspondingly, there are also quadratically divergent contributions to the Higgs mass, proportional to $[\partial^2\, \mathrm{Str}\, \mathcal{M}^2/\partial\phi^2]$. Since in the SM both quantities are generically non-vanishing, the natural scale of the SM Higgs mass and of the corresponding VEV [if $SU(2)\times U(1)$ is broken] is the ultraviolet cut-off scale, e.g. the Planck scale: a ratio $m_Z/M_{\mathrm{P}} \sim 10^{-16}$ is unstable versus perturbative quantum corrections.

A partial solution of the gauge hierarchy problem is provided by the Minimal Supersymmetric Standard Model (MSSM). In this model [4], supersymmetry breaking is parametrized by a collection of explicit but soft mass parameters, such that, denoting spin-0 fields with the generic symbol z, $\mathrm{Str}\, \mathcal{M}^2(z) = constant$, and there are no field-dependent quadratic divergences. The scale M_{SUSY} of the explicit MSSM mass terms acts as an effective cut-off scale for the SM, and a sufficient condition to solve the gauge hierarchy problem is to have $M_{\mathrm{SUSY}} \lesssim 1$ TeV. Usually, the soft mass terms are further constrained by assuming, as boundary conditions at some grand-unification scale $M_{\mathrm{U}} \sim 10^{16}$ GeV, a universal scalar mass m_0, a universal gaugino mass $m_{1/2}$, and a universal cubic scalar coupling A. It should be kept in mind, however, that such assumptions are not based on fundamental symmetry principles, and that different sets of assumptions can also give phenomenologically viable models.

In addition to the stabilization of the hierarchy, another attractive feature of the MSSM is the possibility of describing the spontaneous breaking of the electroweak gauge symmetry as an effect of radiative corrections. Thanks to the quantum corrections associated with the large top-quark Yukawa coupling, the effective potential of the MSSM gives a phenomenologically acceptable vacuum, with $SU(2)_L \times U(1)_Y$ broken down to $U(1)_{em}$ and m_Z naturally of order M_{SUSY}. For appropriate numerical assignments of the boundary conditions, one obtains a mass spectrum compatible with present experimental data.

Besides these virtues, the MSSM has a very unsatisfactory feature: the numerical values of its explicit mass parameters must be arbitrarily chosen 'by hand' (the fact that also the gauge and Yukawa couplings must be chosen 'by hand' is as unsatisfactory as in the Standard Model). This means a certain lack of predictivity, and in particular does not provide any dynamical explanation for the origin of the hierarchy $M_{\mathrm{SUSY}} \ll M_{\mathrm{P}}$, which is just assumed to be there. To go further, one must have a model for spontaneous supersymmetry breaking in the fundamental theory underlying the MSSM. The only possible candidate for such a theory is $N = 1$ supergravity, where, in contrast with the case of global supersymmetry, the spontaneous breaking of local supersymmetry is not incompatible with vanishing vacuum energy. For spontaneous breaking on a flat background, the order parameter is the gravitino mass, $m_{3/2}$, and all the explicit mass parameters of the MSSM are calculable (but model-dependent) functions of $m_{3/2}$.

When discussing the spontaneous breaking of $N = 1$ supergravity, one is faced with some hierarchy problems that are as serious as the gauge hierarchy problem of the SM. In $N = 1$ supergravity, the gravitino mass $m_{3/2}$ depends on the VEVs of the scalar fields, some of which typically have masses of order $m_{3/2}$ or smaller, and there are in general field-dependent quadratically divergent contributions to the effective potential. This implies that a small ratio $m_{3/2}/M_{\rm P}$ is generically unstable versus perturbative quantum corrections. In addition, consistency with the flat background, explicitly assumed in the standard formalism of Poincaré supergravity, asks for the absence of $\mathcal{O}(m_{3/2}^2 {M_{\rm P}}^2)$ contributions to the vacuum energy when discussing physics at scales $Q \sim m_Z \lesssim m_{3/2}$. Already at the classical level, this is a highly non-trivial requirement, since the scalar potential of $N = 1$ supergravity is not positive-semidefinite. This problem is partially solved by the so-called 'no-scale' models [5,6], which naturally fit in the effective supergravity theories derived from classical four-dimensional vacua of the heterotic superstring. In these models, the classical potential is manifestly positive-semidefinite, and all its minima correspond to broken supersymmetry and vanishing vacuum energy, with the gravitino mass sliding along an approximately flat direction [5]. The fact that the supersymmetry breaking scale is classically undetermined provides the possibility of fixing it via quantum corrections. If the latter, as computed in the fundamental quantum theory of gravity, do not introduce $\mathcal{O}(m_{3/2}^2 {M_{\rm P}}^2)$ contributions to the effective potential, then one may obtain an exponentially suppressed gravitino mass, $m_{3/2} \sim \exp[-\mathcal{O}(1)/\alpha] M_{\rm P}$, as a result of the logarithmic quantum corrections in the low-energy effective supergravity theory [6].

2. LHC supergravity models

At the level of the effective $N = 1$ supergravity, the viability of the above program (and of other scenarios for the generation of the hierarchy $m_{3/2} \ll M_{\rm P}$) is plagued by the possible existence of quadratically divergent contributions to the vacuum energy, proportional to

$$\mathrm{Str}\, \mathcal{M}^2(z,\overline{z}) = 2\, Q(z,\overline{z})\, m_{3/2}^2(z,\overline{z})\,, \tag{2}$$

where [7], using here and in the following the supergravity convention $M_{\rm P} = 1$, and assuming for simplicity F-breaking along a gauge singlet direction,

$$Q(z,\overline{z}) = N_{TOT} - 1 - \mathcal{G}^I(z,\overline{z})\, [R_{I\overline{J}}(z,\overline{z}) + F_{I\overline{J}}(z,\overline{z})]\, \mathcal{G}^{\overline{J}}(z,\overline{z})\,, \tag{3}$$

$$R_{I\overline{J}}(z,\overline{z}) \equiv \partial_I \partial_{\overline{J}} \log\det \mathcal{G}_{M\overline{N}}(z,\overline{z})\,, \qquad F_{I\overline{J}}(z,\overline{z}) \equiv \partial_I \partial_{\overline{J}} \log\det[\mathrm{Re}\, f_{ab}(z)]^{-1}\,. \tag{4}$$

To interpret the previous formulae, we recall that the $N = 1$ supergravity Lagrangian is determined by two arbitrary functions: the Kähler function $\mathcal{G}(z,\overline{z}) = K(z,\overline{z}) + \log|w(z)|^2$, where K is the Kähler potential, whose second derivatives determine the kinetic terms for the fields in the chiral supermultiplets, and w is the superpotential; the gauge kinetic function $f_{ab}(z)$, which determines the kinetic terms for the fields in the vector supermultiplets, and in particular the gauge coupling constants $g_{ab}^{-2} = \mathrm{Re}\, f_{ab}$. In eqs. (3) and (4), derivatives of the Kähler function are denoted by $\partial\mathcal{G}/\partial z^I \equiv \partial_I \mathcal{G} \equiv \mathcal{G}_I$ and $\partial\mathcal{G}/\partial\overline{z}^{\overline{I}} \equiv \partial_{\overline{I}}\mathcal{G} \equiv \mathcal{G}_{\overline{I}}$, the Kähler metric is $\mathcal{G}_{I\overline{J}} = \mathcal{G}_{\overline{J}I}$, and the inverse Kähler metric $\mathcal{G}^{I\overline{J}}$ is used to define $\mathcal{G}^I \equiv \mathcal{G}^{I\overline{J}}\mathcal{G}_{\overline{J}}$ and $\mathcal{G}^{\overline{I}} \equiv \mathcal{G}_J \mathcal{G}^{J\overline{I}}$. It is important to observe that both $R_{I\overline{J}}$ and $F_{I\overline{J}}$ do not depend on the superpotential, but only depend on the metrics for the chiral and gauge superfields. This allows for the possibility that, for special geometrical properties of these two metrics, the dimensionless quantity $Q(z,\overline{z})$ may turn out to be field-independent and hopefully vanishing.

In order to appreciate the geometrical meaning of the vanishing of $Q(z,\overline{z})$, we present here a simple working example [1]. Consider a model containing $N_{TOT} \equiv N_c + 3$ chiral superfields, three gauge singlets (T, U, S) and N_c charged fields C_i $(i = 1, \ldots, N_c)$, with a gauge kinetic function $f_{ab} = \delta_{ab} S$, a Kähler function

$$\mathcal{G} = -3\log(T + \overline{T} - C_i\overline{C}_i) - k\log(U + \overline{U}) - \log(S + \overline{S}) + \log|w(C,U,S)|^2 \,, \qquad (5)$$

and a superpotential $w(C,U,S)$, which depends non-trivially on all fields apart from the singlet field T. One can easily prove that, thanks to the field identity $\mathcal{G}^T\mathcal{G}_T \equiv K^T K_T \equiv 3$, the scalar potential of such a model is automatically positive semidefinite, with a flat direction along the T-field. As long as there are field configurations for which $w \neq 0$ with $\mathcal{G}_S = \mathcal{G}_U = \mathcal{G}_C = 0$, there are minima that preserve the gauge symmetry but break supersymmetry with vanishing vacuum energy and $\mathcal{G}_T \neq 0$. The gauge coupling constant at the minimum is fixed to the value $g^2 = (\mathrm{Re}\, S)^{-1}$, and the VEV of the U field is also fixed by the minimization condition, whereas the gravitino mass $m_{3/2}^2 = |w|^2/[(S+\overline{S})(T+\overline{T})^3(U+\overline{U})^k]$ is classically undetermined, sliding along the T flat direction. To compute $Q(z,\overline{z})$ in this model, it is sufficient to realize that the Ricci tensors for the three factor manifolds have the simple expressions $(I = 0, 1, \ldots, N_c)$:

$$R_{I\overline{J}} = \frac{N_c+2}{3}\mathcal{G}_{I\overline{J}} \,, \qquad R_{S\overline{S}} = 2\mathcal{G}_{S\overline{S}} \,, \qquad R_{U\overline{U}} = \frac{2}{k}\mathcal{G}_{U\overline{U}} \,, \qquad (6)$$

from which one finds, by just applying eqs. (3) and (4), that $Q(z,\overline{z}) \equiv 0$ at all minima of the potential along the flat direction T, independently of the details of the superpotential.

The previous example can be generalized [1] to the case of a supergravity model containing N_{TOT} fields $z^I \equiv (z^\alpha, z^i)$ and described, for small field fluctuations around $\langle z^i \rangle = 0$, by the Kähler function

$$\mathcal{G} = -\log Y(r^\alpha) + \sum_A K^A_{i_A\overline{j}_A}(r^\alpha) z^{i_A}\overline{z}^{\overline{j}_A} + \frac{1}{2}\sum_{A,B}\left[P_{i_A j_B}(r^\alpha) z^{i_A} z^{j_B} + \mathrm{h.c.}\right] + \log|w(z^i)|^2 \,, \quad (7)$$

depending on the fields z^α only via the real combinations $r^\alpha \equiv z^\alpha + \overline{z}^{\overline{\alpha}}$. The Kähler potential and the gauge kinetic function are assumed to have the scaling properties

$$r^\alpha Y_\alpha = 3Y \,, \qquad (8)$$

$$r^\alpha K^A_{i_A \overline{\jmath}_A \, \alpha} = \lambda_A K^A_{i_A \overline{\jmath}_A} \, , \tag{9}$$

$$r^\alpha P_{i_A j_B \, \alpha} = \frac{\lambda_A + \lambda_B}{2} P_{i_A j_B} \, , \tag{10}$$

$$r^\alpha (\mathrm{Re}\, f_{ab})_\alpha = \lambda_f \mathrm{Re}\, f_{ab} \, , \tag{11}$$

where it is unambiguous to define $Y_\alpha \equiv \partial Y/(\partial r^\alpha) \equiv \partial Y/(\partial z^\alpha) \equiv \partial Y/(\partial \overline{z}^{\overline{\alpha}})$, etc. If there are field configurations such that $w \neq 0$ with $\mathcal{G}_i = 0$, there are supersymmetry-breaking minima with classically vanishing vacuum energy, and eqs. (3) and (4) give

$$Q = \sum_A (1 + \lambda_A) \, n_A - n - \lambda_f d_f - 1 \, , \tag{12}$$

where n is the number of z^α fields, $\sum_A n_A + n = N_{TOT}$, and d_f is the dimension of the gauge group. From eq. (12) we can immediately read the contributions to Q from all multiplets, once their scaling weights are given: the requirement that $Q = 0$, which completes the definition of the LHC models, amounts to a field-independent but highly non-trivial constraint.

3. Mass terms in LHC models

In the case of LHC models, the general supergravity mass formulae and the resulting expressions for the MSSM mass parameters undergo dramatic simplifications [1] (similar predictions were derived, for special goldstino directions and under slightly different assumptions, in ref. [8]).

Since the spin-0 fields z^α in the supersymmetry-breaking sector are assumed here to be gauge singlets with interactions of gravitational strength, they have always masses $\mathcal{O}(m_{3/2}^2/M_{\rm P})$, i.e. in the 10^{-3}–10^{-4} eV range if the gravitino mass is at the electroweak scale, with interesting astrophysical [9] and cosmological [10] implications, including a number of potential phenomenological problems. After subtracting the goldstino, their spin-1/2 partners χ^α all have masses equal to the gravitino mass.

For the gaugino masses one finds that, if there is unification of the gauge couplings, $(\mathrm{Re}\, f)_{ab} = \delta_{ab}/g_U^2$, then

$$m_{1/2}^2 = \lambda_f^2 \, m_{3/2}^2 \, , \qquad (\lambda_f = 0, 1) \, . \tag{13}$$

As for the spin-1/2 fermions χ^i, we should distinguish two main possibilities. Those in chiral representations of the gauge group, such as the quarks and the leptons, cannot have gauge-invariant mass terms. Those in real representations of the gauge group, such as the Higgsino fields of the MSSM, can have both a 'superpotential' mass, proportional to $w_{i_A j_B}$, and a 'gravitational' mass, proportional to $P_{i_A j_B}[1 + (\lambda_A + \lambda_B)/2]$. Both these terms can in principle contribute to the superpotential 'μ-term' of the MSSM, and to the associated off-diagonal (analytic-analytic) scalar mass term m_3^2. Writing as usual $m_3^2 \equiv B\mu$, one obtains

$$B = (2 + \lambda_{H_1} + \lambda_{H_2}) m_{3/2} \tag{14}$$

in the first case, and

$$B = \left(2 + \frac{\lambda_{H_1} + \lambda_{H_2}}{2}\right) m_{3/2} \tag{15}$$

in the second case.

Moving further to the spin-0 bosons z^i in chiral representations (squarks, sleptons, ...), they can only have diagonal (analytic-antianalytic) mass terms, of the form

$$(m_0^2)_A = (1 + \lambda_A) m_{3/2}^2 \, . \tag{16}$$

Similarly, a general formula can be obtained for the coefficients of the cubic scalar couplings,

$$(A)_{i_A j_B k_D} = (3 + \lambda_A + \lambda_B + \lambda_D) m_{3/2} \, . \tag{17}$$

The previous discussion should have clarified some important features of LHC models: the MSSM mass terms are predicted, as functions of the gravitino mass, by simple formulae involving the approximate scaling weights; the MSSM μ-term can originate from the Kähler potential, and is naturally of the order of the gravitino mass; some desirable universality properties of the MSSM soft mass terms are not automatic, but can be ascribed to some universality properties of the corresponding scaling weights.

4. Superstring-derived LHC models

At the pure supergravity level, the assumptions defining the LHC models might appear plausible, but they certainly are not compulsory. It is then remarkable that, if one considers the effective supergravities [11] corresponding to the known four-dimensional superstring models [12], in the appropriate limit one obtains precisely the desired scaling properties. The candidate z^α fields are the singlet moduli fields: the universal 'dilaton-axion' multiplet S, which entirely determines, at the classical level, the gauge kinetic function $f_{ab} = \delta_{ab} S$, and the other moduli that parametrize the size and the shape of the internal compact space and are usually denoted by the symbols T_i and U_i. In the limit where the T_i and/or U_i moduli are large with respect to the string scale, the Kähler manifold for the chiral superfields displays the desired properties, with well-defined scaling weights of the Kähler metric with respect to the real combinations $(T_i + \overline{T}_i)$ and $(U_i + \overline{U}_i)$. These scaling properties are due to the discrete target-space duality symmetries [13] of four-dimensional superstrings, and the scaling weights are nothing but the modular weights with respect to the moduli fields that participate in the supersymmetry-breaking mechanism: in the limit of large moduli, non-trivial topological effects on the world-sheet are exponentially suppressed and can be neglected, and the discrete duality symmetries are promoted to accidental scaling symmetries of the kinetic terms in the effective supergravity theory.

The discrete target-space dualities are symmetries of the full Kähler function $\mathcal{G}$. Under a generic duality transformation, of the form $z^\alpha \longrightarrow f(z^\alpha)$, the Kähler potential transforms as $K \longrightarrow K + \phi + \overline{\phi}$, where ϕ is an analytic function of the moduli fields z^α. The fact that duality is a symmetry then implies a definite transformation property for the superpotential, $w \longrightarrow e^{-\phi} w$. This in turn puts very strong restrictions on the superpotential modifications that can be used to describe spontaneous supersymmetry breaking in the effective supergravity theory. As for the origin of these superpotential modifications, two types of mechanisms for supersymmetry breaking have been considered so far in the framework of four-dimensional string models. The first one corresponds to exact tree-level string solutions, in which supersymmetry is broken via orbifold compactifications [14]. The second one is based on the assumption that supersymmetry breaking is induced by non-perturbative phenomena, such as gaugino condensation, at the level of the string effective field theory [15].

In the string models with tree-level supersymmetry breaking [14], the superpotential modifications in the large-moduli limit are fully under control, since in that case the explicit form of the one-loop string partition function is known, and one can derive the low-energy effective theory without making any assumption. One obtains automatically the desired scaling properties of the kinetic terms, which in some cases can produce an LHC model. In this class of models, the large-moduli limit is a necessity, since for values of the moduli close to their self-dual points there are singularities, induced by some winding modes that become massless and then tachyonic. In the large-moduli limit, we can disregard the effects of these extra states, excluding them from the effective field theory, and the superpotential modification associated with supersymmetry breaking is not manifestly covariant with respect to target-space duality. On the other hand, the Kähler potential maintains the same expression as in the case of exact supersymmetry, with the desired scaling properties that can produce an LHC supergravity model. Another important property of this class of models is the fact that, in order to have $m_{3/2} \lesssim 1$ TeV, some internal radius must be pushed to very large values. By dimensional analysis, one would expect that huge threshold corrections to the coupling constants, due to the infinite tower of Kaluza-Klein excitations, spoil the perturbative expansion just above the compactification scale, but in the framework of string theories this problem can be avoided.

In the case of non-perturbative supersymmetry breaking [15], in the absence of a second-quantized string formalism one can assume that, at the level of the effective supergravity, the super-Higgs mechanism is induced by a superpotential modification that preserves target-space duality. Unfortunately, the form of the superpotential modification cannot be uniquely fixed by the requirement that it is a modular form of appropriate weight. However, another important constraint comes from the physical requirement that the potential must break supersymmetry and generate a vacuum energy at most $\mathcal{O}(m_{3/2}^4)$ in the large moduli limit. This is not the case for the models of supersymmetry breaking having minima of the effective potential for values of the internal moduli all close to some self-dual point, and making use of the Dedekind function η in the superpotential modification: either they do not break supersymmetry or they do so with a large cosmological constant, in contradiction with the assumption of a constant flat background. On the other hand, superpotential modifications such that $w \longrightarrow \text{constant} \neq 0$ for $z^\alpha \to \infty$ give rise to supersymmetry-breaking minima of the effective potential corresponding to a vacuum energy $\mathcal{O}(m_{3/2}^4)$ and z^α field configurations far away from the self-dual points.

5. Dynamical determination of m_Z and $m_{3/2}$

The effective low-energy theories of the LHC supergravity models contain, besides the states of the MSSM, the additional light scalars z^α in the supersymmetry-breaking sector, with interactions of gravitational strength, whose VEVs control the sliding gravitino mass. This allows for a dynamical determination of both m_Z and $m_{3/2}$, via the logarithmic quantum corrections associated with renormalizable MSSM interactions.

The conventional treatment of radiative symmetry breaking [4] can be briefly summarized as follows. As a starting point, one chooses a set of numerical input values for the independent model parameters at the unification scale $Q = M_{\rm U}$: the soft masses $(m_0, m_{1/2}, A, m_3^2)$, the superpotential mass μ, the unified gauge coupling α_U, and the third-generation[1] Yukawa couplings $(\alpha_t, \alpha_b, \alpha_\tau)$. One then evolves all the running parameters down to a low scale $Q \sim M_{\rm SUSY}$, according to the appropriate renormalization group equations (RGE), and considers the renormalization-group-improved tree-level potential

$$V_0(Q) = m_1^2 v_1^2 + m_2^2 v_2^2 + 2 m_3^2 v_1 v_2 + \frac{{g'}^2 + g^2}{8} \left(v_2^2 - v_1^2 \right)^2 + \Delta V_{cosm} \, . \tag{18}$$

In eq. (18), ΔV_{cosm} stands for a Higgs-field-independent contribution to the vacuum energy (cosmological term). The minimization of the potential in eq. (18), with respect to the dynamical variables $v_1 \equiv \langle H_1^0 \rangle$ and $v_2 \equiv \langle H_2^0 \rangle$, is straightforward: for appropriate numerical assignments of the boundary conditions, one obtains a phenomenologically acceptable vacuum, with $SU(2)_L \times U(1)_Y$ broken down to $U(1)_{em}$ and a mass spectrum compatible with present experimental data.

Here we regard the MSSM as the low-energy effective theory of an LHC supergravity model, where the gravitino mass $m_{3/2}$ cannot be determined at the classical level, and there are no quantum corrections to the effective potential carrying positive powers of the cut-off scale $M_{\rm P}$. Even if more general LHC models can be constructed, we assume that some non-perturbative dynamics fixes the VEVs of the moduli associated with α_U and $M_{\rm U}$, and the following boundary conditions on the MSSM mass parameters

$$m_{1/2} = \xi_1 \cdot m_{3/2} \, , \quad m_0 = \xi_2 \cdot m_{3/2} \, , \quad A = \xi_3 \cdot m_{3/2} \, , \quad m_3 = \xi_4 \cdot m_{3/2} \, , \quad \mu = \xi_5 \cdot m_{3/2} \, , \tag{19}$$

where the scaling weights with respect to target-space duality fix the ξ parameters to constant numerical values $\mathcal{O}(1)$ or smaller. Then all the moduli dependence of the MSSM mass parameters is encoded in the gravitino mass $m_{3/2}$, which should be considered as an extra dynamical variable, in addition to the Higgs VEVs v_1 and v_2. If we take the low-energy limit and neglect the interactions of gravitational strength, we can formally decouple the supersymmetry-breaking sector and recover the MSSM. Quantum effects in the underlying fundamental theory, however, would induce a cosmological term in the

[1] For the purposes of the present paper, mixing effects and all other Yukawa couplings can be neglected.

resulting MSSM effective potential; for LHC models, this term contains no positive powers of $M_{\rm P}$ and must therefore be proportional to $m_{3/2}^4$:

$$\Delta V_{cosm} = \eta \cdot m_{3/2}^4 \,, \tag{20}$$

obeying a boundary condition $\eta(M_{\rm U}) = \eta_0$. We stress that, in contrast with conventional treatments, in the present context we are forced to include the cosmological term, since the gravitino mass is not taken as an external parameter, but rather as a dynamical variable.

According to our program, one should minimize the effective potential of the MSSM not only with respect to the Higgs fields, but also with respect to the new dynamical variable $m_{3/2}$, keeping (for the moment) the values of α_U, $M_{\rm U}$, $\vec{\xi}$, η_0 and $(\alpha_t^U, \alpha_b^U, \alpha_\tau^U)$ as external input data. As in the standard approach, the role of radiative corrections is crucial in developing a non-zero value for the Higgs VEVs at the minimum. Quantum corrections to the classical potential are summarized, at the one-loop level, by $V_1 = V_0(Q) + \Delta V_1(Q)$, where

$$\Delta V_1(Q) = \frac{1}{64\pi^2} {\rm Str}\, \mathcal{M}^4 \left(\log \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) = \sum_i \frac{n_i m_i^4}{64\pi^2} \left(\log \frac{m_i^2}{Q^2} - \frac{3}{2} \right) \,, \tag{21}$$

and $V_0(Q)$ is the tree-level potential, eq. (18), expressed in terms of renormalized fields and parameters at the scale Q. The RGE for the new dimensionless coupling of the theory, the coefficient η of the cosmological term, reads

$$\frac{d\eta}{dt} = \frac{1}{32\pi^2} \left[\frac{{\rm Str}\, \mathcal{M}^4}{m_{3/2}^4} \right]_{v_{1,2}=0} \,, \qquad (t \equiv \log Q) \,, \tag{22}$$

and plays an important role in the determination of the supersymmetry-breaking scale: the MSSM particle content is such that $(\eta - \eta_0)$ is always driven towards negative values at sufficiently low scales, and the gravitino mass dynamically relaxes to a value closely related to the scale at which η turns from positive to negative. The desired hierarchy can be generated for values of η_0 between zero and $\mathcal{O}(100)$, depending on the values of the $\vec{\xi}$ parameters.

To illustrate the main point of our approach, it is convenient to choose, as independent variables, the supersymmetry-breaking scale $m_{3/2}^2$ and the dimensionless ratios $\hat{v}_i \equiv (v_i/m_{3/2})$ $(i = 1, 2)$. Then the minimization condition of the one-loop effective potential with respect to $m_{3/2}$ can be written in the form

$$m_{3/2}^2 \frac{\partial V_1}{\partial m_{3/2}^2} = 2 V_1 + \frac{{\rm Str}\, \mathcal{M}^4}{64\pi^2} = 0 \,. \tag{23}$$

The minimization conditions with respect to the variables $\hat{v}_i$ are completely equivalent to the ones that are usually considered in the MSSM, when the supersymmetry-breaking scale is a fixed numerical input. A general study of the MSSM predictions, as functions of the boundary conditions $\vec{\xi}$, η_0, $\alpha_{t,b,\tau}^U$, can be performed numerically. Some illustrative numerical results, for a particularly simple choice of boundary conditions, can be found in ref. [2].

6. Dynamical determination of Yukawa couplings

We can extend the previous approach by assuming that also $\alpha^U_{t,b,\tau}$ are dynamical variables [2] (see also [16]), in analogy to what was done before for the supersymmetry-breaking scale $M_{\rm SUSY} \sim m_{3/2}$. The main motivation for this proposal comes from four-dimensional superstrings, where all the parameters of the effective low-energy theory are related to the VEVs of some moduli fields. If the dynamical mechanism that breaks supersymmetry fixes the gauge coupling constant α_U to a given numerical value at $M_{\rm U}$, but leaves a residual moduli dependence of the Yukawa couplings, along some approximately flat direction, then also the latter should be treated as dynamical variables in the low-energy effective theory. This means that the effective potential of the MSSM should also be minimized with respect to the moduli on which the Yukawa couplings depend.

It is a well-known fact that, in general four-dimensional string models, tree-level Yukawa couplings are either vanishing or of the order of the unified gauge coupling. Concentrating for the moment on the top Yukawa coupling, one expects a tree-level relation of the form

$$\alpha^U_t = c_t \alpha_U \, , \tag{24}$$

where c_t is a model-dependent group-theoretical constant of order unity. At the one-loop level, both gauge and Yukawa couplings receive in general string threshold corrections [17], induced by the exchange of Kaluza-Klein and winding states, whose masses depend on the VEVs of some moduli fields. One can then consider two main possibilities.

If the top Yukawa coupling receives a string threshold correction identical to the one of the gauge coupling, the unification condition (24) is preserved. In this case the non-perturbative phenomena, which we have assumed to determine α_U, also fix the value of α^U_t; the latter is no longer an independent parameter, and one can perform the analysis described in the previous paragraph with one parameter less. In particular, the structure of the RGE for α_t is such that its numerical value at the electroweak scale is always very close to its effective infrared fixed point [18], $\alpha_t \simeq 1/(4\pi)$.

If eq. (24) receives non-trivial threshold corrections, with additional moduli dependences besides the combination appearing in the gravitino mass, it is plausible to assume that also some of the extra moduli correspond to approximately flat directions, after the inclusion of the non-perturbative physics that breaks supersymmetry and fixes the value of the unified gauge coupling constant. Then minimization with respect to the moduli relevant for the low-energy theory always admits solutions corresponding to the maximum allowed value for α_t. Strictly speaking, this is excluded by the requirement of perturbative unification, $\alpha^U_t < 1$: the minimum value of the effective potential must correspond to the case in which α_t has the largest value permitted by its moduli dependence, which is typically very close to the effective infrared fixed point.

Both situations described above are very interesting, since they give the numerical prediction that, at low energy, $m_t \sim M_t^{IR} \sin\beta$, where $M_t^{IR} \simeq 190$ GeV (with an uncertainty of roughly 10% due to the error on α_3, threshold and higher-loop effects, etc.), and $(1/\sqrt{2}) < \sin\beta < 1$ (the actual value being determined by the form of the boundary conditions at $M_{\rm U}$), thus in the range at present allowed by experimental data. In the second case, the reason of the attraction of $\alpha_t(m_{3/2})$ towards the infrared fixed point is the particular structure of the effective potential, after the minimization with respect to the supersymmetry-breaking scale. Indeed, eq. (23) can be rewritten as

$$V_1|_{min} = -\frac{1}{128\pi^2} {\rm Str}\, \mathcal{M}^4(Q) = \frac{1}{128\pi^2}(-m_t^2 C_t^2 - \ldots) \, , \tag{25}$$

where

$$C_t^2 = 12 \left[m_{Q_3}^2 + m_{U_3}^2 + \frac{m_Z^2}{2}\cos 2\beta + \left(A_t + \frac{\mu}{\tan\beta} \right)^2 \right] \, . \tag{26}$$

Equation (25) looks unbounded from below in the variable α_t, but the actual bound is set by the effective infrared fixed point, which therefore corresponds to the deepest minimum of the effective potential, if permitted by the structure of the moduli space of the underlying string theory.

Equation (25) can be easily generalized [3] to the case in which also α_b^U and α_τ^U are considered as dynamical variables:

$$V_1|_{min} = \frac{1}{128\pi^2}(-m_t^2 C_t^2 - m_b^2 C_b^2 - m_\tau^2 C_\tau^2 + \ldots) \, , \tag{27}$$

where C_b^2 and C_τ^2 have expressions similar to that of C_t^2. In this case, the leading dependence of V_1 on the Yukawa couplings $(\alpha_t, \alpha_b, \alpha_\tau)$ would attract the latter close to an effective infrared fixed surface; minimization of the vacuum energy along this surface can be used to determine the expectation values of the individual couplings, which can produce acceptable values of the third-generation fermion masses in a wide range of the MSSM parameter space.

As a final remark, we should point out that this dynamical determination of the low-energy Yukawa couplings is most naturally embedded in LHC models, but is in principle possible also in models where both the supersymmetry-breaking scale $m_{3/2}$ and the unified gauge coupling α_U are fixed by Planck-scale physics, and only the Yukawa couplings are controlled by the VEVs of singlet scalar fields along approximately flat directions of the effective potential.

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